Regular Separability of One Counter Automata

Wojciech Czerwiński

Sławomir Lasota

Separability

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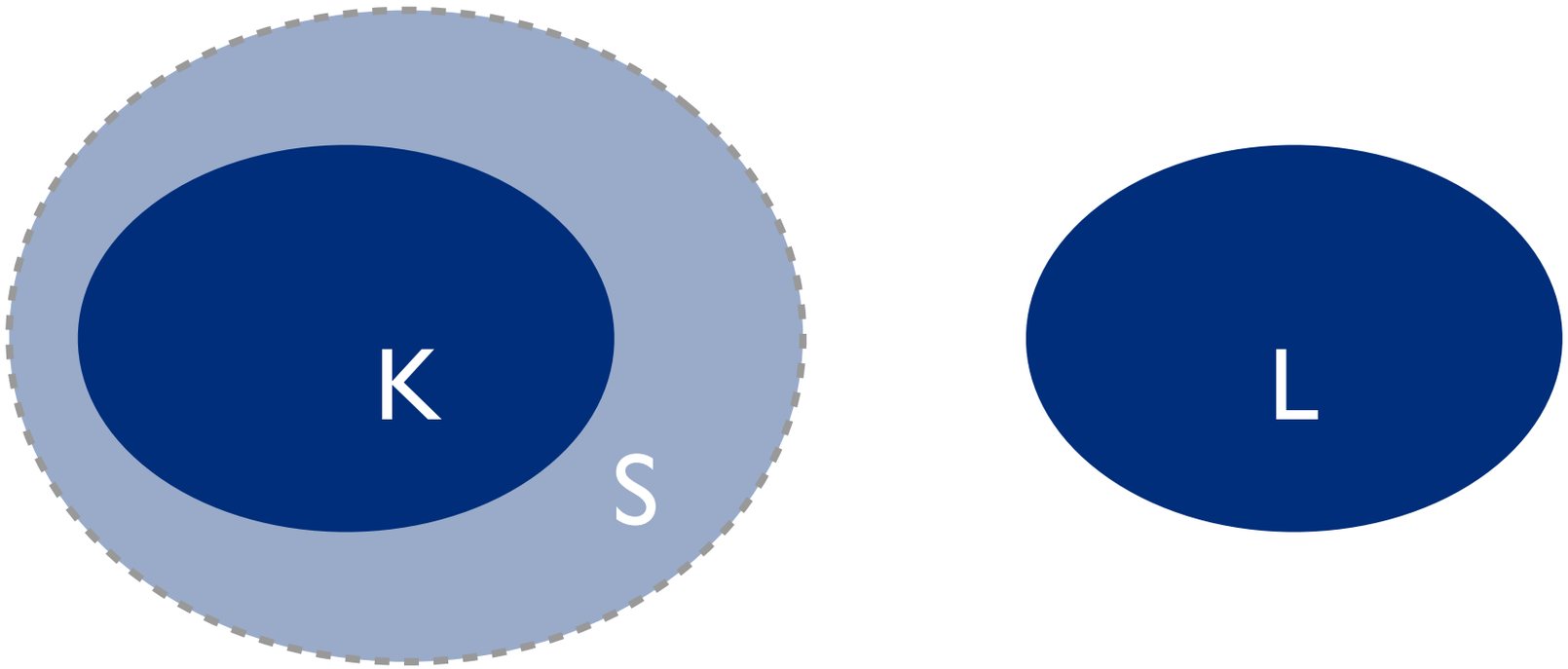

