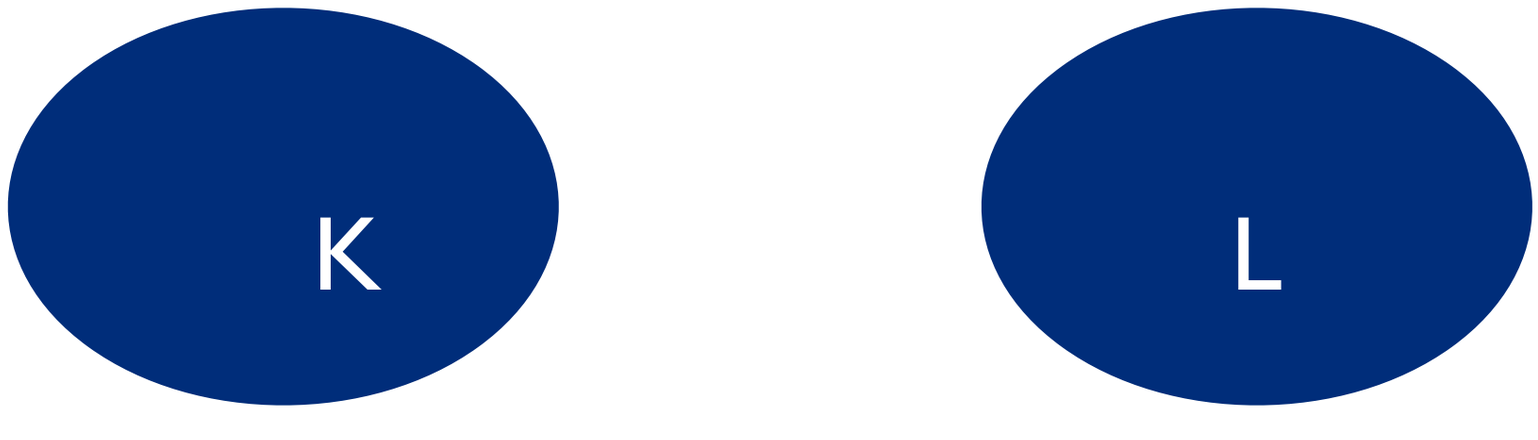
Regular Separability of One Counter Automata

Wojciech Czerwiński

Sławomir Lasota

Separability

Separability



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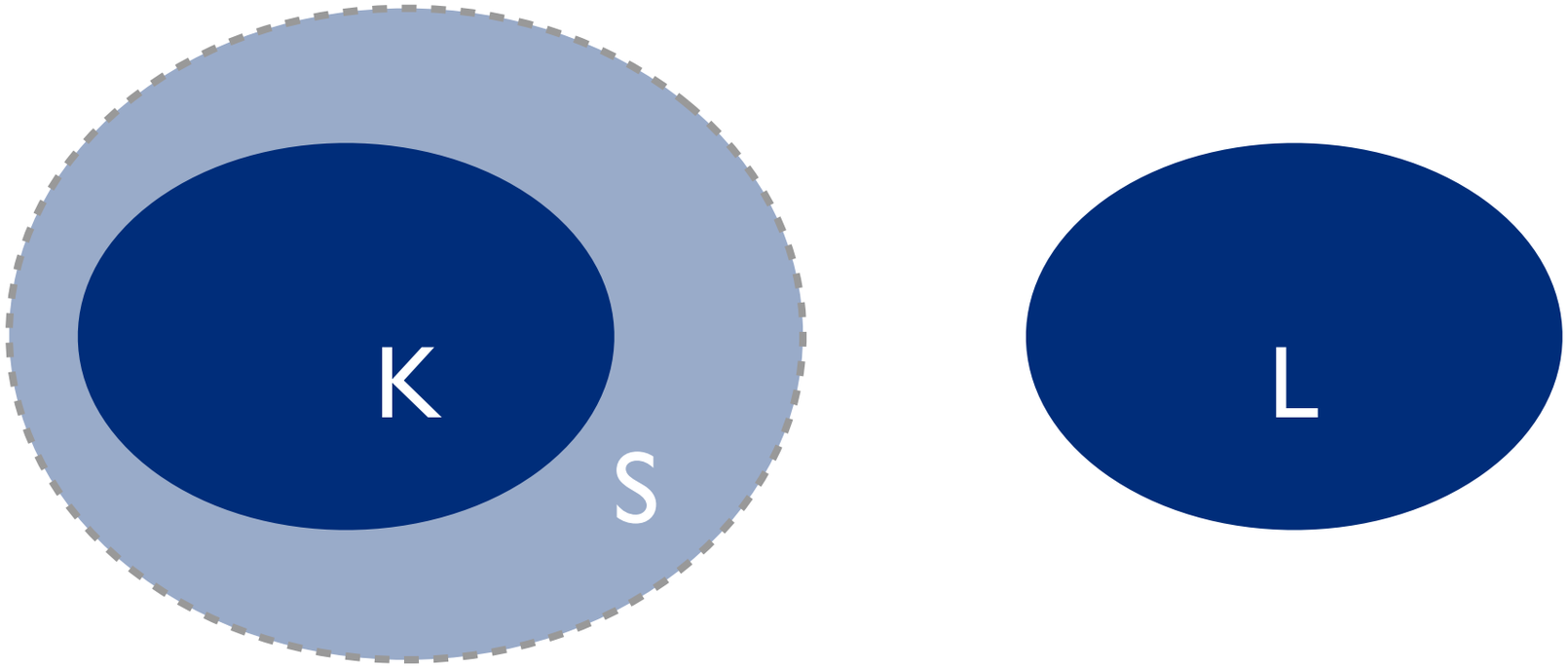

