Knowledge Representation: Logic

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What is Knowledge Representation?

- Expressing knowledge of certain domain in an explicit, organized and computer-readable manner.
- Knowledge Representation is making use of logic and ontology to construct computable models for given domains.
  - **Logic** provides the formal structure and rules of inference
  - **Ontology** defines the kinds of things
  - **Computable models** (e.g. relational database model) facilitate the implementation of logic and ontology in computers.
- Application examples
  - database designing
  - object methods
  - natural language processing
  - knowledge engineering

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Knowledge Representation in logic

- Logic gives processable form to all the information that can be precisely expressed in any other language.
- Logic allows to express all the information that can be stored in computer memory.
- If some knowledge is not logic-conveyable it cannot be processed by computers no matter the notation.

Every car has 4 wheels.

Representation in propositional logic:

$$p$$

- the simplest representation form
- no details about the cars, the wheels, the number 4 and their interrelationships

The loss of details may be an advantage in some applications.

First Order Logic

- Let us consider the following syllogism (ancient inference rule)

  Every car has 4 wheels.
  Some Corvettes are cars.
  Therefore, some corvettes have 4 wheels.

- We can express it in logic in the following way

$$(\forall x)(\text{car}(x) \Rightarrow \text{fourWheeler}(x))$$
$$(\exists x)(\text{Corvette}(x) \wedge \text{car}(x))$$
$$(\exists x)(\text{Corvette}(x) \wedge \text{fourWheeler}(x))$$

- Notice that sentences of the type *Every A is B* are translated to

$$\forall x(A(x) \Rightarrow B(x))$$

  and sentences of the type *Some A are B* are translated to

$$\exists x(A(x) \wedge B(x))$$

Choice of Predicates

- The predicate „fourWheeler" doesn't convey the number 4 nor its relation to wheels.
- We can replace it with a two-place predicate:

$$\text{numberOfWheeles}(x, n),$$

which means „the number of wheels of x is $n$ór „x has n wheels".

- The sentence *Every car has 4 wheels* has the logical form

$$(\forall x)(\text{car}(x) \Rightarrow \text{numberOfWheeles}(x, 4))$$

which may be read as *For every x, if x is a car, the number of wheels of x is 4*

Choice of Predicates

- In the predicate

$$\text{numberOfWheeles}(x, n)$$

  x refers to cars, n refers to numbers but no variable refers to wheels.
- The names of the predicates are but meaningless labels for the computer system.
- If the notion of a wheel is important for some application, a more detailed selection of predicates is necessary.

Choice of Predicates

- Let us consider the predicates:

car(x)	x is a car
wheel(x)	x is a wheel
part(x, y)	y is a part of x
set(s)	s is a set
count(s, n)	The count of elements in s is n
member(x, s)	x is a member of the set s

- The car formula then becomes:

$$(\forall x)(\mathrm{car}(x) \Rightarrow (\exists s)(\mathrm{set}(s) \wedge \mathrm{count}(s, 4)$$

$$\wedge(\forall w)(\mathrm{member}(w, s) \Rightarrow (\mathrm{wheel}(w) \wedge \mathrm{part}(x, w)))))$$

- the formula may be read:
  For every x, if x is a car, then there exists s, where s is a set of count 4 and for every w, if w is a member of s, then w is a wheel and w is a part of x.

Definitions

- The formula
$$(\exists s)(\mathrm{set}(s) \wedge \mathrm{count}(s, 4)$$
$$\wedge(\forall w)(\mathrm{member}(w, s) \Rightarrow (\mathrm{wheel}(w) \wedge \mathrm{part}(x, w)))))$$
describes the property of having 4 wheels.
- We can use it to define the predicate fourWheeler(x):
$$\mathrm{fourWheeler}(x) = (\exists s)(\mathrm{set}(s) \wedge \mathrm{count}(s, 4)$$
$$\wedge(\forall w)(\mathrm{member}(w, s) \Rightarrow (\mathrm{wheel}(w) \wedge \mathrm{part}(x, w)))))$$
- A definition is a notational short-cut and it corresponds to a subroutine or a macro in programming languages.
- x — the free variable in the defining formula — is called the formal parameter.
- While expanding a definition we use α-conversion: renaming local variables to unique ones.