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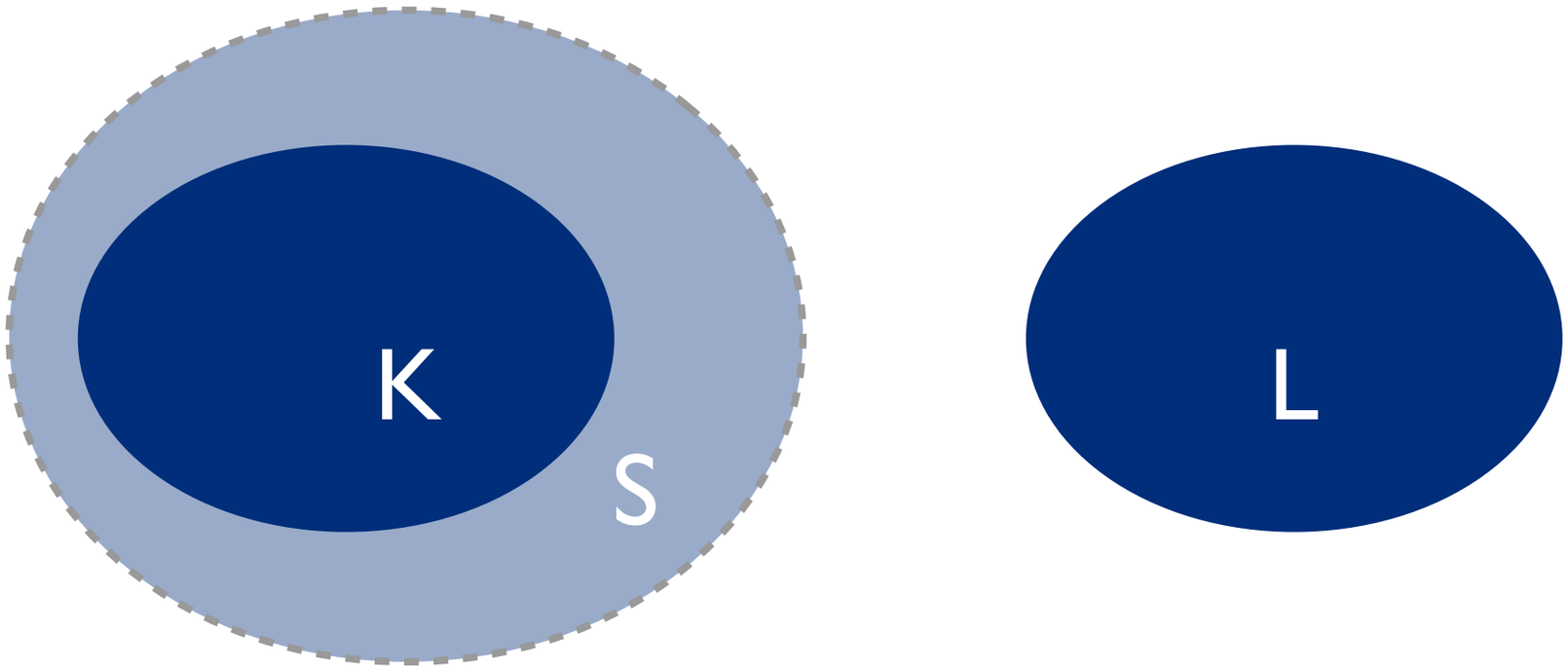
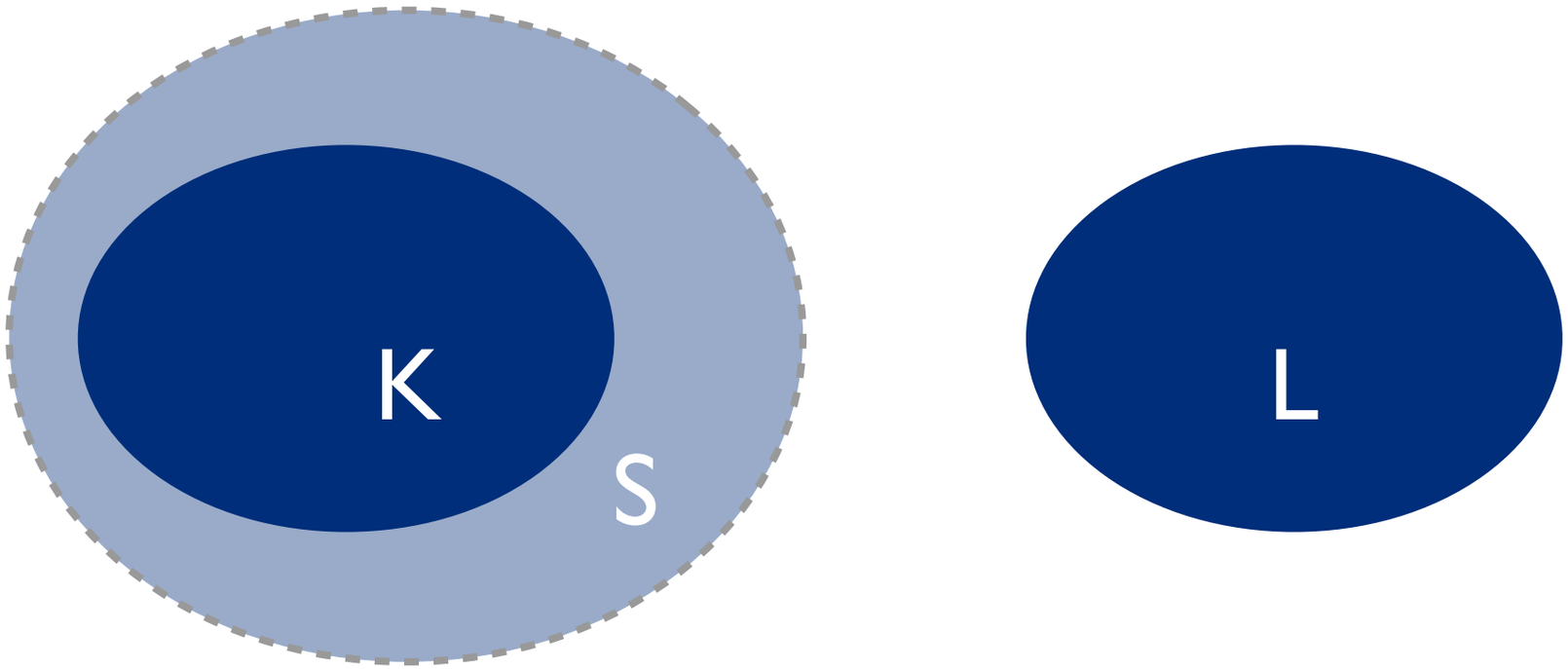


S *separates* K and L

Separability



S *separates* K and L

K and L are *separable by* family F
if some S from F separates them

General problem

F separability of G

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Given: two languages K and L from family G

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Given: two languages K and L from family G

Question: are K and L separable by some language from family F

History of separability

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- regular separability of CFL is undecidable (Szymański, Williams '76)

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- regular separability of visibly pushdown languages is undecidable (Kopczyński '15)

Motivation

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- this talk: one counter languages

One counter automata

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configuration: state + one nonnegative counter

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language: single source and target configuration (assume counter 0), every transition labelled

Problem(s)

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Given: two one counter nets (automata) A and B

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Question: are L(A) and L(B) regular separable?

