Branching Bisimilarity on Normed BPA Processes is in NEXPTIME

Wojciech Czerwiński

Petr Jančar

Bisimulation

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Given labelled (multi)graph

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Bisimulation - equivalence on the set of nodes

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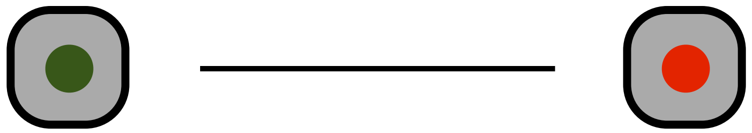

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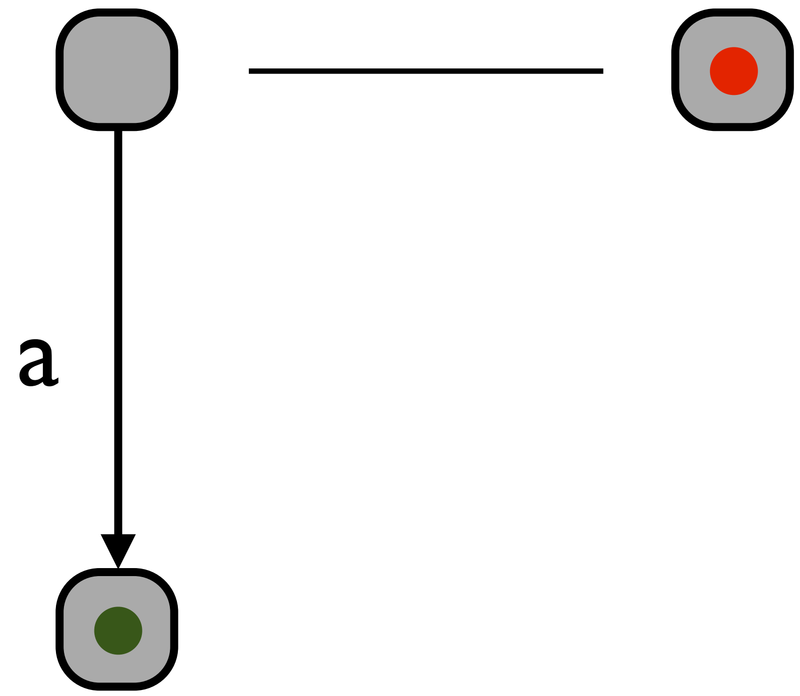

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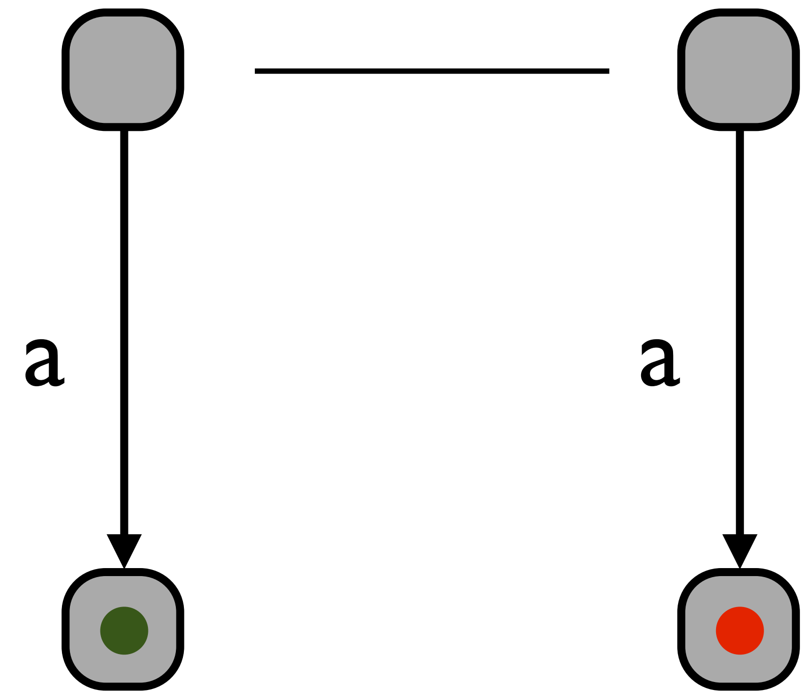

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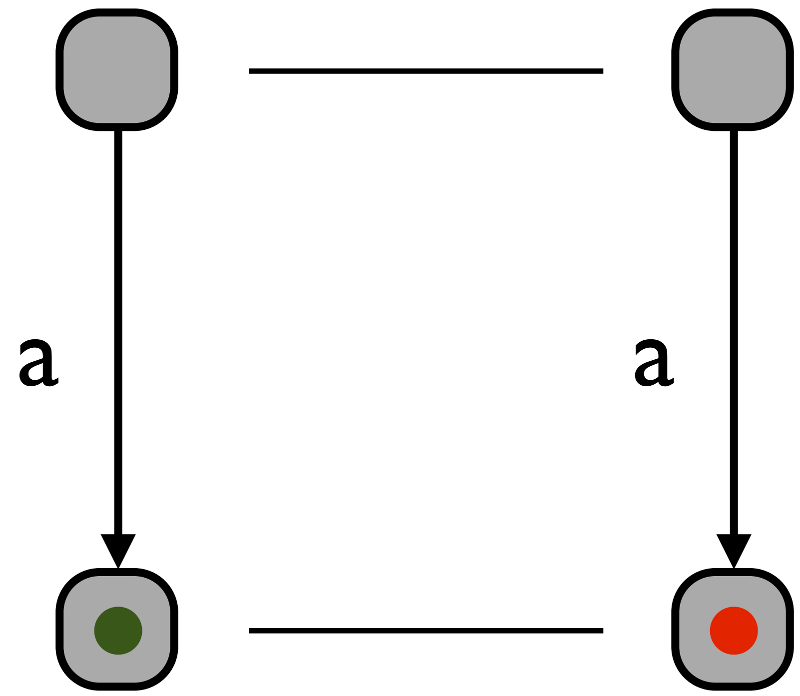

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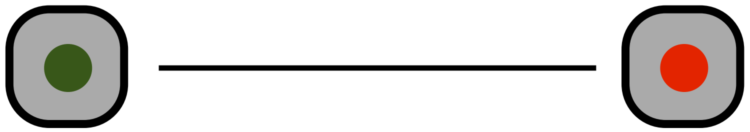

