Knowledge representation: Conceptual graphs, mental entities and inference about beliefs.

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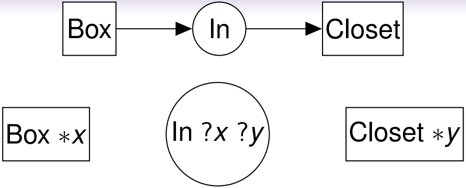
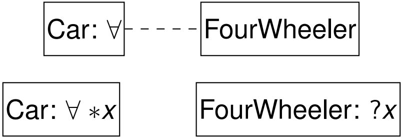
Outline

Variables



- The symbols $*x$ and $*y$ are *defining labels*. They indicate the places where variables are introduced.
- The symbols $?x$ and $?y$ are *bound labels*. They indicate the use of variables.

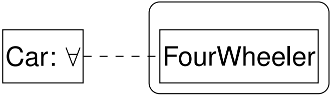


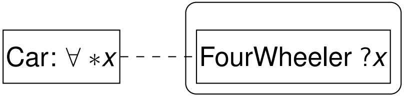
$$(\forall x : \mathrm{Car})\mathrm{fourWheeler}(x)$$

Negation

- We introduce negation by placing the formula being negated in a special context:

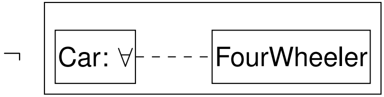
$$(\forall x : \mathrm{Car})\neg \mathrm{fourWheeler}(x)$$





- Negation can also be indicated in the following way:

$$\neg(\forall x : \mathrm{Car})\mathrm{fourWheeler}(x)$$



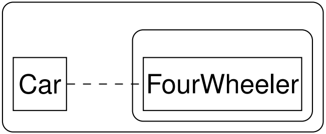
Elimination of ∀

- Conjunction (∧), negation (¬) and the existential quantifier (∃) may be used to express all other logical operators.
- The sentence

  $$(\forall x : \mathrm{Car})\mathrm{fourWheeler}(x)$$

  is equivalent to

  $$\neg(\exists x : \mathrm{Car})\neg\mathrm{fourWheeler}(x)$$



- Graphs without any universal quantifiers are semantically unambiguous.

Implication

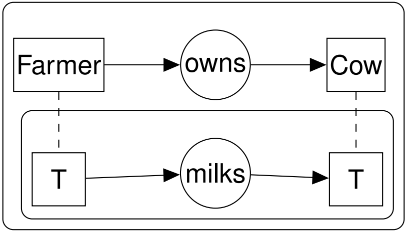
The sentence

If a farmer owns a cow he milks it.

$$(\forall x : \text{Farmer})(\forall y : \text{Cow})(\text{owns}(x, y) \Longrightarrow \text{milks}(x, y))$$

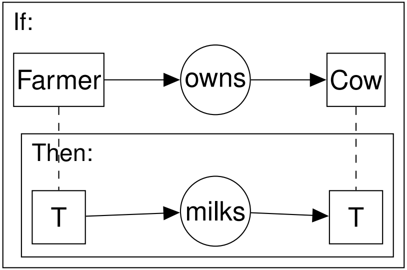
is equivalent to

$$\neg(\exists x : \text{Farmer})(\exists y : \text{Cow})(\text{owns}(x, y) \wedge \neg\text{milks}(x, y))$$



Implication

Implication is overtly represented by the If-Then context:



The representation above is closer to the original

If a farmer owns a cow he milks it.

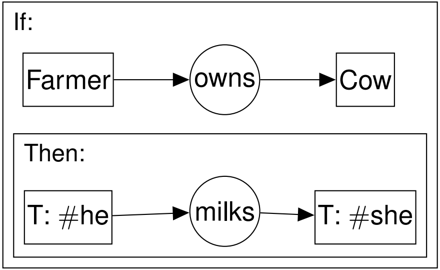
then the logical formula

$$(\forall x : \mathrm{Farmer})(\forall y : \mathrm{Cow})(\mathrm{owns}(x, y) \Longrightarrow \mathrm{milks}(x, y))$$

Coreference

If a farmer owns a cow he milks it.

- In the course of analysis of the original sentence we first obtain the intermediary form:



- And then we will seek for the objects referred to by the indicators #he and #she.
- It is a case of coreference resolution.