Main result

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Theorem:

Regular separability of
languages of one counter nets
is decidable

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Remark: even PSPACE-complete

Another result

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Proof idea: approximants

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For OCN A and integer n define automaton A_n such that:

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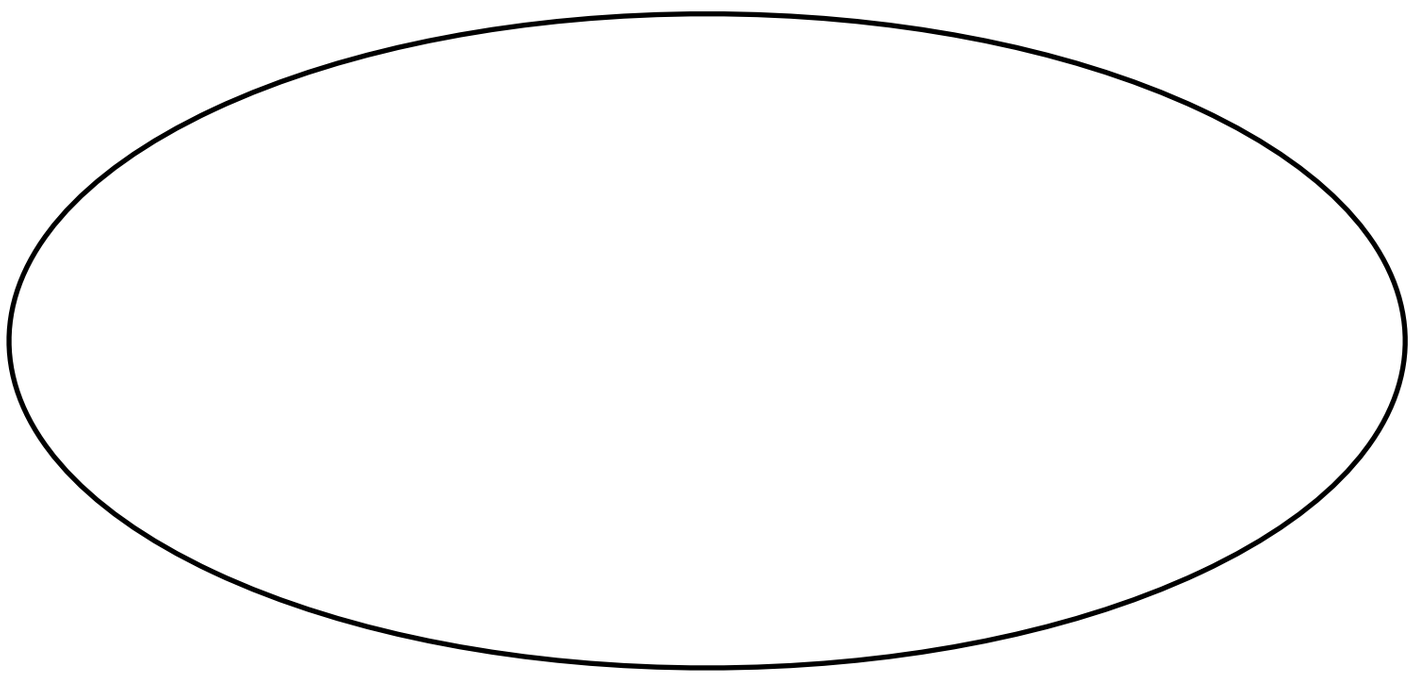

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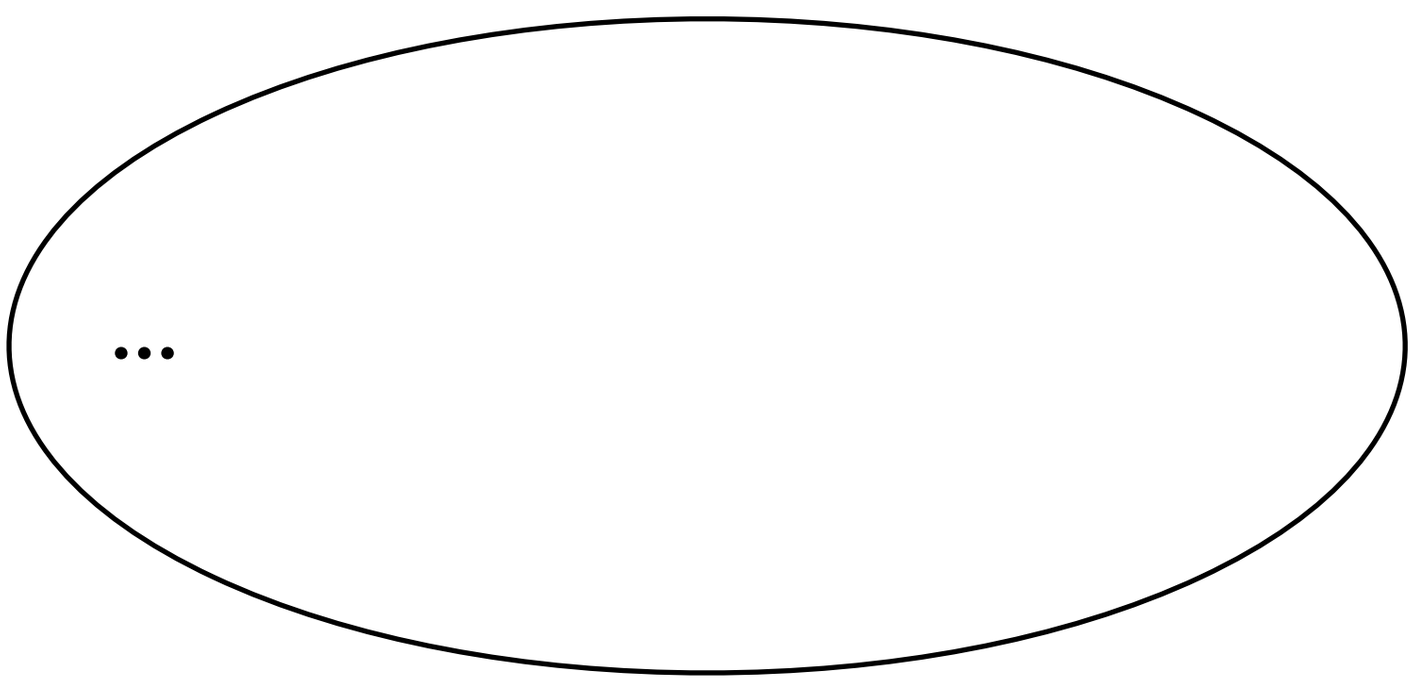

