

# The (Almost) Complete Guide to Tree Pattern Containment

Wojciech Czerwiński

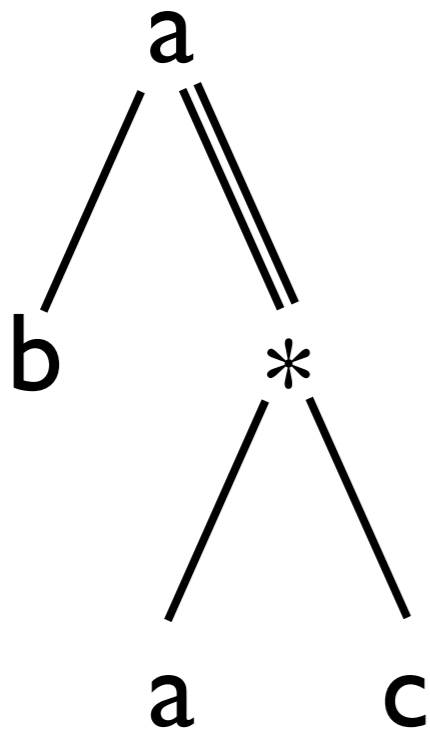
Wim Martens

Paweł Parys

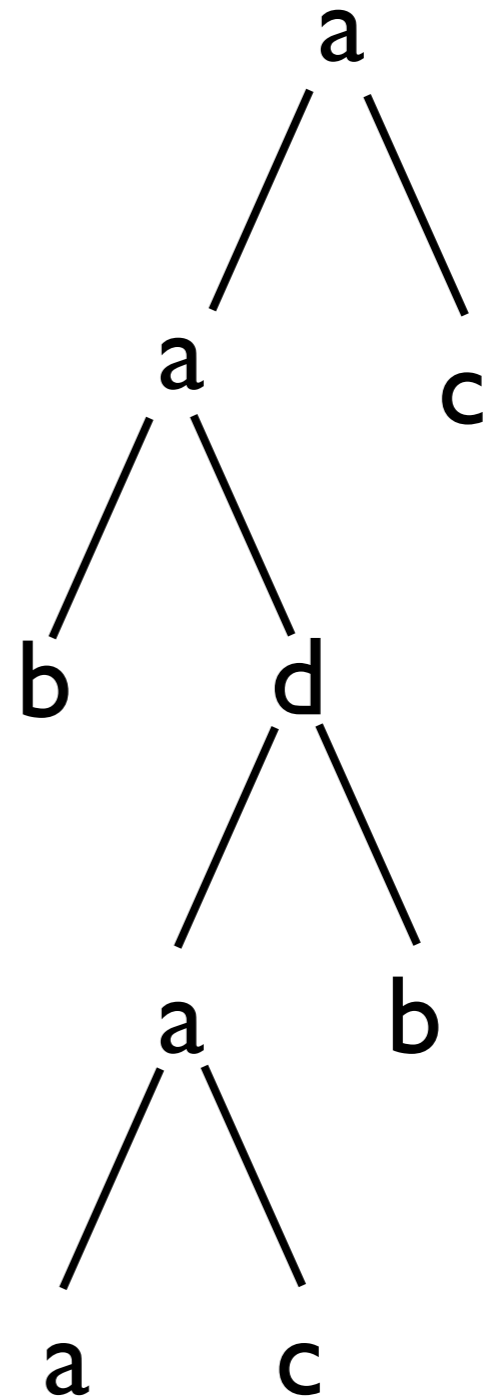
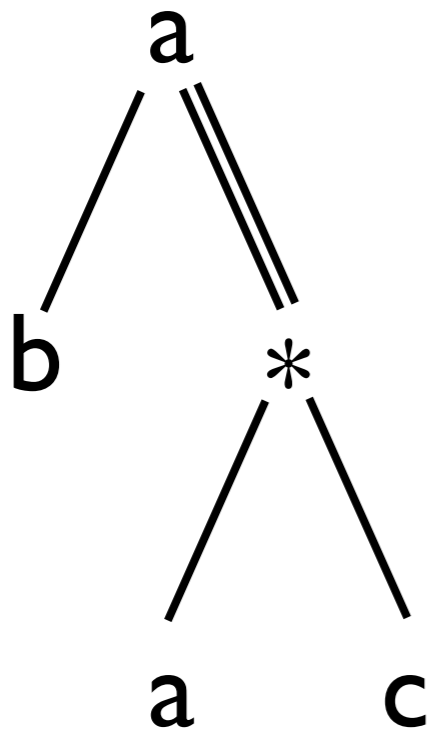
Marcin Przybyłko

