Separability of Reachability Sets of Vector Addition Systems

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Separability

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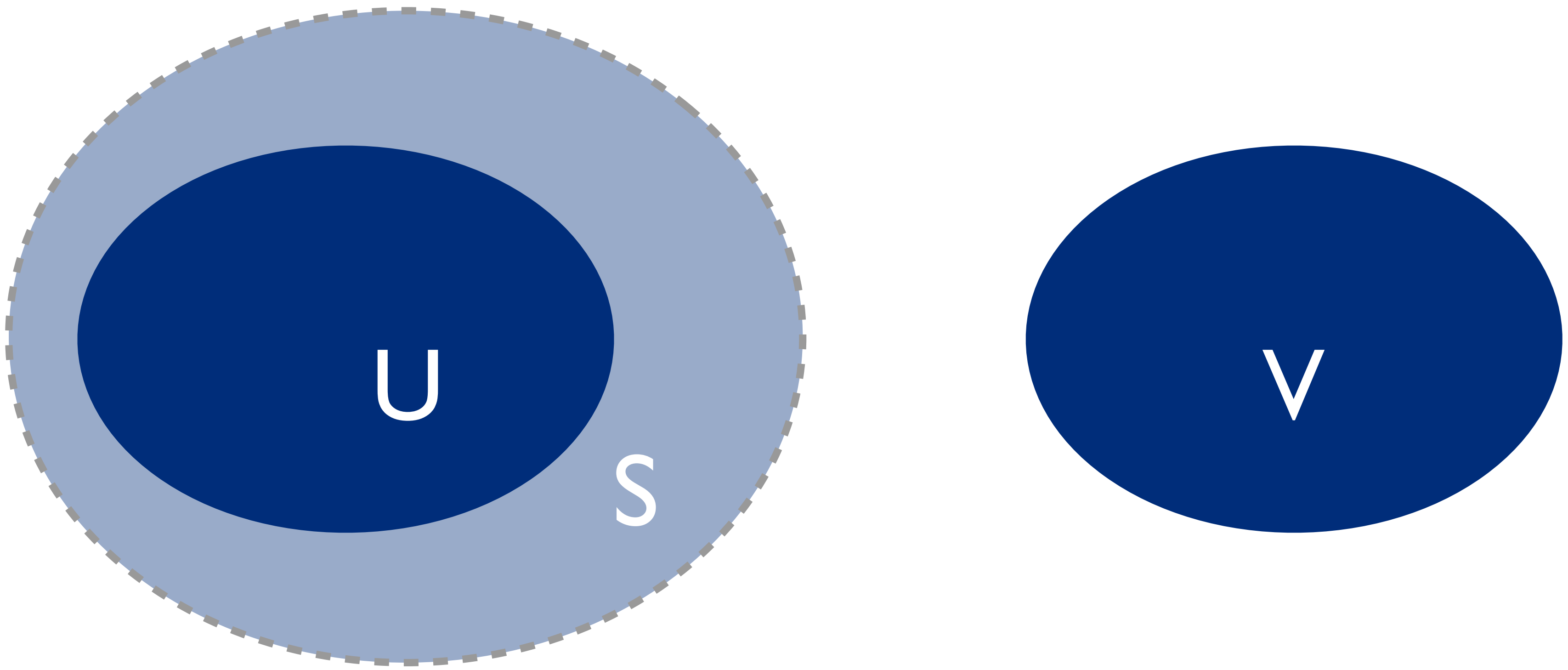