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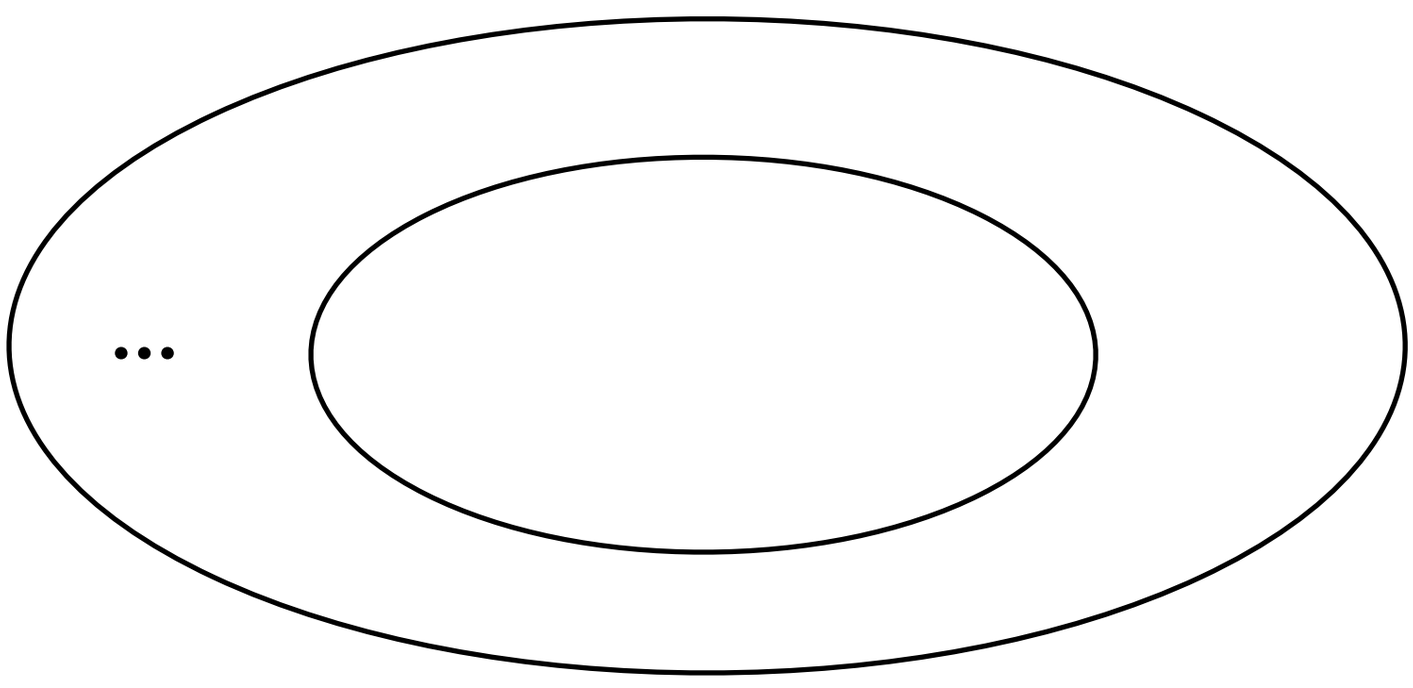

