

Efficient Separability
of Regular Languages
by Subsequences and Suffixes

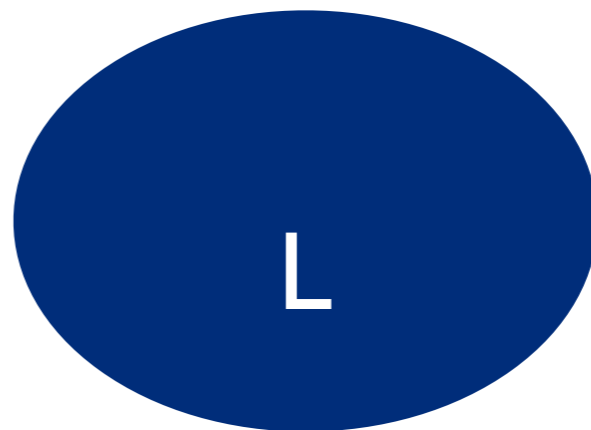
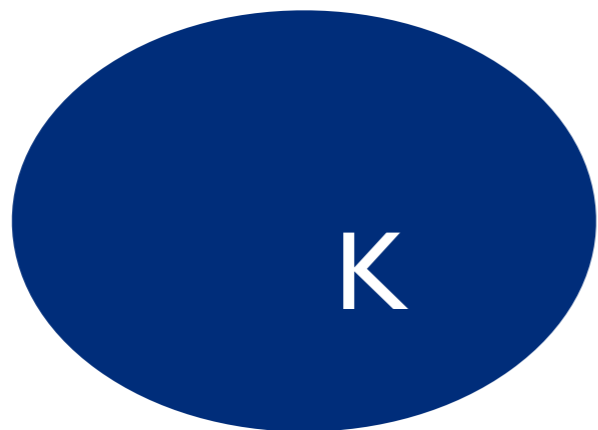
Wojciech Czerwiński

Tomáš Masopust

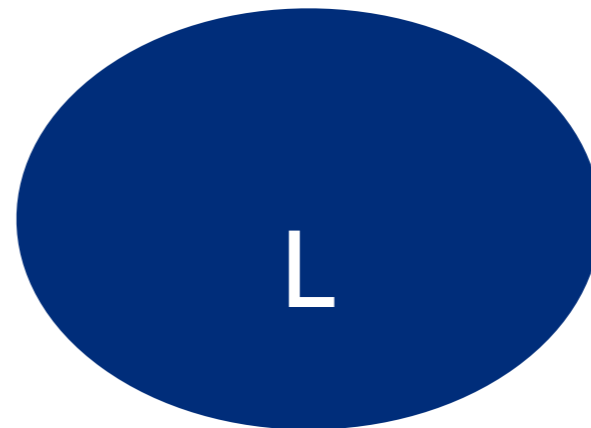
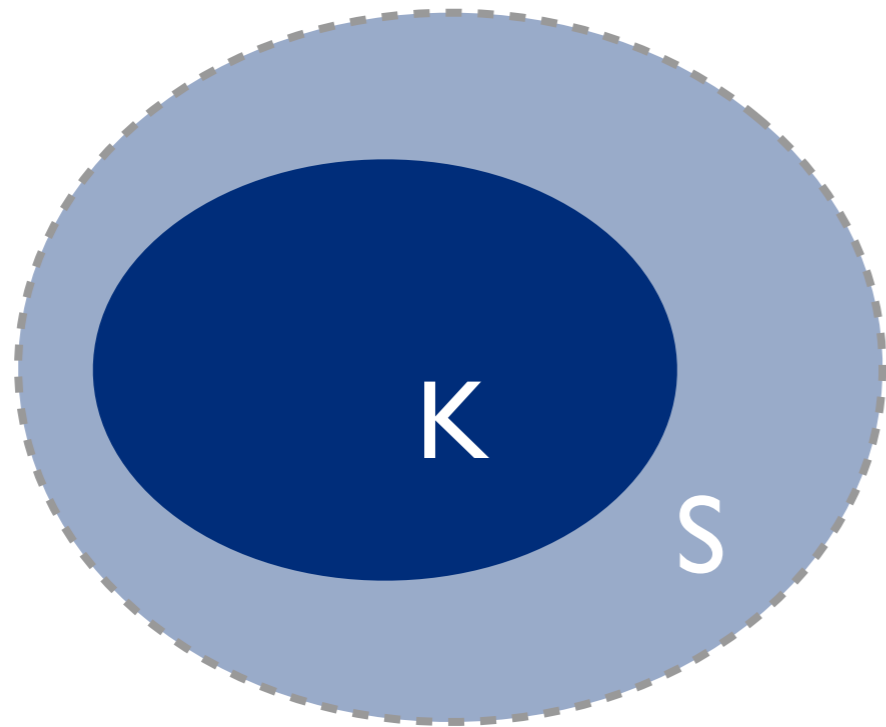
Wim Martens