Alternative

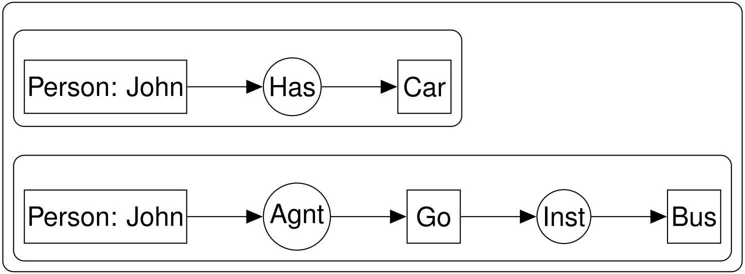
We exploit the tautology.

$$p \vee q \iff \neg(\neg p \wedge \neg q)$$

John has a car or John goes by bus.

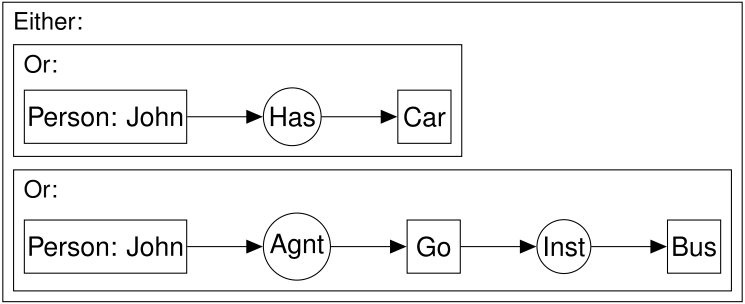
is changed to

It is not the case that John has no car and does not go by bus.



Alternative

Alternative is represented by an Either-Or context:

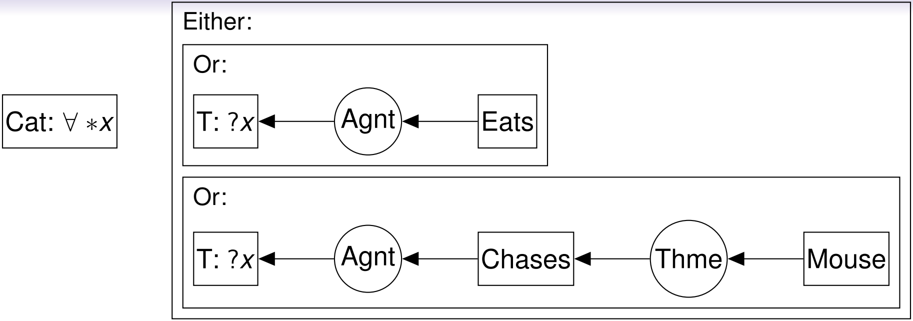


Since it is always the case that

$$p_1 \vee p_2 \vee \cdots \vee p_n \iff \neg(\neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n)$$

we can put any number of Or contexts in one Either context.

Either-Or Semantics

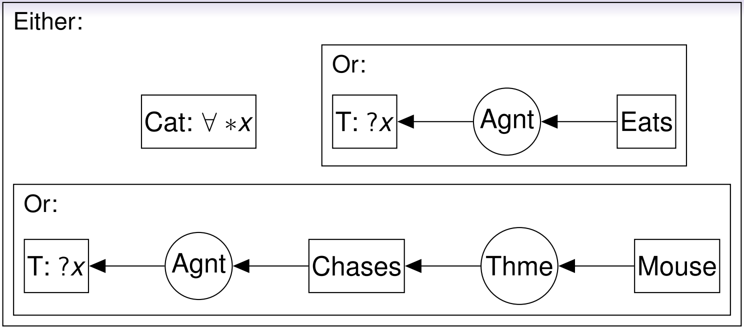


$$(\forall x : \text{Cat})\neg(\neg((\exists a : \text{Eats})\text{agnt}(a, x)) \wedge$$
$$\neg((\exists b : \text{Chases})(\exists y : \text{Mouse})\text{agnt}(b, x) \wedge \text{thme}(b, y)))$$

or equivalently

$$(\forall x : \text{Cat})(((\exists a : \text{Eats})\text{agnt}(a, x)) \vee$$
$$((\exists b : \text{Chases})(\exists y : \text{Mouse})\text{agnt}(b, x) \wedge \text{thme}(b, y)))$$

Either-Or Semantics



$$\neg(\forall x : \mathrm{Cat})\big(\neg\big((\exists a : \mathrm{Eats})\mathrm{agnt}(a, x)\big)\wedge$$