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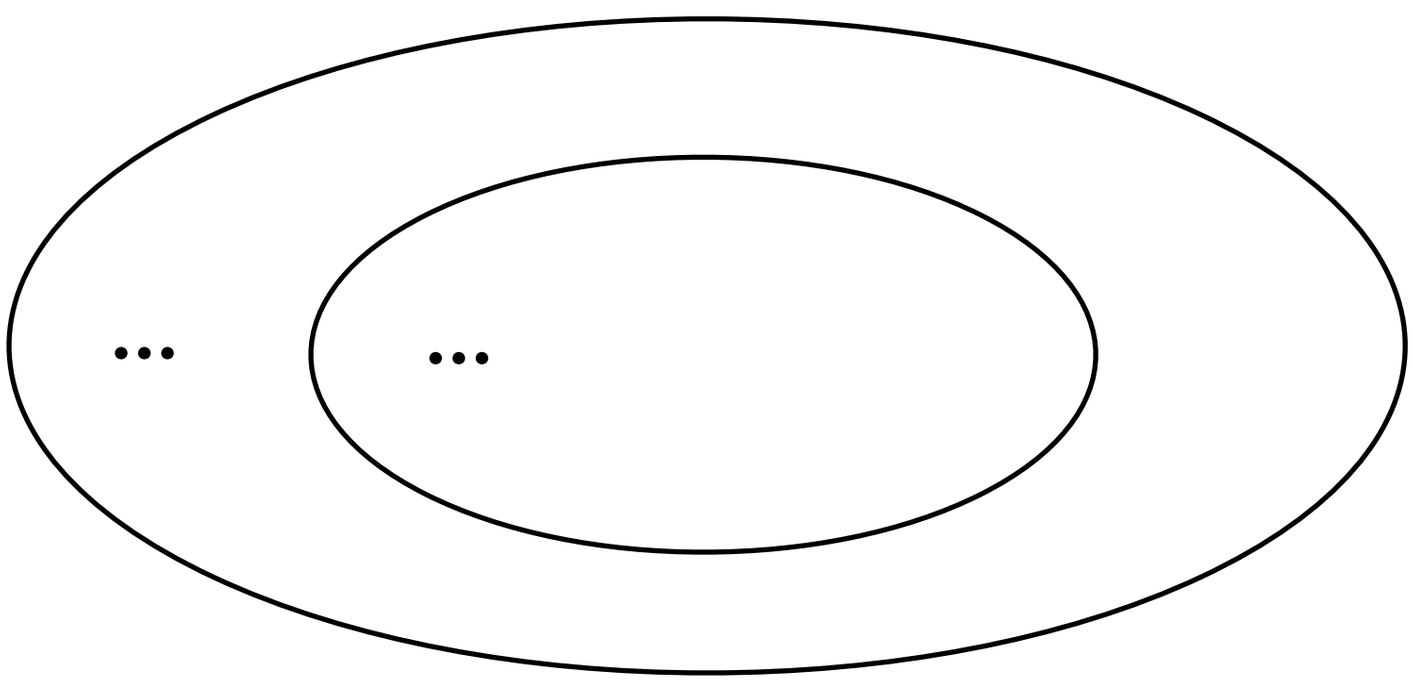

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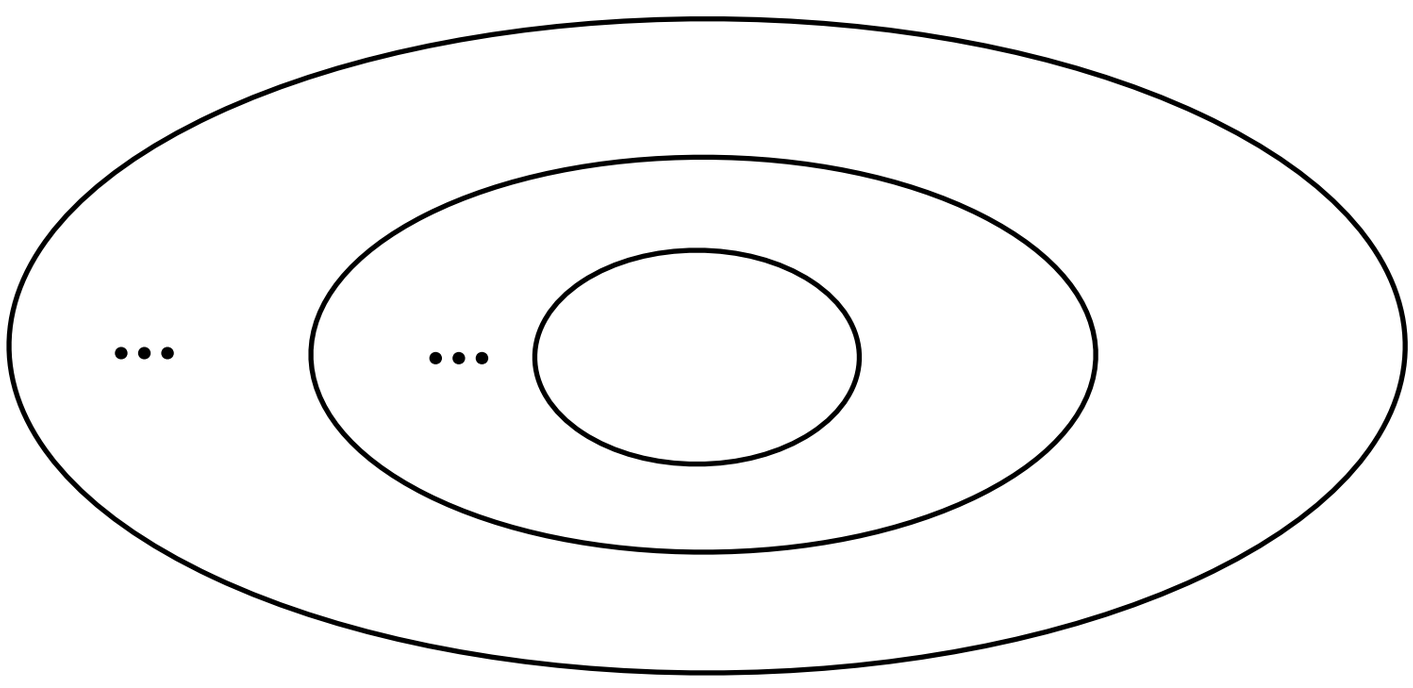

