Universality checking for unambiguous Vector Addition Systems with States

Wojciech Czerwiński

Diego Figueira

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Plan

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- basic notions

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- motivation

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- results summary

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- ExpSpace-hardness

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- ExpSpace algorithm

Unambiguity

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universality for URA (2ExpSpace)

Universality

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Emptiness not easier

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Equivalence is easier

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Inclusion is easier

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Inclusion is easier

First step: universality problem

Which system?

Which system?

Universality in unambiguous case is:

Which system?

Universality in unambiguous case is:

in NC^2 for finite automata

Which system?

Universality in unambiguous case is:

in NC^2 for finite automata

in 2ExpSpace for register automata

Which system?

Universality in unambiguous case is:

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not investigated for:

Which system?

Universality in unambiguous case is:

in NC^2 for finite automata

in 2ExpSpace for register automata

not investigated for:

pushdown automata

Which system?

Universality in unambiguous case is:

in NC^2 for finite automata

in 2ExpSpace for register automata

not investigated for:

pushdown automata

counter automata

What was known

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Universality is:

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- decidable for OCN and VASS (wqo)

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Acceptance by states, ε-transitions allowed

Results

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1) VASS is ExpSpace-complete

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3) 1-VASS is coNP-hard, binary

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The universality problem in unambiguous case for

1) VASS is ExpSpace-complete

2) d-VASS is PSpace-complete for $d \geq 2$, binary

3) 1-VASS is coNP-hard, binary

4) d-VASS is in NC^2, NL-hard, $d \geq 1$, unary

ExpSpace-hardness

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B_1 accepts words of length < N

B_2 accepts words of length ≥ N

ExpSpace algorithm

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Main idea: values bigger than doubly exponential do not matter

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For uVASS A construct 2exp size UFA B such that

A universal $\iff$ B universal

ExpSpace algorithm

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For uVASS A construct 2exp size UFA B such that A universal $\Longleftrightarrow$ B universal

Use NC^2 algorithm to check universality of B

Profiles

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N-profile of a vector: value x become $\min(x,N)$

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Lemma 2: For any universal uVASS there is 2exp-size number N such that any two reachable configurations of the same N-profile have the same set of accepting runs.

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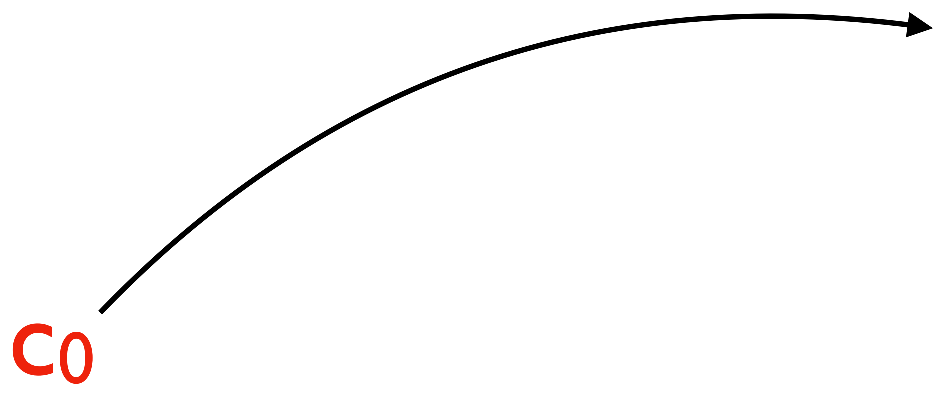