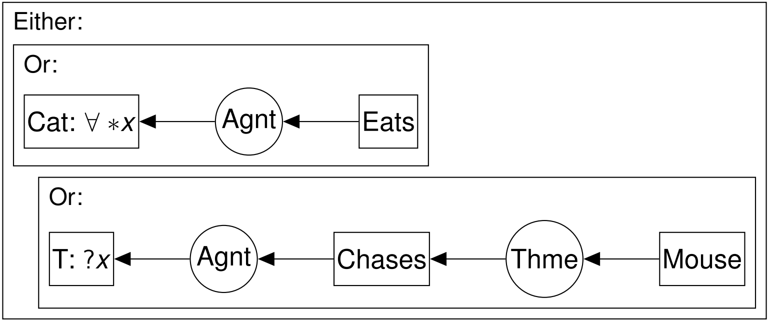
$$\neg\big((\exists b : \mathrm{Chases})(\exists y : \mathrm{Mouse})\mathrm{agnt}(b, x) \wedge \mathrm{thme}(b, y)\big)\big)$$

or equivalently

$$(\exists x : \mathrm{Cat})\big(\big((\exists a : \mathrm{Eats})\mathrm{agnt}(a, x)\big)\vee$$

$$\big((\exists b : \mathrm{Chases})(\exists y : \mathrm{Mouse})\mathrm{agnt}(b, x) \wedge \mathrm{thme}(b, y)\big)\big)$$

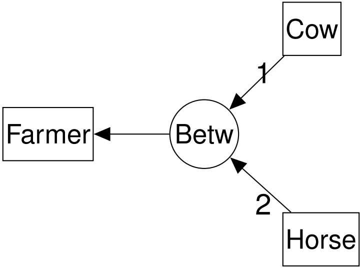
Either-Or Semantics



- the graph above is incorrect.
- It cannot be translated into a logical formula.
- x is used in a context separate form the one that defines it.

Polyadic relations

A farmer stands between a cow and a horse.



$$(\exists x : \text{Farmer})(\exists y : \text{Cow})(\exists z : \text{Horse})\text{betw}(y, z, x)$$

- The arrow starting at the circle indicates the last argument of the relation.
- The rest of the arguments is represented by arrows ending in the circle and labeled with numbers giving their order.

Outline

Main example

- Let us consider a robot looking for the exit of a maze.
- the robot can perform actions of moving in the maze.
- It constructs a model of the world in the course of exploring it (it gets to know the maze).
- It acts intentionally: Undertaking of actions is a consequence of that it didn't reach the goal yet.
- It plans its actions by conjoining actions into blocks and defining intermediary goals, eg. it may want to escape a dead end.

- Let us now assume the maze is a fragment of a web-based game world.
- The robot is a player who plays the game.

Perception

The robot constructs a model of the surrounding world.

- This means that the robot has some data structure in which it stores information delivered to it.
- The game engine delivers some pieces of information about the world that correlate to **sensations** registered by humans.
- The robot processes those pieces of information into the form of concepts storable in its data structure, a process called **perception**.
- It then adds the pieces of information to the data structure, **memorizes** them.
- What concepts are storable in the world model is determined by the ontology.

Propositions

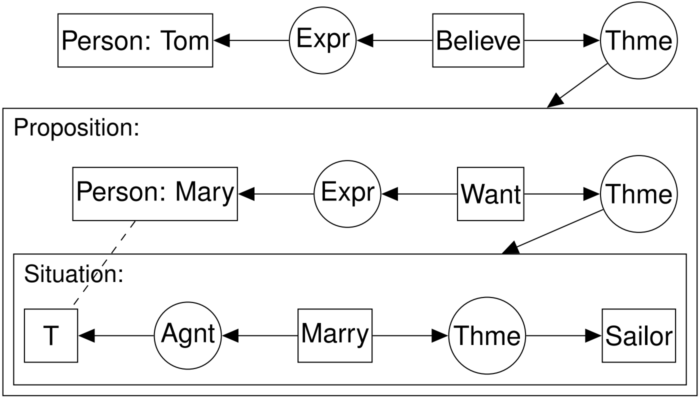
- For the data structure modelling the world it may be assumed that the maze walls are segments memorized as coordinates of their end points in a chosen coordinate system.
- the structure can be expressed with:
  - a list of end point pairs;
  - logical formula;
  - conceptual graph;
  - natural language expression;
- The data structure modeling the world is a set of **propositions**, beliefs that the robot holds of the world.