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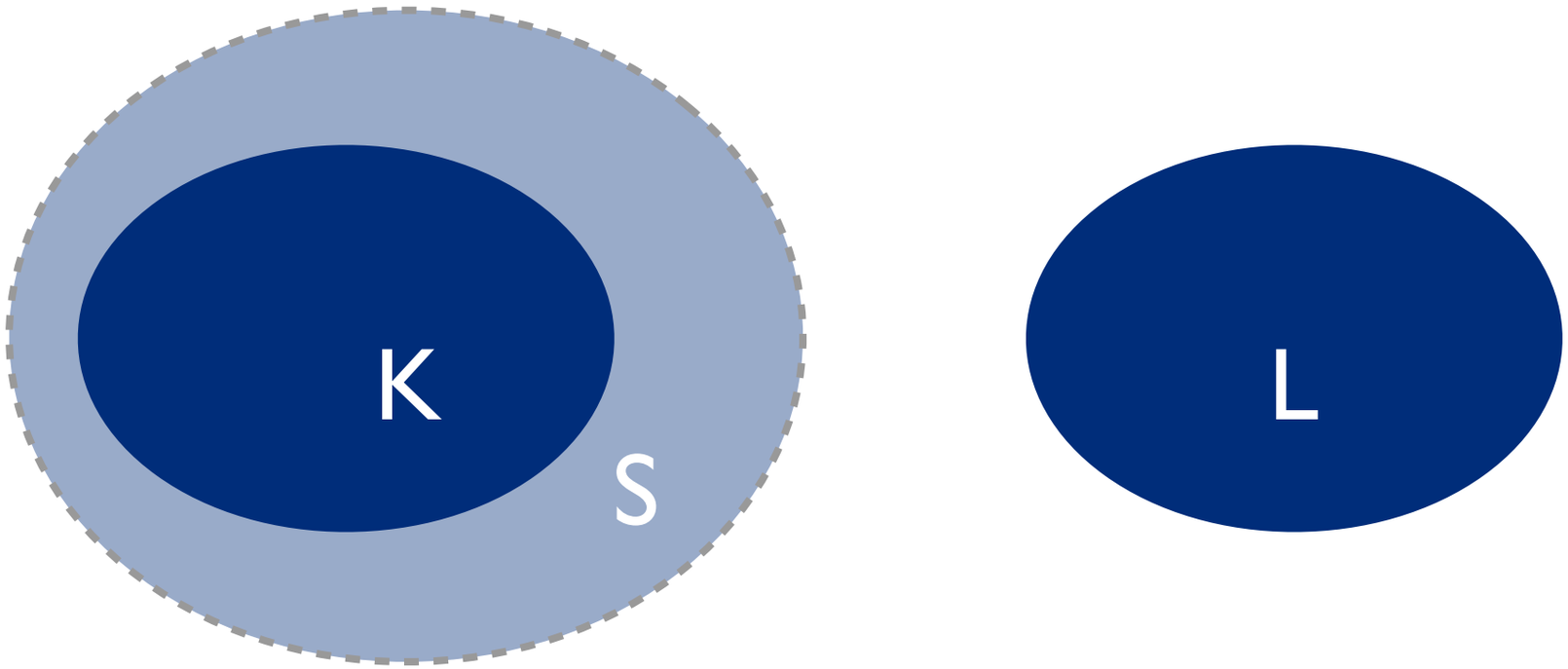