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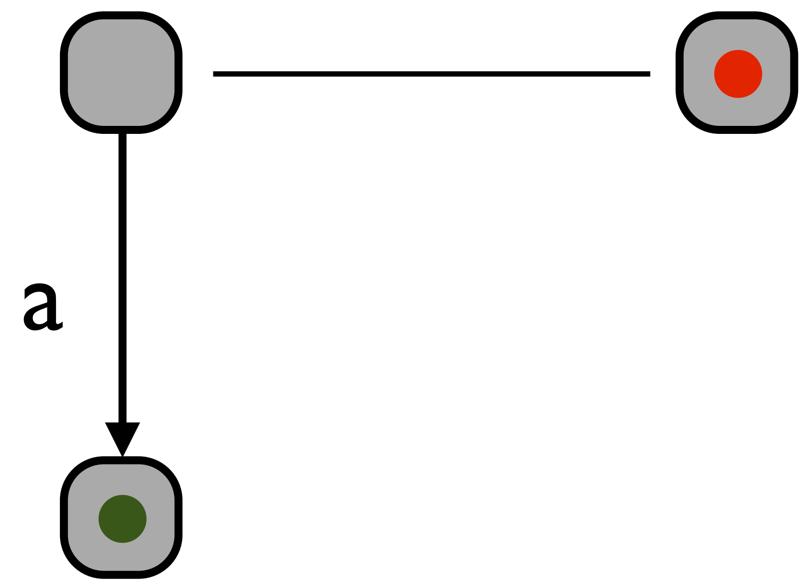

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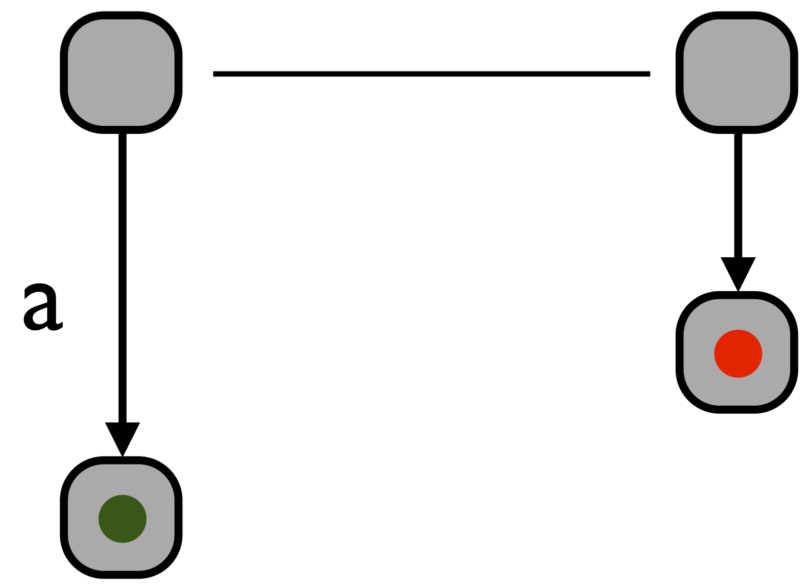

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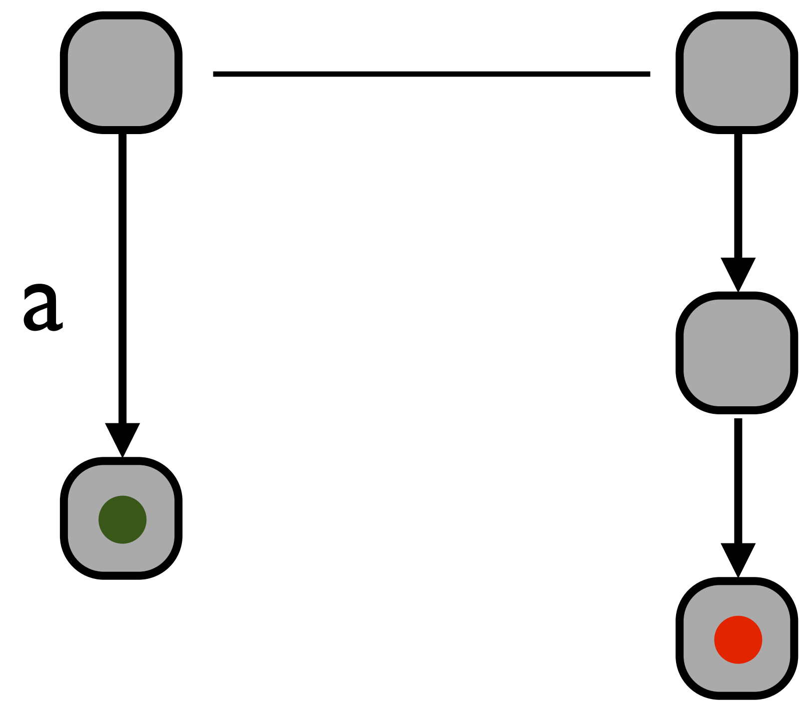