# Tree Pattern

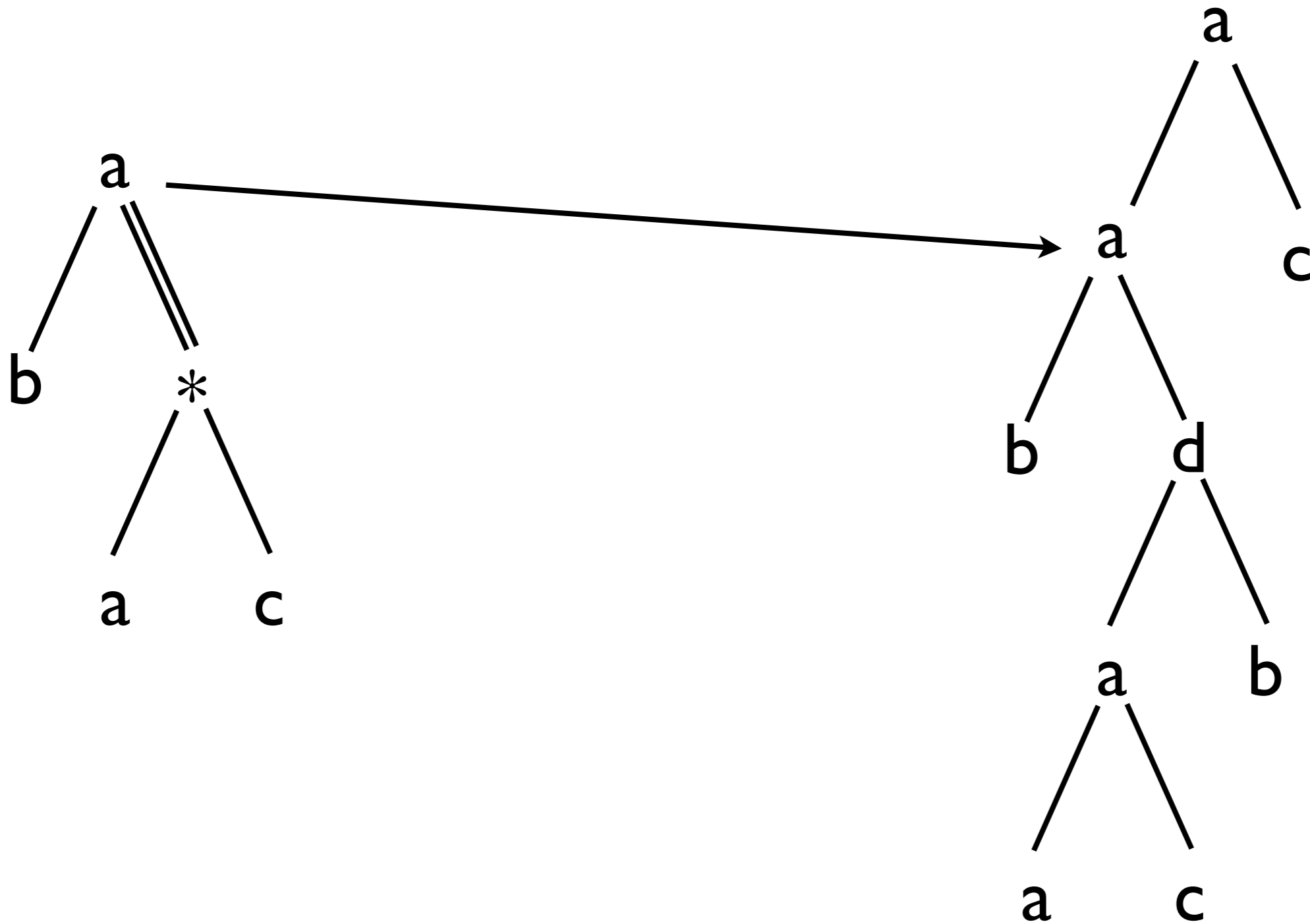
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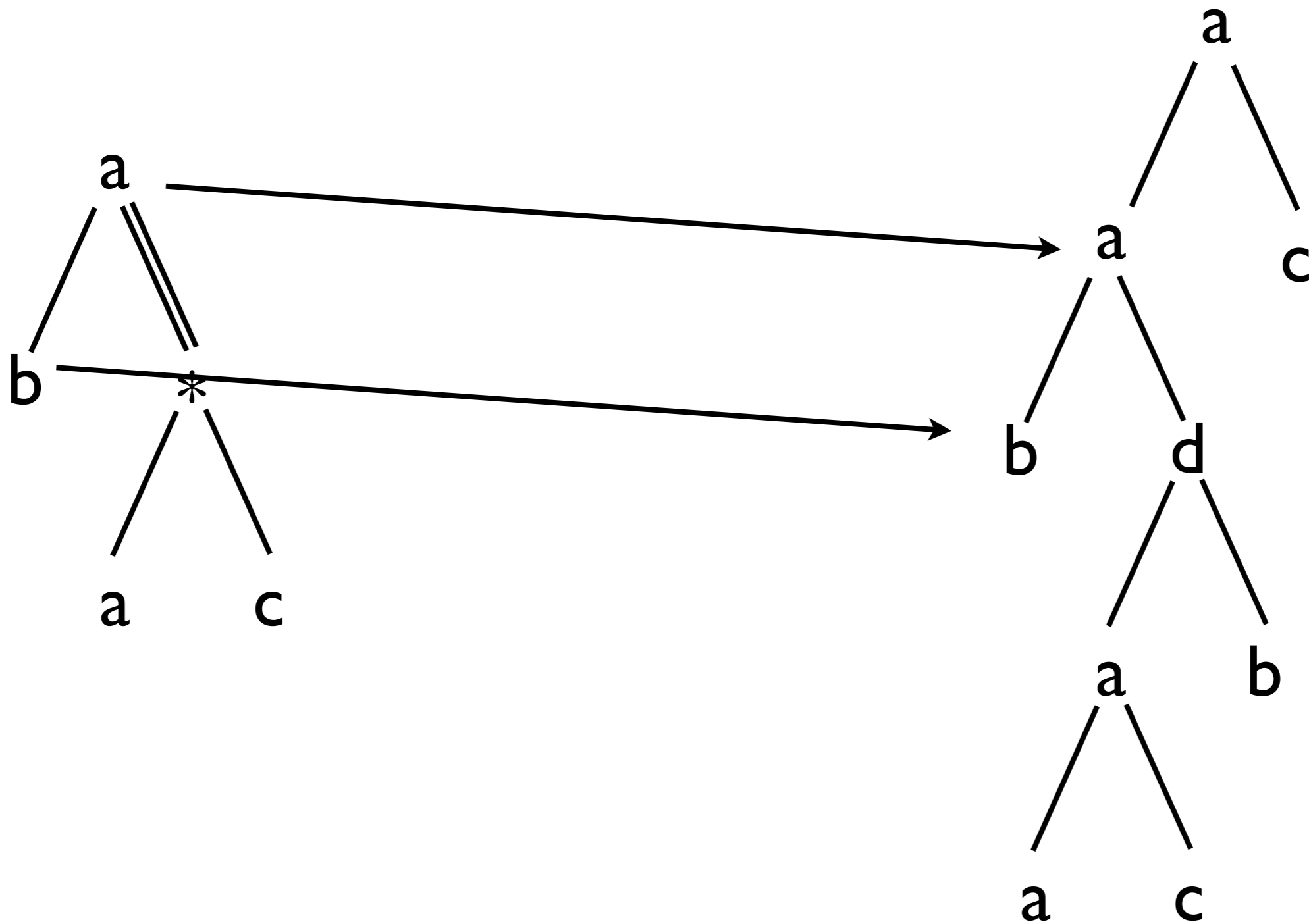
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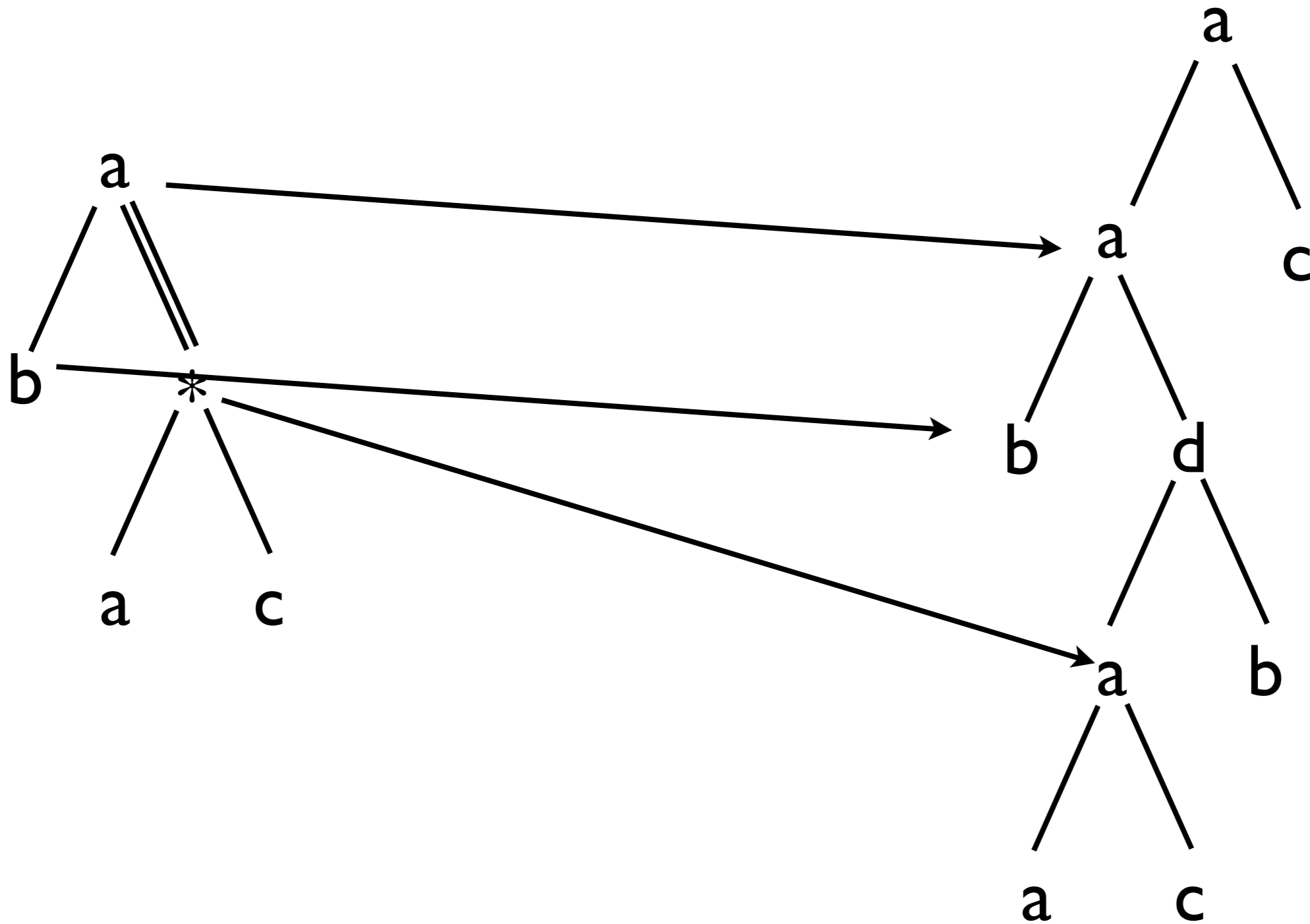