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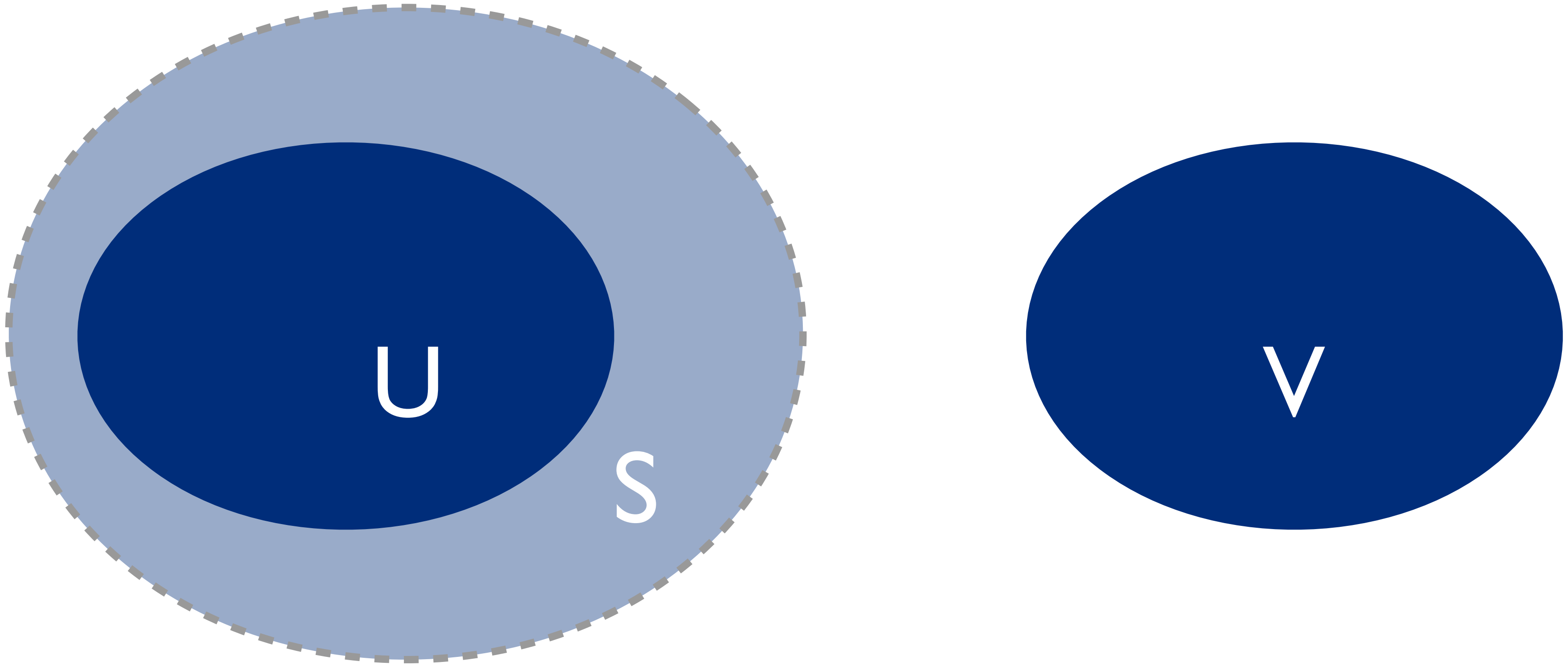

S *separates* U and V

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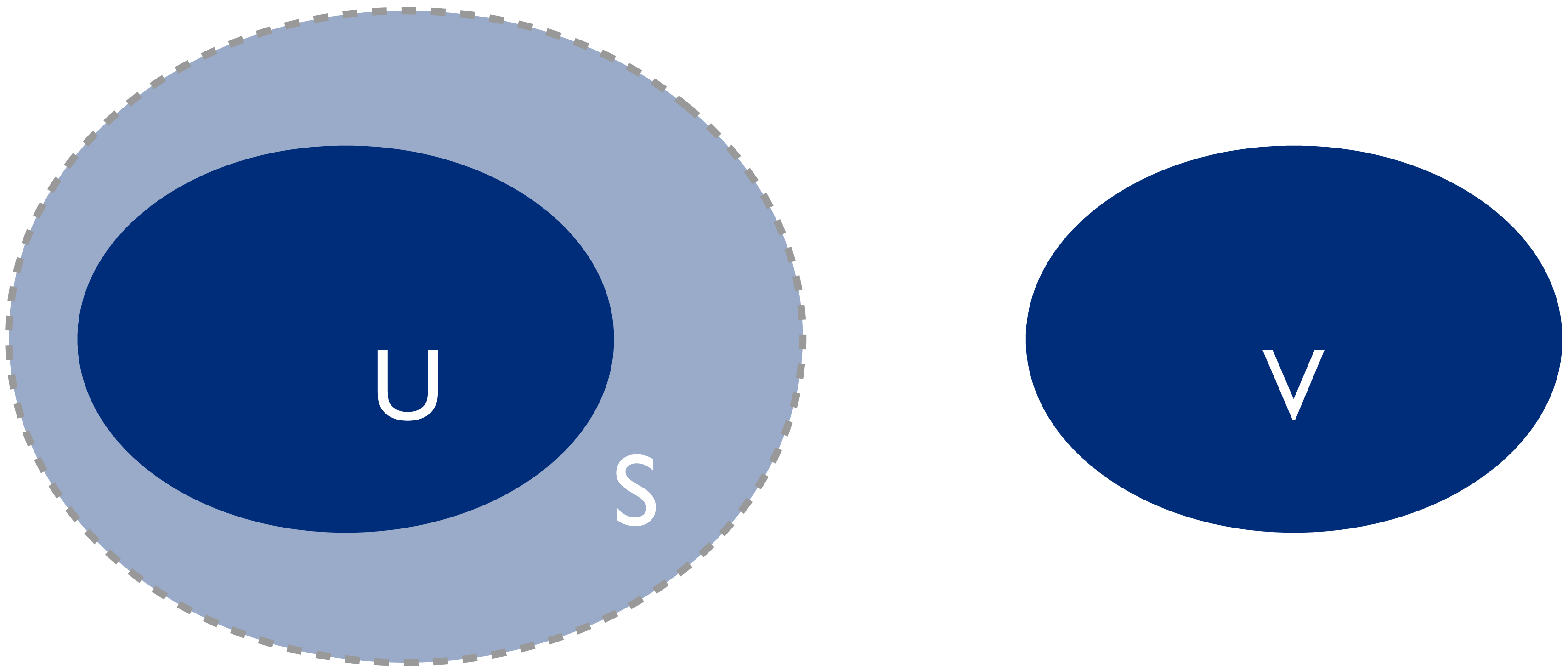