Separability

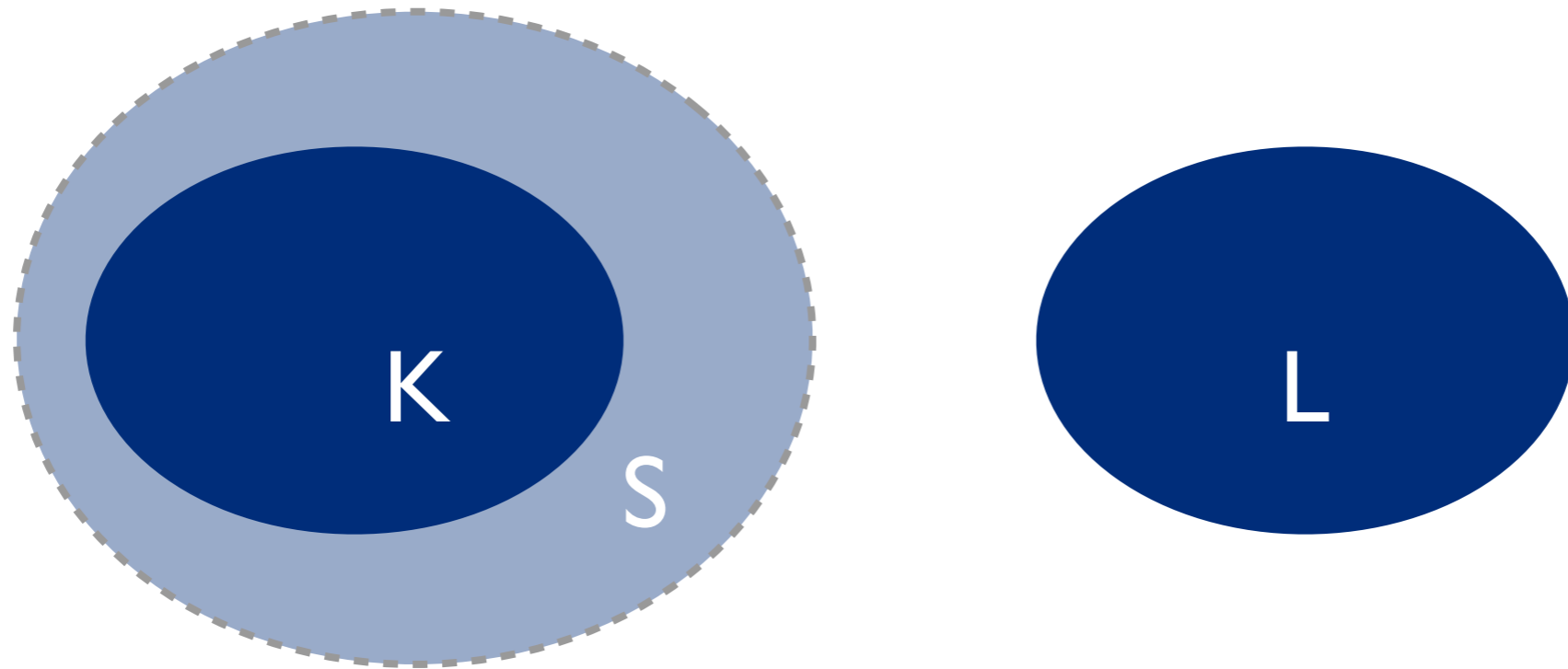
Separability



Separability

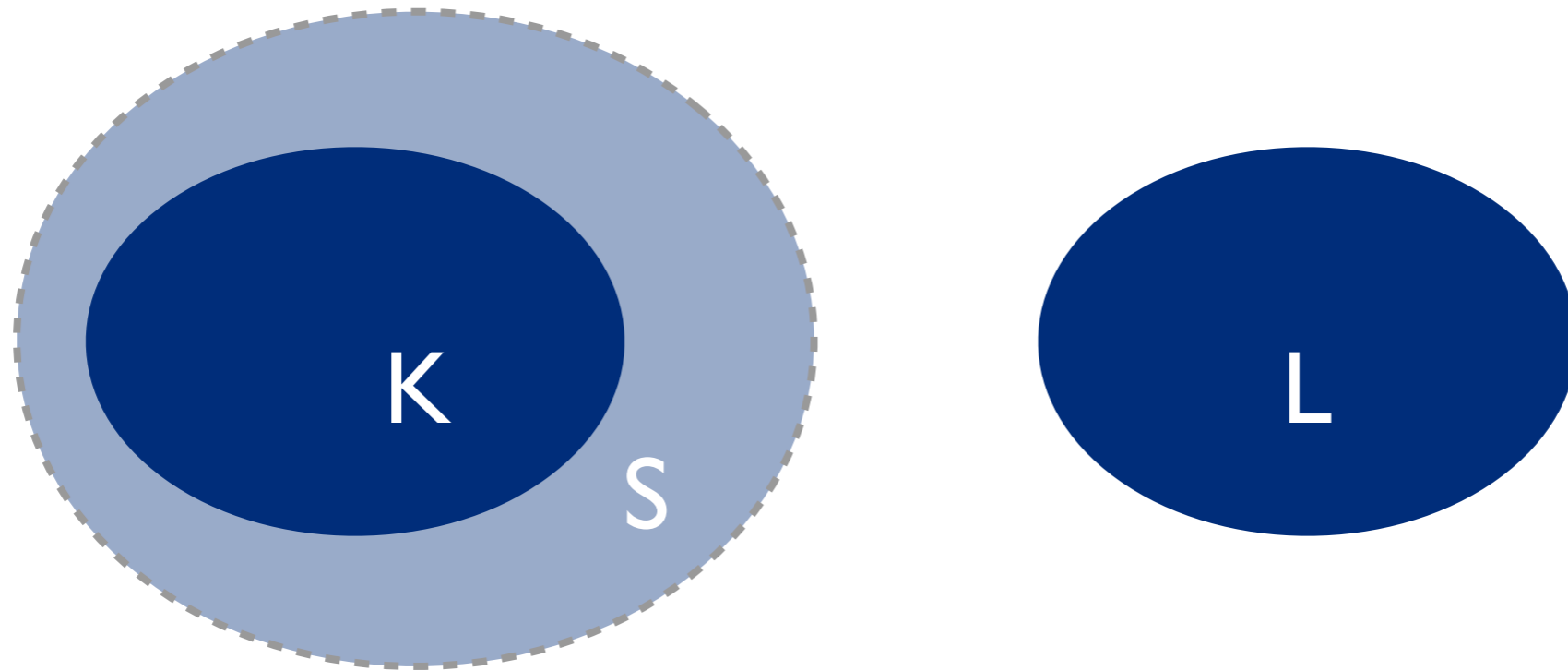


Separability



S separates K and L

Separability



S separates K and L

K and L are *separable* by family F
if some S from F separates them

Problem

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Given: nondeterministic automata for
languages K and L

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Question: are K and L separable by Piecewise Testable Languages (PTL)?

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piece language

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$$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$$

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piecewise testable language

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Given: nondeterministic automata for languages K and L

Question: are K and L separable by Piecewise Testable Languages (PTL)?

piece language

$$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$$

piecewise testable language

bool. comb. of pieces

What is known?

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- Simon 1975, piecewise testable = j-trivial

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- Stern 1985, deciding if a language is piecewise testable is in PTIME

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- Stern 1985, deciding if a language is piecewise testable is in PTIME
- Almeida, Zeitoun 1997, exponential algorithm for separability by PTL

First main result

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Theorem:

Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

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obtained independently
by Place, van Rooijen, Zeitoun
MFCS '13

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Second main result

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For **any** two word languages K and L the following conditions are equivalent:

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For **any** two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

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For **any** two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

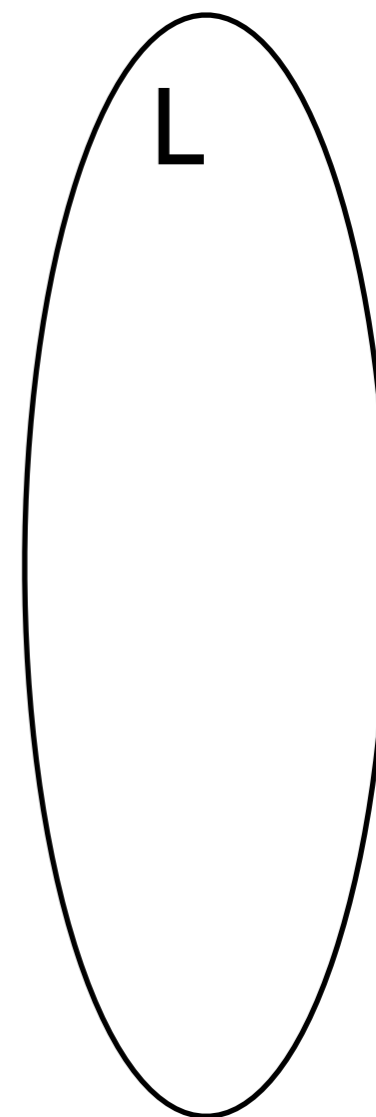
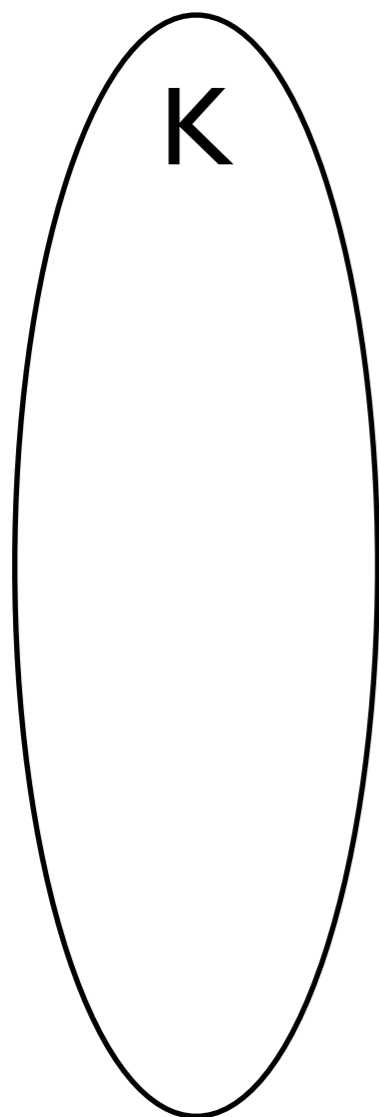
Infinite zigzag

Infinite zigzag

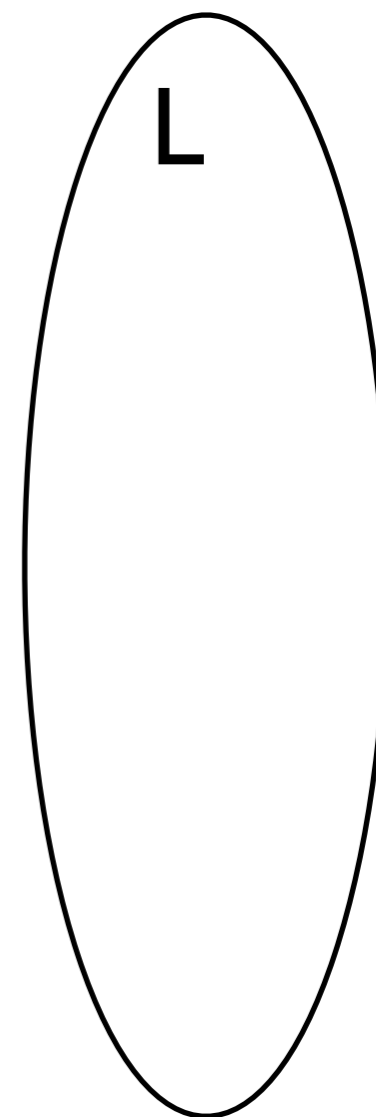
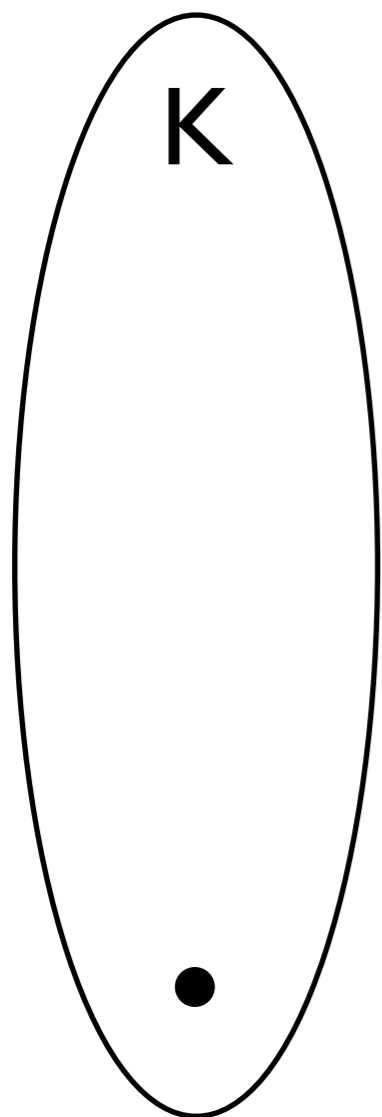
abcd \preceq dbabacbdb

