Star exercises - series I
Languages, automata and computations II

Deadline: January 11, 2019

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1. UFA on finite words

A finite automaton $\mathcal{A}$ is unambiguous if for every word accepted by $\mathcal{A}$ there is exactly one accepting run of it. Decide whether there is a polynomial P such that for every nondeterministic finite automata $\mathcal{A}$ and $\mathcal{B}$ over Σ with at most n states, such that $L(\mathcal{B}) = \Sigma^* \setminus L(\mathcal{A})$, there is an unambiguous finite automaton $\mathcal{C}$ accepting $L(\mathcal{A})$ with at most $P(n)$ states. **Remark:** We don't know the solution.

2. UFA on infinite trees

Decide the same question as in exercise 1., but for Büchi automata over infinite words. **Remark:** We don't know the solution.

3. Top-down tree automata

Is the following problem decidable: given a regular language of finite trees over some ranked alphabet, is this language recognizable by some deterministic top-down tree automaton?

4. Distance automata with more counters

Consider the following extension of a distance automaton. Instead of having a set of costly transitions, we have two counters $\{1, 2\}$ and each transition is labelled by an instruction from the following toolkit:

- do nothing;
- increment counter 1;
- reset counter 1;
- reset counter 1 and increment counter 2;
- reset both counters.

The value of a run is the biggest value attained by any counter. Prove that limitedness is decidable for these automata, using the limitedness game.

5. Separation

Prove that the following problem is decidable:

- **Input:** Regular word languages $L, K \subseteq \Sigma^*$, given say by deterministic automata.
- **Question:** Is there a language of star height 1 which contains L but is disjoint with K? A language of star height 1 is a language which can be defined by a regular expression, without complement, where the Kleene star is allowed, but it cannot be nested.

As a hint, use the previous exercise.