S *separates* U and V

U and V are *separable by* family F
if some S from F separates them

Vector Addition System

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initial vector v in N^d

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set of transitions T in Z^d

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move: from u to u+t
if u+t in N^d

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initial vector v in N^d

set of transitions T in Z^d

move: from u to $u+t$
if $u+t$ in N^d

reachability set: vectors in N^d reachable
from v by a sequence of moves

General problem

F-separability of G

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Given: two sets U and V from family G

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Question: are U and V separable by some set from family F

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- regular-separability of CFL is undecidable (Szymański, Williams '76)

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- regular-separability of visibly pushdown languages is undecidable (Kopczyński '15)

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- conjecture: decidable for VAS-languages
- our (second) problem: equivalent to reg.-sep. for commutative closures of VAS-languages

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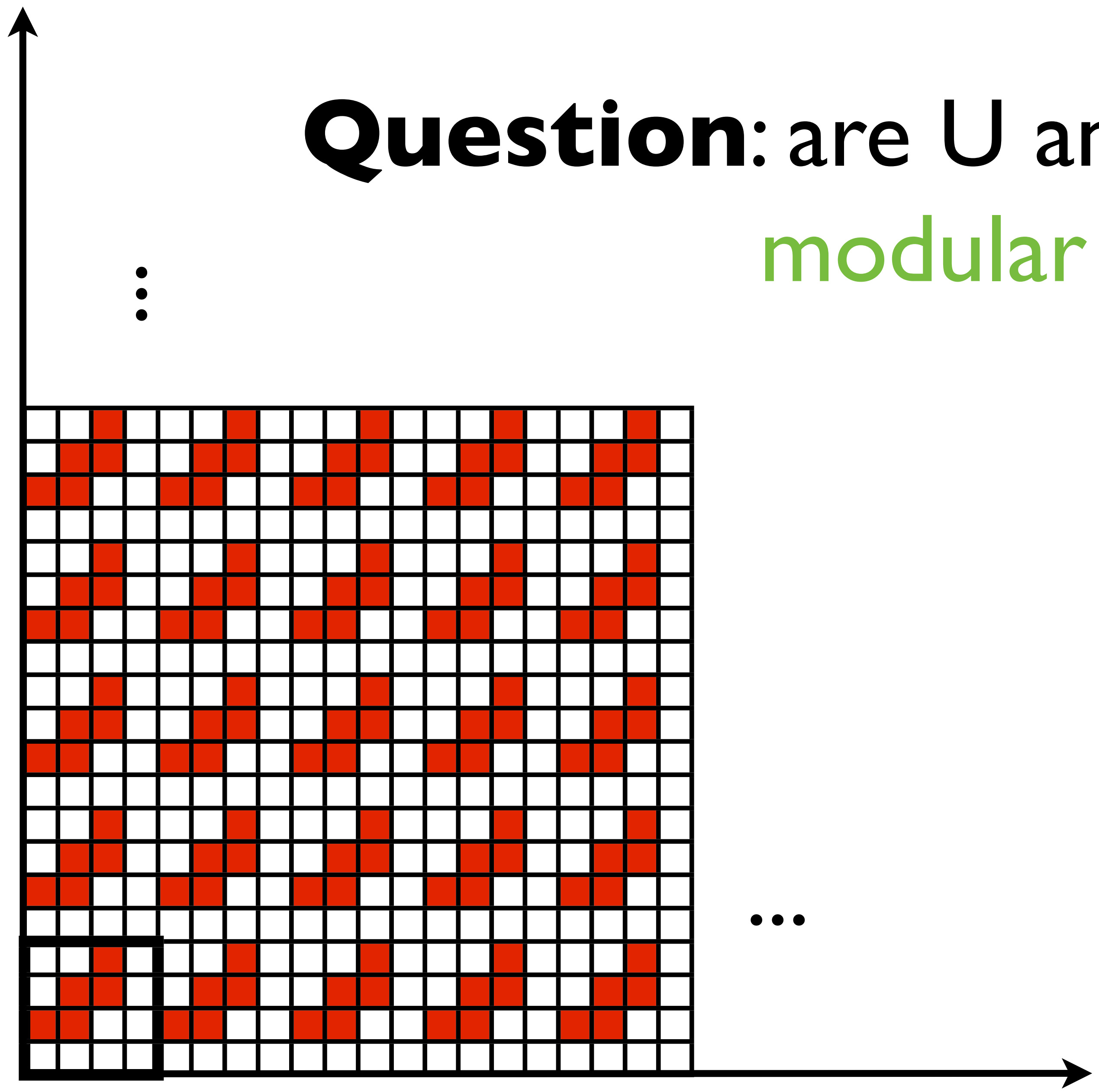

