

Efficient Separability
of Regular Languages
by Subsequences and Suffixes

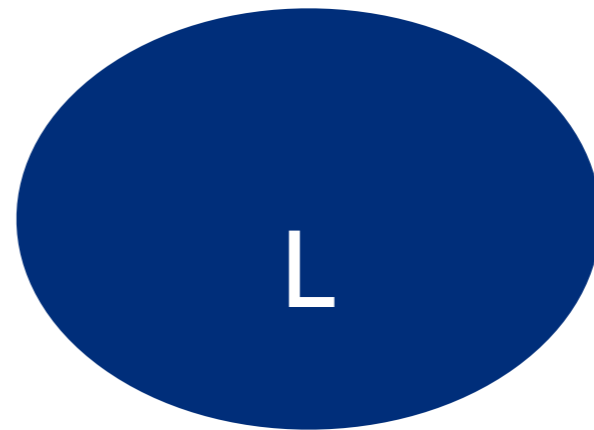
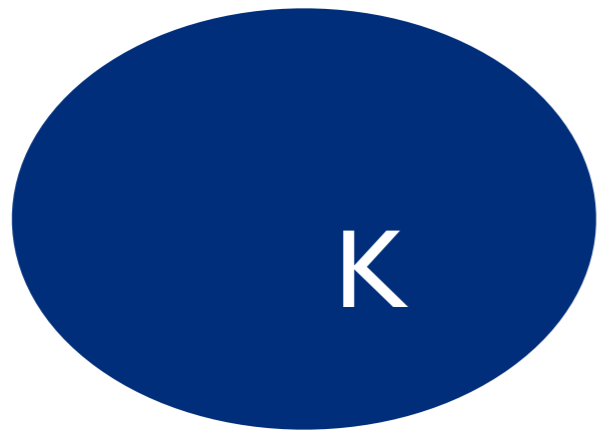
Wojciech Czerwiński

Tomáš Masopust

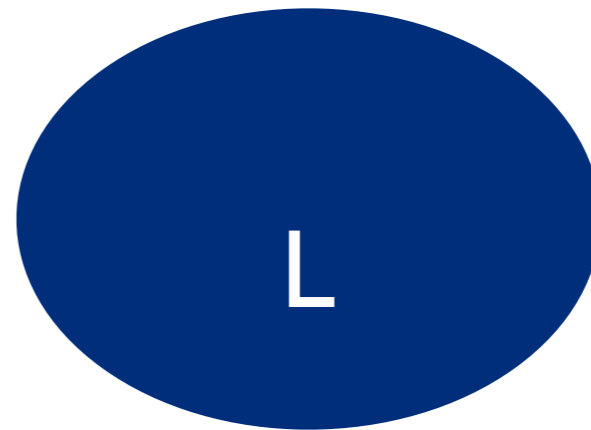
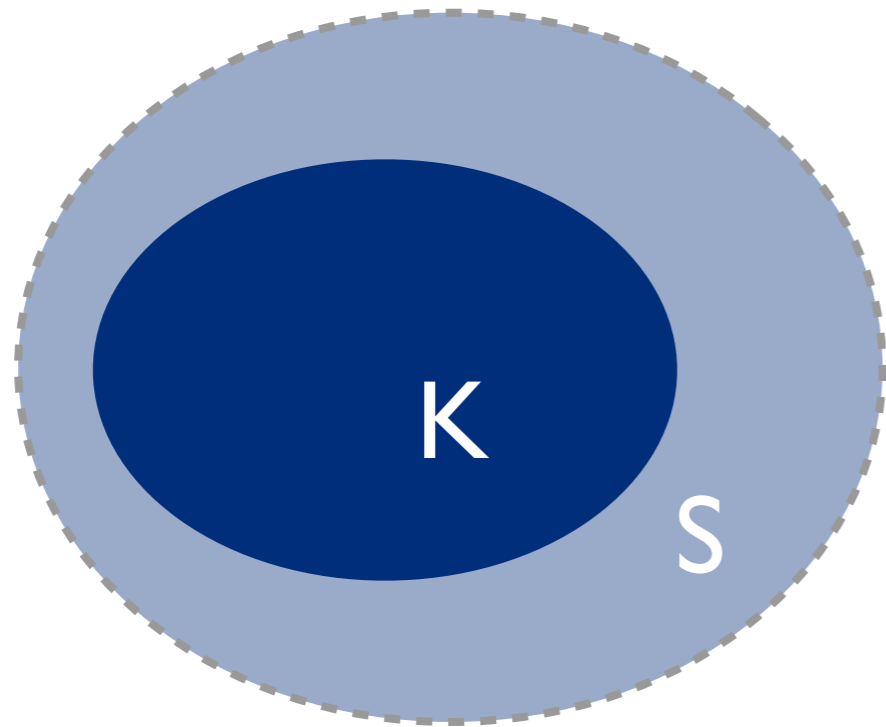
Wim Martens