S *separates* K and L

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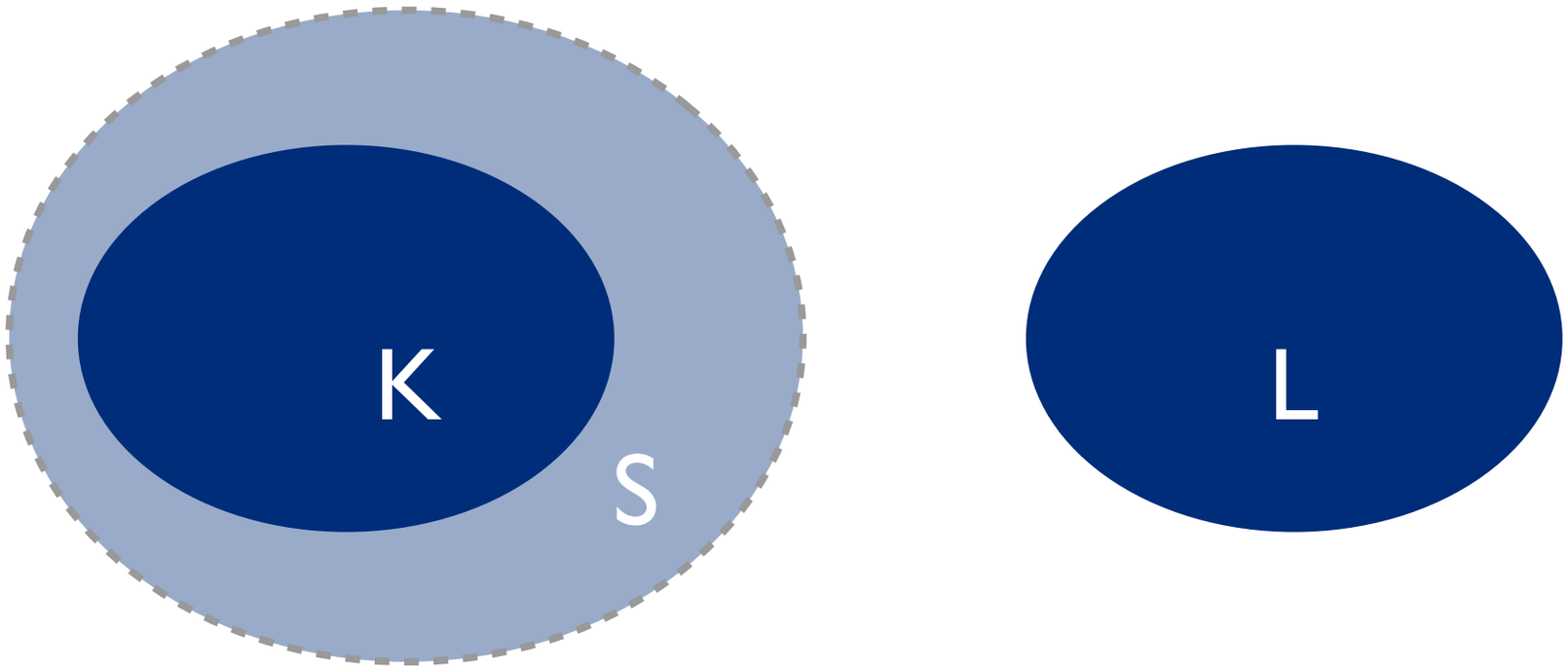

S *separates* K and L

K and L are *regular separable*
if some regular S separates them

One counter automata

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configuration: state + one nonnegative counter

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transition: based on state chooses new state and counter change (in binary)

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language: single source and target configuration
(assume counter 0), every transition labelled

Problem(s)

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Given: two one counter nets (automata) A and B

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Question: are L(A) and L(B) regular separable?