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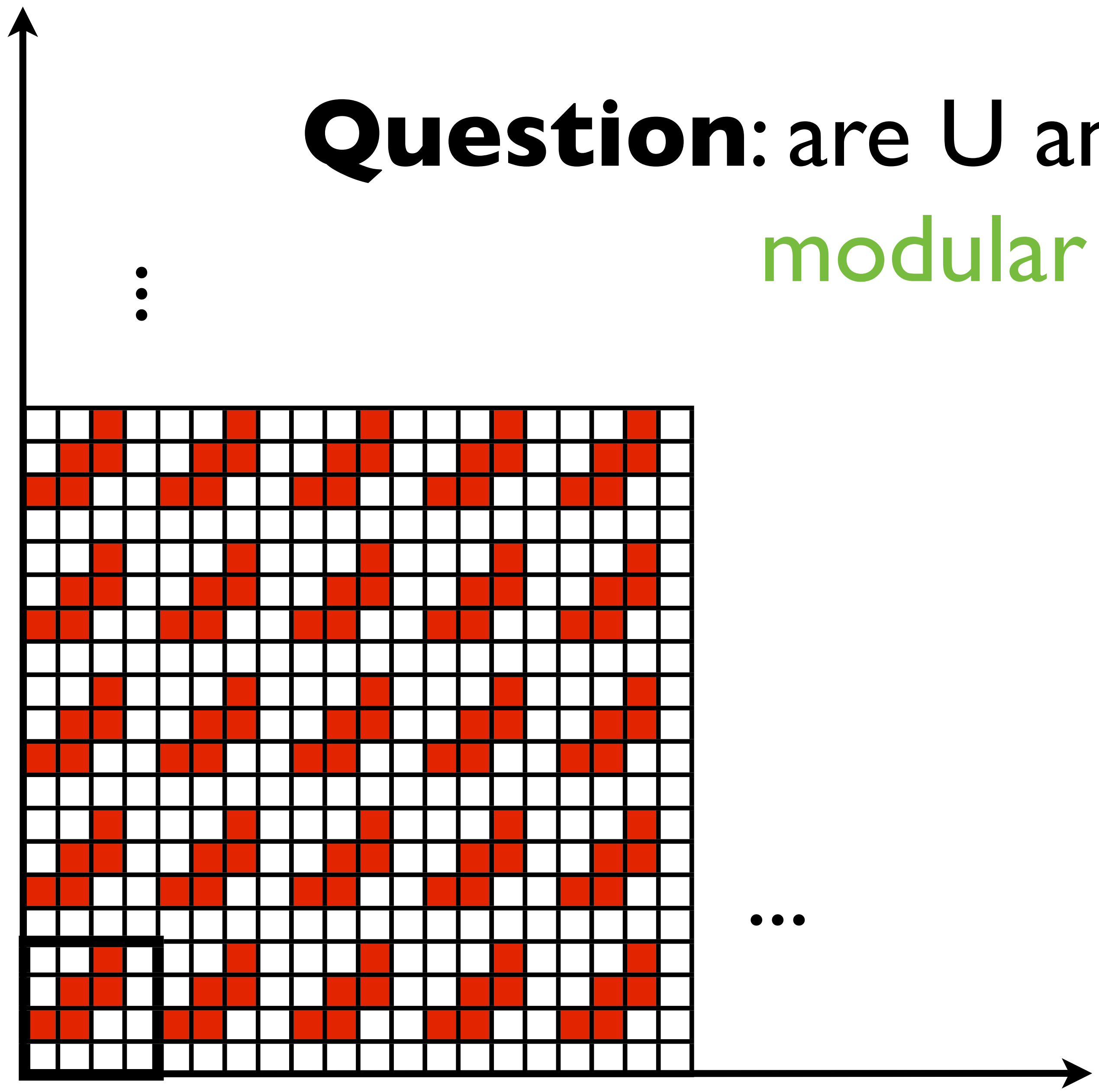