Propositional attitudes

- **Propositional attitudes** are the robots attitudes toward the propositions.
- Propositions that make up the model of the robot's world are its **beliefs**.
- We express them with the verbs *to believe that*, *to think that*.
- The robot *acts intentionally* so it has a belief that describes a state of the world which is the aim.
- **Desire** is the will to have the world in accordance with this belief.
- The desire may be expressed with the verb *to want*.
- Eg. *The robot wants to find the way out from the maze*.

Propositional attitudes

Tom believes that Mary wants to marry a sailor.

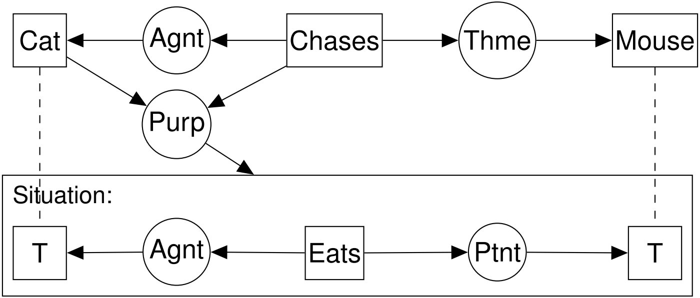


Propositional attitudes: intentions

- In order to quench its desire the robot performs a sequence of actions.
- For each of them it expects a particular effect, some situation to happen and it also desires this situation to happen
- Thus the robot has **Intention** to realize the aim expressed by the proposition.
- The intention has an aspect of realization, it connects the mental realm with the physical one.
- The intention is expressed with an adverbial.

Propositional attitudes: intentions

Cat chases a mouse to eat it.



- The intention connects the agent, the action and the situation.
- We represent it with a trinary relation **Purp**.

Propositional attitudes: expressions

- Propositional attitudes can be expressed.
- Some examples are: promises, statements, lies, declarations.
- An **Expression** is a proposition expressed by means of some medium.
- Expressions consist of symbols of objects existing in the world and symbols of relations that bind them.
- The medium can be a natural language, the language of logic, a conceptual graph, a scheme, etc.

Main example continued

- The state of affairs in the game is stored in the memory of the game engine.
- We can describe it with a box of the type situation that contains a conceptual graph.
- The robot has a fragmentary and possibly wrong picture of the state of affairs.
- Its picture of the world will be described with a box of the type proposition that contains a conceptual graph representing the content if its data structure.
- This proposition will be the object of the robot's beliefs.

Agent

- A subject, entity, that has propositional attitudes, acts intentionally.
- An agent has intends determined by the intentions.
- An agent does not need to be conscious.
- Intentional agents are: a man, an organization, a society, a computer program.
- On the basis of the intentions the agent composes a **plan** that is a structure consisting of actions and other plans and it executes it.
- It also believes things about the results of its actions.
- Even some conscious agents (people) can realize desires and intentions they don't realize.

Mental entities

- The mental is described by analogy to the physical world.
- **Mental entities**, entities constant over time are beliefs and propositional attitudes.
- **Mental processes** (thinking, memorizing, imagining, etc.) are operations on mental objects.
- **Mental states** are counterparts for physical situations.
- **Mental representations** are signs that replace the elements of the outer world (eg. surrogates).

Epistemic roles

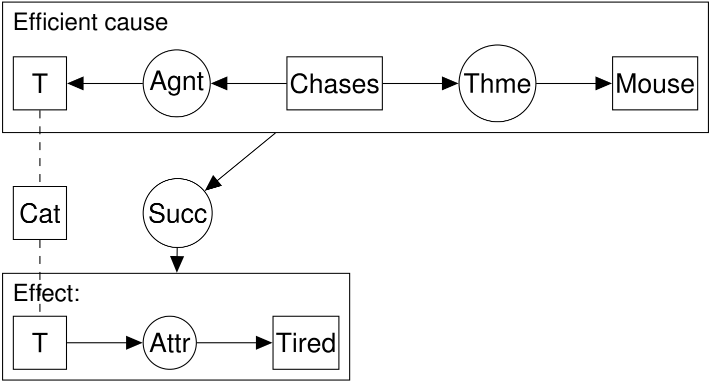
- **Epistemic Roles** are roles of beliefs and situations in processes of inferring or reasoning.
- They also determine the truth — the relation of the proposition to the world.
- Propositions also play epistemic roles and they are objects of propositional attitudes.
- **Observations** are beliefs resulting from perception (processing of sensations). They are to correspond to some perceived situation.
- **Expectation** understood as hope or anticipation is a role of a situation that is the object of an intention.