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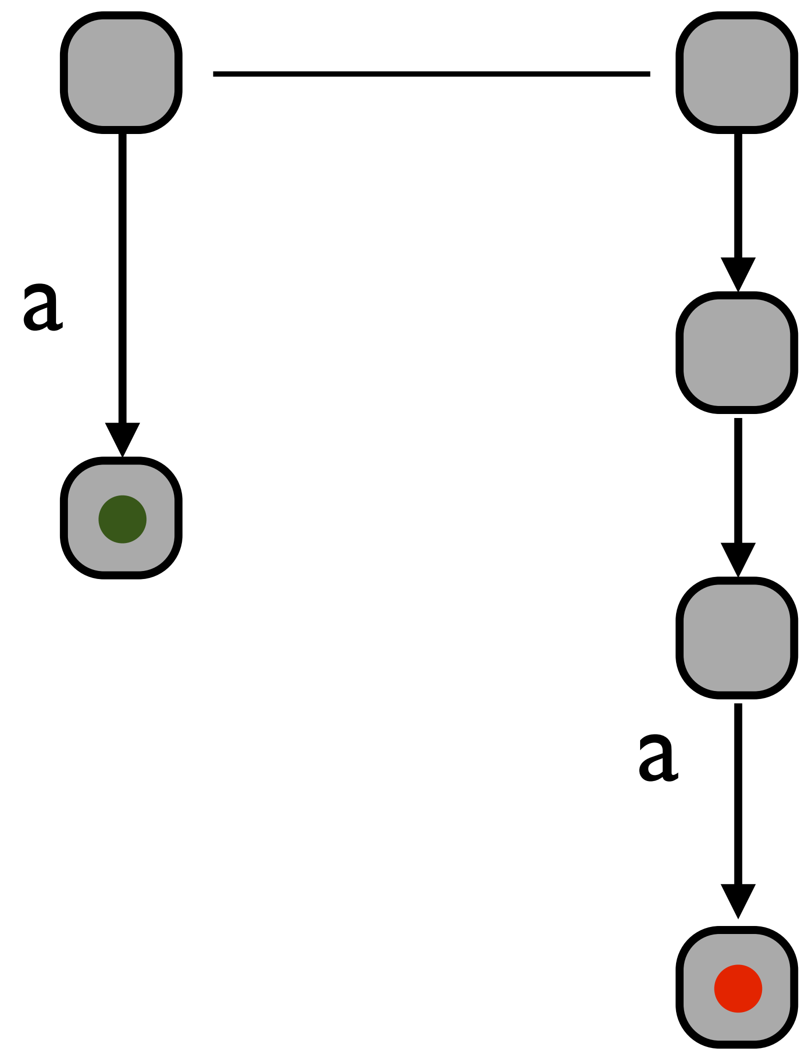

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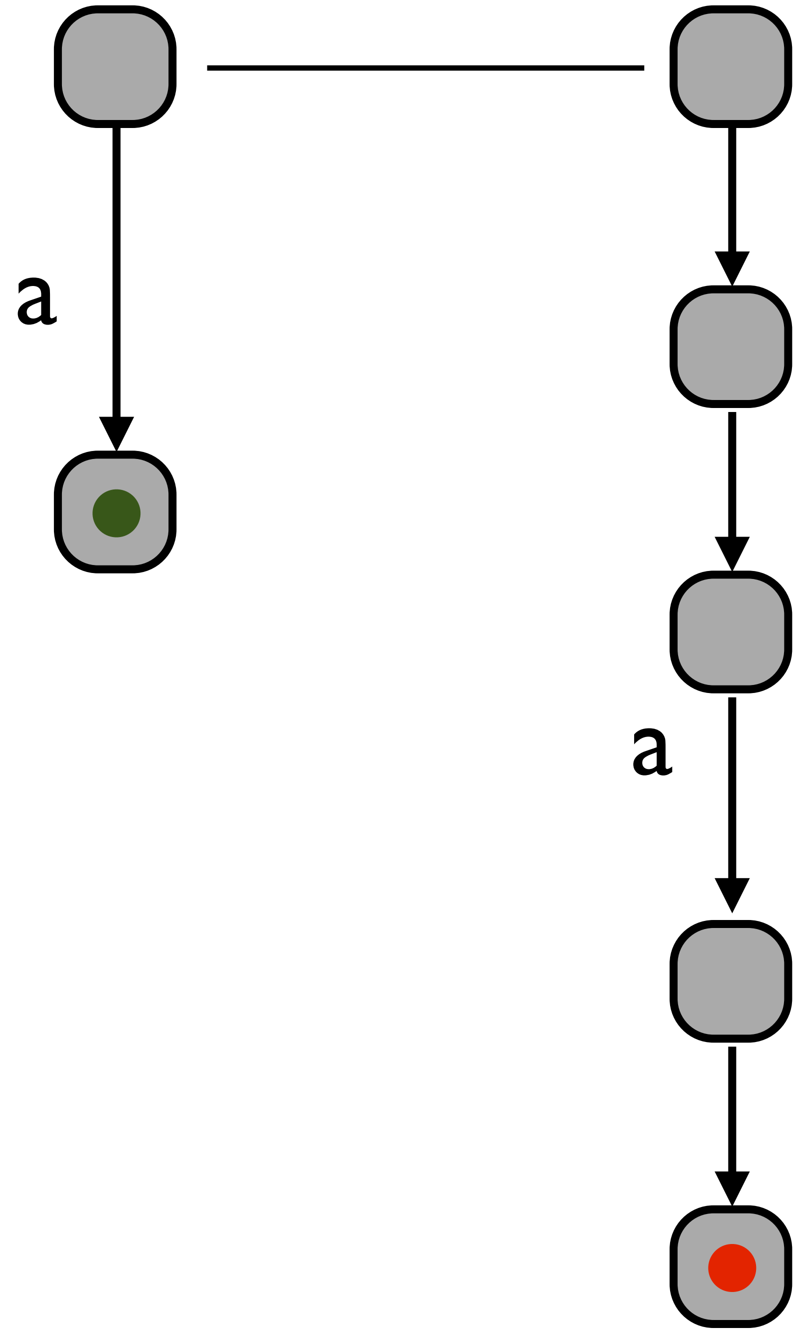