Separability

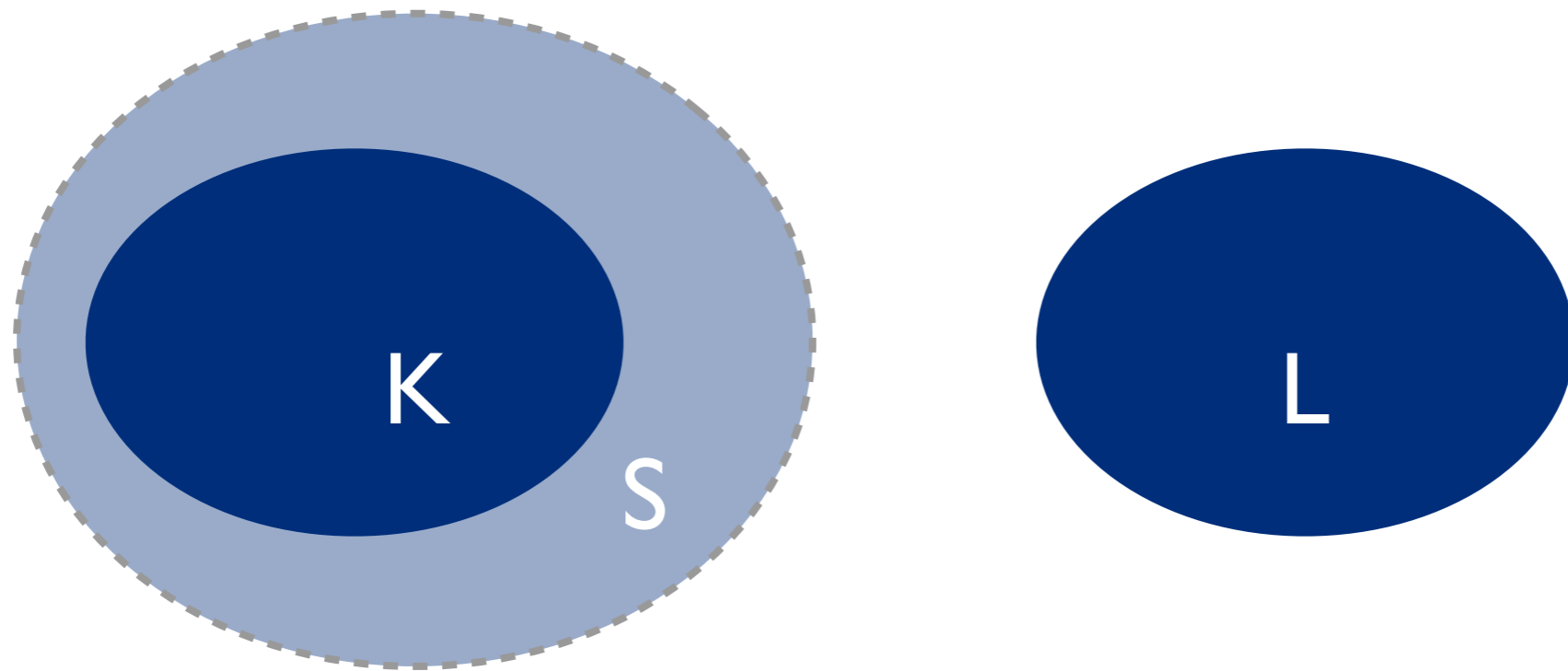
Separability



Separability

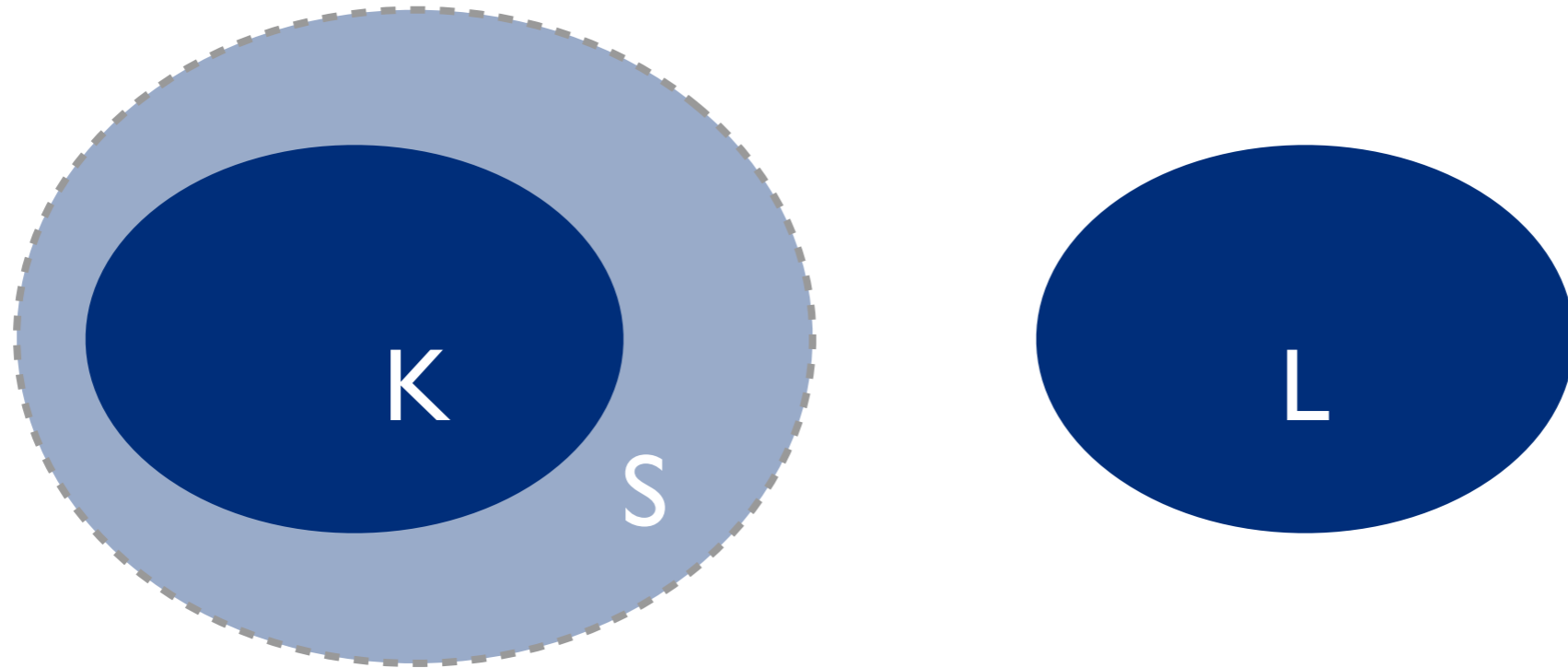


Separability



S separates K and L

Separability



S separates K and L

K and L are *separable by family F*
if some S from F separates them

Problem

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Given: nondeterministic automata for
languages K and L

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Question: are K and L separable by Piecewise Testable Languages (PTL)?

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piece language

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piecewise testable language

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piece language

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piecewise testable language

bool. comb. of pieces

What is known?

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- Simon 1975, piecewise testable = j-trivial

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- Stern 1985, deciding if a language is piecewise testable is in PTIME
- Almeida, Zeitoun 1997, exponential algorithm for separability by PTL

First main result

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Theorem:

Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

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obtained independently
by Place, van Rooijen, Zeitoun
MFCS '13

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Second main result

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For **any** two word languages K and L the following conditions are equivalent:

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1) K and L are separable by piecewise testable languages

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For **any** two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

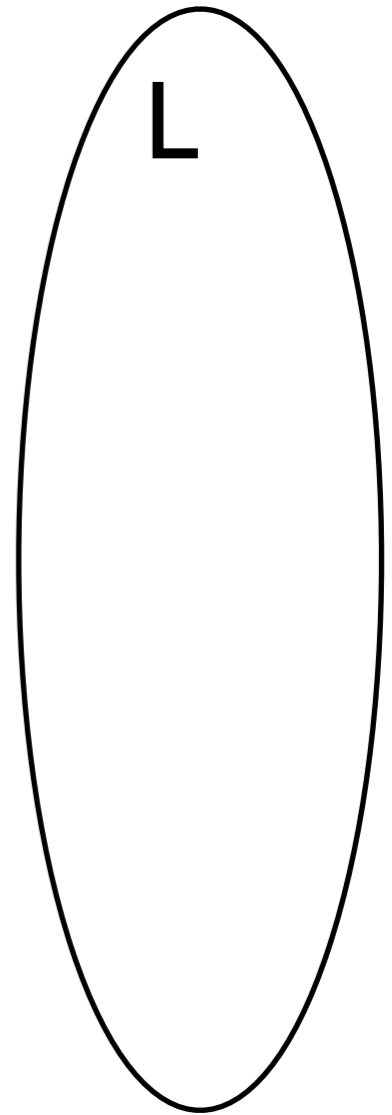
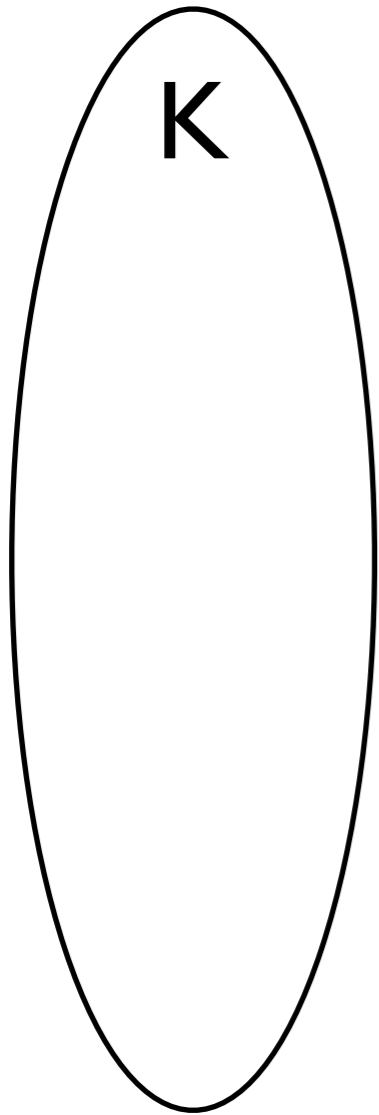
Infinite zigzag

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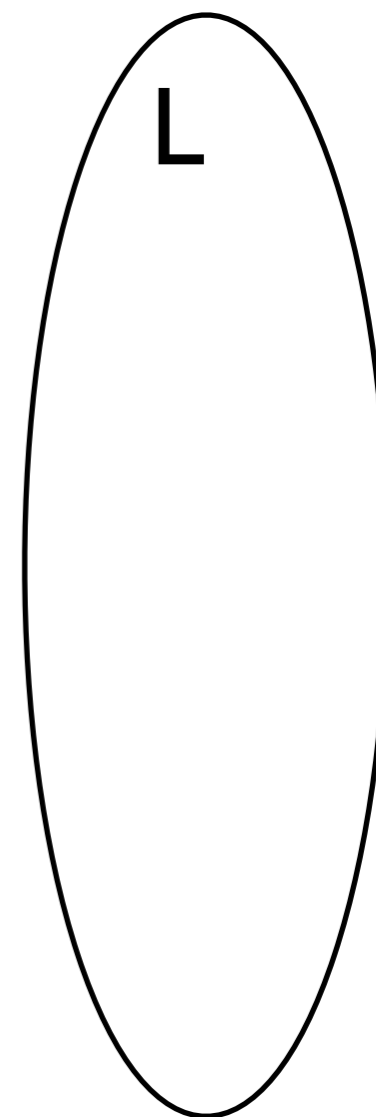
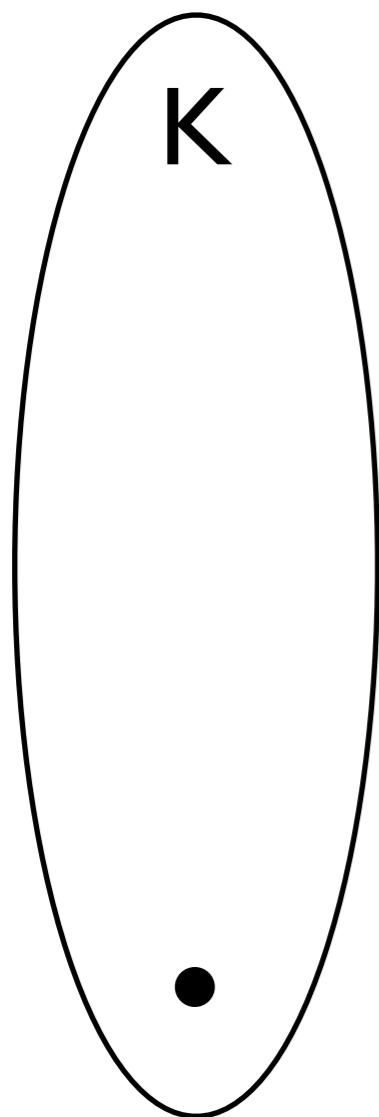
abcd ≍ dbabacbdb