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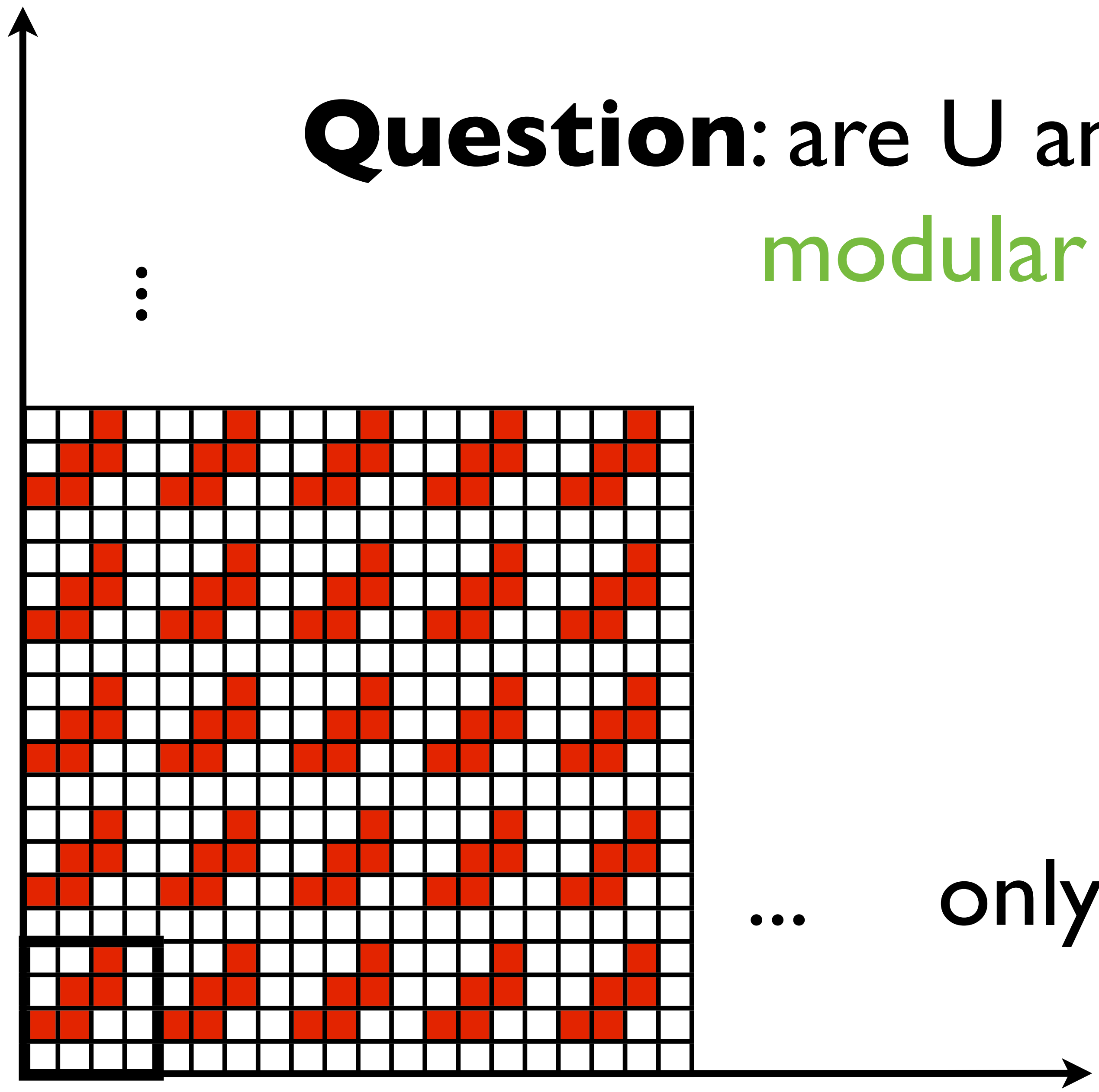