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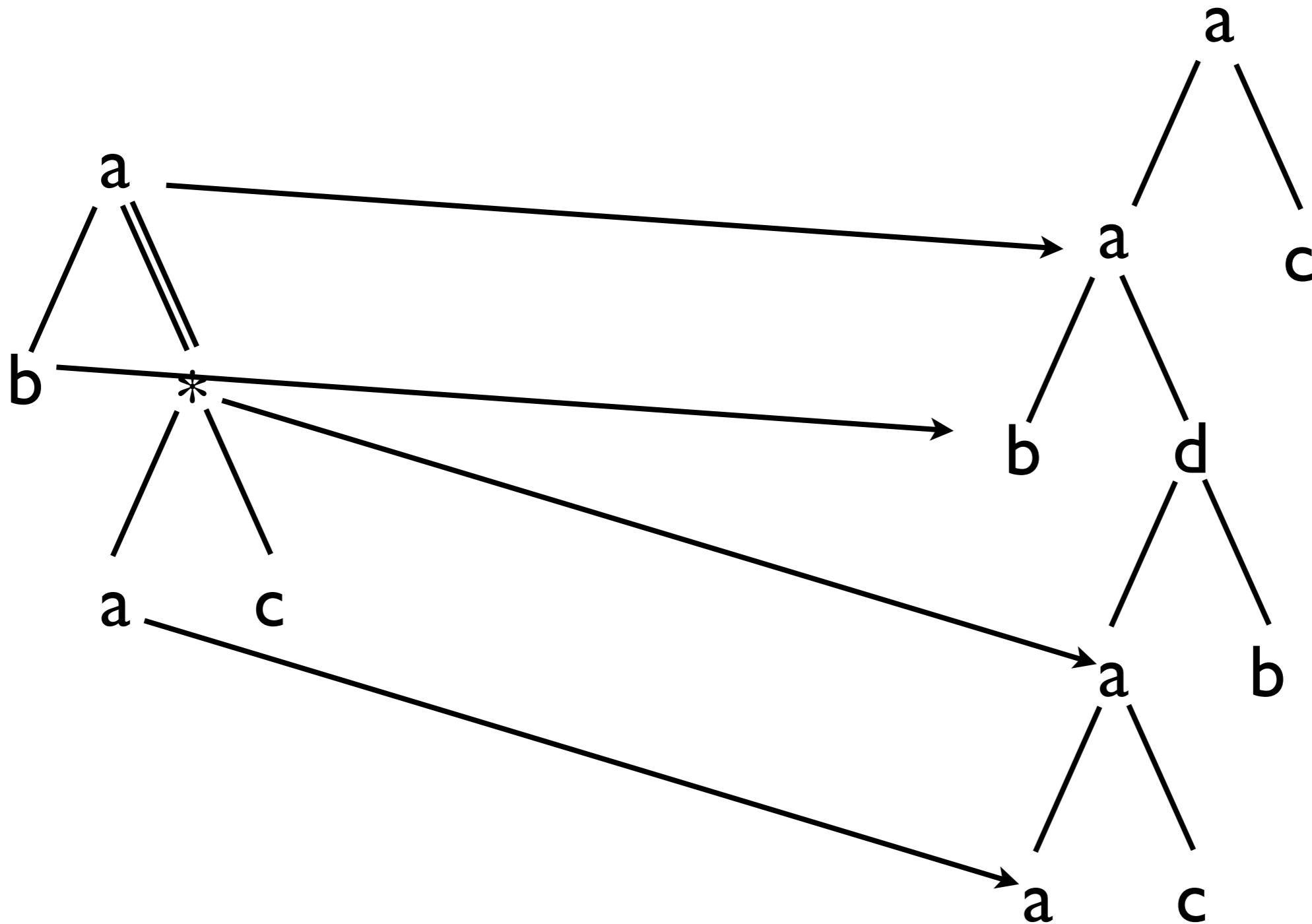
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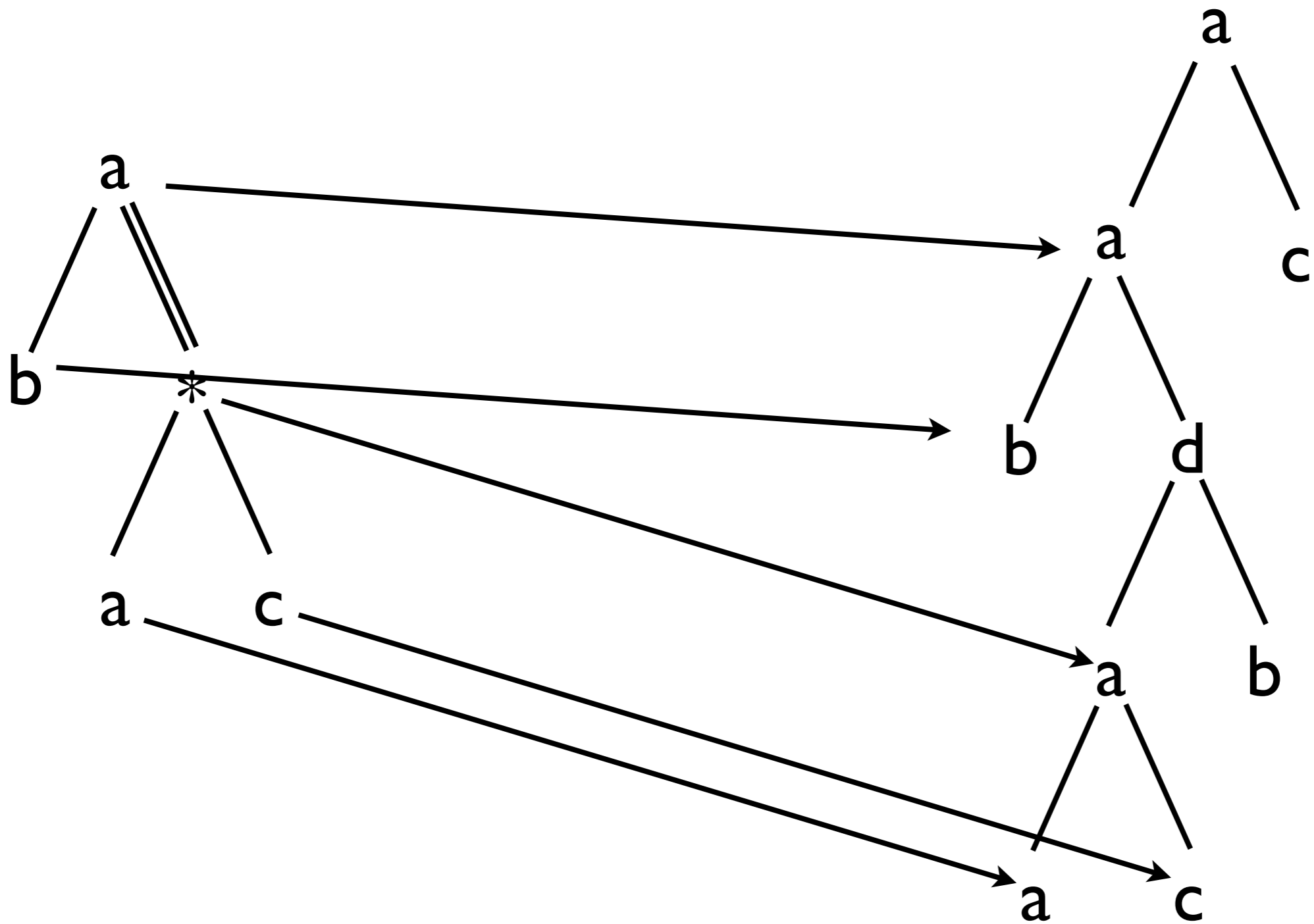