Infinite zigzag

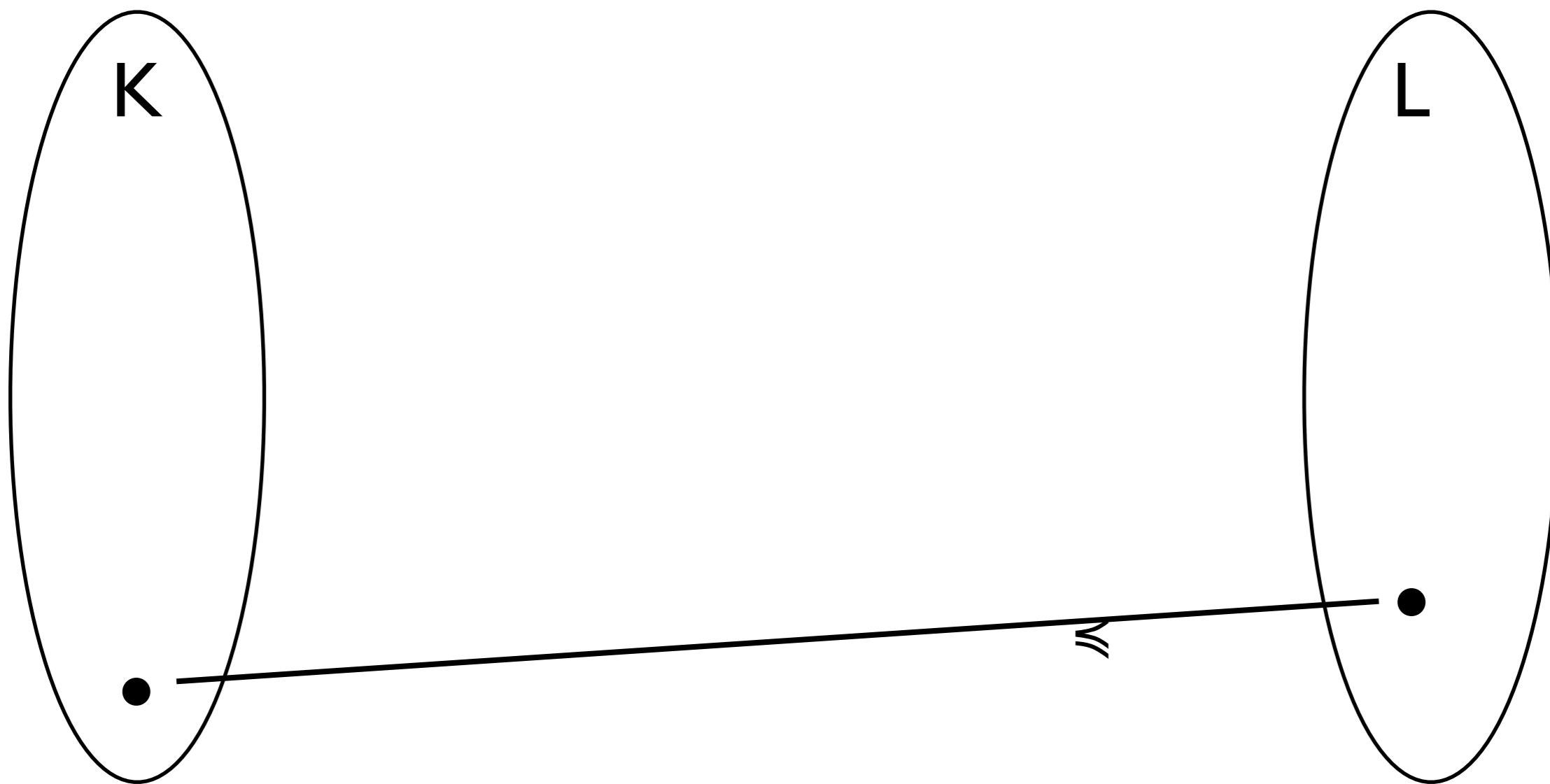
Infinite zigzag



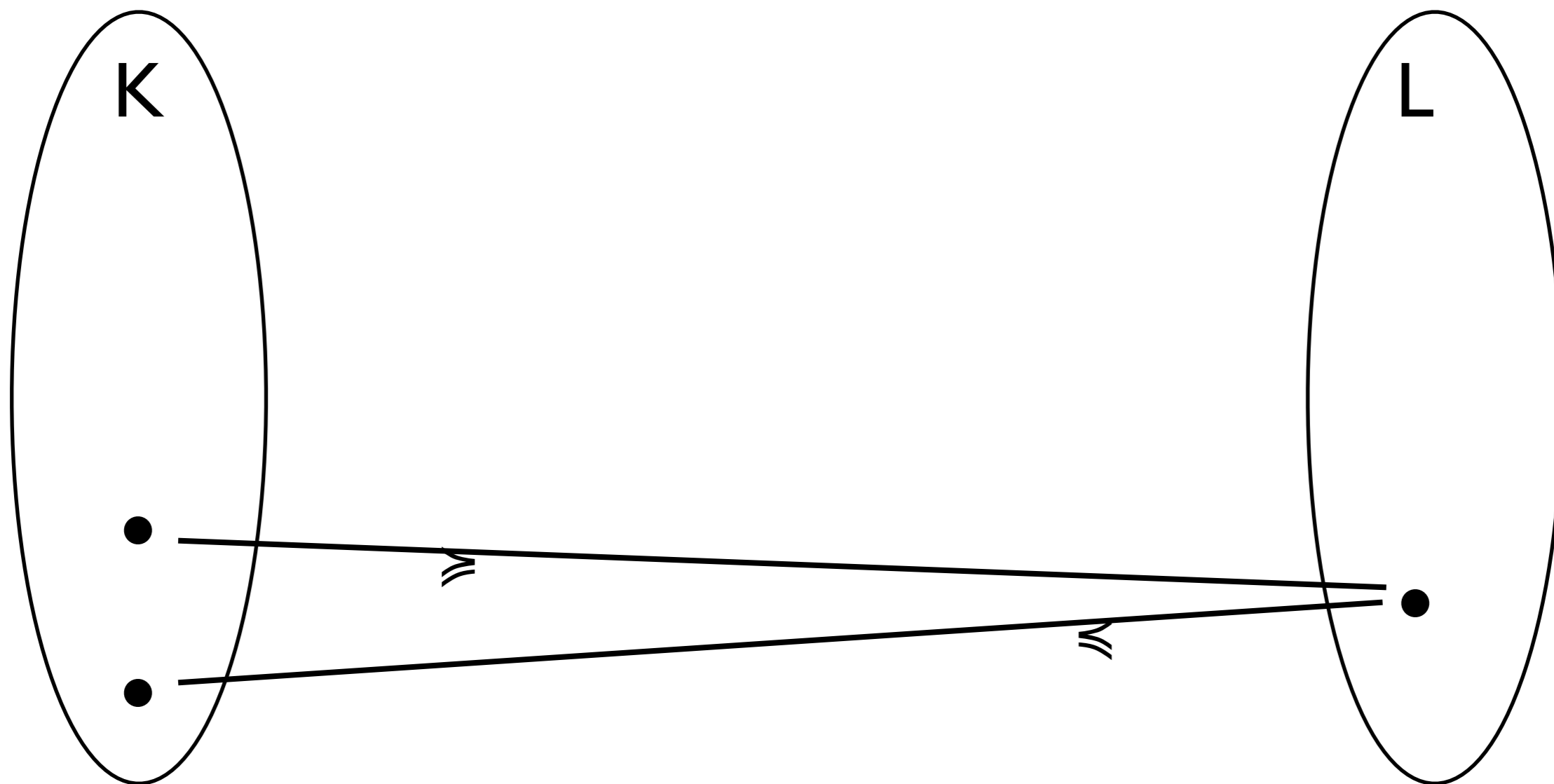
Infinite zigzag



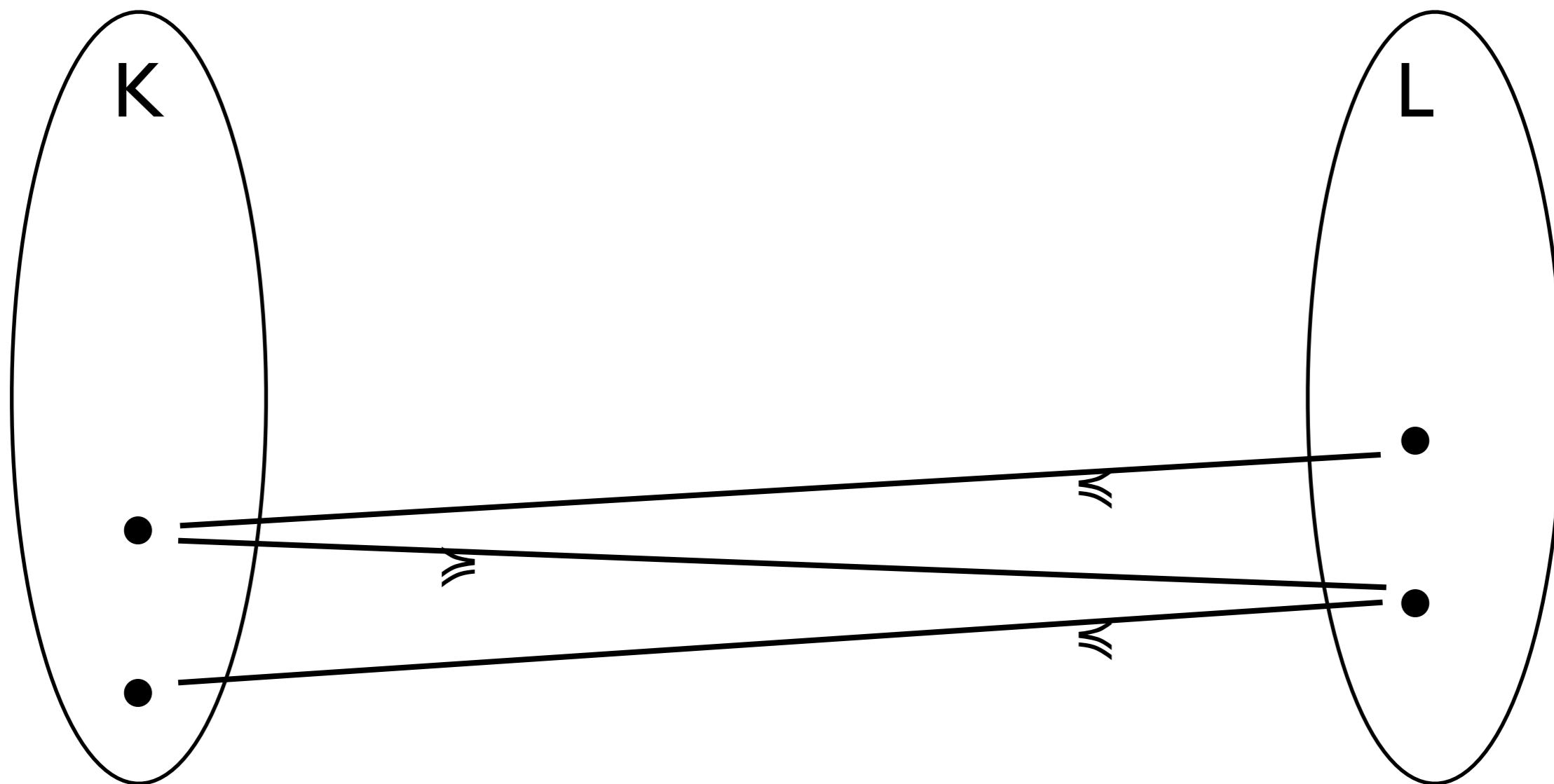
Infinite zigzag



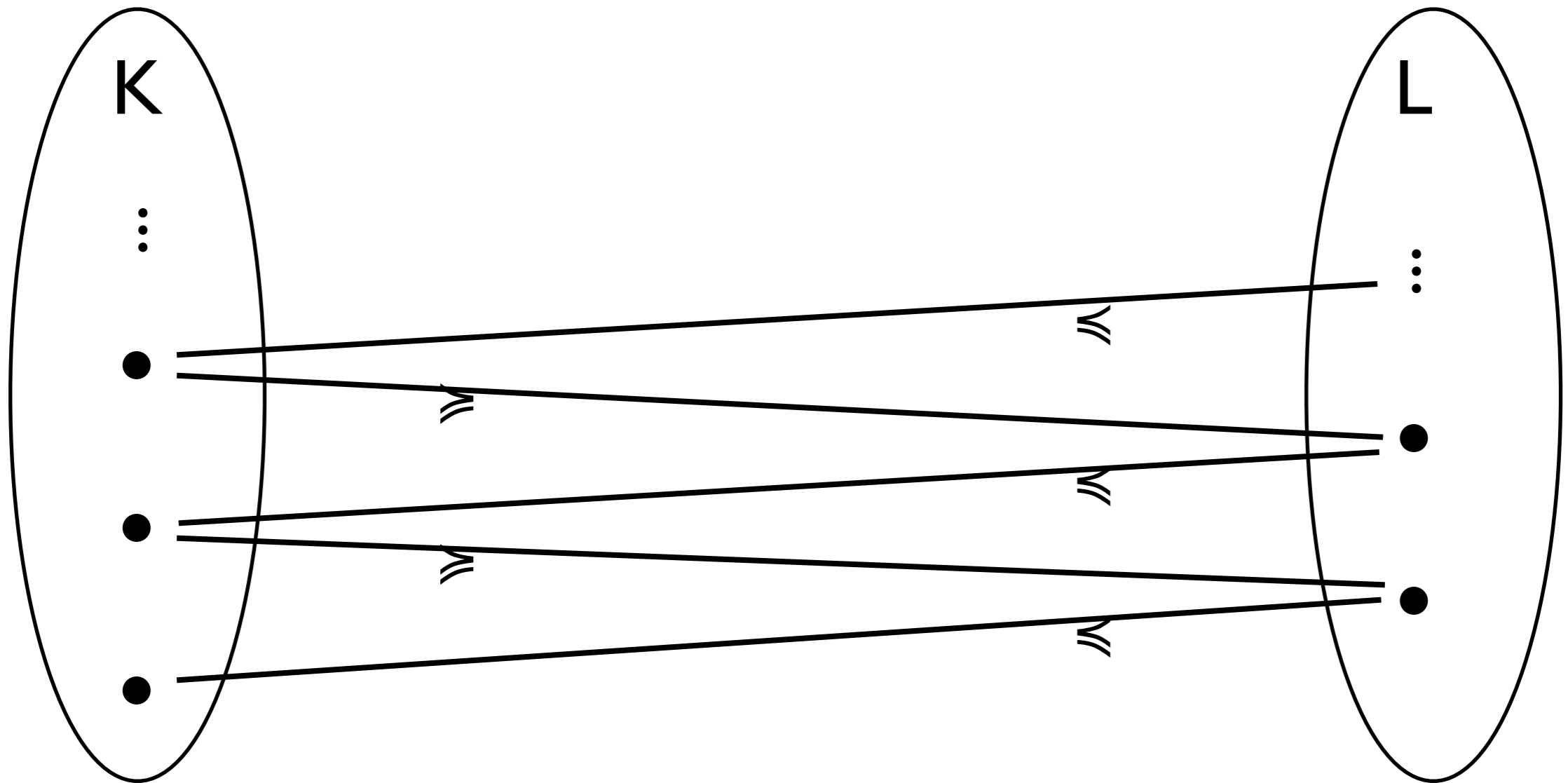
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For **any** two word languages K and L the following conditions are equivalent:

- 1) K and L are separable by piecewise testable languages
- 2) there is no infinite zigzag between K and L
- 3) K and L are layered separable by pieces

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$$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$$