Epistemic roles

- **Fact**: a belief corresponding a an existing situation, eg. *A corpse is lying on the street.*.
- **Efficient cause** (causa efficiens; LKIF: Cause): A process that brings about a situation: *He was stabbed with a knife*.
- **Final cause** (causa finalis; LKIF: Reason): the purpose for the situation to take place (*He was robbed.*).
- The final cause is closely connected to the intention and expectation.
- In everyday speech the cause and the reason seem synonymic.
- **Effect**: the chronologically second member of the cause-effect relation, the result of an action.

Epistemic roles

A cat chased a mouse and became tired.



Outline

Belief modeling

John does not know how to integrate,
but he is convinced that he know how to integrate.

- Two levels: objective one and the level of John's beliefs.
- Two worlds: the objective world and the world as John sees it.
- John is not omniscient, so his beliefs do not describe the world precisely.
- Moreover, John may be wrong: his beliefs may contradict the facts.

Possible worlds

- Let $\mathcal{K}$ be a set of entities called *possible worlds*.
- There is a *privileged world* w_0 that represents the real world.
- *Accessibility relation* $R(u, v)$ connects possible worlds.
- *Evaluation* $\Phi(w, p)$ maps a proposition p and a possible world w.
- By the notion of possible world we can describe a static situation with agent who posses beliefs concerning the state of world surrounding him.
- Predicate believes(a, p) means that agent a believes that proposition p is true.

$$\Phi(w, \text{believes}(a, p)) \iff (\forall v \in \mathcal{K})(\Phi(v, p) \Leftrightarrow R(w, v))$$

- Relation R point out all worlds that agent considers as possible for a given actual world.

Situations

- Situations are fragments of worlds chosen according to observer intention.
- For example in the actual world *John does not know how to integrate*
- We may create a situation s, which will be the world w_0 cut to this fact.
- Situations are not spatial sections of world.
- We extend relation Φ so that it will connects situations with facts.

$$\Phi(s, \text{"John does not know how to integrate"})$$

Contexts

- Some people regard that a set of possible worlds is an idea of questionable meaning.
- Instead of assuming the possible world believed by John, we may define the set of propositions that define that world and state that John believes that these propositions are true in actual world.
- This way beliefs become propositions wrapped in contexts that define their roles.
- Similarly, we may interpret situations as contexts that consists of proposition describing world.
- It allows us to abandon the concept of possible worlds for the benefit of
  - one changing world, governed by laws of logic and
  - approximate, composed of logic formulas, descriptions of this world in agents minds.

John observes

John does not know how to integrate,
but he is convinced that he know how to integrate.
In addition, John believes that someone who know how to integrate,
will pass the colloquium on integration.

- Let $U(x)$ denote that x know how to integrate,
- and $Z(x)$ denote that x passed the colloquium on integration.

$$\Phi(s_1, \neg U(\text{John}) \wedge \text{believes}(\text{John}, U(\text{John}) \wedge (\forall x)U(x) \Rightarrow Z(x)))$$

John learns from the USOS that he did not pass the colloquium on integration.

- We have here a fact and an observation:

$$\Phi(s_2, \neg U(\text{John}) \wedge \neg Z(\text{John}) \wedge \text{believes}(\text{John}, U(\text{John}) \wedge (\forall x)U(x) \Rightarrow Z(x))$$

$$\wedge \text{observes}(\text{John}, \neg Z(\text{John})))$$

John revises his beliefs

$$\Phi(s_2, \neg U(\text{John}) \wedge \neg Z(\text{John}) \wedge \text{believes}(\text{John}, U(\text{John}) \wedge (\forall x) U(x) \Rightarrow Z(x))$$

$$\wedge \text{observes}(\text{John}, \neg Z(\text{John})))$$

- Let us assume that John is a rational agent, that is, he does not allow his beliefs to be contradictory.
- Let's also assume that John can deduce in logic in order to notice a contradiction.
- As a result of the observation John revises his beliefs:

$$\Phi(s_3, \neg U(\text{John}) \wedge \neg Z(\text{John}) \wedge \text{believes}(\text{John}, \neg Z(\text{John}) \wedge (\forall x) U(x) \Rightarrow Z(x)))$$

- Agent makes conclusions on the base of his beliefs. In order to conclude, he must believe in the relationship between his skills and the colloquium results.