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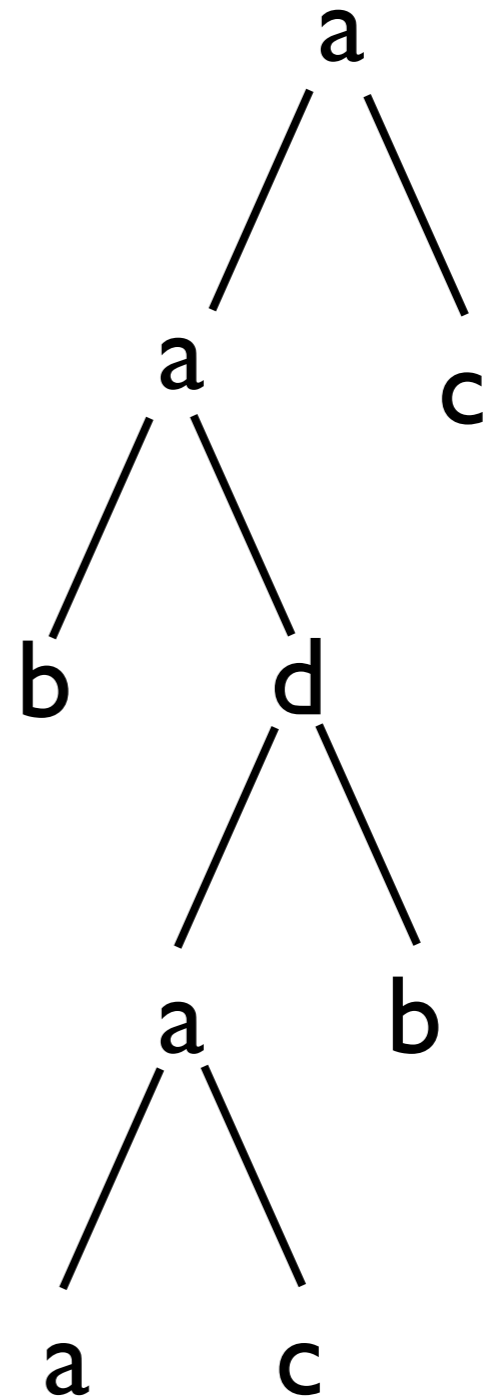
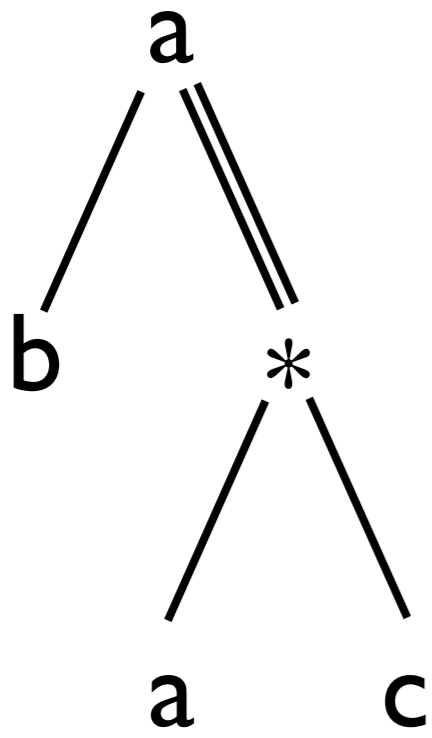