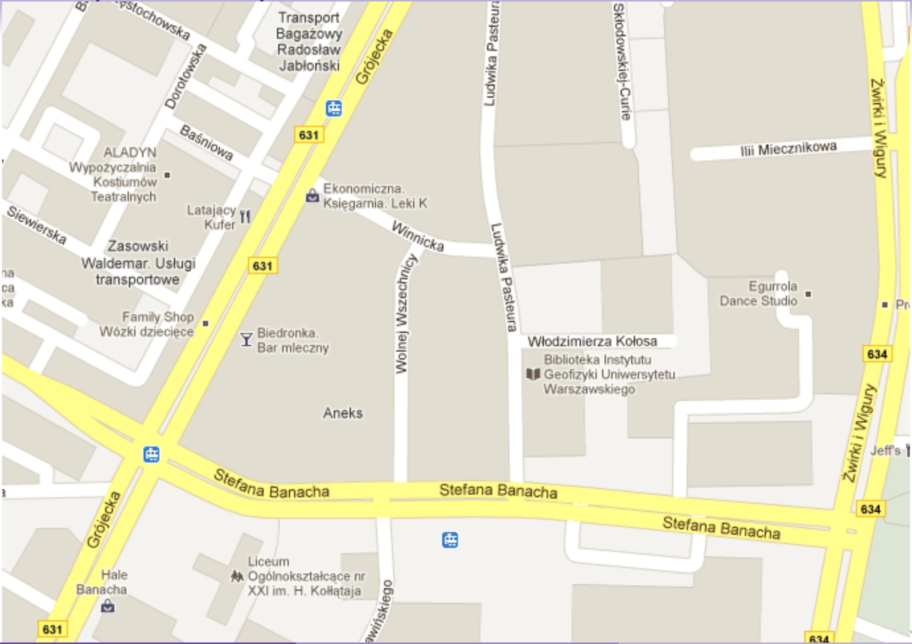
Logic and Ontology

- Even simple sentences of a natural language convey much hidden information.
- Giving all the details leads to difficulties in finding the key information while reading.
- The overflow of the details is partially caused by making all the variables explicit.
- the level of detail depends on the choice of predicates, i.e. the choice of an ontology.
- The predicates in an ontology may be divided in two classes:
  - the domain-dependent predicates: $\text{car}, \text{wheel}(x)$
  - the domain-independent predicates: $\text{part}(x, y), \text{set}(s), \text{count}(s, n), \text{member}(x, s)$

Special languages

- Subsets of logic with their own notation and built-in ontology.
- Examples: musical notation, timetables, maps, plans, schemes.
- Shorter and more readable than logic (and natural language).
- Logic is an ontologically neutral notation that can be adapted to any subject by adding one or more domain-dependent predicates.
- Special languages may be translated to logical formulas in order to automatically process the content they express.

Example: A map



Example: A map

- Streets may be represented as segments of straight lines and arcs.
- Streets intersect at cross-roads and make a graph.
- Street names are edge labels in the graph.
- There are also points such as restaurants, bus stops etc.
- A structural implementation of the map would be done by creating a graph of streets etc. Adding a new component to the map requires some modifications to be introduced to the data structure, so the code must be rewritten.
- The elements of the map could be associated with object classes divided into point-like, line-like and so on. Addition of a new component may be then achieved by adding a new subclass, but it can be impossible, for example for street names.
- We may as well express the content of the map using logic and add new components by introducing new predicates.

Existential-Conjunctive Logic

- Allows to convey facts about particular objects.
- Suffices as the representation for most of the special languages.
- Represents the information stored in both relational and object-oriented database systems.
- It cannot represent any generalizations, negations, implications, or alternatives.
- An example of generalization: The rule *A bus stop must be located an a street.*.