Belief revision

- Beliefs are propositions (sentences in logic), on the basis of which the agent models the world
- The belief system is a theory based on these propositions.
- We assume that the agent is rational, that is, his beliefs constitute a consistent theory.
- We also assume that the agent has a logical omniscience, i.e. he can determine all the logical conclusions of his beliefs.
- The agent acquires new information that forces him to change beliefs.
- In particular, new information may contradict his previous beliefs.
- The agent must give up some of the previous beliefs in such situation.
- The belief revision operation is expressed with the $*$ operator
- For a given theory (set of propositions) K and φ — the proposition being acquired — operation $K * \varphi$ generates a new set of propositions.

Notation

- L be the formal language governed by logic, in our case FOL.
- Let Γ be the set of some sentences of L.
- $\Gamma \vdash \varphi$ means that φ may be proved assuming that sentences from Γ are true.
- We denote by $Cn(\Gamma)$ the set of all logical consequences of Γ, i.e.

$$Cn(\Gamma) = \{\psi \in L : \Gamma \vdash \psi\}$$

- We denote the set of all theories of L by $\mathbb{K}_L$.
- For the theory K we define

$$K + \Gamma = Cn(K \cup \Gamma)$$

- For a sentence φ notation $K + \varphi$ is an abbreviation of $K + \{\varphi\}$

Postulates for Belief Revision (Alchourron, Gardenfors, Makinson)

Belief Revision function $* : \mathbb{K}_L \times L \to \mathbb{K}_L$ satisfies the following axioms:

$$
\begin{array}{ll}
(K*1) & K*\varphi \text{ is a theory of } L \\
(K*2) & \varphi \in K*\varphi
\end{array}
$$

- Postulate $(K*1)$ says that the agent, remains logically omniscient after she revises her beliefs.
- Postulate $(K*2)$ that the new information φ should always be included in the new belief set.
- $(K*2)$ places enormous faith on the reliability of φ.
- The new information is perceived to be so reliable that it prevails over all previous conflicting beliefs, no matter what these beliefs might be.

Postulates for Belief Revision continued

$$(K*3) \qquad K*\varphi \subseteq K+\varphi$$
$$(K*4) \qquad \text{If } \neg\varphi \notin K, \text{ then } K+\varphi \subseteq K*\varphi$$

- $(K*3)$ and $(K*4)$ viewed together state that whenever the new information φ, does not contradict the initial belief set K there is no reason to remove any of the original beliefs at all.
- The new belief state $K*\varphi$ will contain the whole of K, the new information φ and whatever follows from the logical closure of K and φ.
- Essentially $(K*3)$ and $(K*4)$ express the notion of minimal change in the limiting case when the new information is consistent with the initial beliefs.

Postulates for Belief Revision continued

$(K * 5)$	If φ is consistent then $K * \varphi$ is also consistent
$(K * 6)$	If $\vdash \varphi \Leftrightarrow \psi$, then $K * \varphi = K * \psi$

- (K $*$ 5) says that the agent should aim for consistency at any cost.
- The only case where it is ”acceptable” for the agent to fail is when the new information in itself is inconsistent (in which case, because of $(K * 2)$, the agent cannot do anything about it).
- (K $*$ 6) is known as the irrelevance of syntax postulate.
- It says that syntax of the new information has no effect on the revision process; all that matters is its content.
- Logically equivalent sentences φ and ψ change o theory K in the same way.

Postulates for Belief Revision continued

$$(K*7) \qquad K*(\varphi \wedge \psi) \subseteq (K*\varphi)+\psi$$

$$(K*8) \qquad \text{If } \neg\psi \notin K*\varphi\text{, then } (K*\varphi)+\psi \subseteq K*(\varphi \wedge \psi)$$

- Postulates (K $*$ 7) and (K $*$ 8) say that for any two sentences φ and ψ,
  - if in revising the initial belief set K by φ one is lucky enough to reach a belief set $K*\varphi$ that is consistent with ψ,
  - then to produce $K*(\varphi \wedge \psi)$ all that one needs to do is to expand $K*\varphi$ with ψ.

  in symbols: $K*(\varphi \wedge \psi) = (K*\varphi)+\psi$.
- The rationale is as follows: $K*\varphi$ is a minimal change of K to include φ and therefore there is no way to arrive ad $K*(\varphi \wedge \psi)$ from K with "less change".
- However, if ψ is consistent with $K*\varphi$, belief revision may be restricted to adding ψ to $K*\varphi$ and closing under logical implications.