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only value modulo K matters

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- Leroux `09: two VAS reachability sets are separable by a semilinear set iff they are disjoint

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- VAS reachability sets may not be semilinear
- Leroux `09: two VAS reachability sets are separable by a semilinear set iff they are disjoint
- so separability by semilinear sets is decidable

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First main result

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Theorem:
Separability of reachability sets of
Vector Addition Systems by modular sets
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linear set = $\{v_0 + n_1 v_1 + \ldots + n_k v_k \mid n_1, \ldots, n_k \in \mathbb{N}\}$

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simple by VAS reachability

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two semiprocedures

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↑ simple by VAS reachability

negative

enumerates special linear sets $L_U \subseteq U$, $L_V \subseteq V$ and checks whether they are modular separable

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two semiprocedures

positive

enumerates and checks numbers K

simple by VAS reachability

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enumerates special linear sets $L_U \subseteq U$, $L_V \subseteq V$ and checks whether they are modular separable

simple by linear algebra

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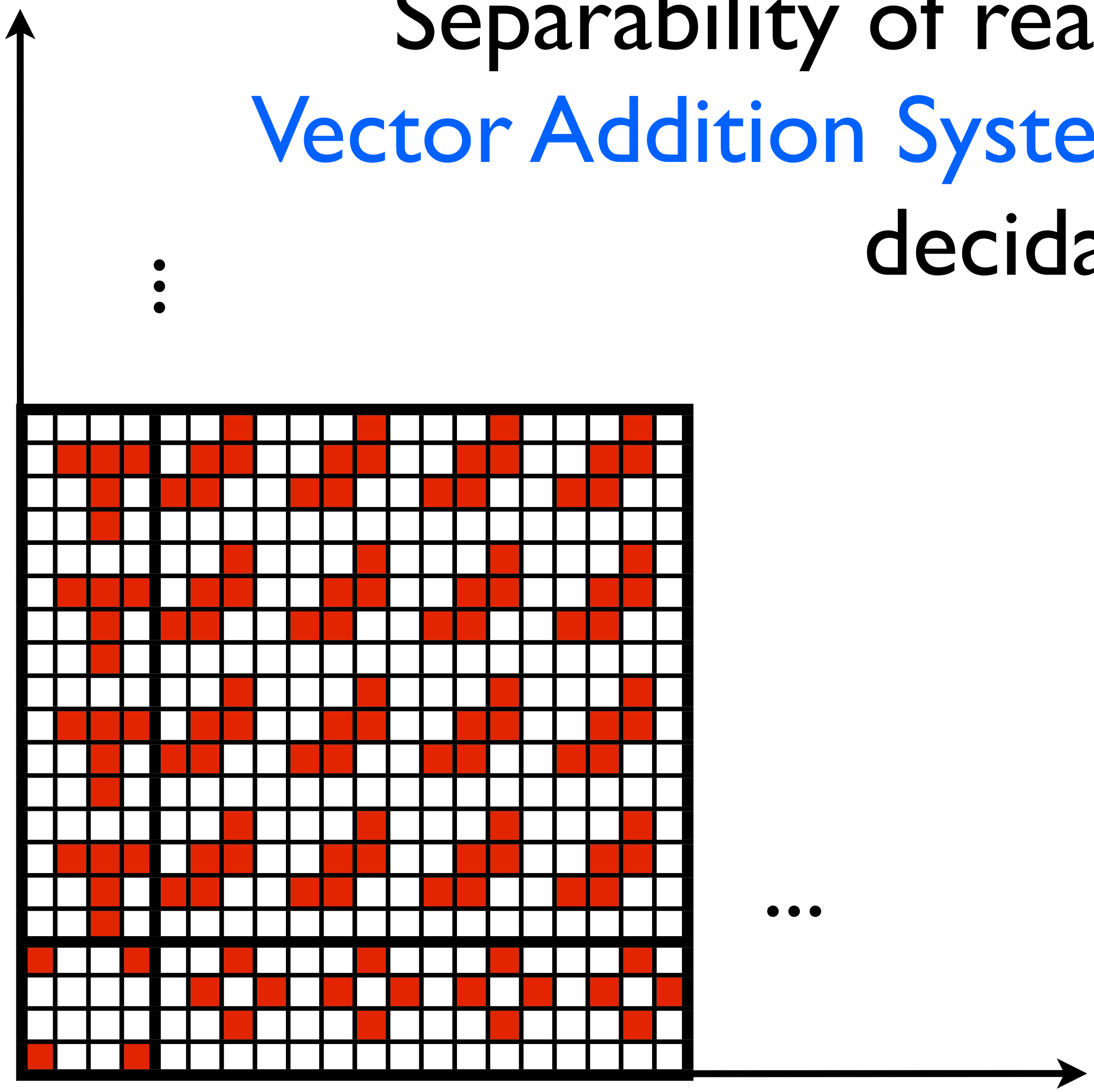

