Unboundedness problems for languages of vector additions systems

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Plan

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- Definitions

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- Idea of our result - decidability tool for VASes

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- Applications

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language: labellings of paths from s to t

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- some problems need adaptations of KLM decompositon

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- prove a general lemma for such properties

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- infinite, unbounded, infix universal

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- need for n-dimensional unboundedness predicates

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- if $p(Inf_n(L_1 L_2 ... L_k))$ then $n = n_1 + ... + n_k$ such that $p(Inf_{n1}(L_1) \times ... \times Inf_{nk}(L_k))$

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- downward closure universal

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Given a VAS language $L \subseteq \Sigma^*$ one can compute a regular language $R \subseteq \Sigma^*$ such that $L \subseteq R$ and for every n-dimensional unboundedness predicate p we have

$$p(\mathrm{Inf}_n(L)) = p(\mathrm{Inf}_n(R))$$

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Corollary:

If one can decide an unboundedness property for n-infixes of regular languages then also for n-infixes of VAS languages

Core lemma

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Given a VAS language $L \subseteq \Sigma^*$ one can compute $m, k \in N$ and regular languages $R_{i,j} \subseteq \Sigma^*$ for $i \in [1,m]$, $j \in [1,k]$

such that

$$L \subseteq \bigcup_i R_{i,1} \ldots R_{i,k}$$

and

$$R_{i,1} \times \ldots \times R_{i,k} \subseteq \mathrm{Inf}_k(L) \text{ for every } i \in [1,m]$$

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- not boundedness is a 1-dim. unboundedness property
- decidable for regular languages, so decidable for VAS languages

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- prefix universality is undecidable

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- the same as $(a^*, b^*, c^*, d^*) \subseteq Inf_4(L)$
- 4-dimensional unboundedness predicate
- decidable for regular, so decidable for VAS languages
- implies computability of downward closures of VAS languages (Habermehl et al. ICALP 2010)

Thank you!