Main result

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Theorem:

Regular separability of
languages of one counter nets
is decidable

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Remark: even PSPACE-complete

Another result

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Theorem:

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Remark: even by any F closed
under boolean combination and containing $w\Sigma^*$

Proof idea: approximants

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Idea: canonical separators

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For OCN A and number n define automaton A_n such that:

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For OCN A and number n define automaton A_n such that:

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3) for m dividing n holds $L(A_m)$ includes $L(A_n)$

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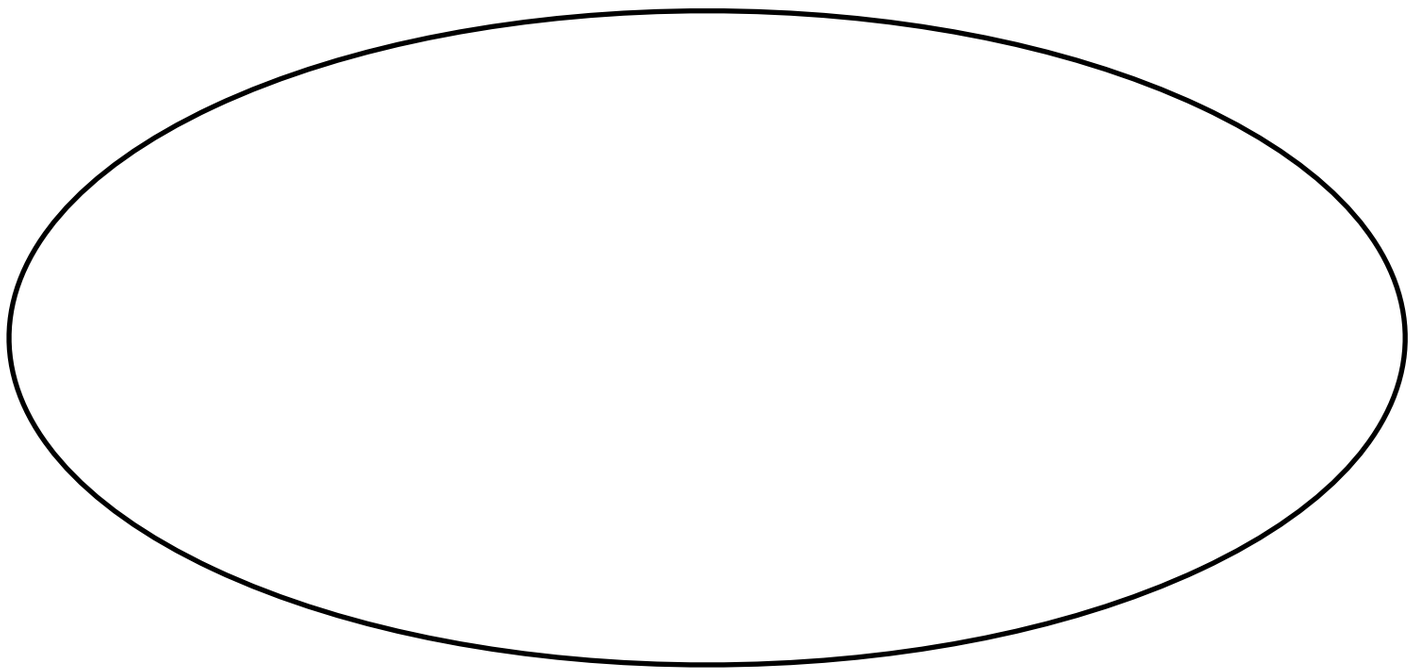