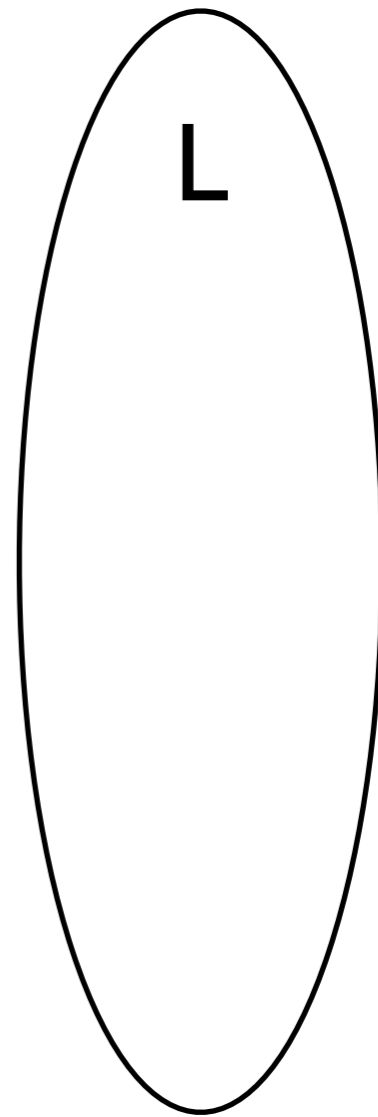
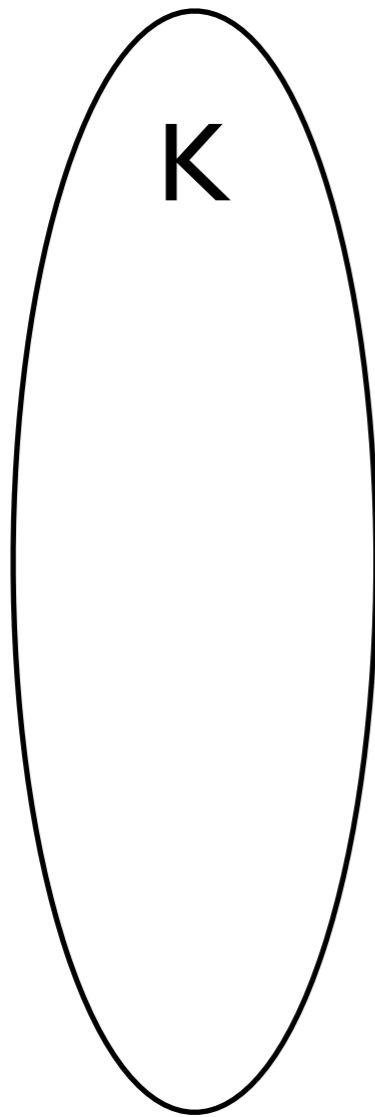