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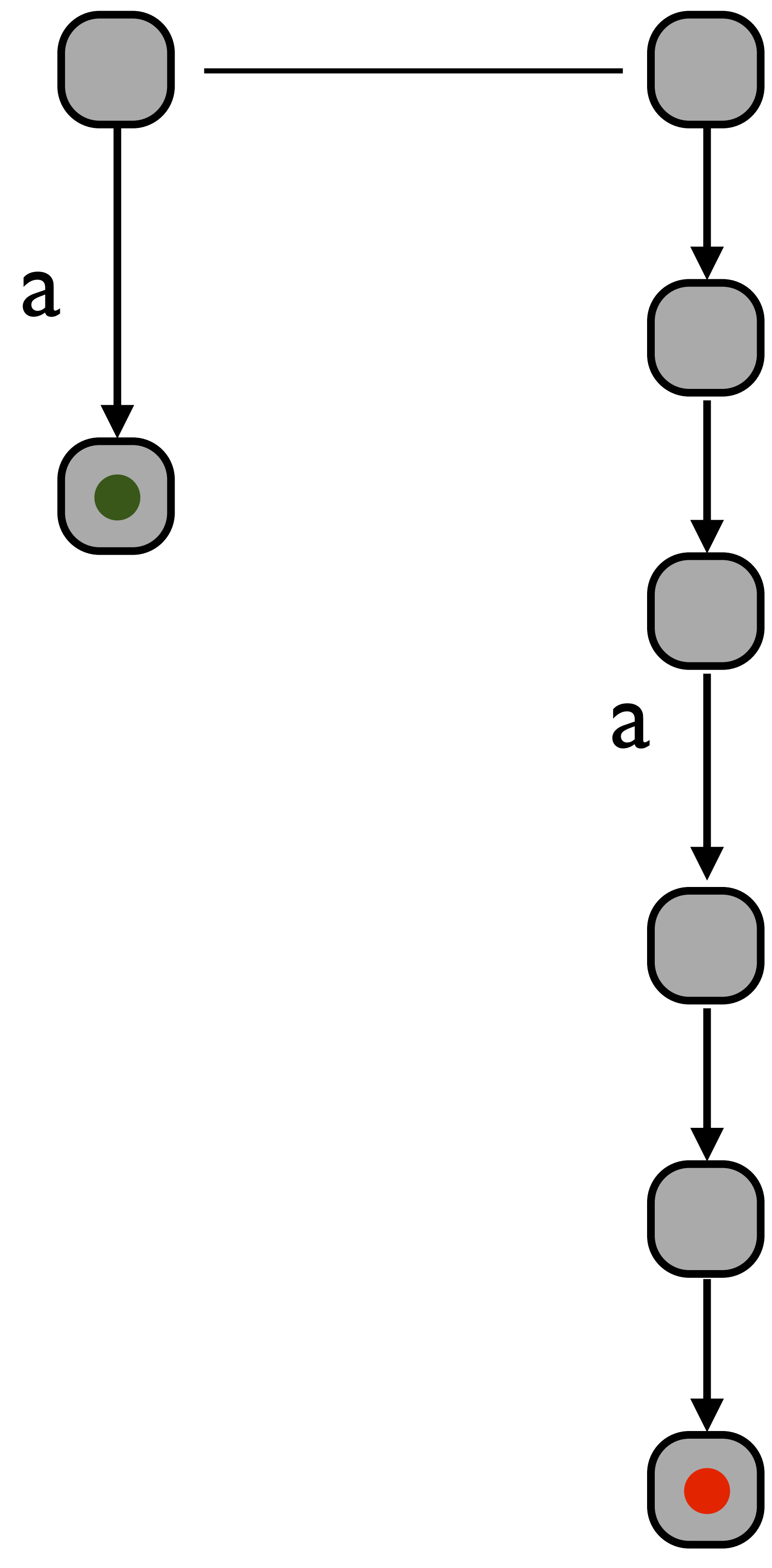

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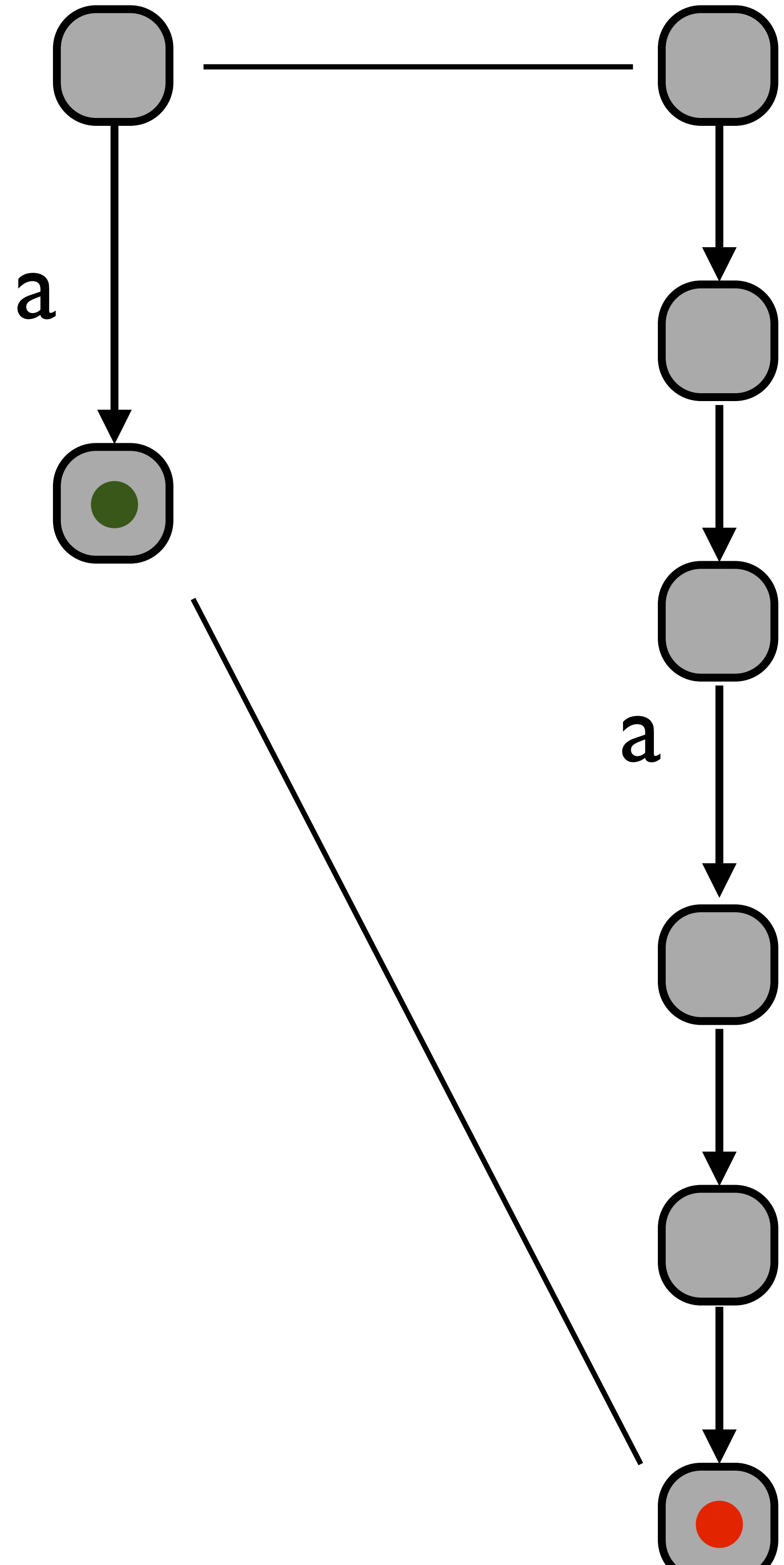