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Varieties of Logic

Logics can differ along the following dimensions

- Syntax: Influences readability, doesn't change the expressive power
- Subset: Possible operators and combinations of operators, e.g. the logic $\exists\wedge$ or propositional calculus
- Proof theory: Restrictions on the permissible proofs (e.g. intuitionistic logic, linear logic), or extensions (e.g. non-monotonic logics); In that cases the semantics follows from the proof theory — opposite to FOL.
- Model theory: Modifications of the notion of truth, e.g. three-valued logic or fuzzy logic.
- Ontology: the set of predicates and axioms we take for basic and domain-independent, e.g. $=$, set theory, time theory
- Metalanguage: The language that is used to talk about another language (e.g. FOL, context-free grammar)

Typed Logic

- We label variables with types:

$$(\forall x : t)P(x) \equiv (\forall x)(t(x) \Rightarrow P(x))$$

$$(\exists x : t)P(x) \equiv (\exists x)(t(x) \wedge P(x))$$

- For example:

Every car has 4 wheels.
$(\forall x : \mathrm{car})\mathrm{fourWheeler}(x)$
Some Corvettes are cars.
$(\exists x : \mathrm{Corvette})\mathrm{car}(x))$
Therefore, some Corvettes have 4 wheels.
$(\exists x : \mathrm{Corvette})\mathrm{fourWheeler}(x)$

Typed Logic

- We can also use special notation for domain-independent predicates.
- The sentence

$$(\forall x)(\text{car}(x) \Rightarrow (\exists s)(\text{set}(s) \wedge \text{count}(s, 4)$$

$$\wedge(\forall w)(\text{member}(w, s) \Rightarrow (\text{wheel}(w) \wedge \text{part}(x, w)))))$$

can be written as

$$(\forall x : \text{car})(\exists s : \text{set})(s@4$$

$$\wedge(\forall w \in s)(\text{wheel}(w) \wedge \text{part}(x, w)))))$$

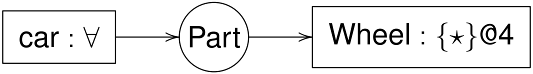
Defining new functions and function closures

- Instead of
$$\text{fourWheeler}(x) =$$
$$= (\exists s : \text{set})(s@4 \wedge (\forall w \in s)(\text{wheel}(w) \wedge \text{part}(x, w)))$$
we can write
$$\text{fourWheeler} =$$
$$= (\lambda x)((\exists s : \text{set})(s@4 \wedge (\forall w \in s)(\text{wheel}(w) \wedge \text{part}(x, w))))$$
- The notation $(\lambda x)\Psi(x)$ is a definition of a function closure, where x is the formal parameter and Ψ is the formula that defines the function.
- Tha application $((\lambda x)\Psi(x))(a)$ generates the formula $\Psi(a)$.
- λ will be used among others for defining the function closures.

$$(\exists x : \text{fourWheeler})\ \text{car}(x)$$

$$(\exists x : (\lambda x)((\exists s{:}\text{set})(s@4 \wedge (\forall w{\in}s)(\text{wheel}(w) \wedge \text{part}(x, w)))))\text{car}(x)$$

Conceptual Graphs



Every car has 4 wheels.

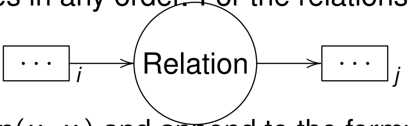
- A notation for logic that eliminates variables
- The boxes are called concepts and circles are called conceptual relations.
- Inside the box is the concept description (a label or a definition) and a colon-separated description of reference which consists of either a label, a quantifier ∀ or a specification of quantity.
- The relations show how the denotations are associated.