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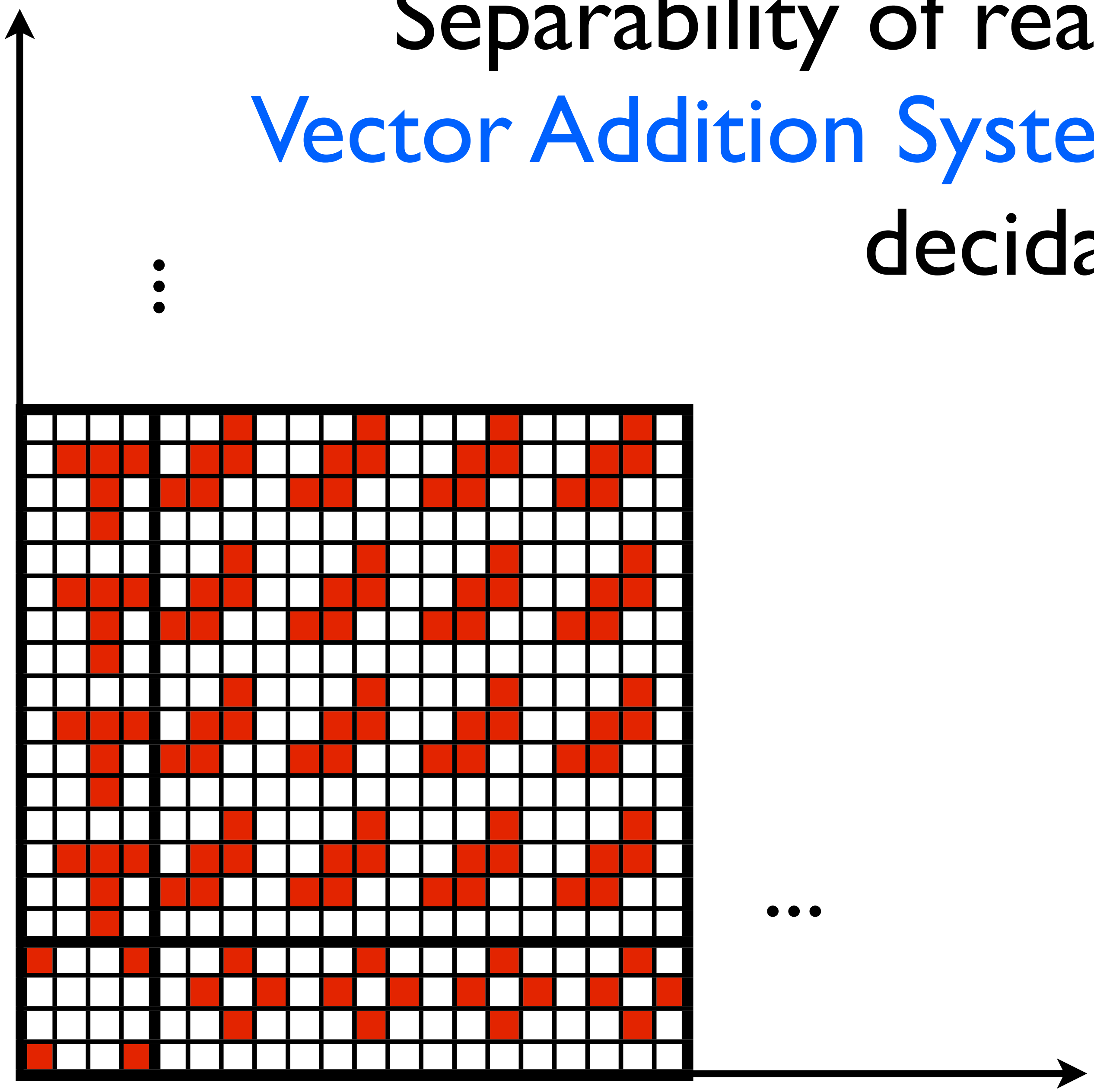