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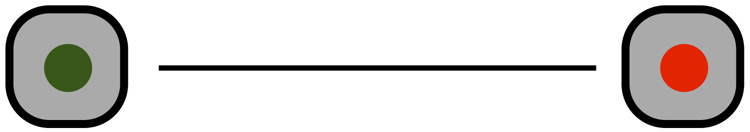

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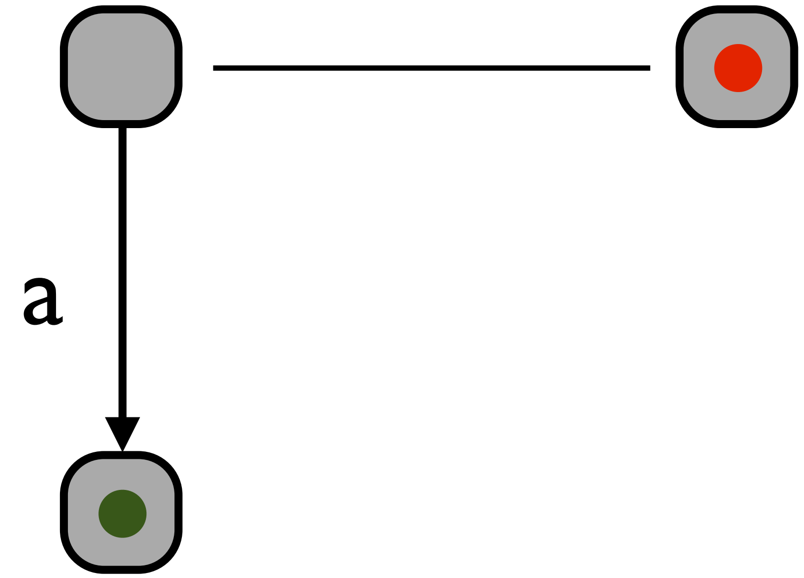

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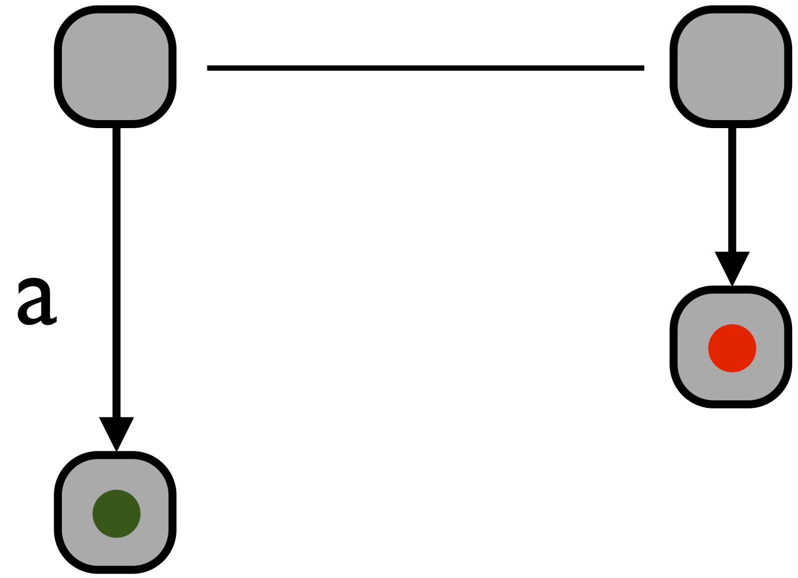

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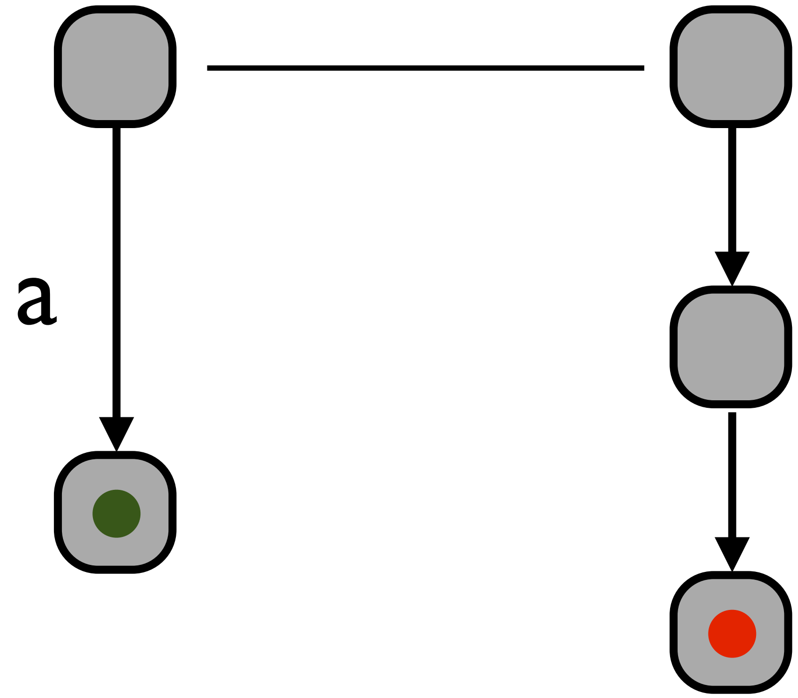