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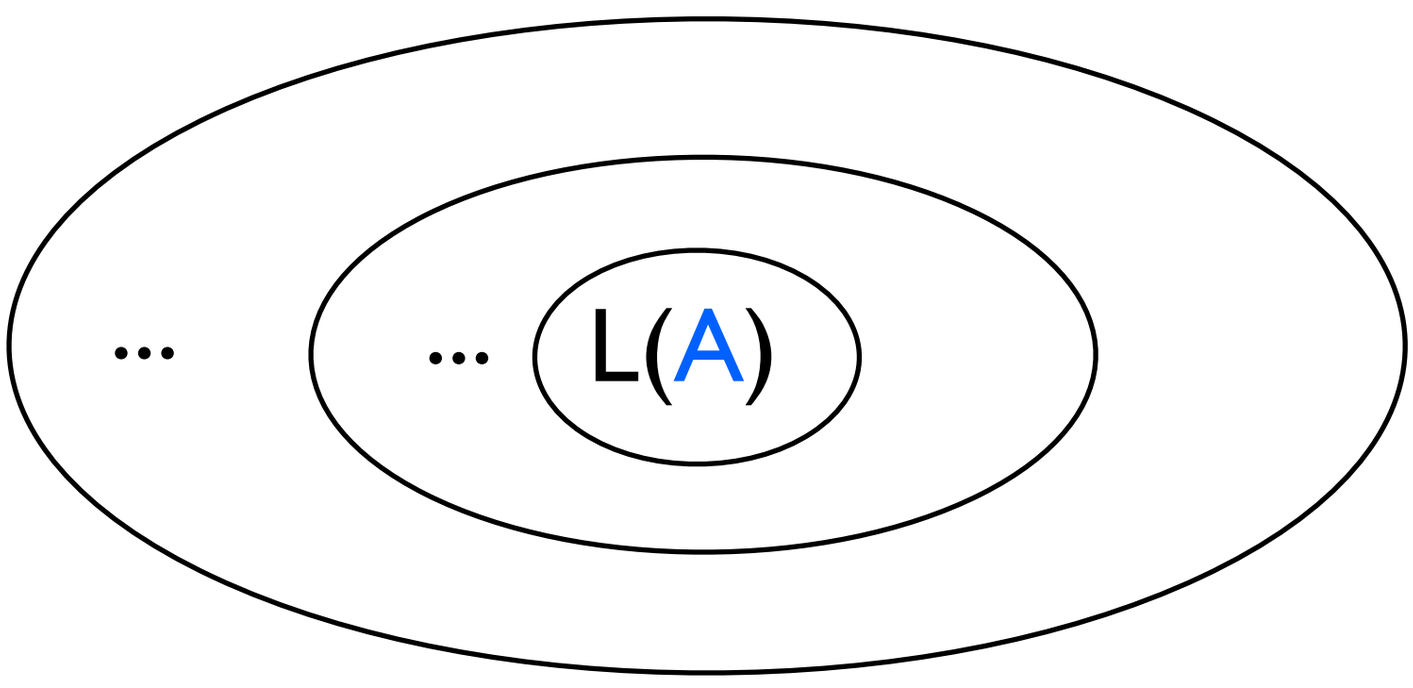

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1) L(A) and L(B) are regular separable

2) there exists n s.t. $L(A_n)$ and $L(B_n)$ disjoint

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Lemma

For two OCNs A and B tfae:

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One direction clear!

Definition

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A - set of states: Q

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A_n - set of states: $Q \times \{0, ..., n-1\} \times \{small, big\}$

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A - set of states: Q

A_n - set of states: Q x {0, ..., n-1} x {small, big}

A_n remembers:

- state of A
- counter value modulo n
- is counter smaller than n? (maybe wrong)

Proof idea

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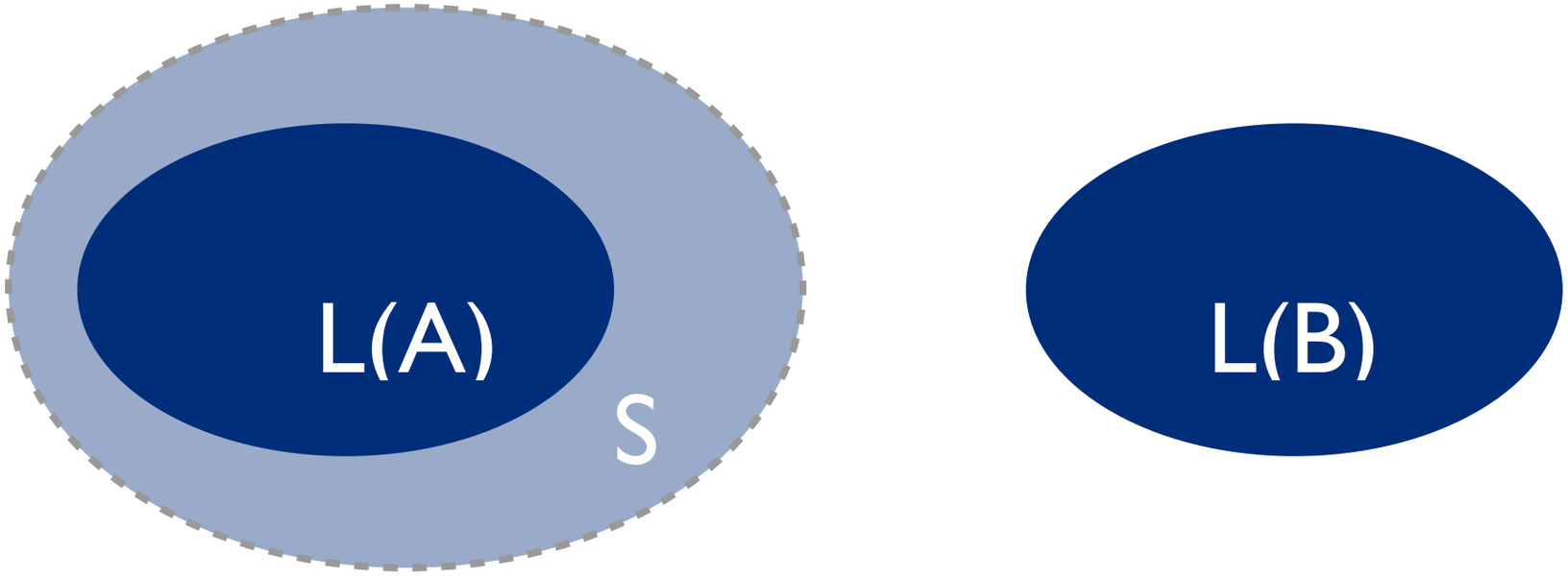