Conceptual Graphs Semantics

- Every box is labeled with a unique natural number.
- A FOL formula is composed step-by-step from the labelled boxes in increasing order:
  - If you see $\boxed{\text{Concept}}_i$, append $(\exists x_i : \text{concept})$ to the formula.
  - $\boxed{\text{Concept} : \forall}_i$ translates to $(\forall x_i : \text{concept})$
  - $\boxed{\text{Concept} : \{\star\}@n}_i$ translates to

$$(\exists s_i : \text{set}) s_i@n \wedge (\forall x_i \in s_i)\text{concept}(x_i)$$

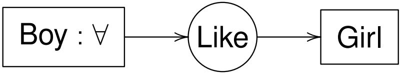
  where i is the label of the box.
- Process the circles in any order. For the relations of the form



  generate $\wedge\text{relation}(x_i, x_j)$ and append to the formula (omit „$\wedge$” at the first occurrence)

Semantic Ambiguity

- If we're restricted to Existential-Conjuntive Logic, no order of predicates and quantifiers does matter, so the translation from a graph to a FOL formula is unambiguous.
- If ∀ occurs, the translation is ambiguous.
- For example



can be interpreted as

$$(\forall y : \text{boy})(\exists x : \text{girl})\text{like}(y, x)$$

or as

$$(\exists x : \text{girl})(\forall y : \text{boy})\text{like}(y, x)$$

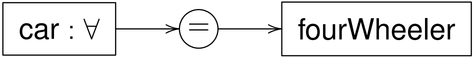
- The same phenomenon occurs in natural languages (*Quantifier Scope Ambiguity*).

Co-reference

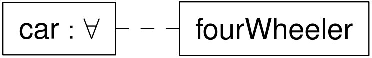
Every car ma 4 wheels.

$(\forall x : \text{car})\text{fourWheeler}(x)$

- In the example above two concepts have the same referent.
- We can express it in a graph as:



- Which corresponds to the formula:
  $(\forall x : \text{car})(\exists y : \text{fourWheeler})x = y$
- We can also use the shorthand notation:



Knowledge Interchange Format (KIF)

- Typed logic with Lisp-style syntax and restricted to ASCII symbols.
- Example

```
(forall (?x car)
  (exists (?s set)
    (and (count ?s 4)
      (forall (?w in ?s) (and (wheel ?w)
            (part ?x ?w))) )))
```

Modal Logics

- Logic with operators that express modality.
- $\Diamond\varphi$ means that φ is possible.
- $\Box\varphi$ means that φ is necessary.
- Modal operators semantics can be summarized as follows:
  - we have a set of possible worlds one of which is designated as the actual world;
  - φ means that φ is true in the actual world;
  - $\Diamond\varphi$ means that there exists a world where φ is true;
  - $\Box\varphi$ means that φ is true in every possible world.
- Different axiomatizations of modality differ in their expressive power.
- In database theory
  - any statement that is stored in the database is assumed true in the actual world
  - constraints are taken to bo necessary true.

Higher order logics

- first order logic allows for quantification over individuals.
- Second order logic allows for quantification over relations on individuals.
- Third order logic allows for quantification over relations on relations on individuals etc. ...
- For example, the axiom of induction for natural numbers:

$$(\forall P : \text{Predicate})((P(0) \wedge (\forall n \in \mathbb{N})(P(n) \Rightarrow P(n+1)) \Rightarrow (\forall n \in \mathbb{N})P(n))$$

- Higher order logics may be defined as FOL with an ontology for relations (eg. ZFC)

Metalanguage

- A language used to talk about a language.
- Example: Natural language when used to define first order logic.
- Let us consider the sentence: *The sentence „It is true that John is tall" means the same as „John is tall"*
- Quotation marks indicate the use of metalanguage, the word „true" as well.
- Thus, whole the utterance belongs to the metametalanguage and the slide that treats about it - to the metametametalanguage.
- We can introduce a special predicate into a logic to play the role of quotation marks and to connect a language with it's metalanguage.