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Layered separability

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S_1

Layered separability

K

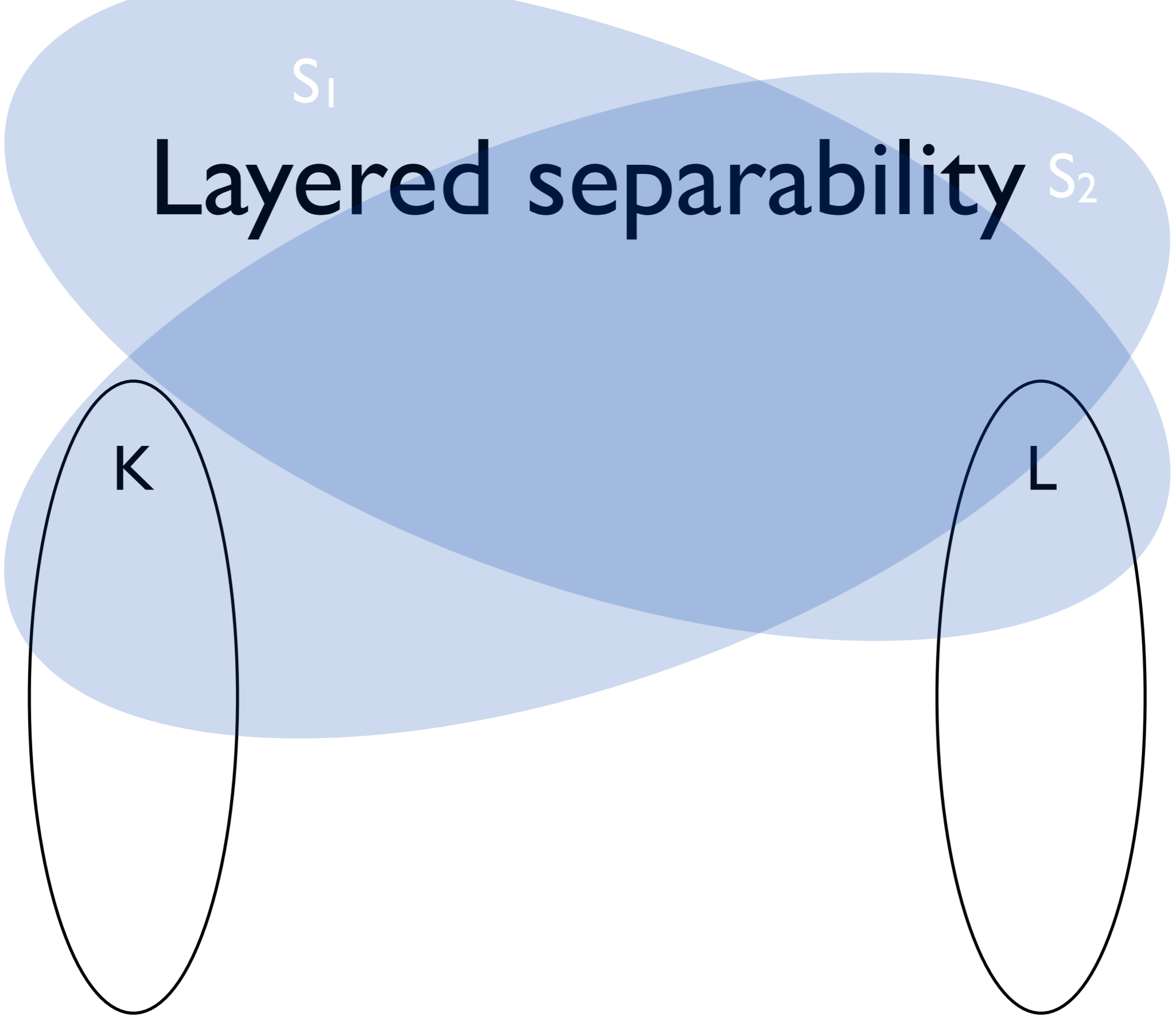
L

S_1

Layered separability S_2

K

L



Layered separability

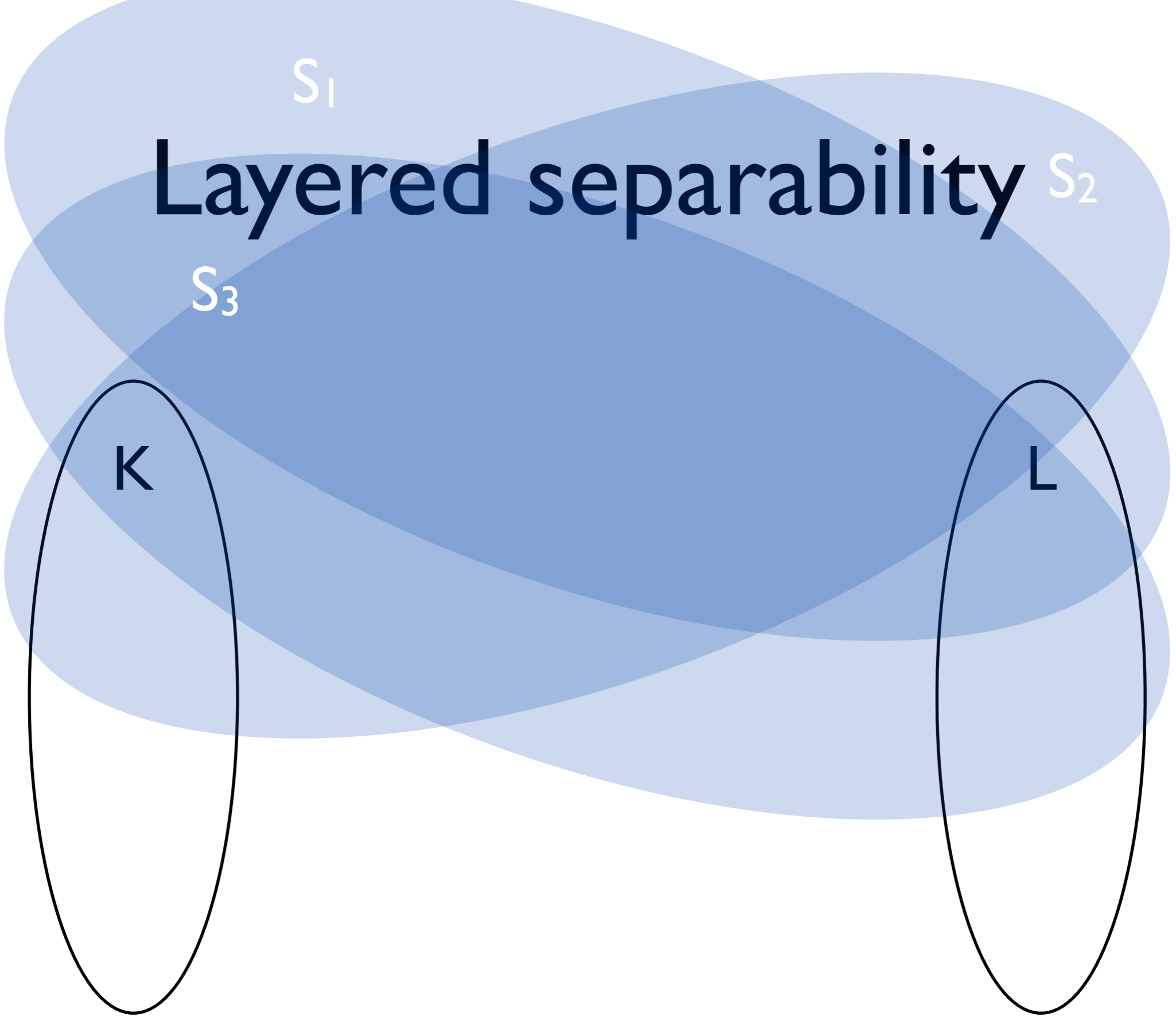
S_1

S_2

S_3

K

L



Layered separability

S_1

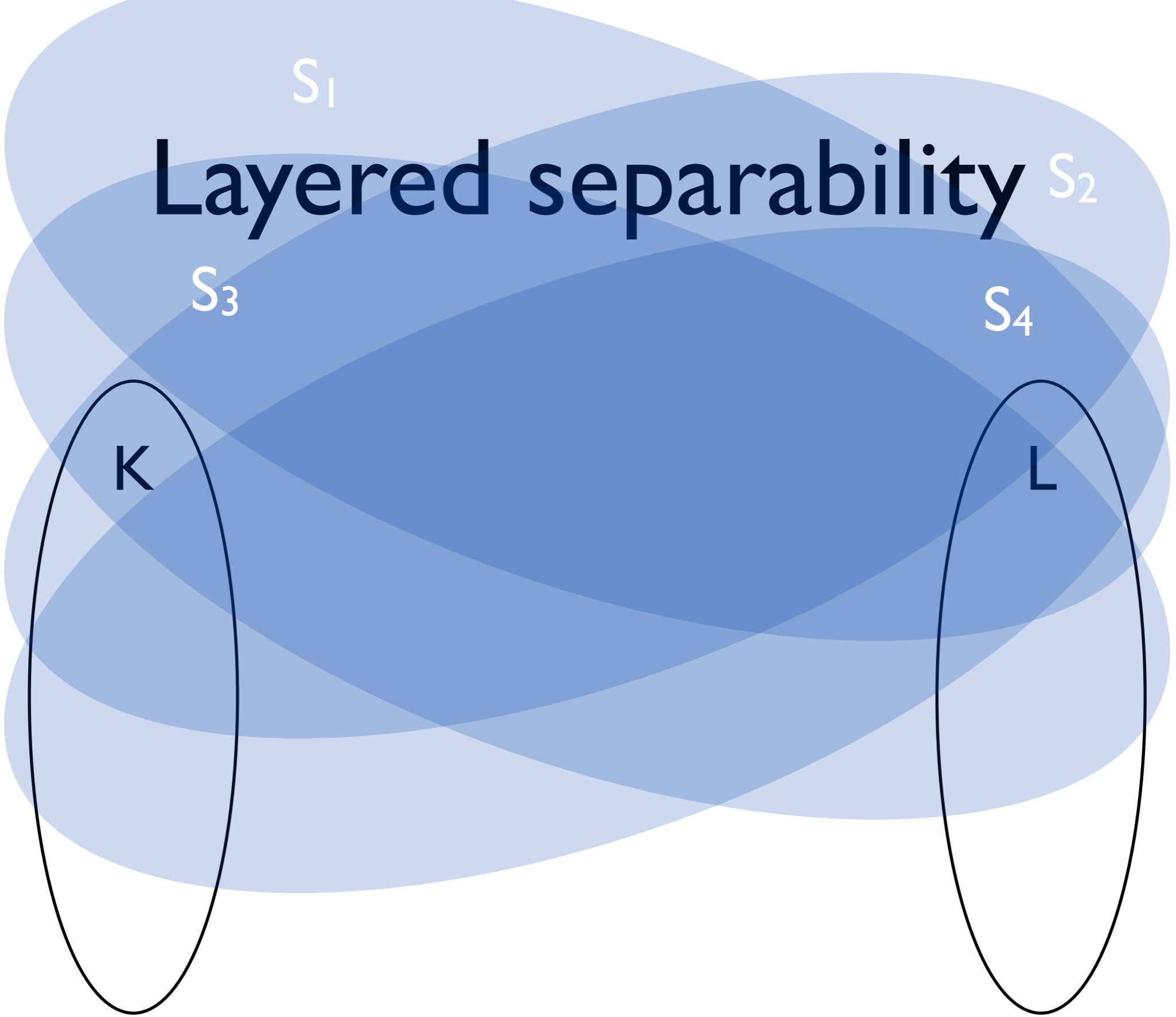
S_2

S_3

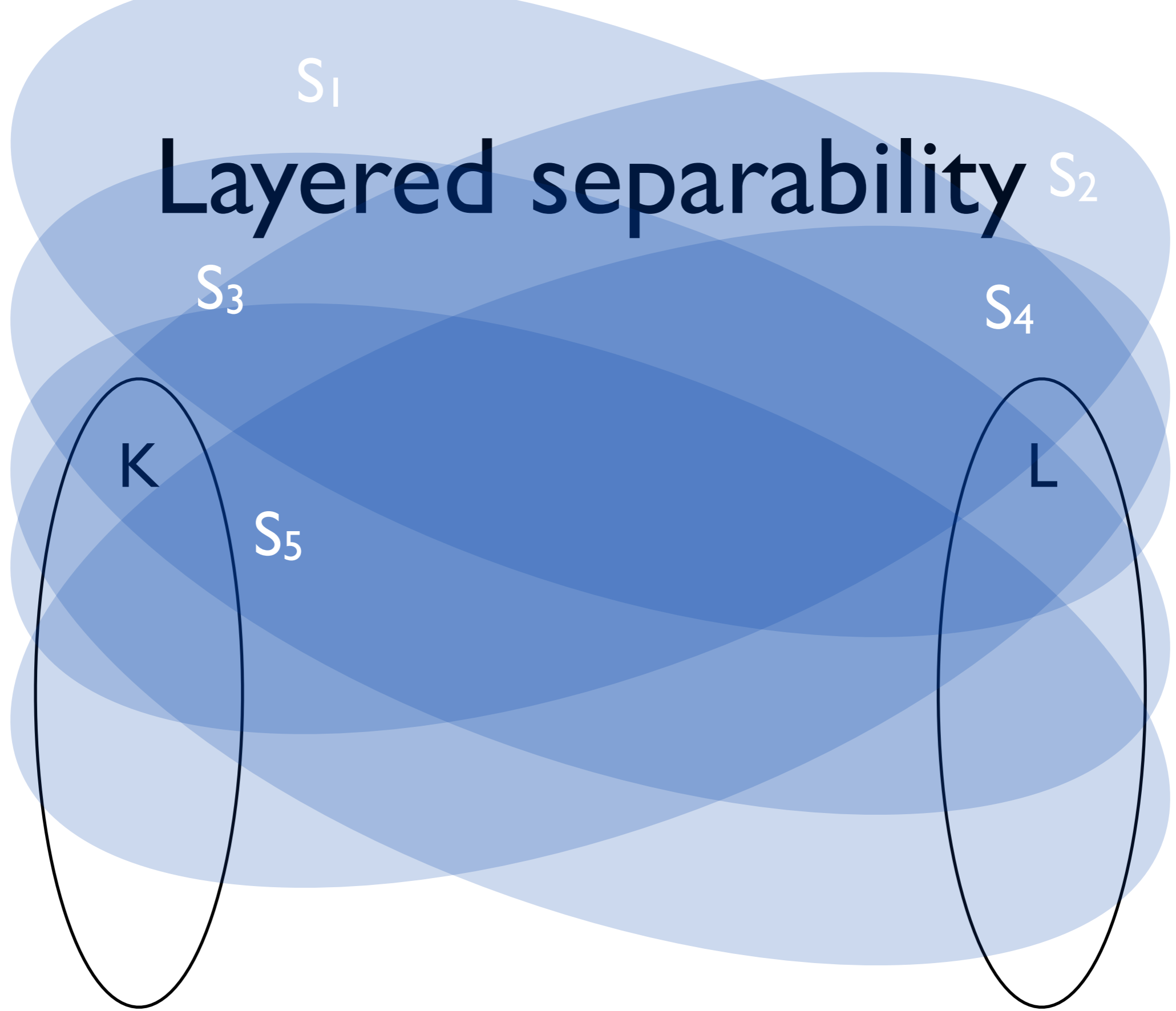
S_4

K

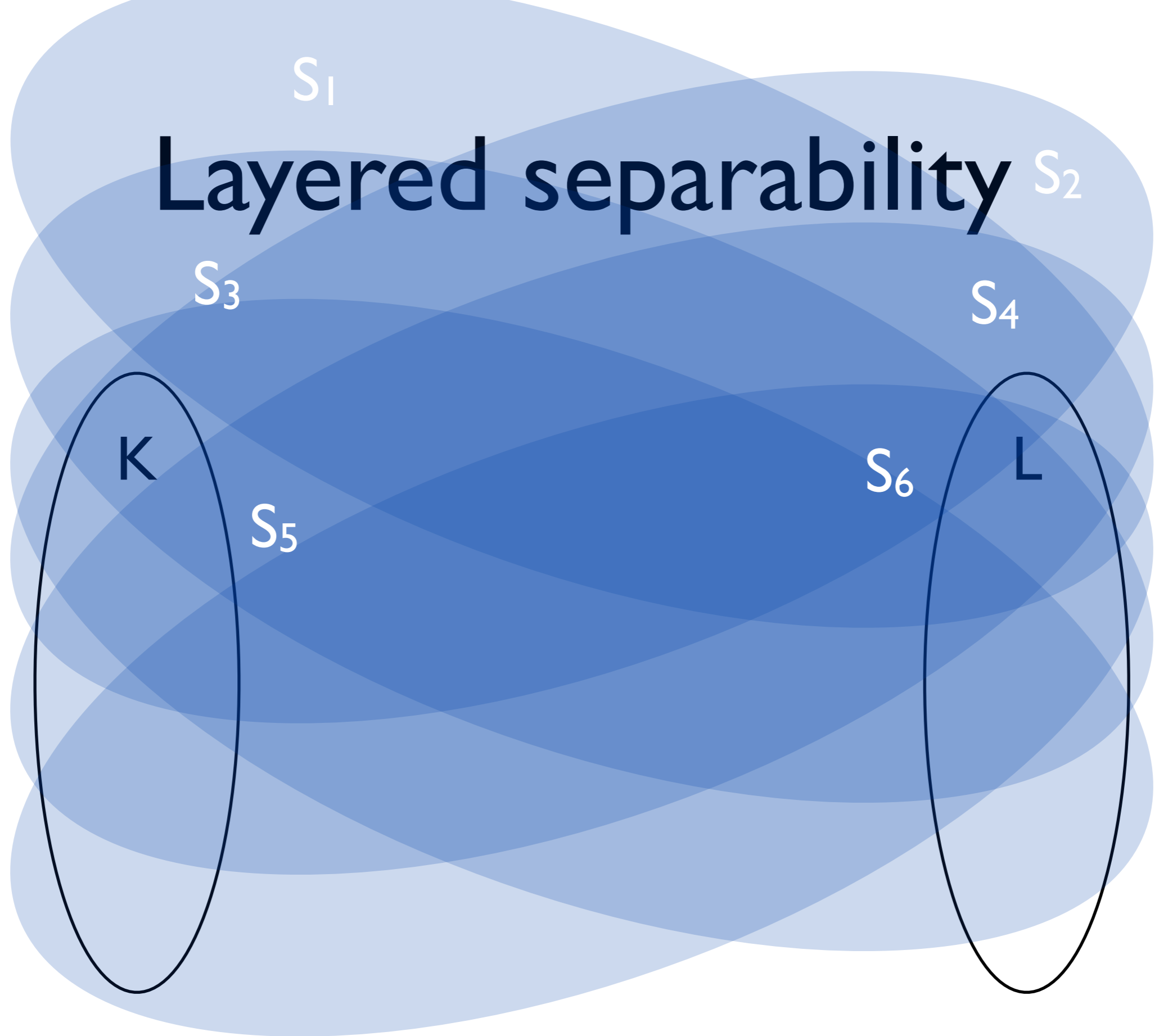
L



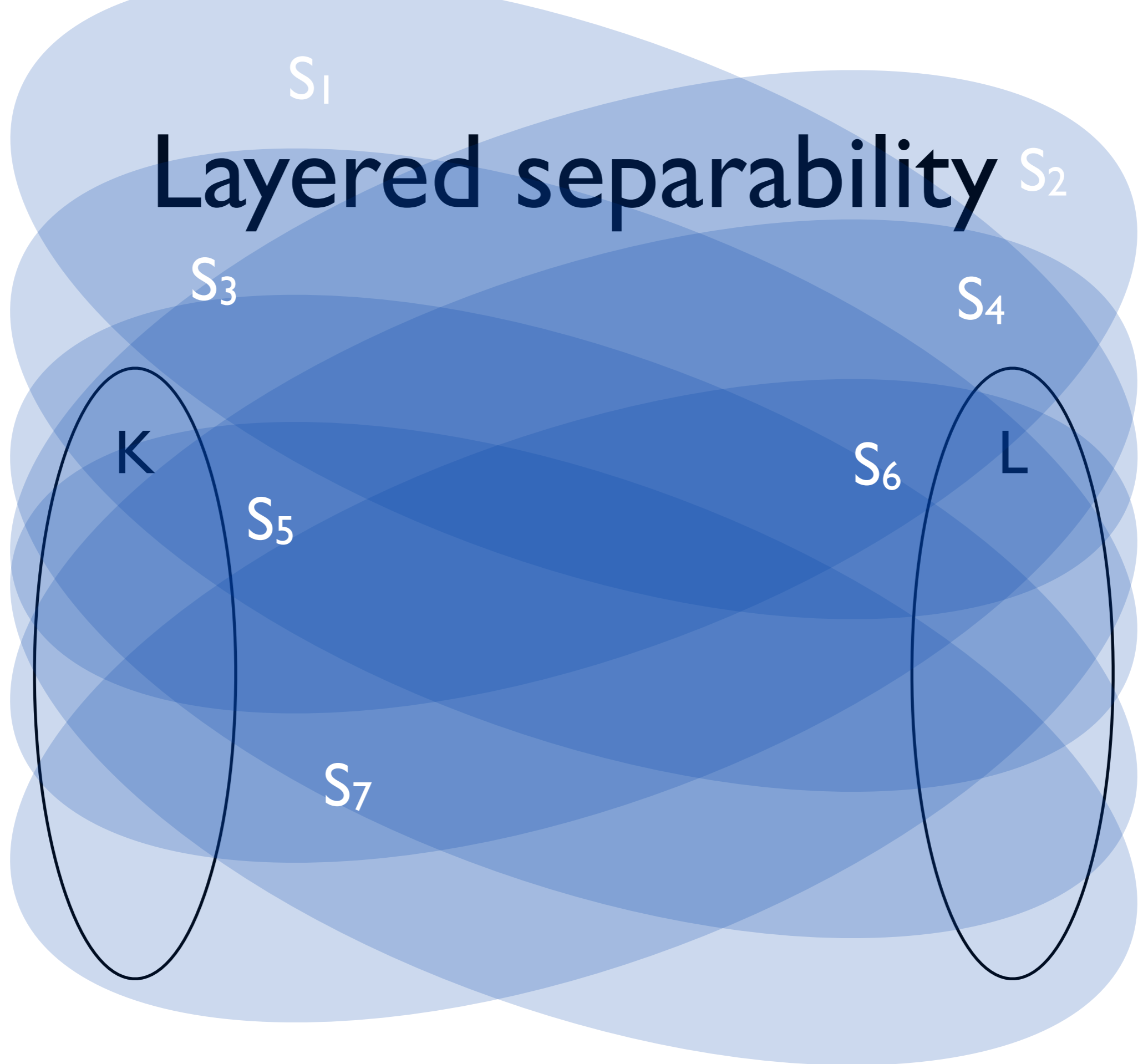
Layered separability