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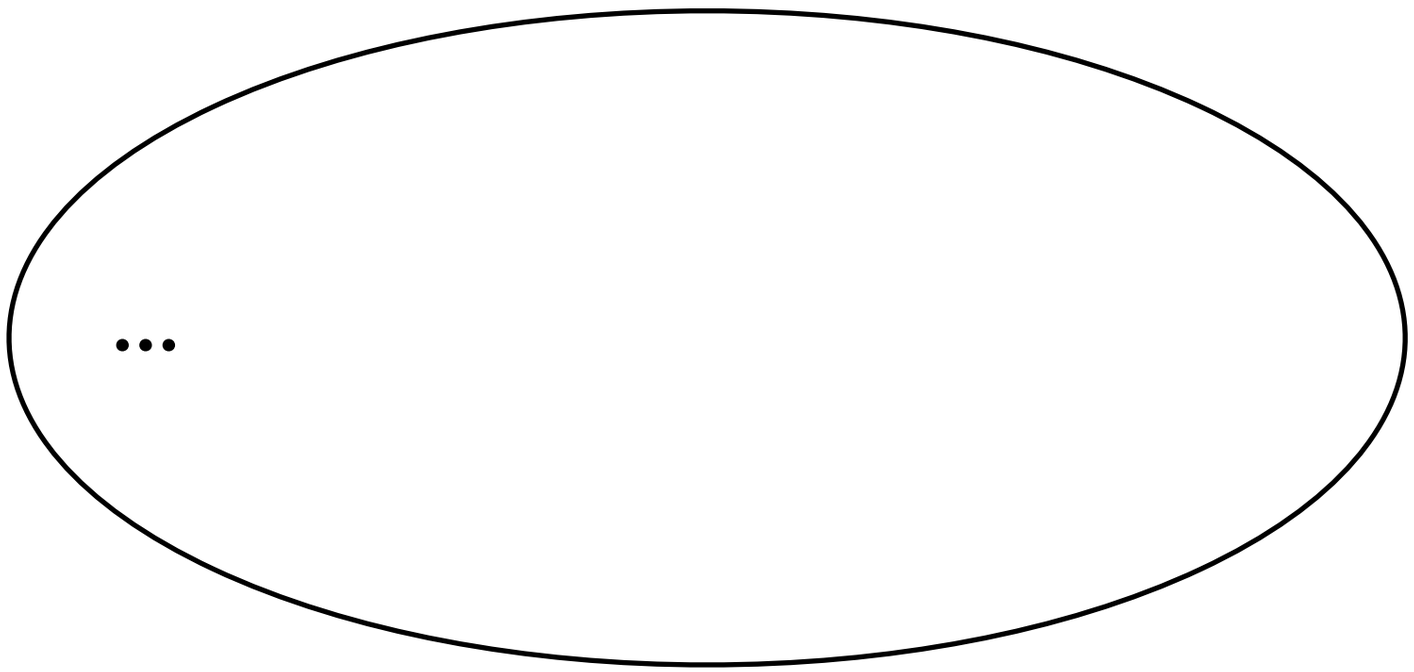