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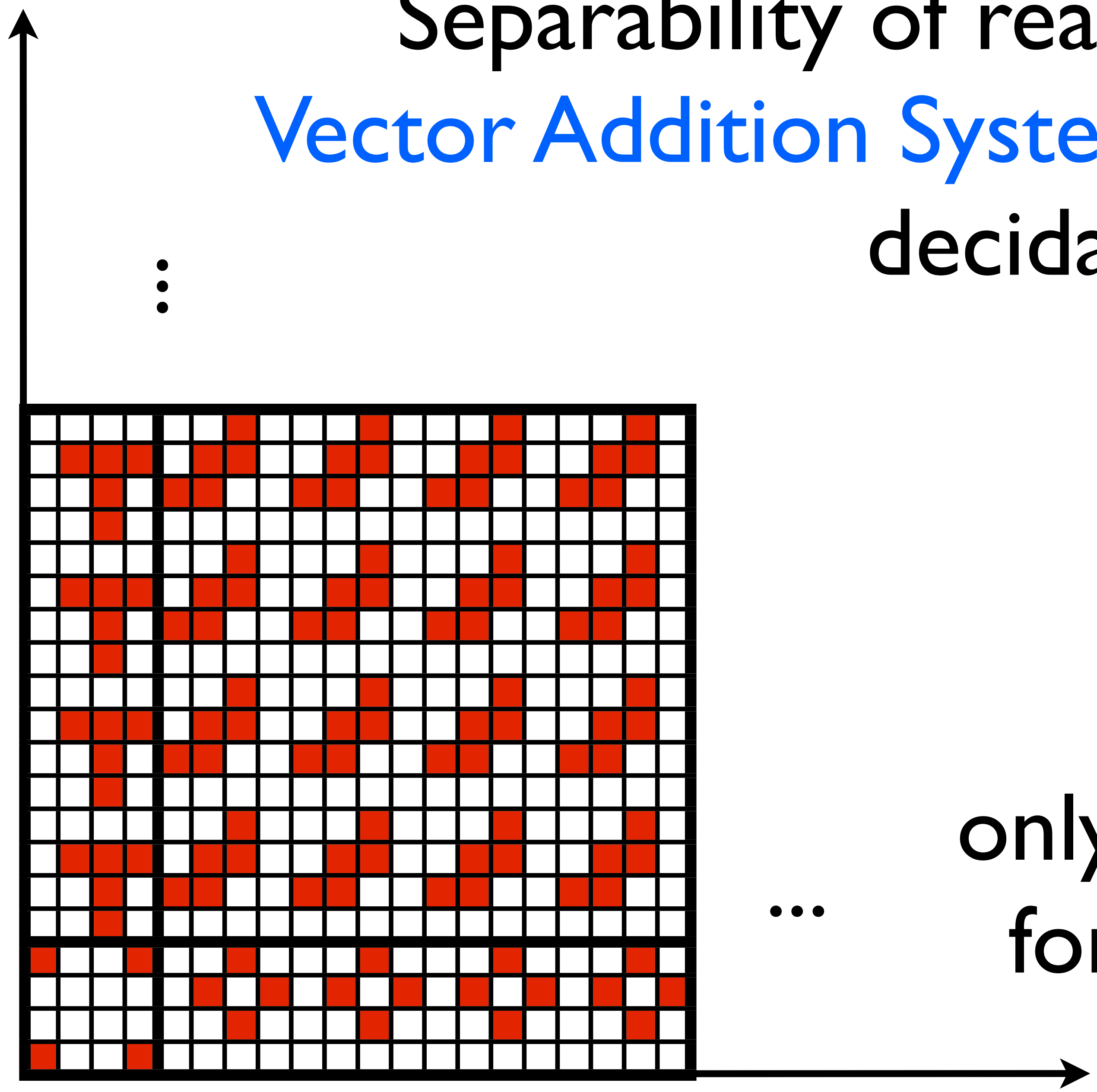

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only value modulo K matters for numbers bigger than K

Thank you!

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