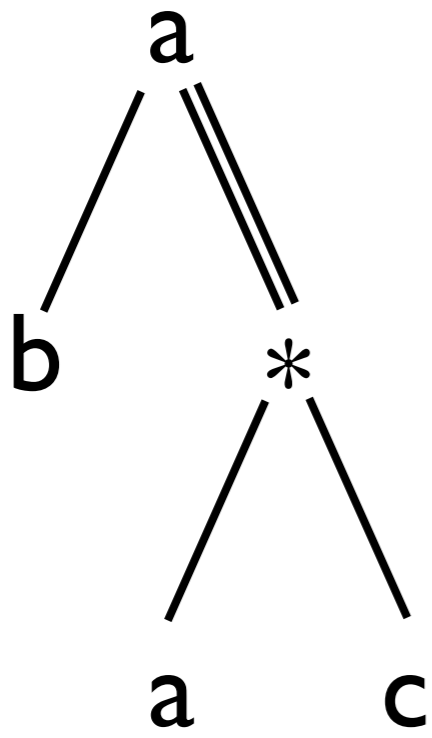
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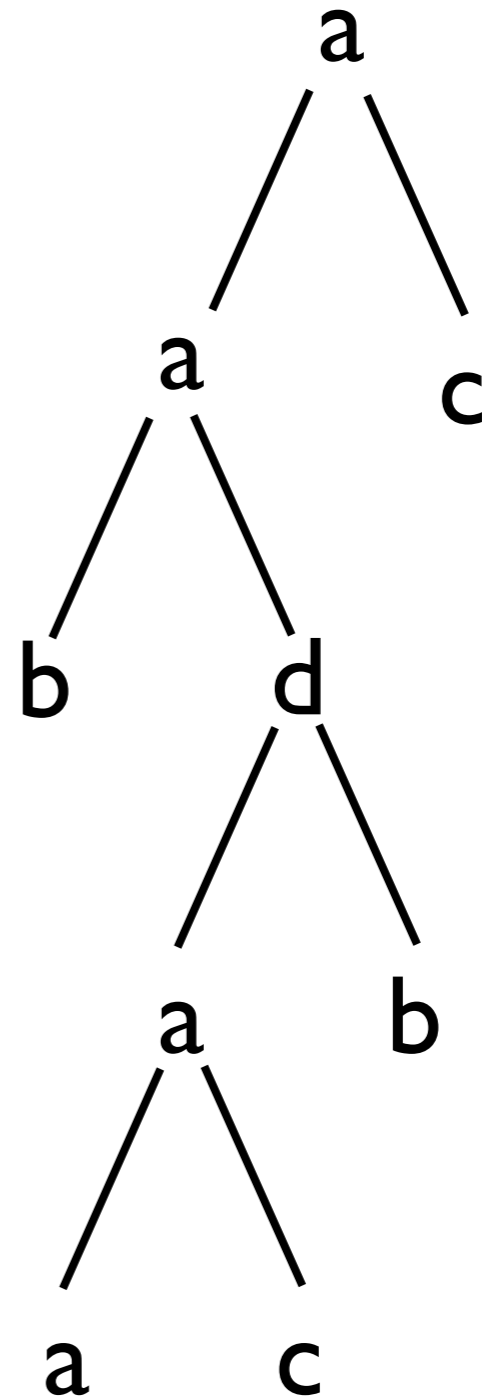
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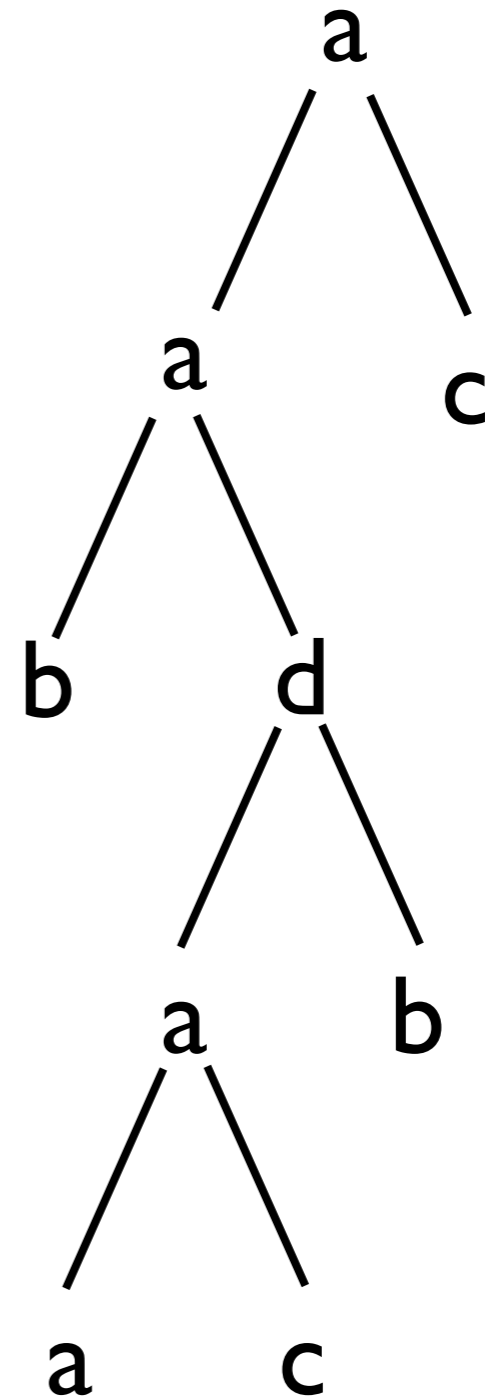
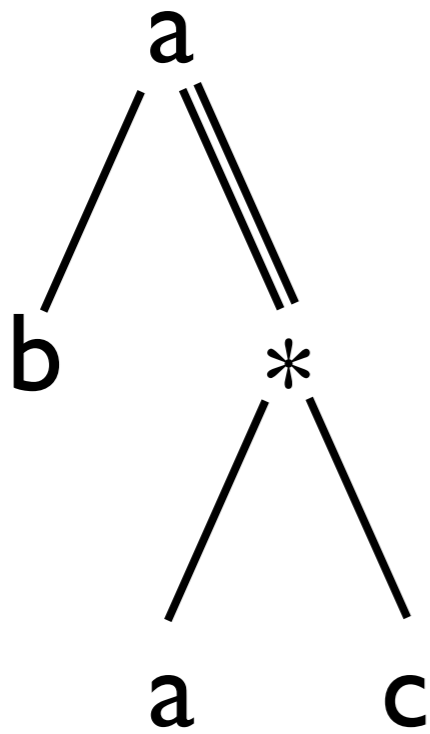
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$L_w(p)$