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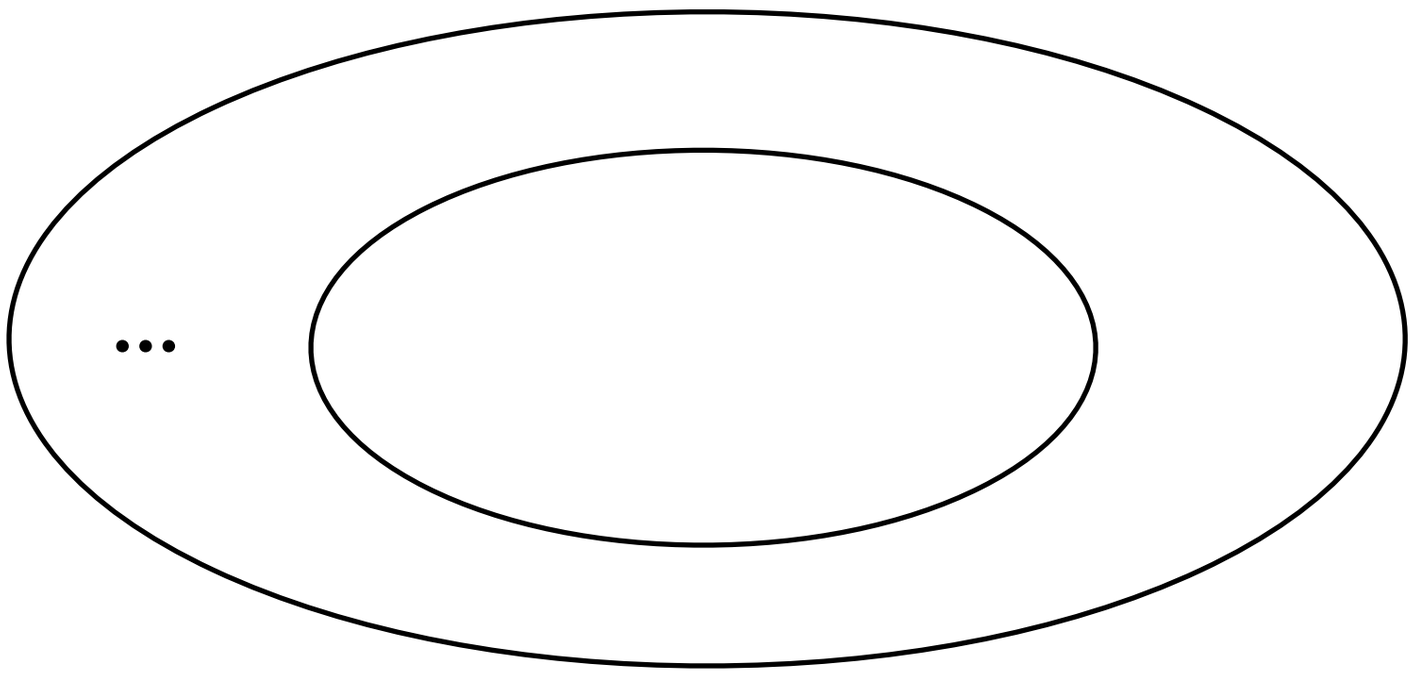

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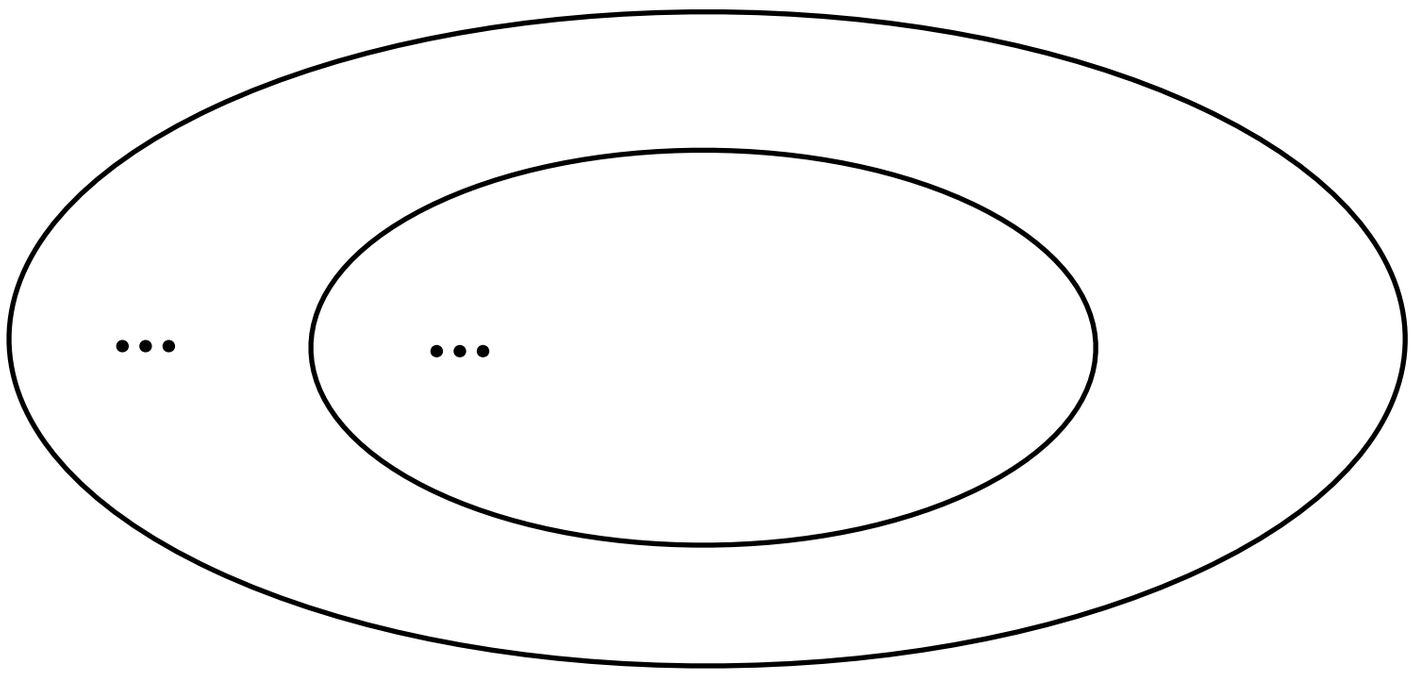