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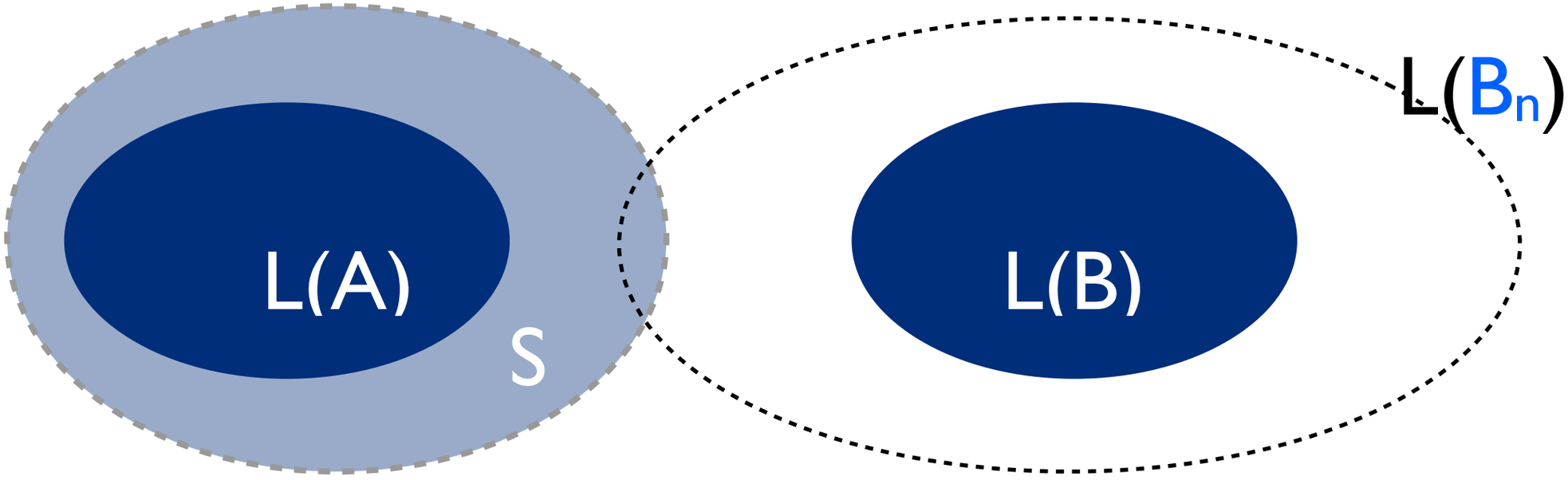

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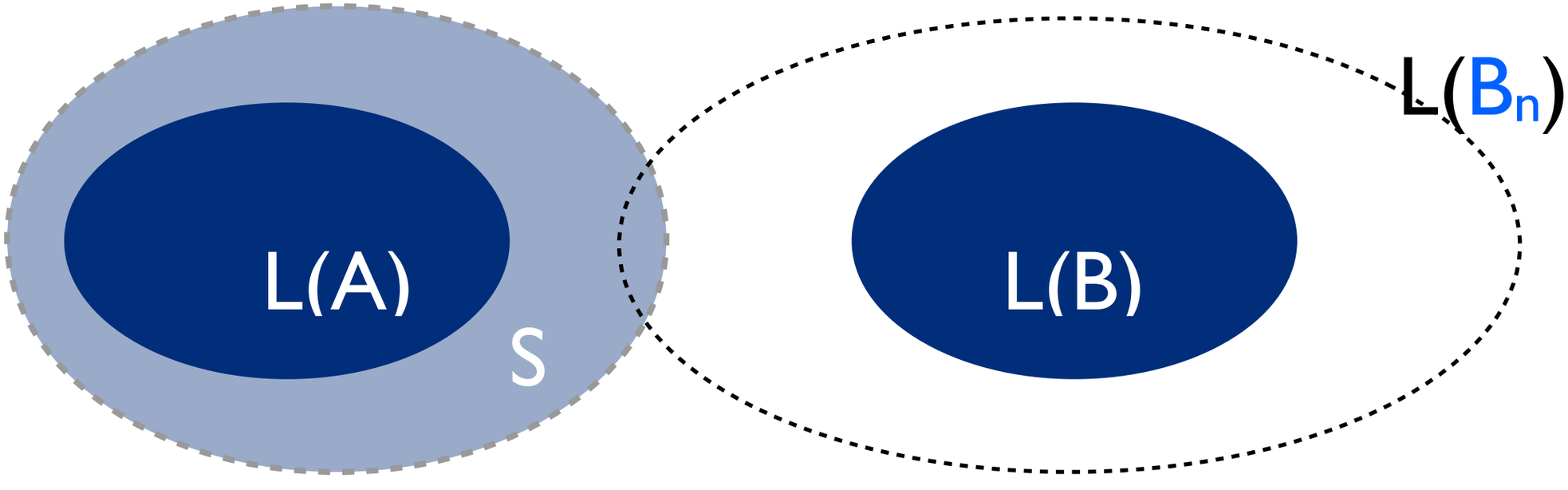