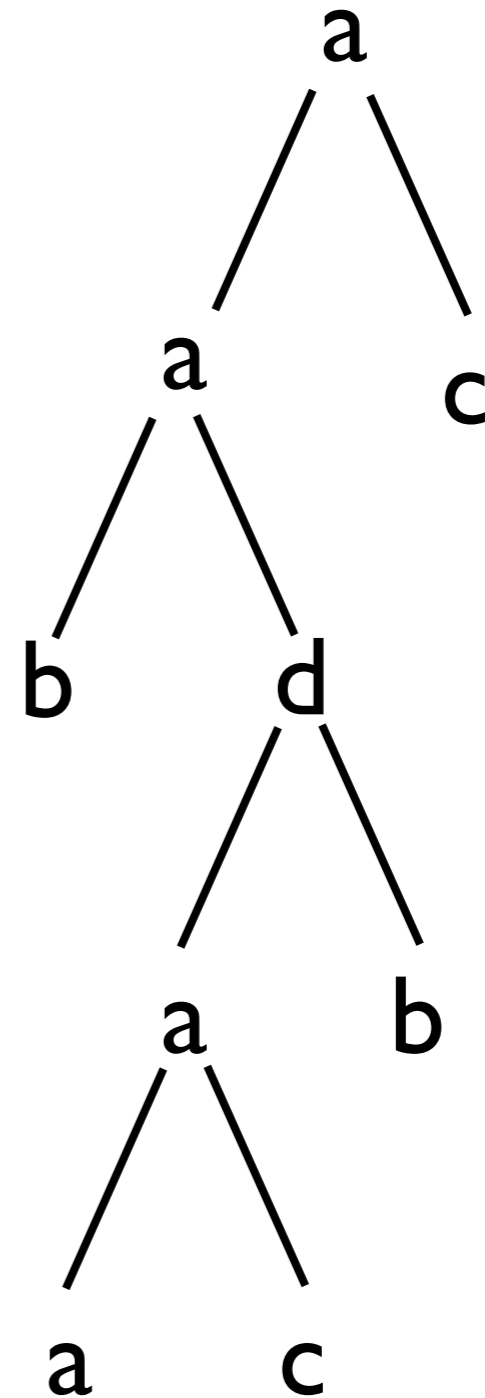
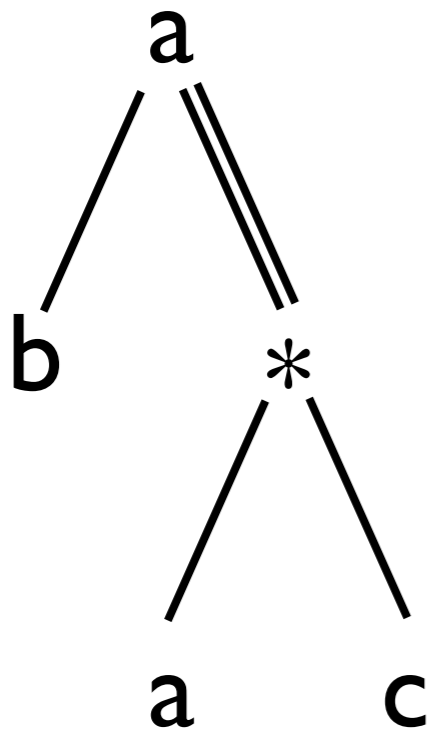