Infinite zigzag

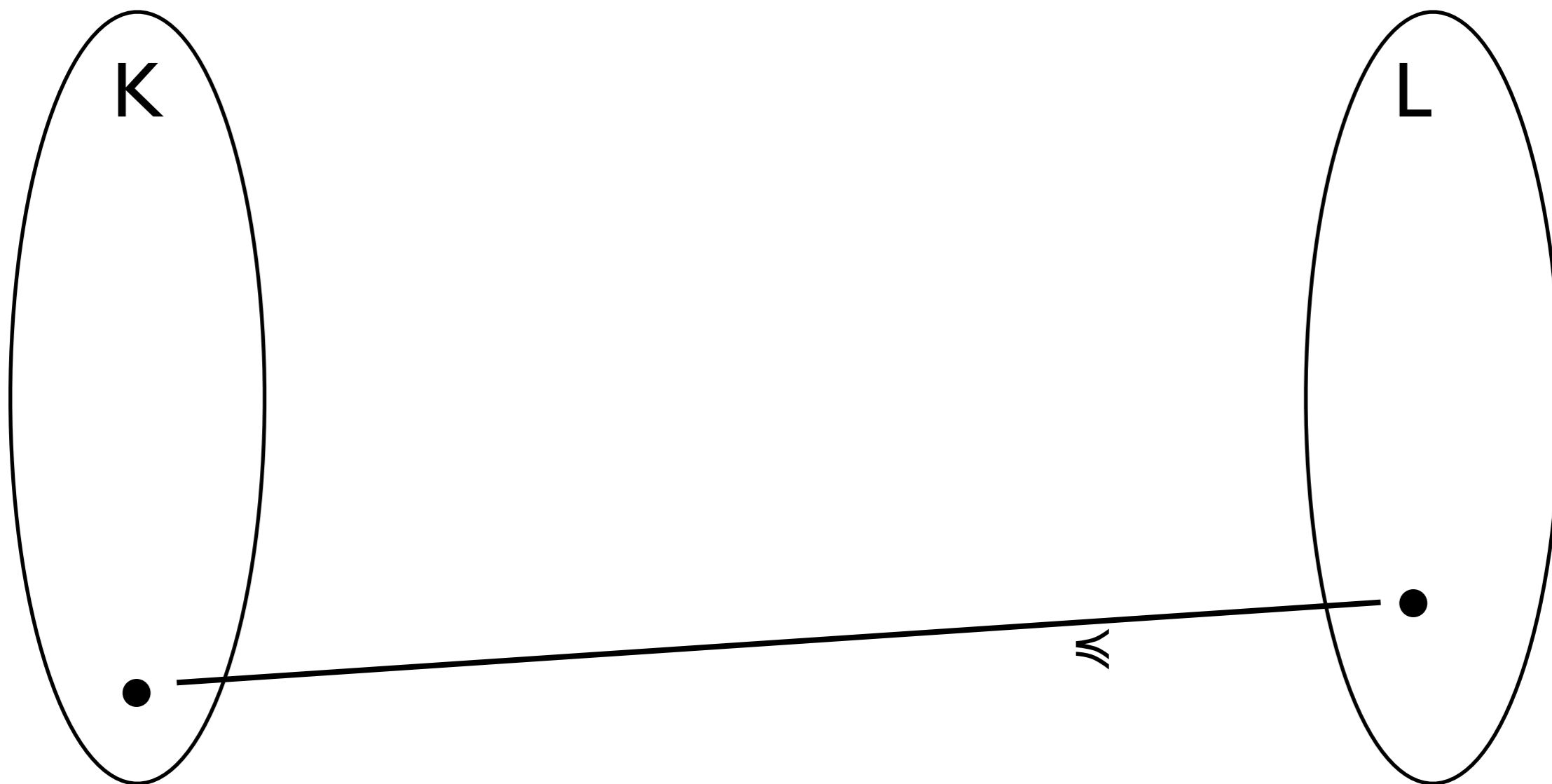
Infinite zigzag



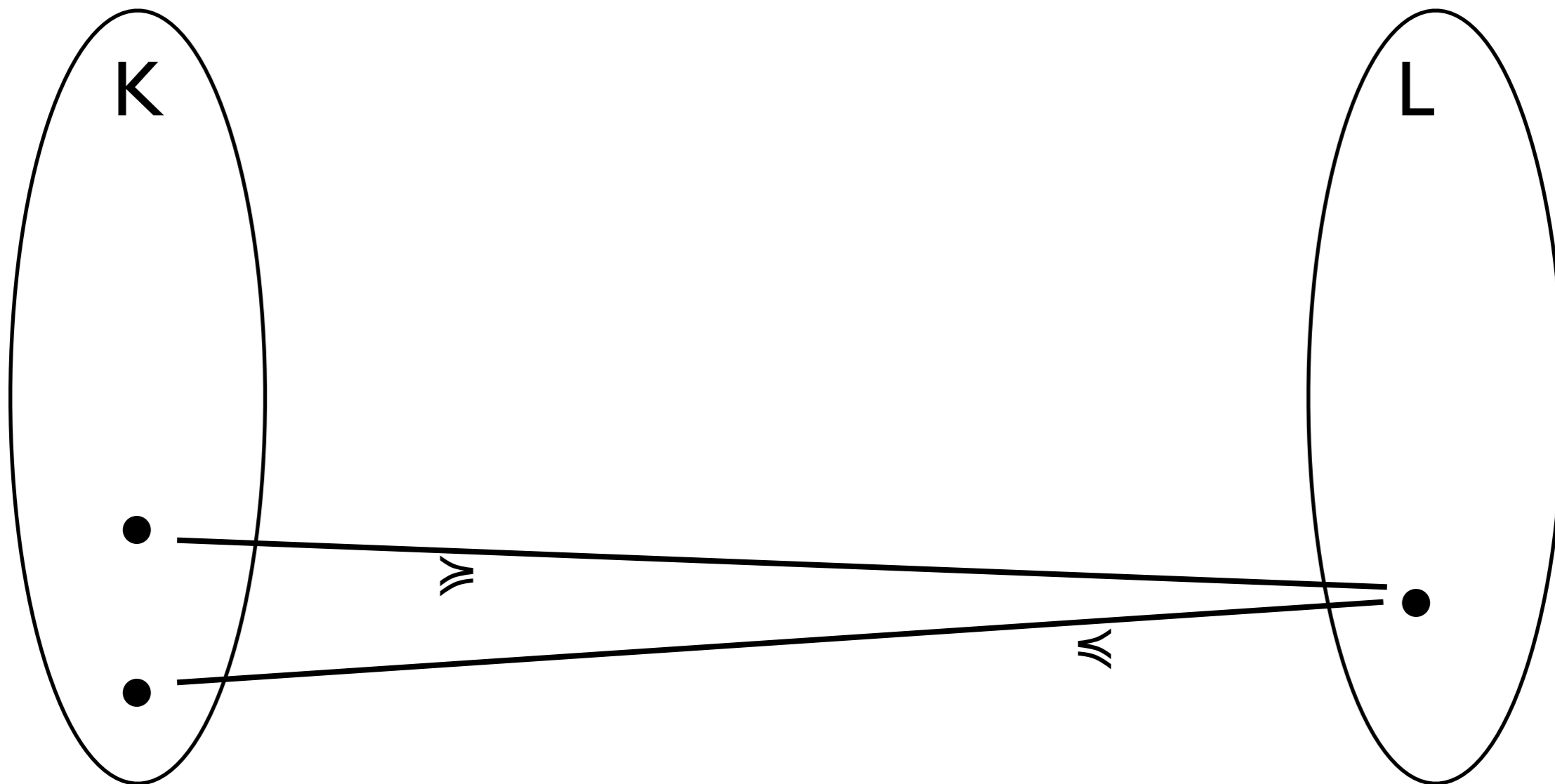
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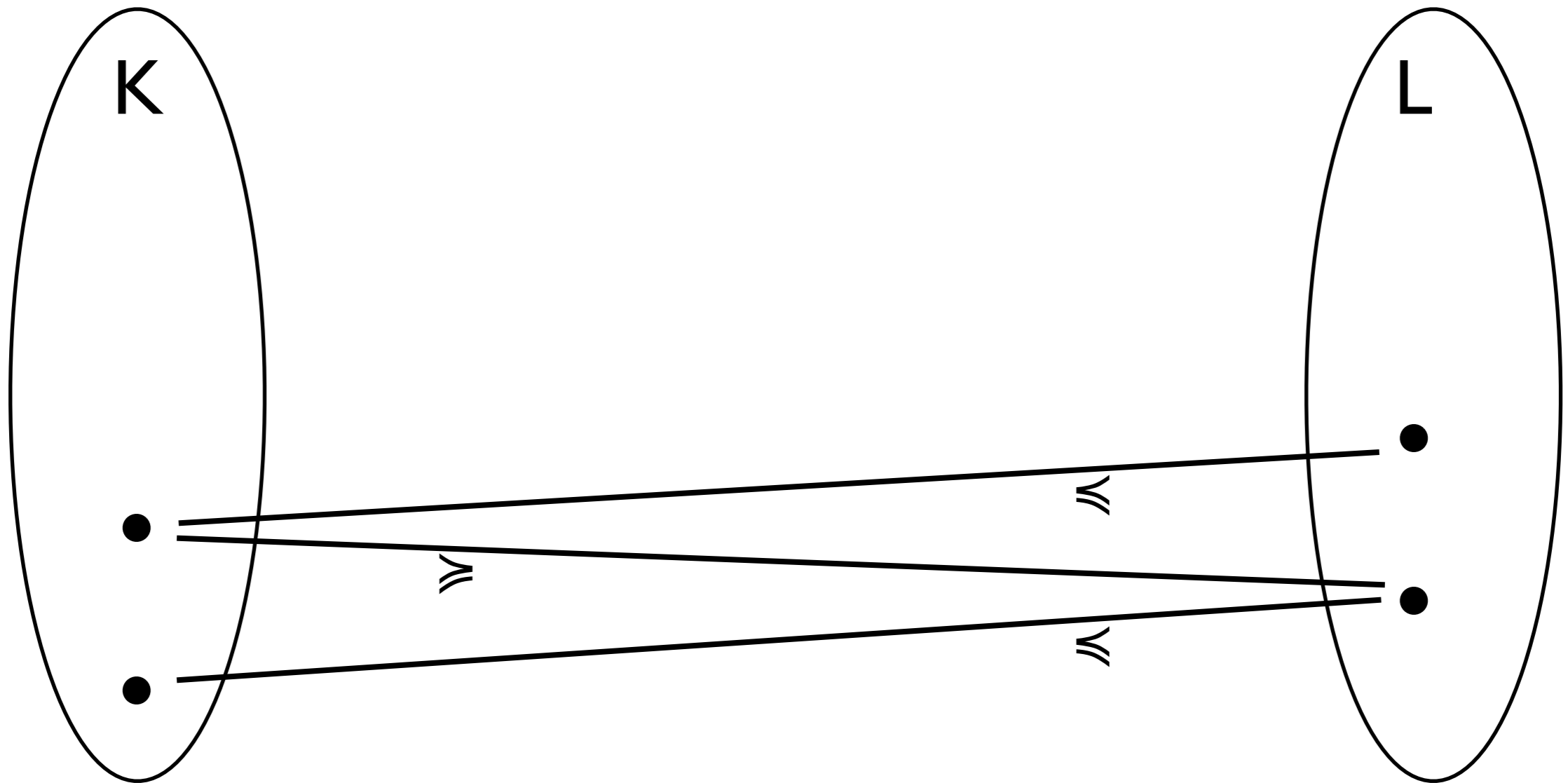
Infinite zigzag