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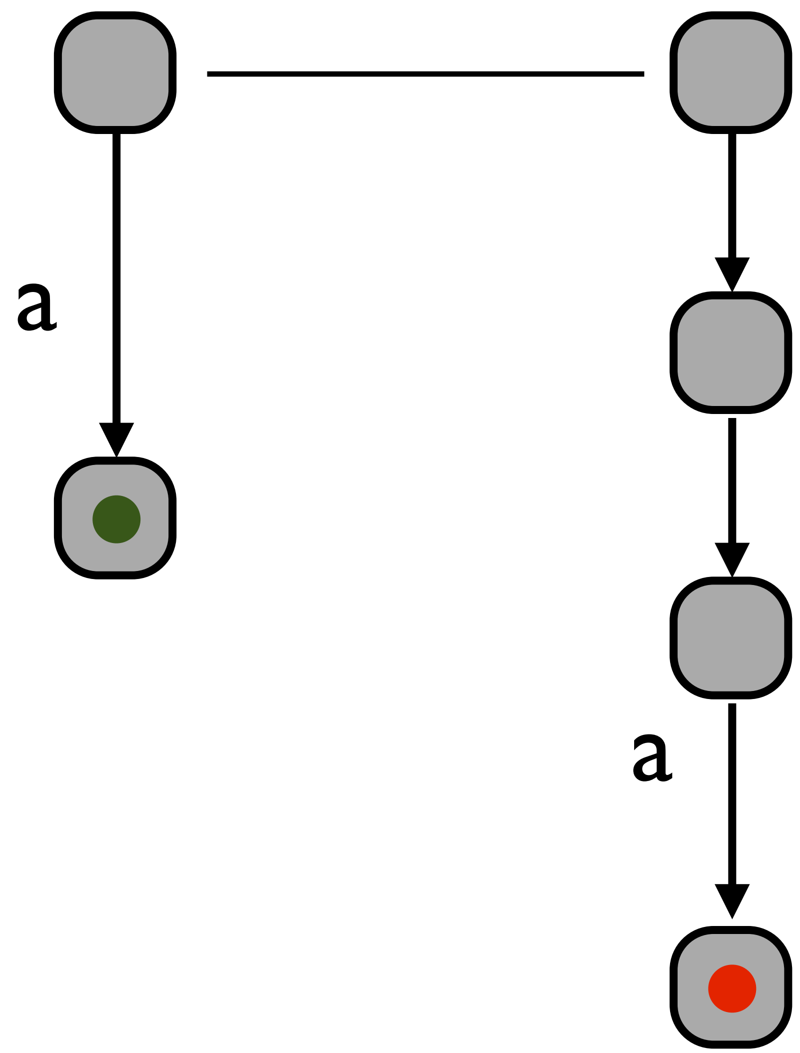

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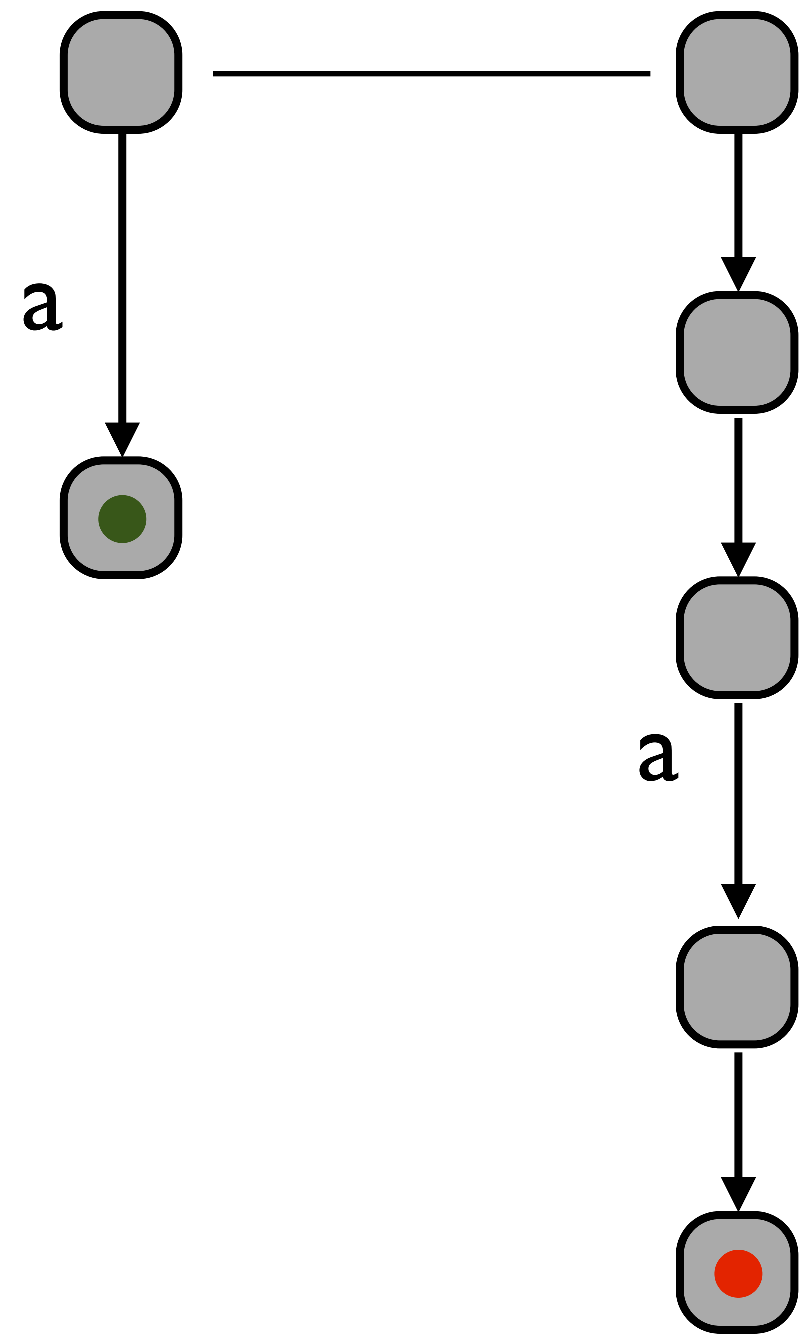