John revises his beliefs

John's beliefs before the announcement of the colloquium results are described be the following theory

$$K = Cn(U(\text{John}), (\forall x)U(x) \Rightarrow Z(x))$$

After reading the results, John learns that the following fact takes place:

$$\varphi = \neg Z(\text{John})$$

This new knowledge drives John to revise his beliefs obtaining the following theory $K * \varphi$. According to the axiom $K * 2$ we have $\varphi \in K * \varphi$. According to the axiom $K * 5$ $K * \varphi$ is consistent. Theory

$$K' = Cn((\forall x)U(x) \Rightarrow Z(x), \neg Z(\text{John}))$$

satisfies the axioms. Theory

$$K'' = Cn(U(\text{John}), \neg Z(\text{John}))$$

also satisfies the axioms.

John revises his beliefs

- Theory

$$K' = Cn((\forall x)U(x) \Rightarrow Z(x), \neg Z(\text{John}))$$

  describes the revision process in which John realized he does not know how to integrate.
- For theory

$$K'' = Cn(U(\text{John}), \neg Z(\text{John}))$$

  John doubted the connection between his skills and the grade.
- Which of the above theories should John gain as a result of belief revision?
- It is also worth to be noted that as a result of revision in the above case John should not accept the sentence ψ meaning *There is an elephant in the room*.

Belief Contraction (Alchourron, Gardenfors, Makinson)

- The process of rationally removing a certain belief from a belief set K.
- Contraction occurs when an agent loses faith in belief φ and decides to give it up.
- Simply taking out φ from K set will not suffice since other sentences that are present K may reproduce φ through logical closure.

Postulates for Belief Contraction

Belief contraction function $\div : \mathbb{K}_L \times L \to \mathbb{K}_L$ satisfies the following postulates:

$(K \div 1)$	$K \div \varphi$ is a theory of L
$(K \div 2)$	$K \div \varphi \subseteq K$
$(K \div 3)$	If $\varphi \notin K$ then $K \div \varphi = K$
$(K \div 4)$	If $\not\vdash \varphi$ then $\varphi \notin K \div \varphi$
$(K \div 5)$	If $\varphi \in K$ then $K \subseteq (K \div \varphi) + \varphi$

- Postulate $(K \div 3)$ says that if φ is not in the initial belief set there is no reason to change anything.
- $(K \div 4)$ tells us that contraction remove φ from the belief set, unless φ is a tautology.
- Postulates $(K \div 5)$ and $(K * 2)$ say that contraction and then addition of φ will give us back the initial theory.
- $(K \div 5)$ realizes the notion of minimal change.

Postulates for Belief Contraction

$$
\begin{array}{ll}
(K \div 6) & \text{If } \vdash \varphi \Leftrightarrow \psi, \text{ then } K \div \varphi = K \div \psi \\
(K \div 7) & (K \div \varphi) \cap (K \div \psi) \subseteq K \div (\varphi \wedge \psi) \\
(K \div 8) & \text{If } \varphi \notin K \div (\varphi \wedge \psi), \text{ then } K \div (\varphi \wedge \psi) \subseteq K \div \varphi
\end{array}
$$

- Postulate $(K \div 6)$ tells us that contraction is not syntax-sensitive.
- The last two postulates relate the individual contraction by two sentences, to the contraction by their conjunction.
- To contract K by $\varphi \wedge \psi$ we need to give up either φ or ψ or both.
- A belief that is given up either by contraction of φ nor by contraction of ψ is not related to them and therefore it is also not related to $\varphi \wedge \psi$.
- Hence, says $(K \div 7)$, by the principle of minimal change it should not be affected by the contraction of $\varphi \wedge \psi$.
- $\varphi \wedge \psi$ implies φ thus contraction of $\varphi \wedge \psi$ should generate greater effect then contraction of φ $(K \div 8)$.

Relation between contraction and revision

Levi Identity:

$$K * \varphi = (K \div \neg\varphi) + \varphi$$

Theorem

Let $\div : \mathbb{K}_L \times L \to \mathbb{K}_L$ *be any function that satisfies the postulates* $(K \div 1)$–$(K \div 8)$*. Then the function* $*$ *produced from* $\div$ *by means of the Levi Identity, satisfies the postulates* $(K * 1)$–$(K * 8)$

Harper Identity:

$$K \div \varphi = (K * \neg\varphi) \cap K$$

Theorem

Let $* : \mathbb{K}_L \times L \to \mathbb{K}_L$ *be any function that satisfies the postulates* $(K * 1)$–$(K * 8)$*. Then the function* $\div$ *produced from* $*$ *by means of the Harper satisfies the postulates* $(K \div 1)$–$(K \div 8)$*.*

Elephant issue

- We will use the Levi identity to describe the result of John's revision process by means of contraction.
- First remove a belief
$$\varphi = \neg Z(\text{John})$$
and obtain, according to $(K \div 2)$ contraction postulate two possible theories
$$Cn((\forall x)U(x) \Rightarrow Z(x))$$
$$Cn(U(\text{John}))$$
- Then we add φ.
- As we can see we are assured then in this way John will not start to believe that there is an elephant in the room.