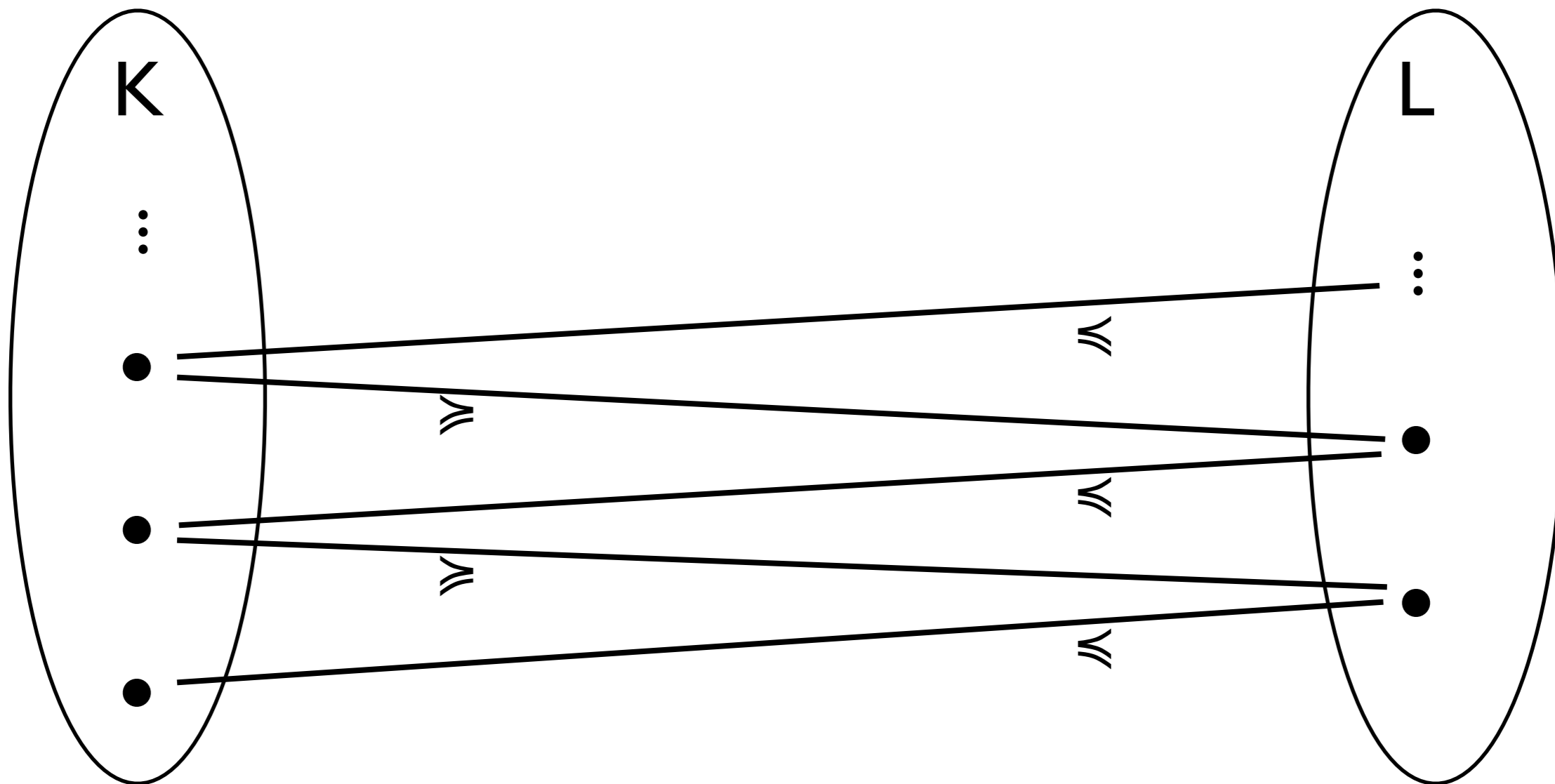
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For **any** two word languages K and L the following conditions are equivalent:

- 1) K and L are separable by piecewise testable languages
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- 3) K and L are layered separable by pieces