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Proper names

- The expression *Fluffy is a cat* may be represented as

$$\text{cat}(\text{Fluffy})$$

- This representation is simple but fallible, as
  - the name is ambiguous, eg. many cats bear the name „Fluffy”.
  - one entity bears many names, eg. a cat called „Fluffy” by one family is known as „Mittens” to another.
- The solution are **surrogates**, i.e. unique identifiers for individuals.
- Surrogates for Polish citizens are their PESEL numbers.
- Proper names become plain attributes of objects:

$$\text{Name}(\text{„Fluffy”}) \wedge (\exists x : \text{Cat})\text{hasName}(x, \text{„Fluffy”})$$

Unique Name Assumption

- If two constants a and b have different names then $a \neq b$.
- KIF has two types of constants with uniqueness assumed for one of the types but not for the other.
- The constants in conceptual graphs are not assumed unique but symbols of the form #number are. They serve as surrogates.
- A node of the form [Concept : Name] is represented in logic as

  Concept(Name)

  and means that an object named „Name" is of type „Concept".
- [Name : „Fluffy"] is the name „Fluffy".
- [Cat : #2563] is a cat under the record number #2563.

Skolemization

- If we introduce a surrogate for each individual of which we know then we can discard all $\exists$ and substitute for all the occurrences of bound variables their surrogates.
- The process is an existential instantiation, also called skolemization.
- Skolemization changes the meaning slightly, because an existentially quantified variable doesn't uniquely indicate an object.
  - If we know that
    $$(\exists x)\mathrm{cat}(x)$$
    then we can answer the question „What is the color of this cat's fur?" by giving the color of any existing cat.
  - But if our knowledge is that
    $$\mathrm{cat}(\#2563)$$
    then the question „What is the color of this cat's fur?" must be answered with the color of the fur of the object indicated by $\#2563$.

Common names

- The sentence *Cats like fish* can be represented in logic by

$$(\forall x : \mathrm{Cat})(\forall y : \mathrm{Fish})\mathrm{like}(x, y)$$

- And *A cat likes a fish* can be represented as

$$(\exists x : \mathrm{Cat})(\exists y : \mathrm{Fish})\mathrm{like}(x, y)$$

- The correlates of common names are types or predicates.

Type „Type”

- First order logic allows to quantify over individuals only.
- In order to quantify over types we introduce the type „Type”.
- The variables of type „Type” are types.
- „Type” is a type of second order.

Cats are mammals. The cat is a mammal.

$$(\forall x : \mathrm{Cat})\mathrm{mammal}(x)$$

Cat is a specie.

$$(\exists x : \mathrm{Specie})(\mathrm{type}(\mathrm{Cat}) \wedge x = \mathrm{Cat})$$

Fluffy is a cat.

$$(\exists x : \mathrm{Cat})x = \mathrm{Fluffy}$$

Fluffy the cat.

$$\mathrm{cat}(\mathrm{Fluffy})$$

Predicate „kind”

- We introduce the predicate „kind(x, t)” which means that object x is of type t.
- Using the predicate „kind” to give types is a process called reification:

$$(\forall x)(\forall y)(\text{kind}(x, \text{Cat}) \wedge \text{kind}(y, \text{Fish})) \Rightarrow \text{like}(x, y)$$

- The names of types may have aliases just as the names of individuals can, so in practice surrogates are used for the names of types as well.
- Now we can define non-empty types by stating that *Every non-empty type contains at least one individual*:

$$(\forall t : \text{Non-EmptyType})(\exists x : \text{Individual})\text{kind}(x, t)$$

Representing Measures

Tom and Sue each earn a salary whose amount is $30.000.

$$(\exists x, y : \text{Earn})(\exists z, w : \text{Salary})$$

$$(\text{Person}(\text{Tom}) \wedge \text{Person}(\text{Sue}) \wedge$$

$$\text{agnt}(x, \text{Tom}) \wedge \text{thme}(x, z) \wedge \text{amount}(z, 30000\$) \wedge$$

$$\text{agnt}(y, \text{Sue}) \wedge \text{thme}(y, w) \wedge \text{amount}(w, 30000\$))$$

- The formula doesn't make it clear that the two salaries are distinct, as indicated by the word „each".
- This can be conveyed in the following ways:
  - append the condition $z \neq w$ to the formula
  - introduce surrogates for the salaries and establish the unique naming convention.