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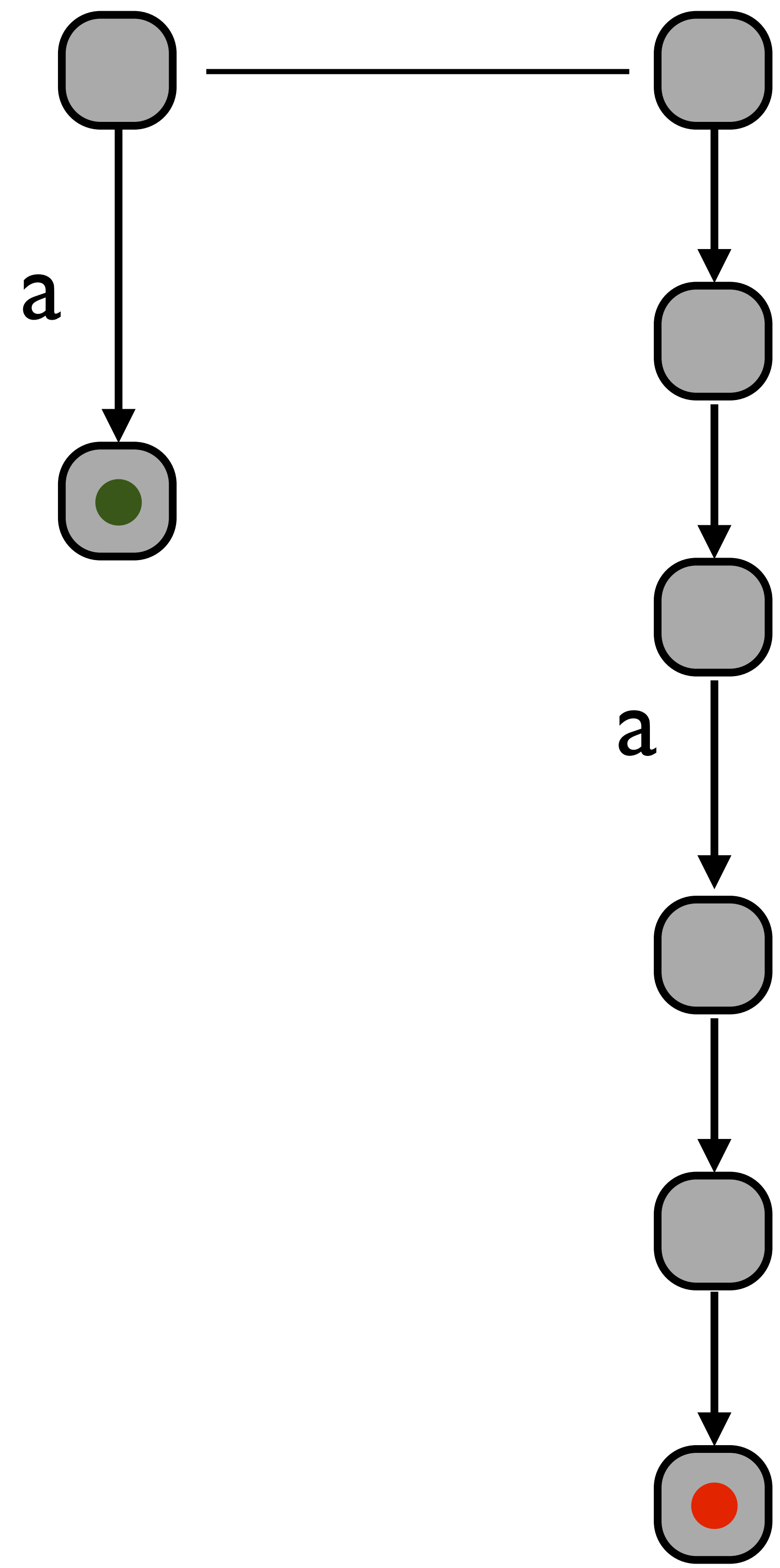

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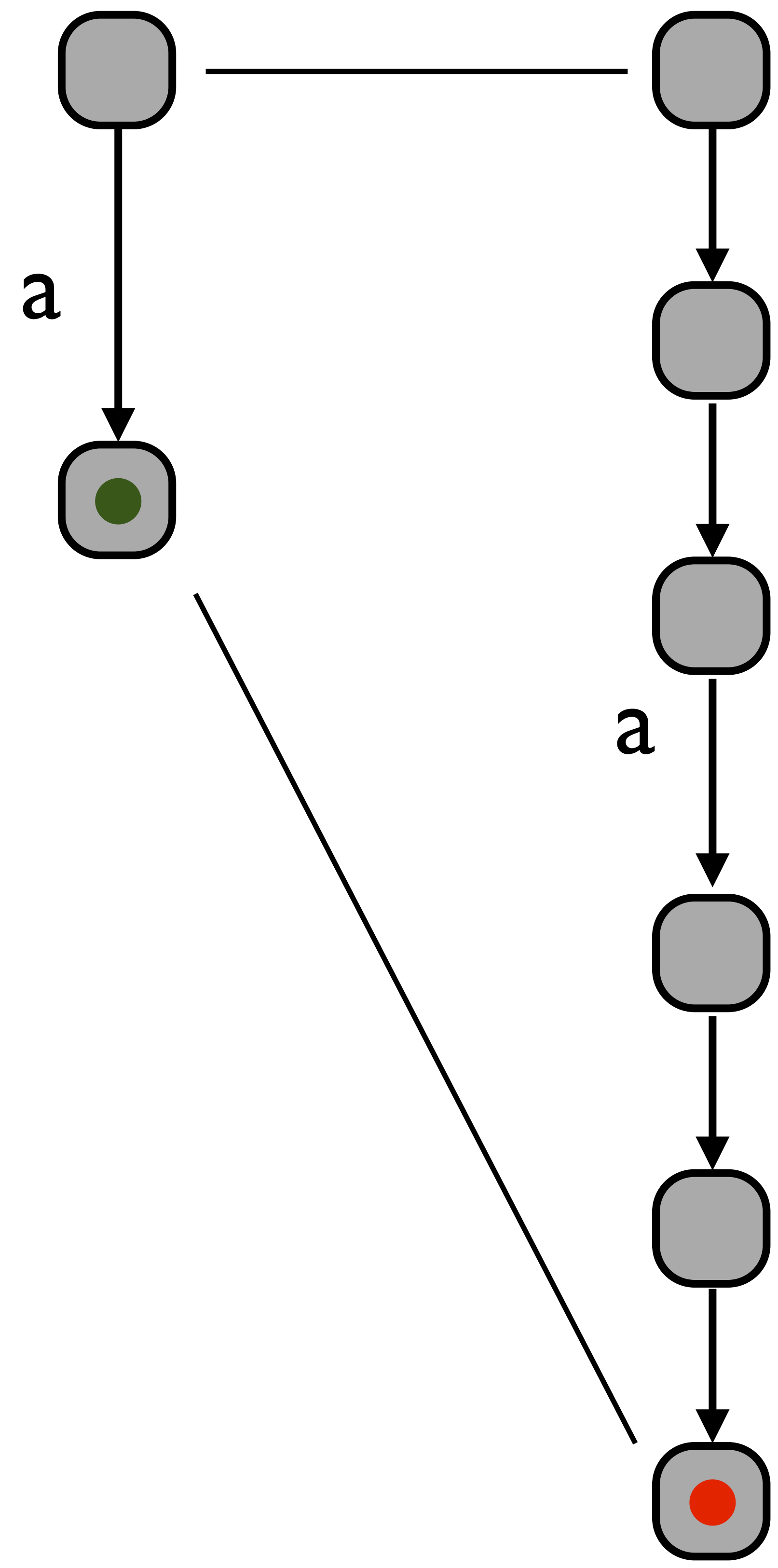