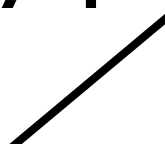
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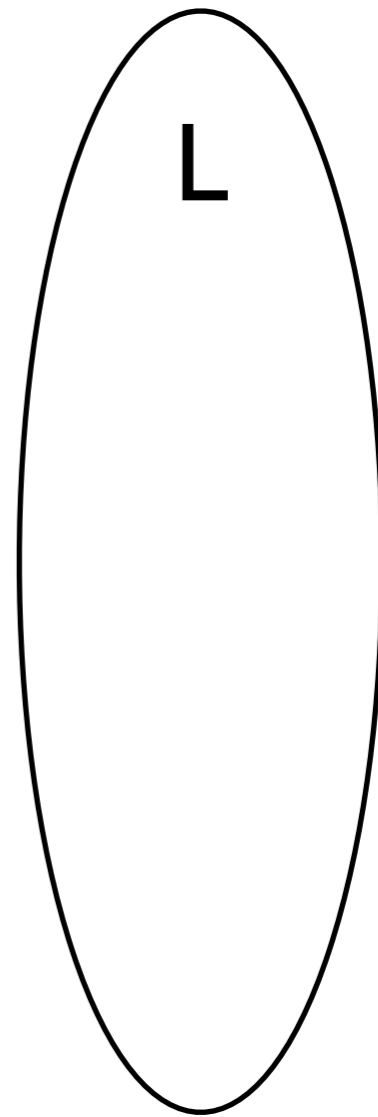
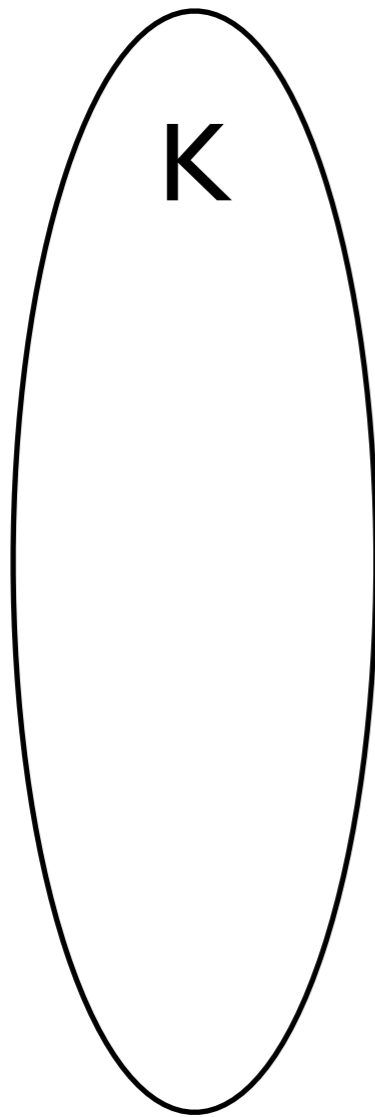
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Layered separability