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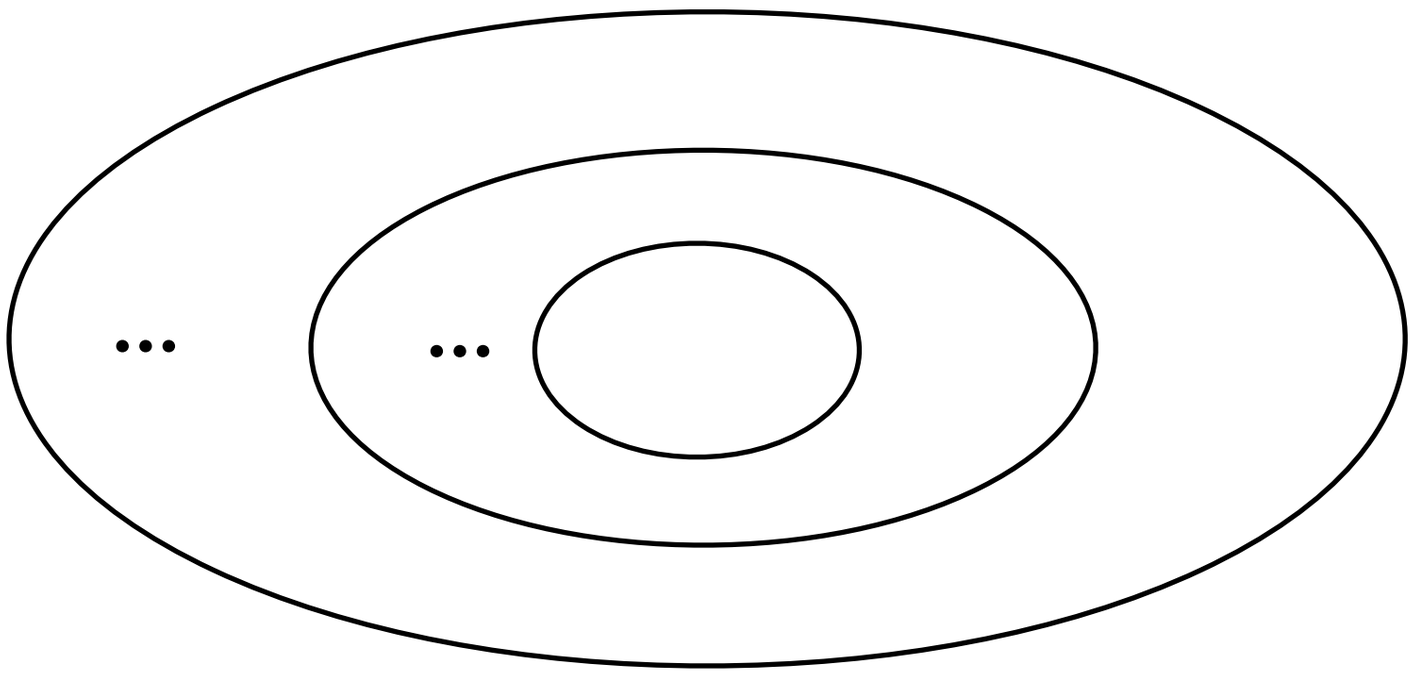