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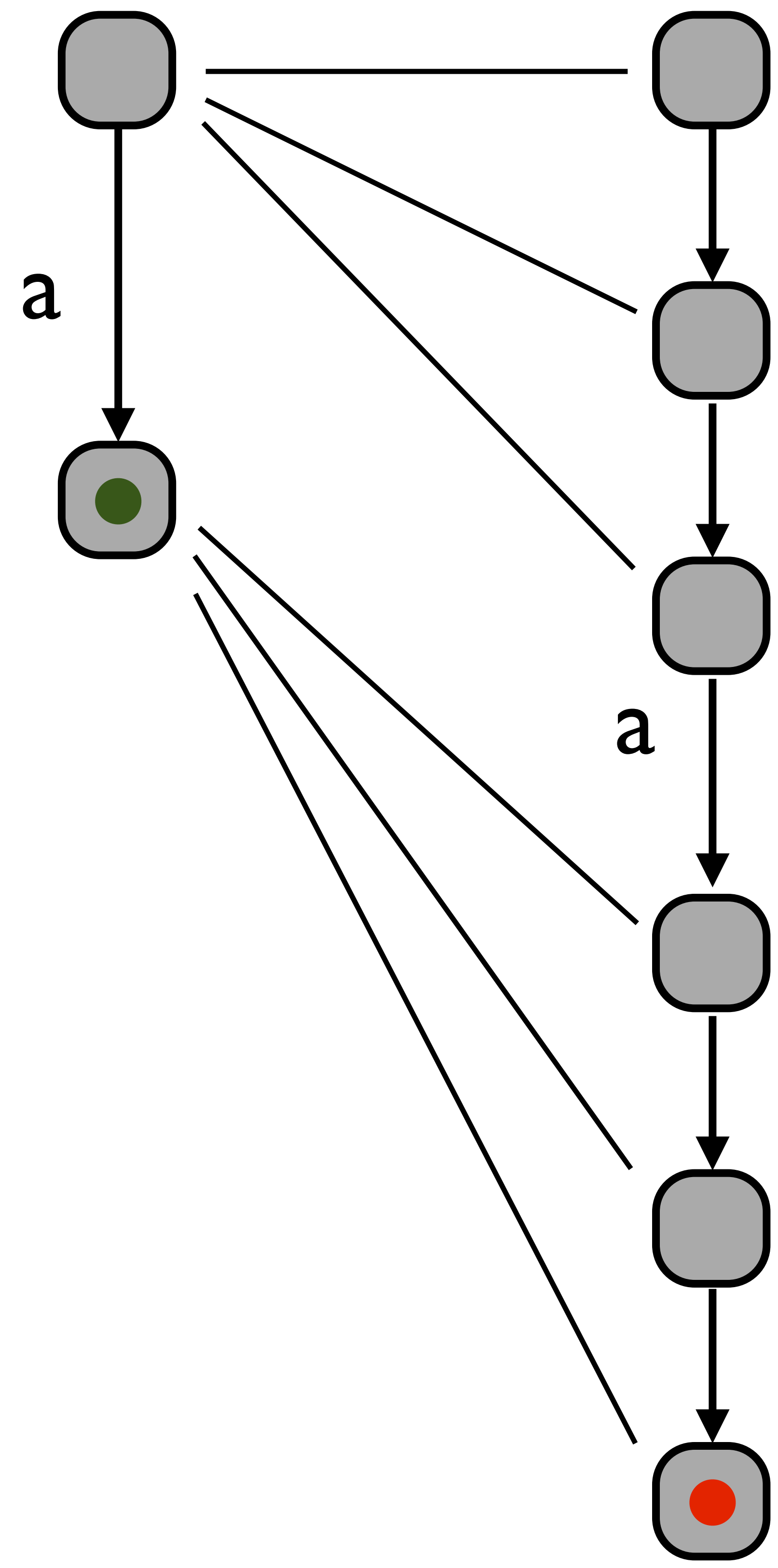

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Normedness: every variable has a path to the empty configuration

Problem

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Given: normed BPA, configurations α and β

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Question: is α branching bisimilar to β?

Main result

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Theorem

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- check whether (α, β) belongs to guessed relation

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- every decomposable has a decomposition into primes
- every configuration - exactly one equivalent configuration using only primes
- bisimilarity = equality of prime forms

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unique decomposition does not work!

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$$A \xrightarrow{a} A \qquad A \xrightarrow{a} \varepsilon$$

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For branching bisimulation

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$$pf(A) = pf(A)\, pf(A) \Rightarrow pf(A) = \varepsilon$$

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some more ideas needed!

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$$\mathrm{RED}(\alpha) = \mathrm{RED}(\beta) \Rightarrow (\gamma\alpha \sim \delta\alpha \Leftrightarrow \gamma\beta \sim \delta\beta)$$

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- relative decomposition works!
- decomposition depends on the suffix
- concretely: on the RED(suffix)
- one decomposition system for every $R \subseteq Var$ is enough
- unique decomposition: still exactly one fully decomposed form!

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- for every $R \subseteq Var$:
  - set of R-primes, R-decomposables and decompositions for them
  - for every $X \in Var$ a rule $R \xrightarrow{X} R'$ $(RED(\alpha) = R \Rightarrow RED(X\alpha) = R')$

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  - check only not too big responses

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- cc-norm does not change too much ⇒ short responses are enough
- correctness possible to verify

Thank you!