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S_1

Layered separability

K

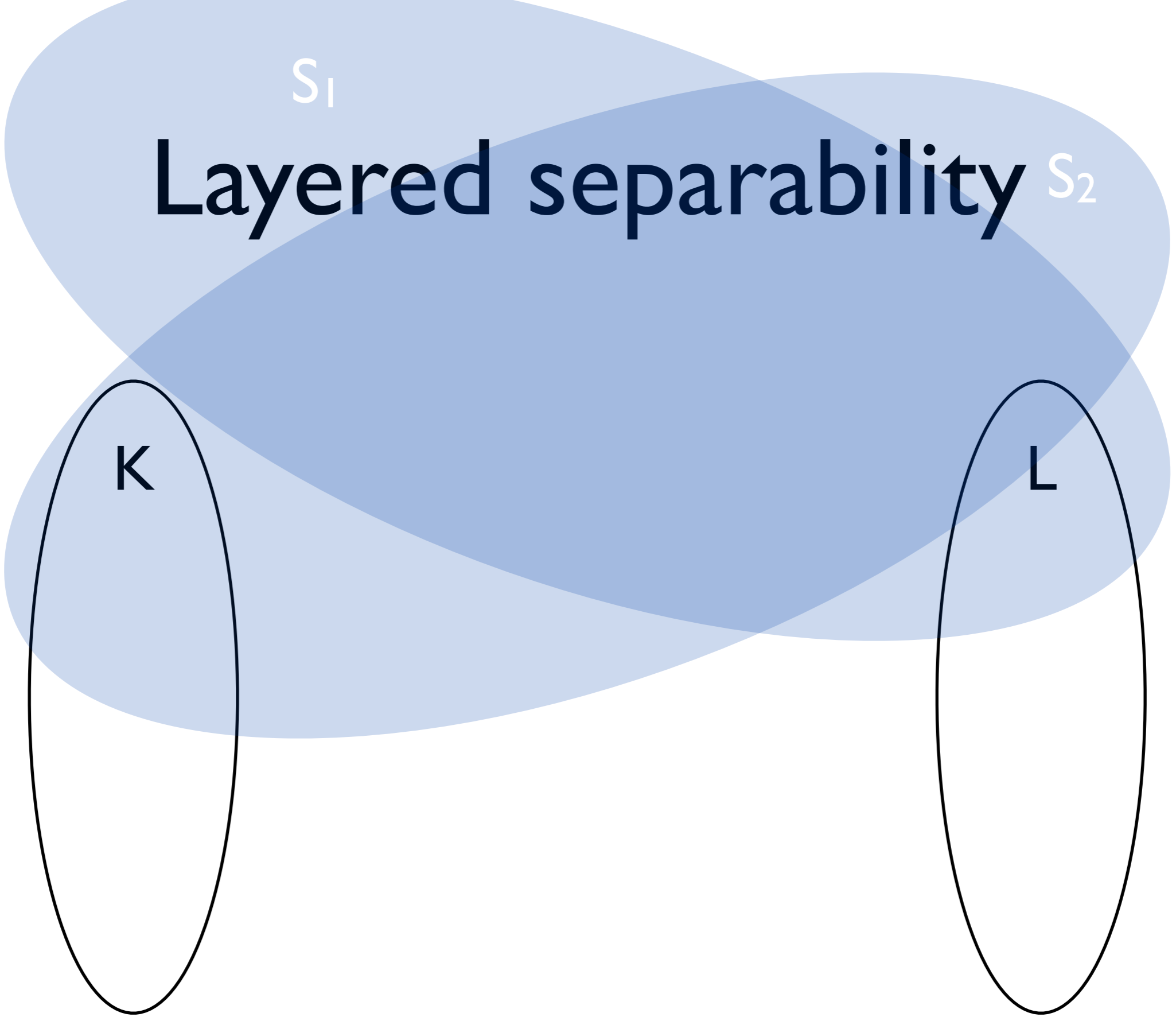
L

S_1

Layered separability S_2

K

L



Layered separability

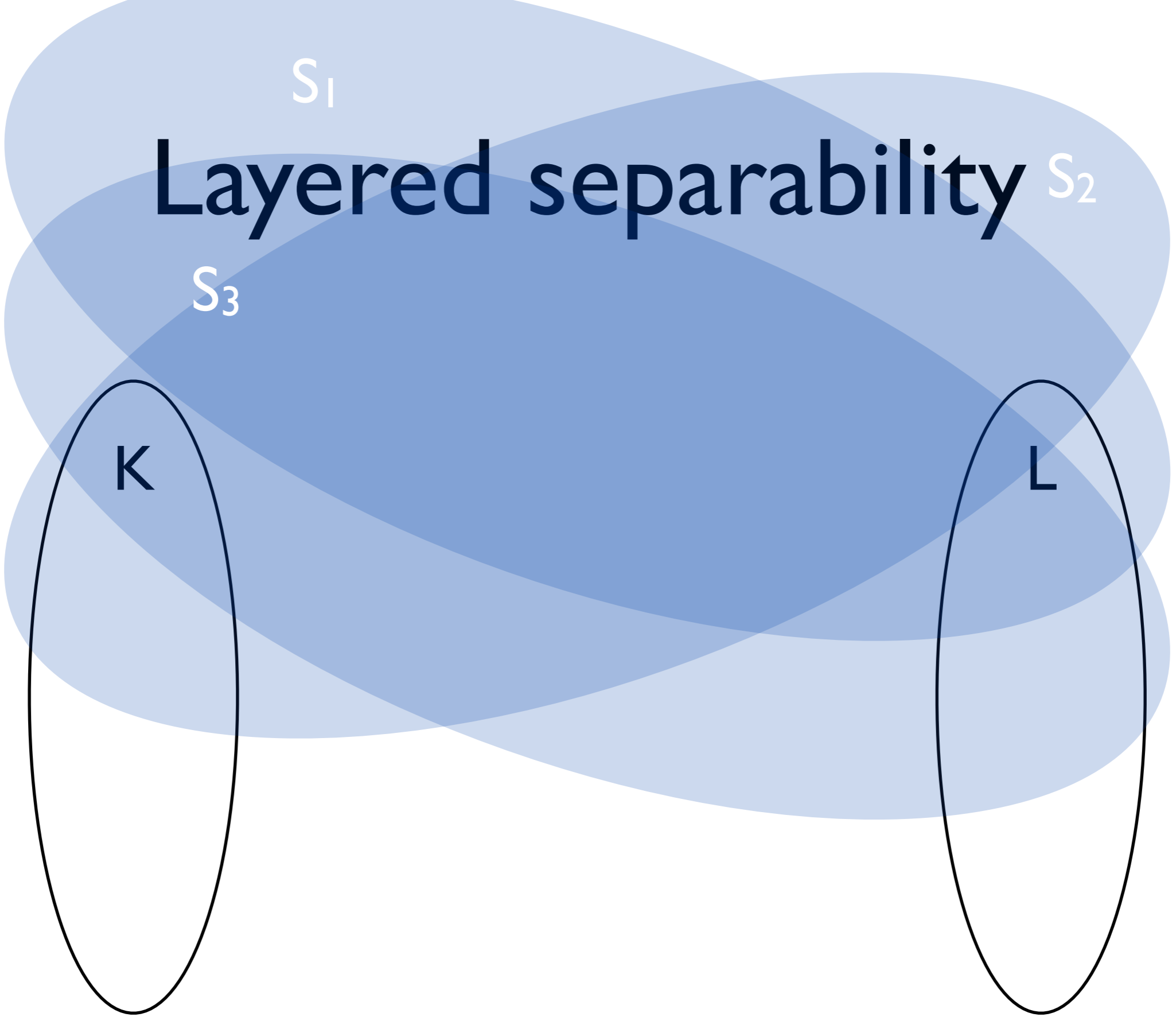
S_1

S_2

S_3

K

L



Layered separability

S_1

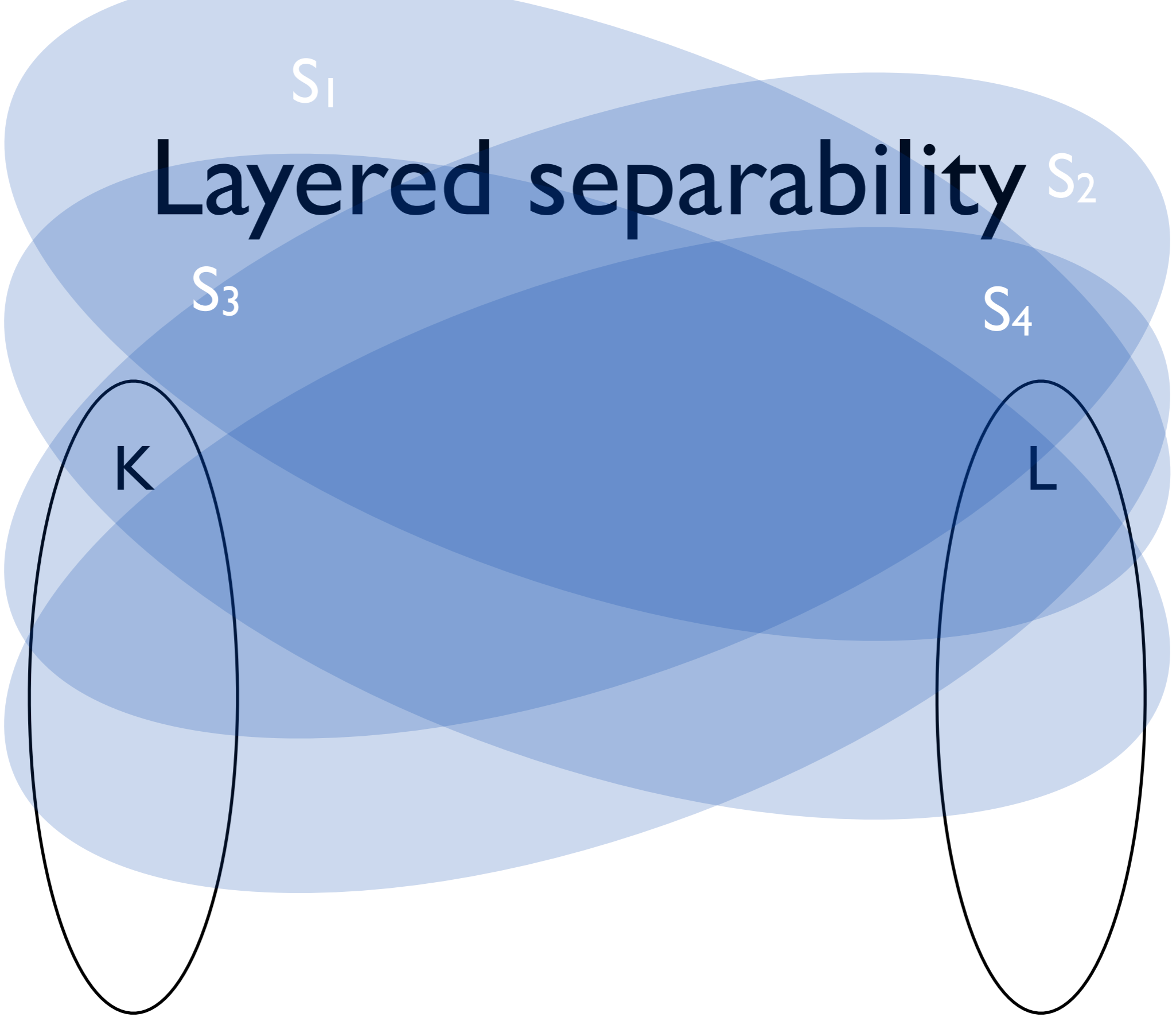
S_2

S_3

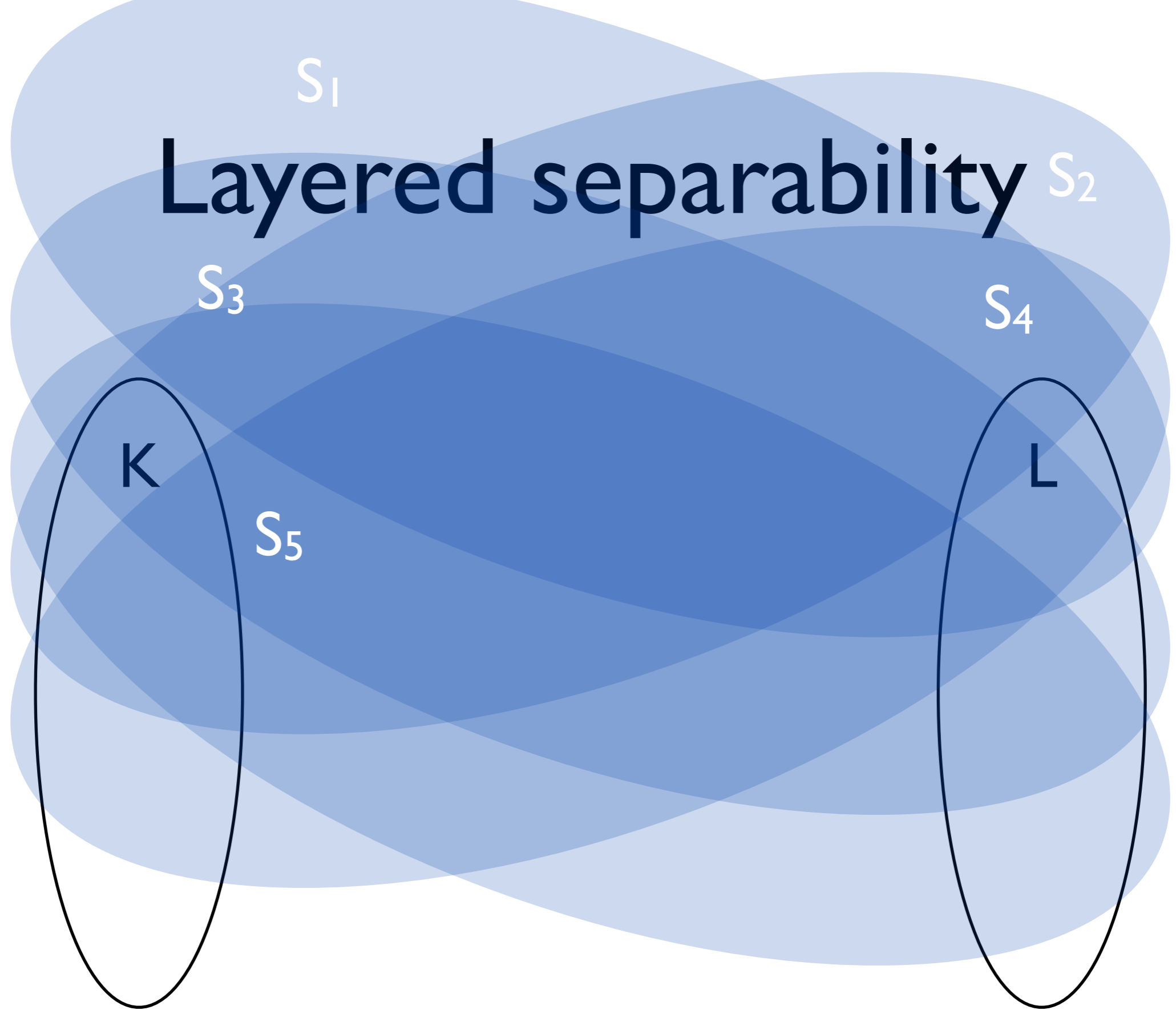
S_4

K

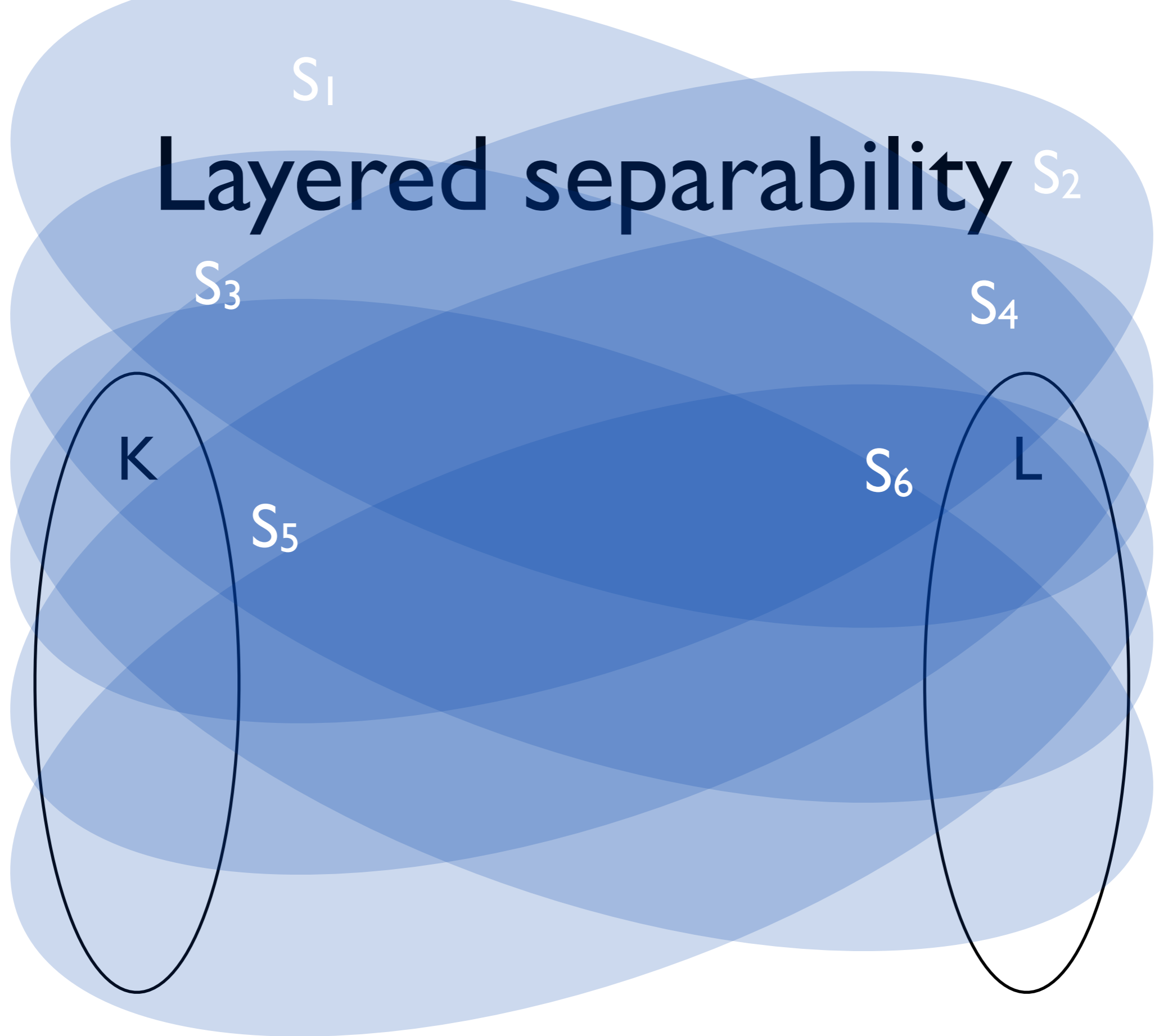
L



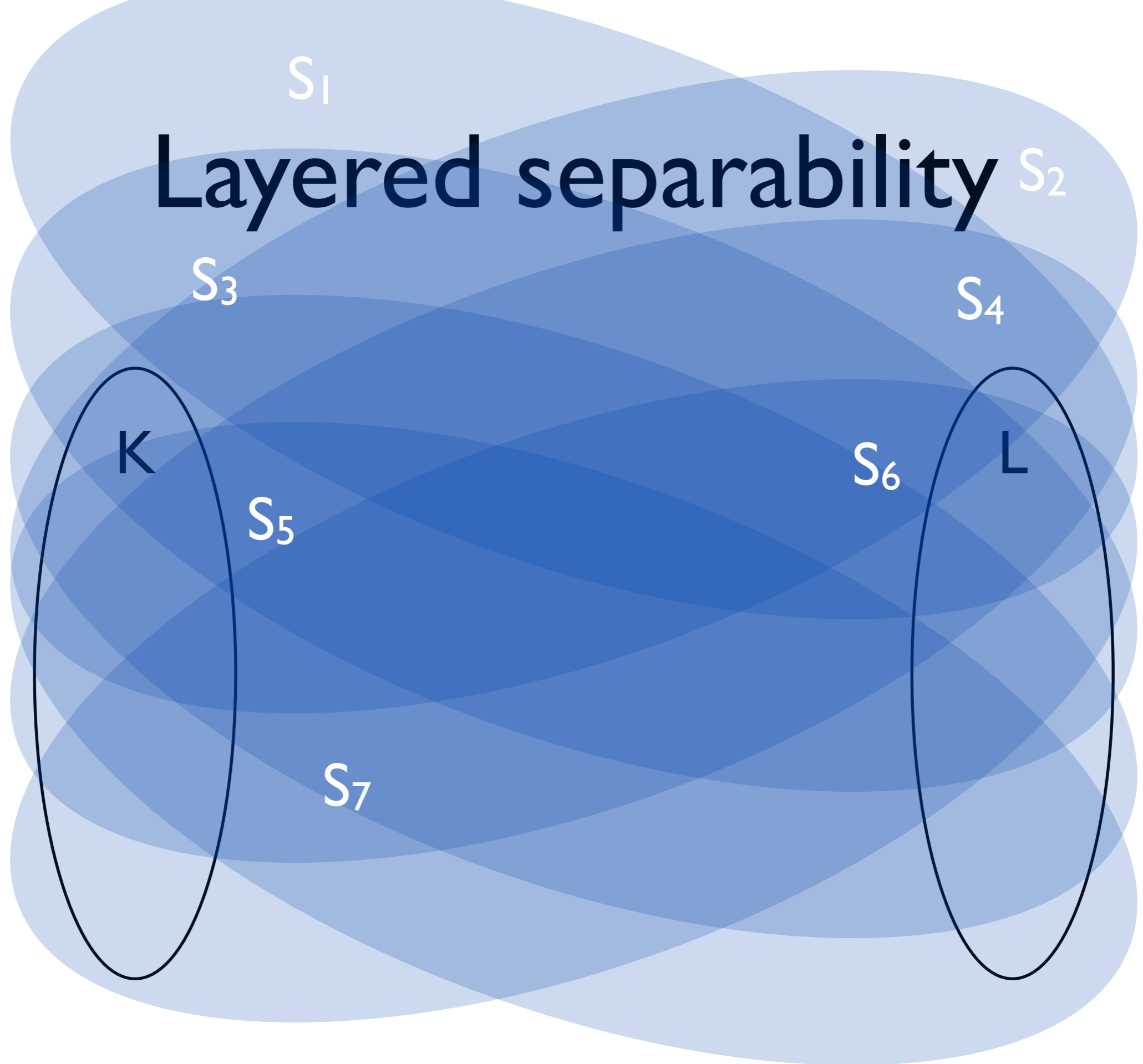
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For **any** two word languages K and L the following conditions are equivalent:

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subsequence order is a WQO (Higman's Lemma)

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For **any** two word languages K and L the following conditions are equivalent:

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Second main result

For **any** two **word** languages K and L the following conditions are equivalent:

- 1) K and L are separable by **piecewise testable languages**
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For **any** two ~~word~~ languages K and L the following conditions are equivalent:

- 1) K and L are separable by **bool. comb. of \preceq -closed languages**
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for any well quasi order \preceq