# Tree Pattern



$L_w(p)$  = set of trees which embed  
**weakly** into  $p$

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$L_s(p)$  = set of trees which embed  
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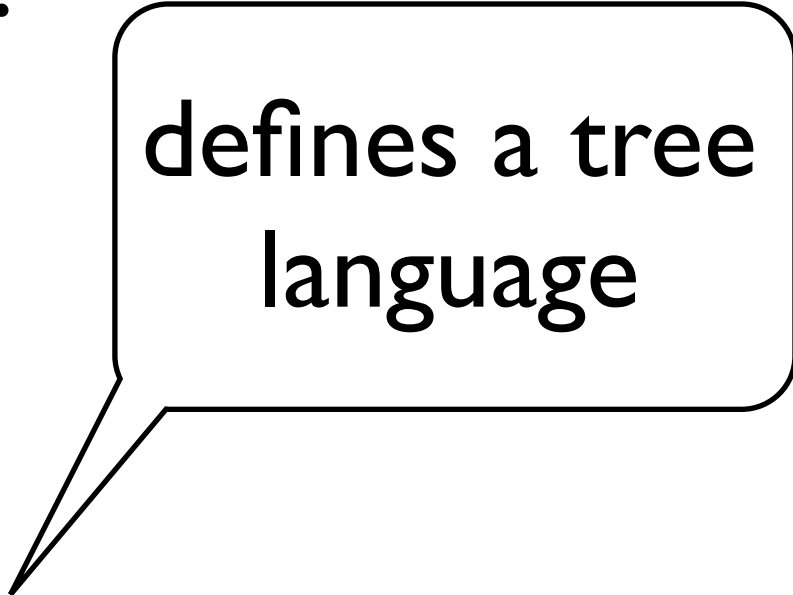
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defines a tree language

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does p enforce q among trees in L(d)?

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- XML with incomplete information (Barcelo et al. '10)
- (some) results transfer to graph structured data



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Goal: uniform overview, solve hard cases,  
track the PTIME borderline

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- a lot of mess in the landscape

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**This talk**



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- $PQ(/, //)$  - no wildcard



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Technique: new triomino-tiling method

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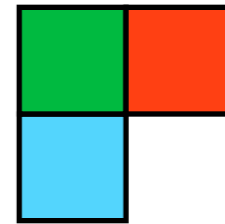
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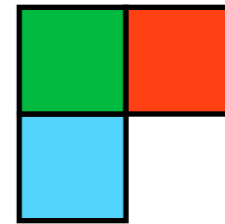


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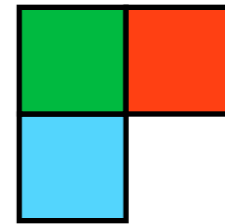


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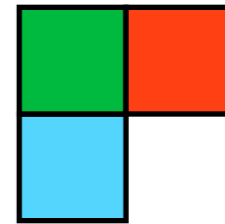
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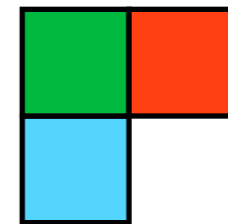


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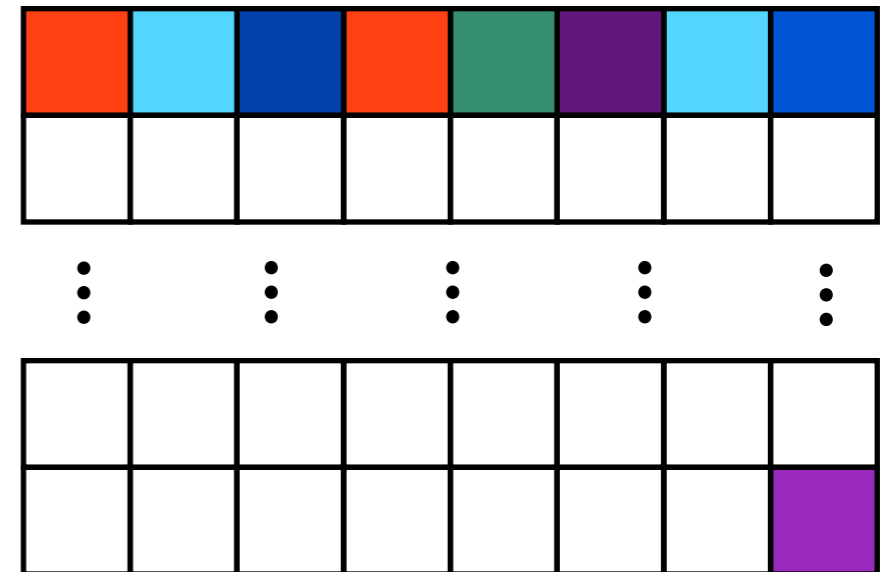
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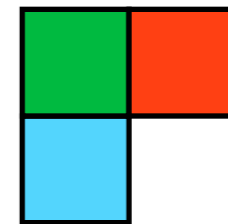