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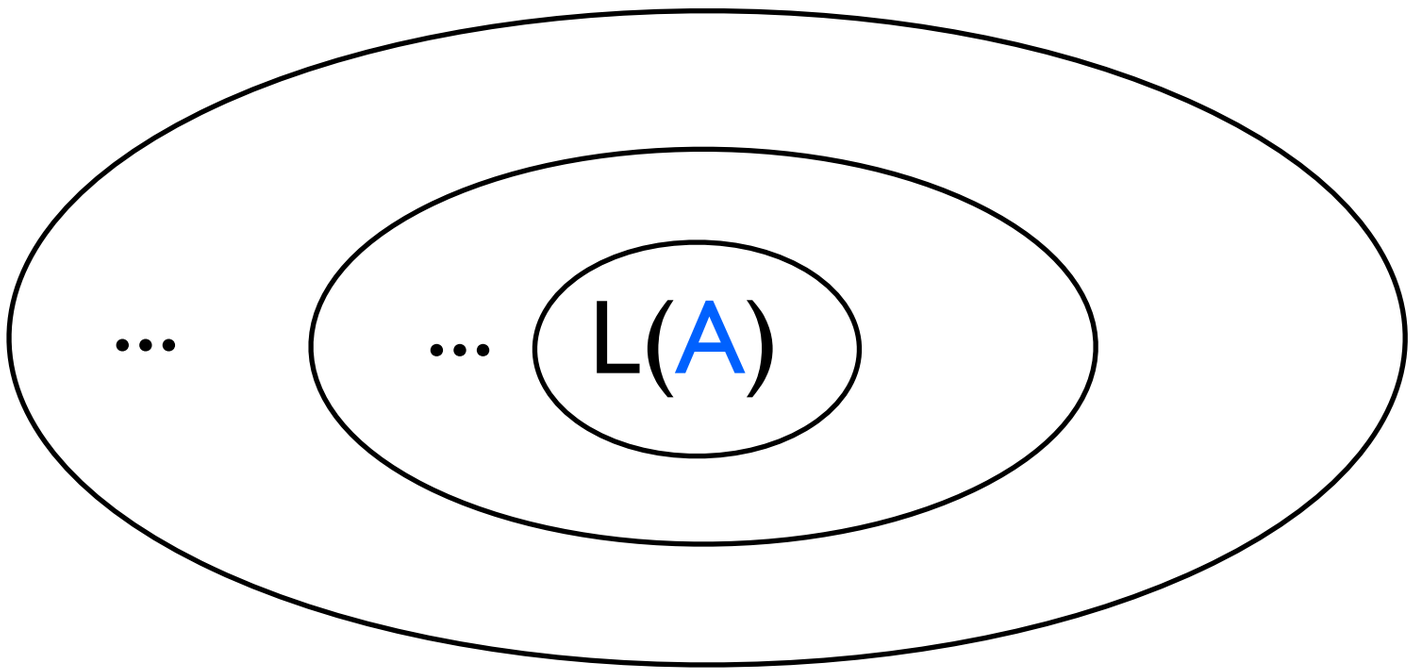

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For two OCNs A and B tfae:

1) L(A) and L(B) are regular separable

2) there exists n s.t. $L(A_n)$ and $L(B_n)$ disjoint

Approximation lemma

Lemma

For two OCNs A and B tfae:

1) L(A) and L(B) are regular separable

2) there exists n s.t. $L(A_n)$ and $L(B_n)$ disjoint

One direction clear!

Algorithm

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Checks whether there is some n such that L(A_n) and L(B_n) disjoint

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Quite standard

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Semininear set techniques - PSPACE

Hardness

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Undecidability and PSPACE-hardness - similar techniques

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Old technique by Hunt

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Reduction from every decidable problem
(of bounded blowup)

Thank you!