Effective characterization

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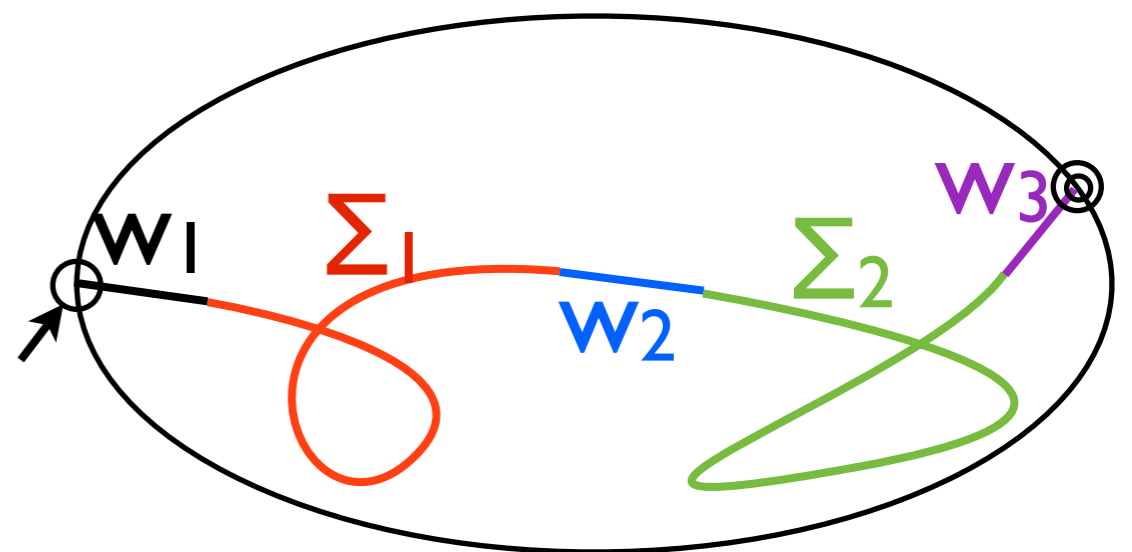
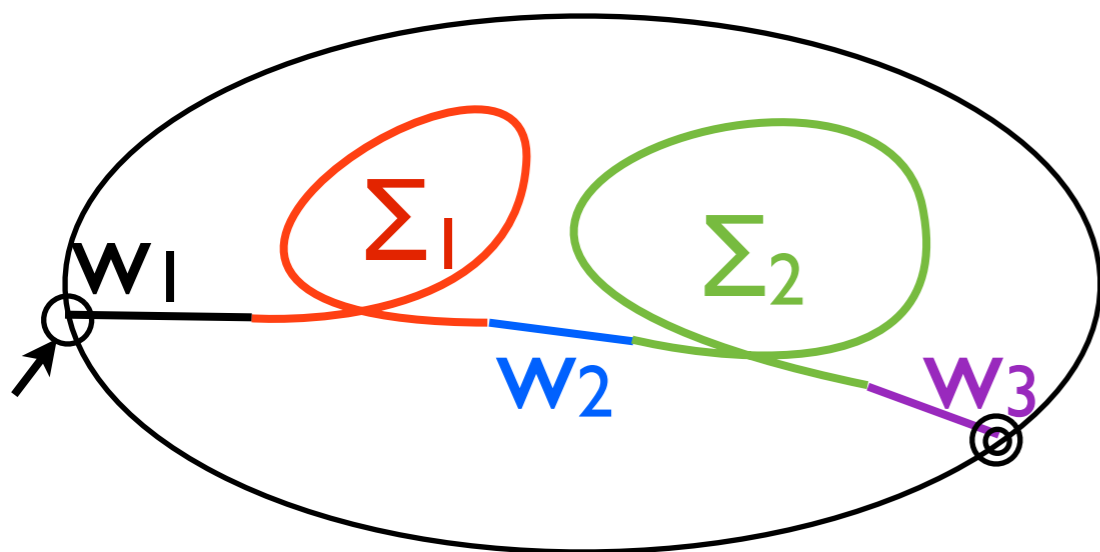
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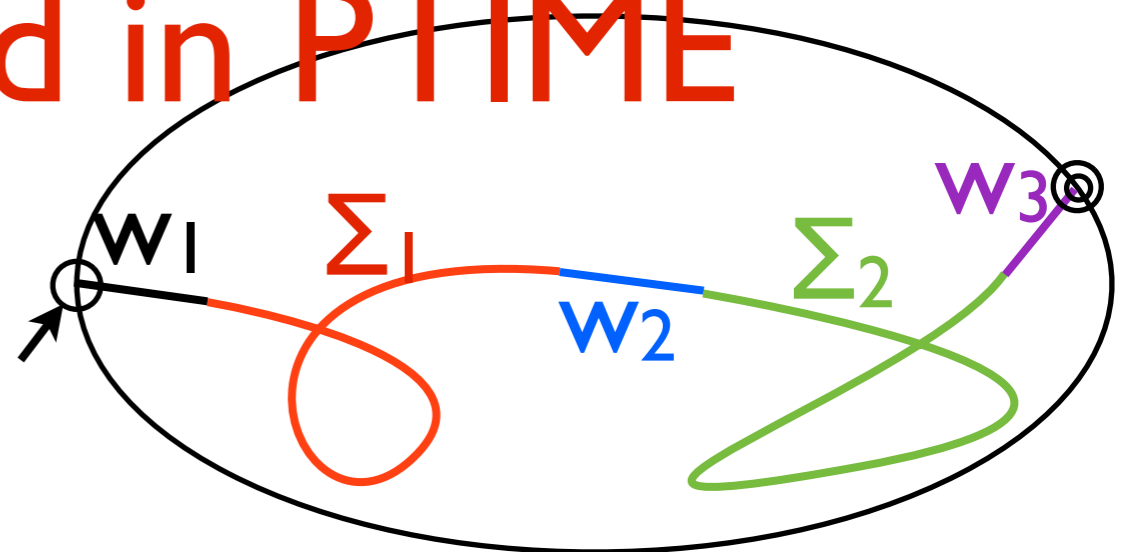
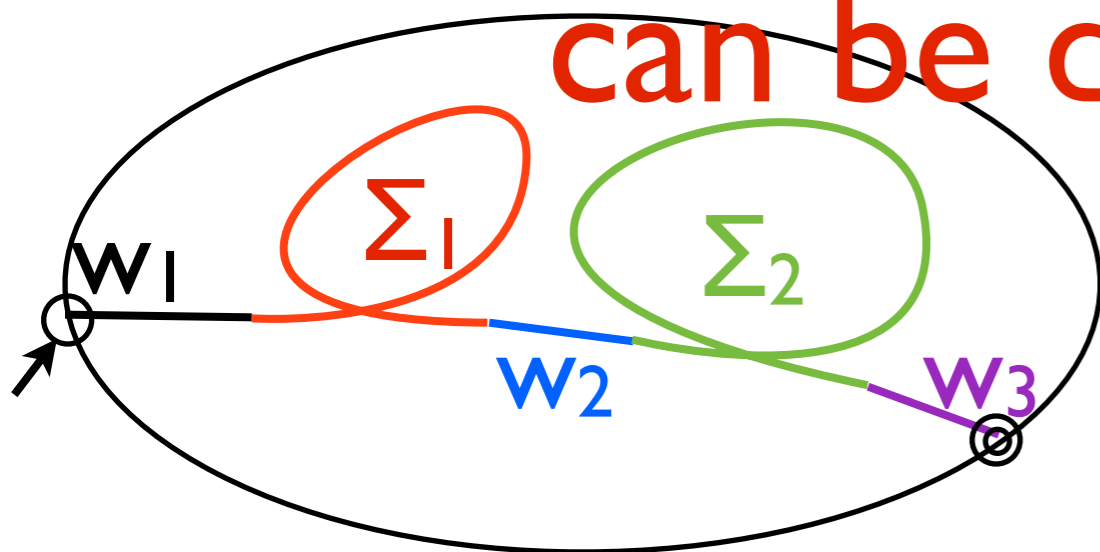


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can be decided in PTIME



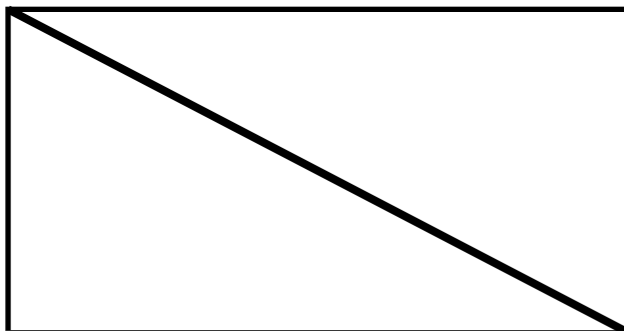
Other variants

Other variants

Separability Problem

Other variants

Separability Problem

Other variants

Separability Problem

pieces			

Other variants

Separability Problem

\	single		
pieces			

Other variants

Separability Problem

/	single	unions	
	pieces		

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			

Other variants

Separability Problem

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$\Sigma^* w$

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Further research

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- computing the separator (not deciding existence)

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- solving a problem efficiently for other classes

Thank you!