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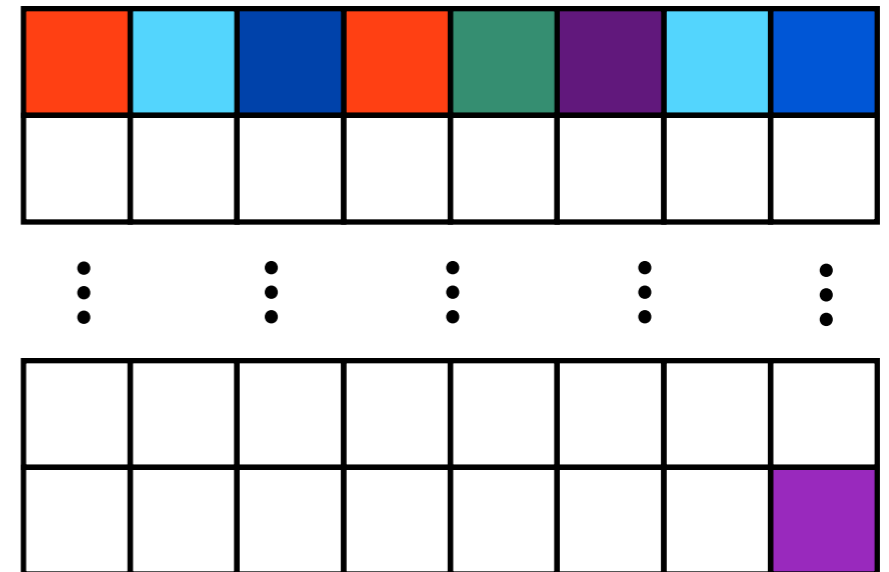
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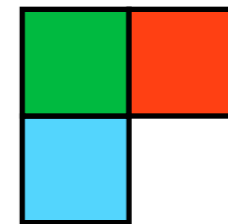
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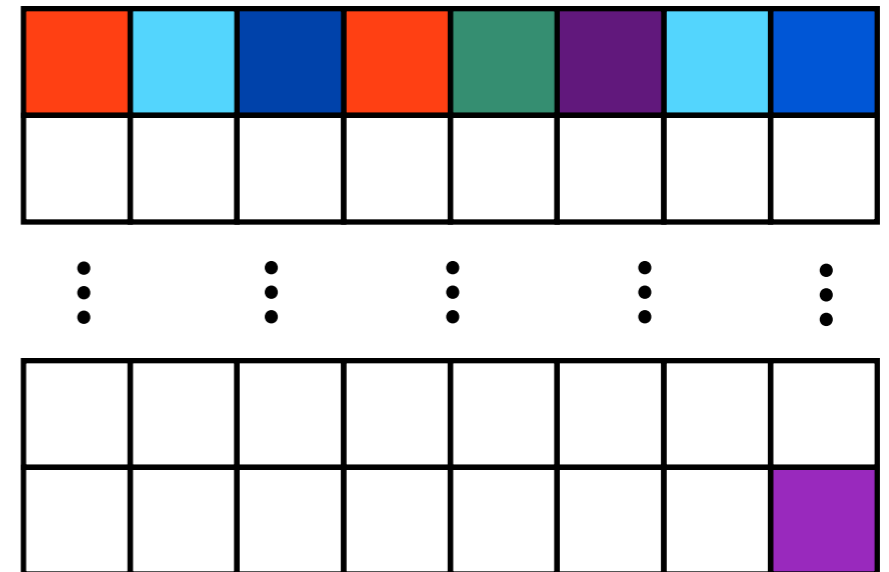


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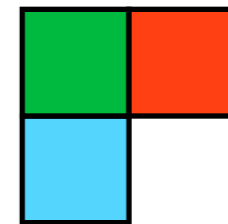
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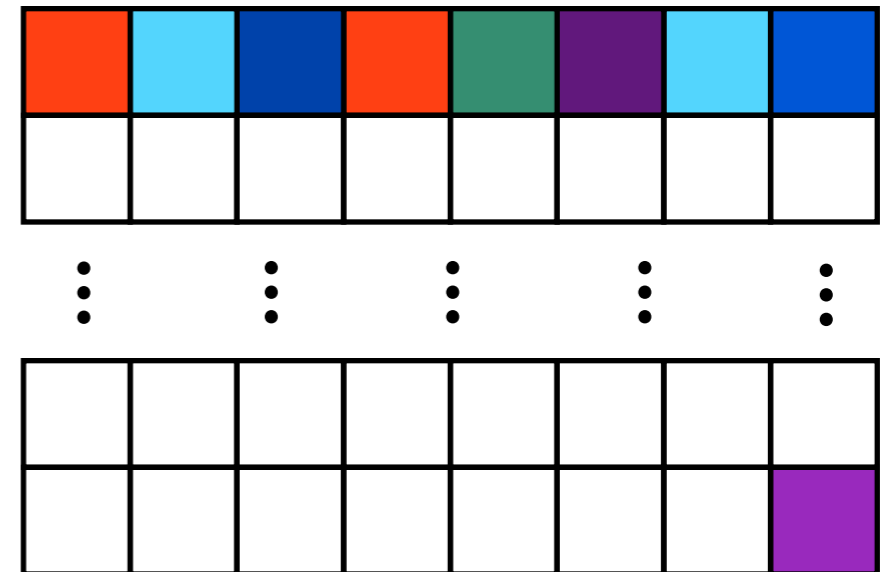
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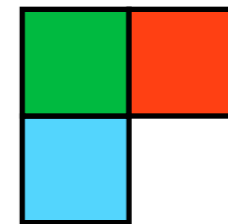
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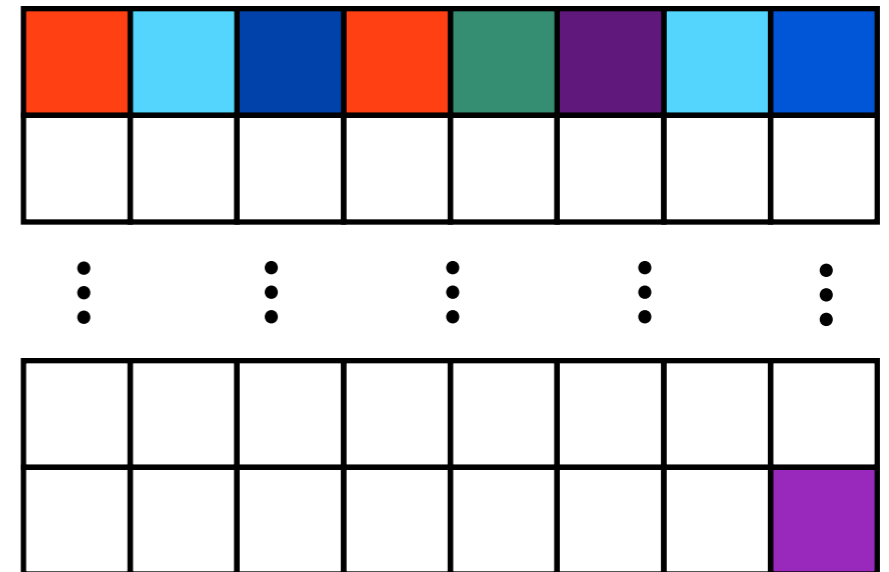


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Right pattern and DTD: implement forbidden triominos via forbidding  $a^{*****}b$

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**open for a decade - now complete  
classification without DTD**

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Message: **such a uniform technique works in all easy cases**

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- probably hard to solve

**Thank you!**