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Effective characterization

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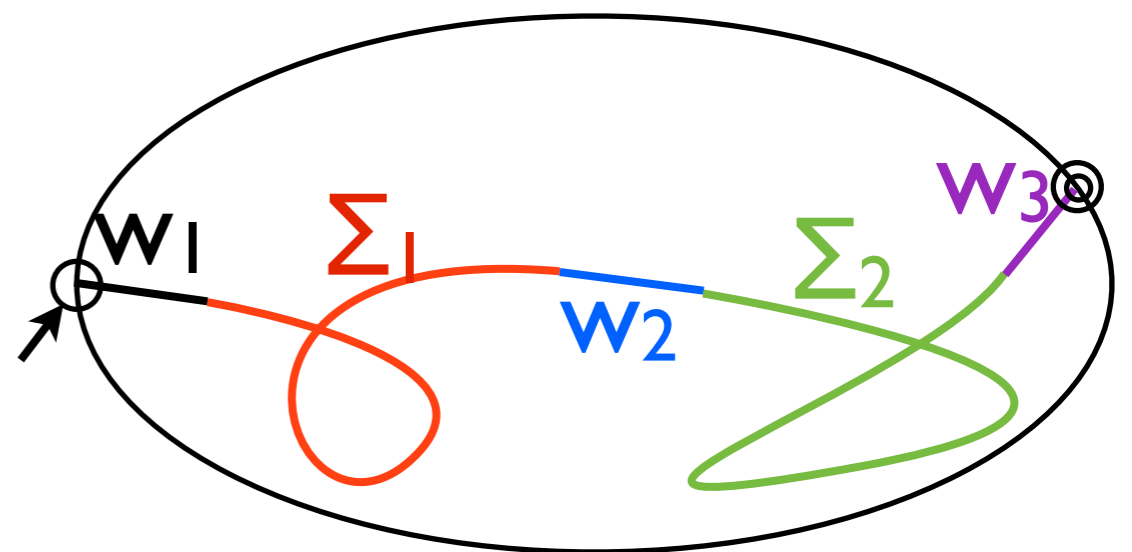
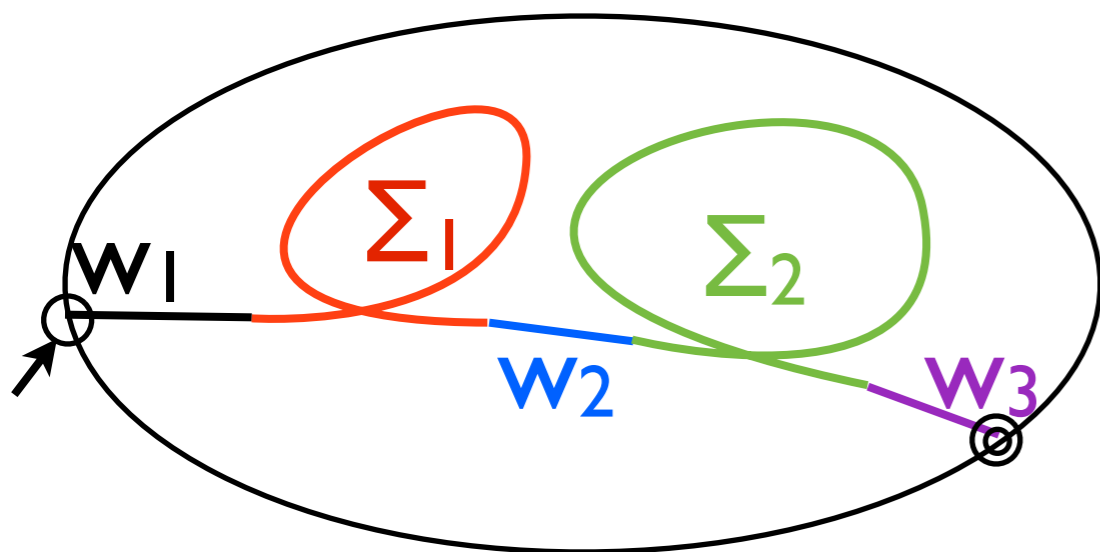
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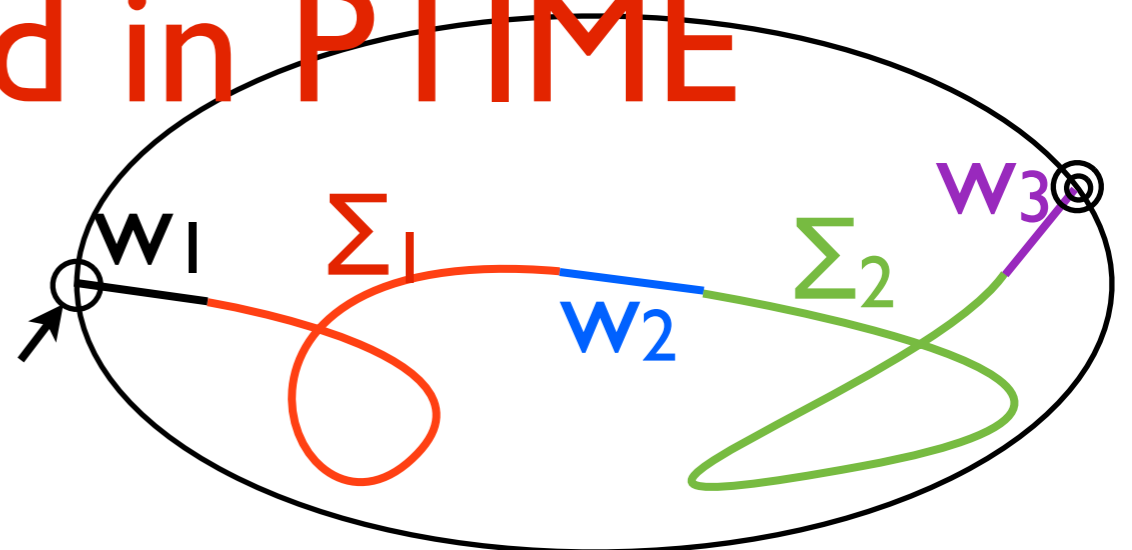
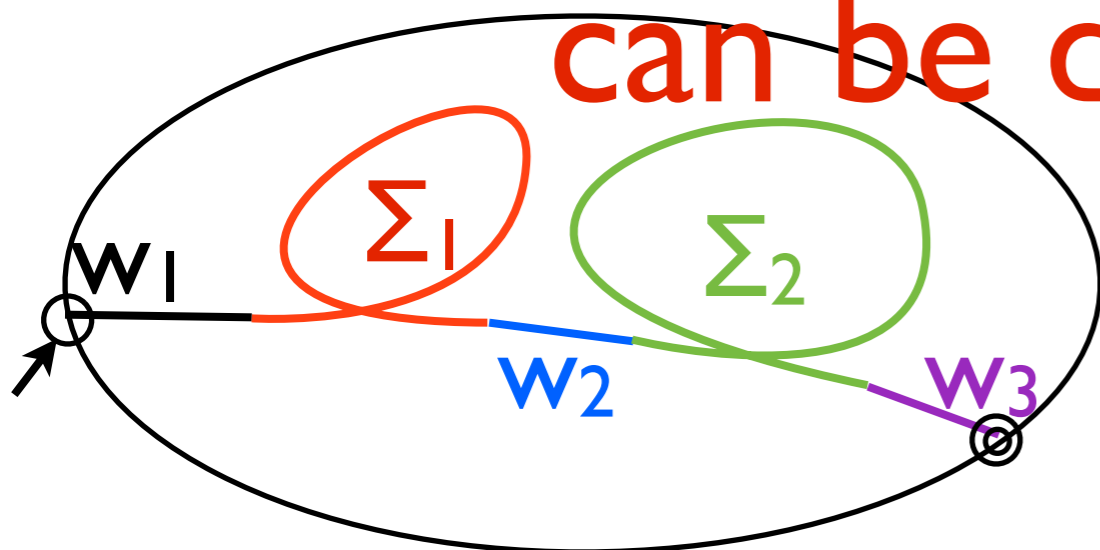


Effective characterization

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can be decided in PTIME



Other variants

Other variants

Separability Problem

Other variants

Separability Problem

Other variants

Separability Problem

pieces			

Other variants

Separability Problem

\	single		
pieces			

Other variants

Separability Problem

\	single	unions	
pieces			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			PTIME

Other variants

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	single	unions	boolean combinations
pieces		PTIME	PTIME

Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME

Other variants

Separability Problem

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pieces	NP-comp.	PTIME	PTIME
suffixes			

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$\Sigma^* w$

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Thank you!