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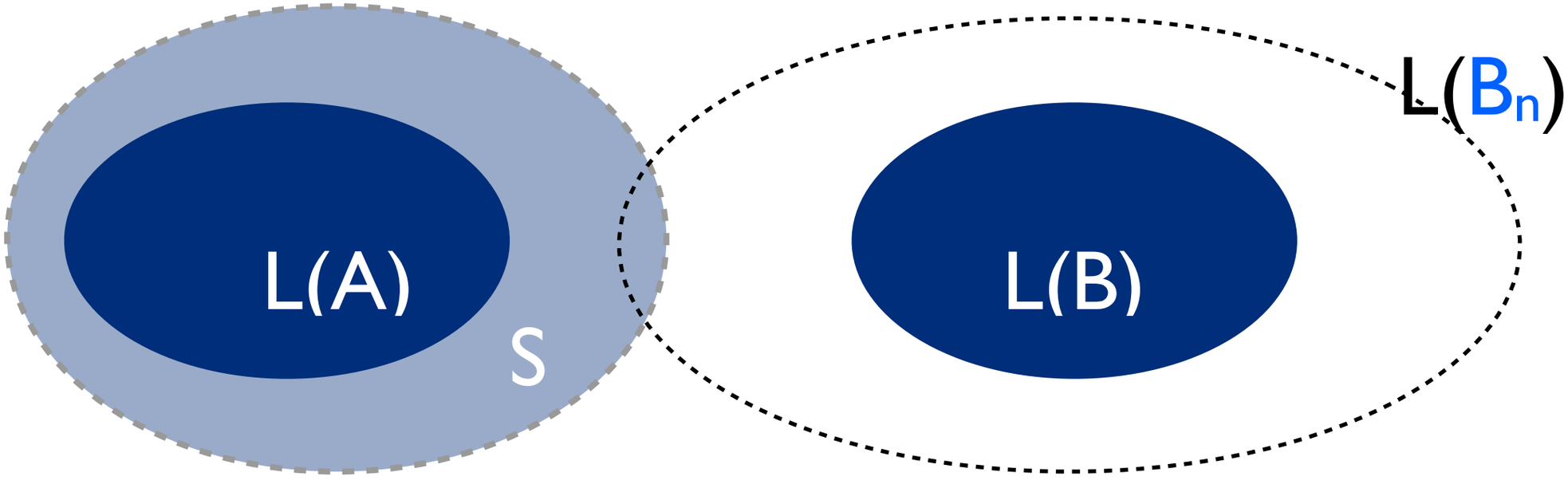

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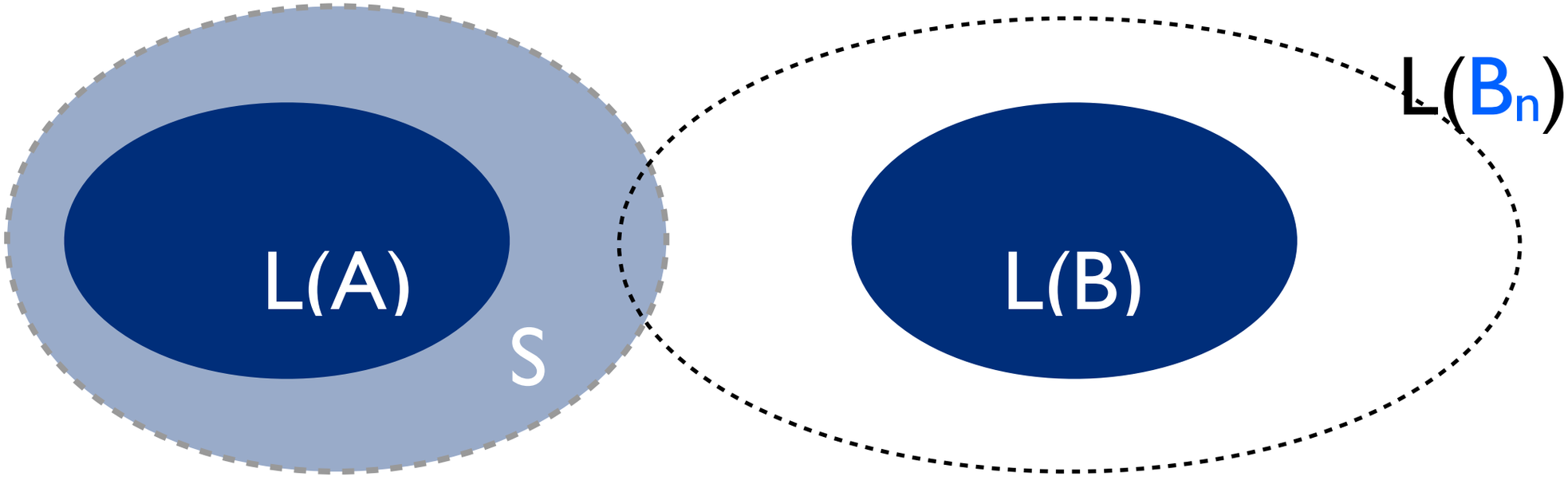

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Similarly $L(A_n)$ is disjoint from compl(S)

Algorithm

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Checks whether there is some n such that L(A_n) and L(B_n) disjoint

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Semininear set techniques - PSPACE

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Undecidability and PSPACE-hardness - similar techniques

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Old technique by Hunt

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Reduction from every decidable problem

Thank you!