Epistemic Entrenchment

- Agent assigns epistemic values to her individual beliefs.
- For example the general rule $(\forall x)U(x) \Rightarrow Z(x)$ may be more important for John that proposition $U(\text{John})$.
- Thus, having to choose between them, John will hold $(\forall x)U(x) \Rightarrow Z(x)$.
- The epistemic entrenchment of a belief is the degree of resistance of that belief to change.
- The more entrenched the belief is, the less likely it is to be swept away during contraction by some other belief.

Axioms for Epistemic Entrenchment

Epistemic Entrenchment is a preorder $\leqslant$ on L satisfying the following conditions:

$$(EE1) \qquad \text{If } \varphi \leqslant \psi \text{ and } \psi \leqslant \chi \text{ then } \varphi \leqslant \chi$$

$$(EE2) \qquad \text{If } \varphi \vdash \psi \text{ then } \varphi \leqslant \psi$$

$$(EE3) \qquad \varphi \leqslant \varphi \wedge \psi \text{ or } \psi \leqslant \varphi \wedge \psi$$

- Axiom (EE1) states that $\leqslant$ is transitive.
- (EE2) says that the stronger a belief is logically, the less entrenched it is.
- If $\varphi \vdash \psi$ holds, we may remove φ and retain ψ, but not the other way.
- Because of logical closure, one cannot give up $\varphi \wedge \psi$ without removing at least one of the sentences φ or ψ (EE3)
- From (EE1)–(EE3) it follows that $\leqslant$ is total.

Axioms for Epistemic Entrenchment

$(EE4)$	When K is consistent:
	$\varphi \notin K$ iff $\varphi \leqslant \psi$ for all $\psi \in L$
$(EE5)$	If $\psi \leqslant \varphi$ for all $\psi \in L$, then $\vdash \varphi$

- Axioms (EE4) says that all non-beliefs (i.e., all sentences that are not in K) are minimally entrenched.
- On the other hand, tautologies are hardest to remove (EE5).

Subjectivity of epistemic entrenchment

- For a fixed belief set there is more then one preorder that satisfies axioms (EE1)–(EE5).
- This is explained by the subjective nature of epistemic entrenchment.
- Interestingly, we are intuitively willing to associate the choice of support relationship with rationality.
- It is more rational for John to decide that he does not know how to integrate than to doubt in the assessment system.

Relationship between epistemic entrenchment and contraction

- Once the epistemic entrenchment chosen by the agent is given, it is possible to determine uniquely result of contraction.
- We may define contraction in terms of epistemic entrenchment as follows:

$$(C-) \qquad \psi \in K \div \varphi \text{ iff } \psi \in K \text{ and either } \varphi < \varphi \vee \psi \text{ or } \vdash \varphi$$

Theorem

- *Let K be a theory of L.*
- *If $\leqslant$ is a preorder in L that satisfies the axioms (EE1)–(EE5) then the function defined by (C-) is a contraction function.*
- *Conversely, if $\div$ is a contraction function, then there is a preorder $\leqslant$ in L that satisfies the axioms (EE1)–(EE5) as well as condition (C-).*

John revises his beliefs using epistemic entrenchment and contraction

$$\varphi = Z(\text{John}), \quad \psi = U(\text{John}), \quad \chi = (\forall x)U(x) \Rightarrow Z(x)$$

$$K = Cn(\psi, \chi) = Cn(\psi, \chi, \varphi, \varphi \vee \psi, \varphi \vee \chi)$$

We assume that $\psi < \chi$, what gives us epistemic entrenchment

$$\varphi \leqslant \psi \leqslant \varphi \vee \psi < \chi \leqslant \varphi \vee \chi$$

We need to calculate $K * \neg\varphi$. According to Levi identity the following holds

$$K * \neg\varphi = (K \div \varphi) + \neg\varphi$$

We know that $K \div \varphi \subseteq K$. Following (C-) we obtain

$$\text{If } \varphi < \varphi \vee \psi \text{ then } \psi \in K \div \varphi$$

$$\text{If } \varphi < \varphi \vee \chi \text{ then } \chi \in K \div \varphi$$