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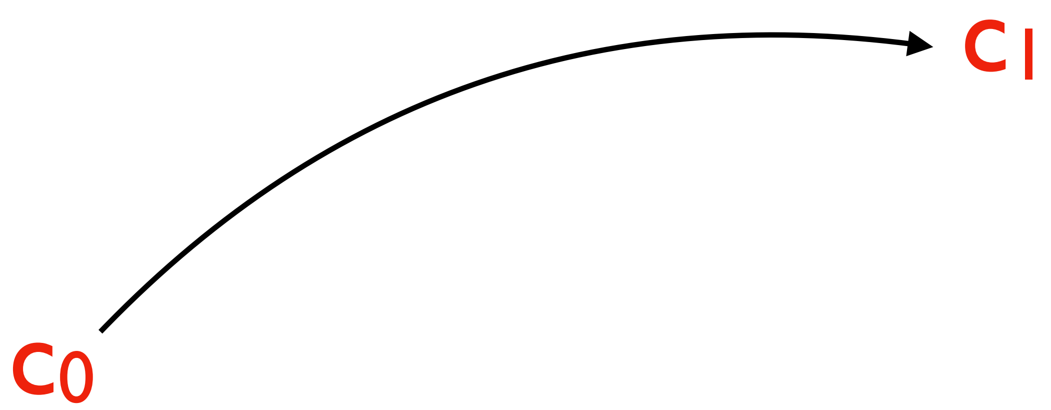

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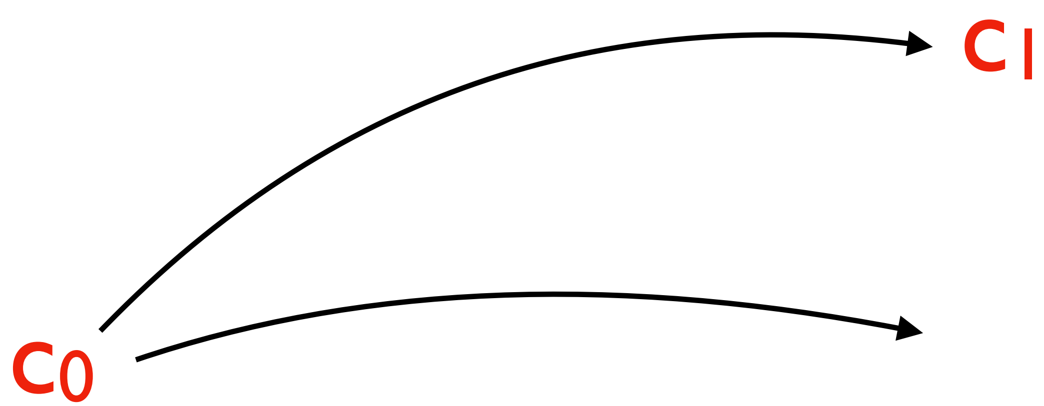

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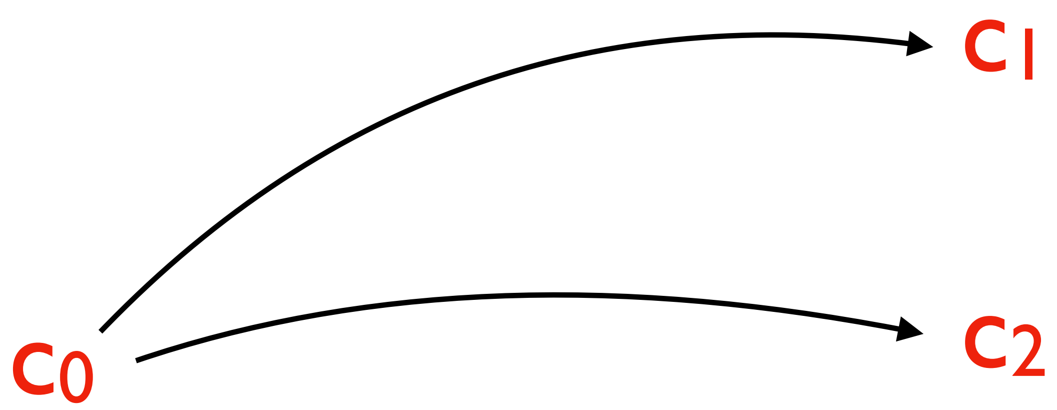

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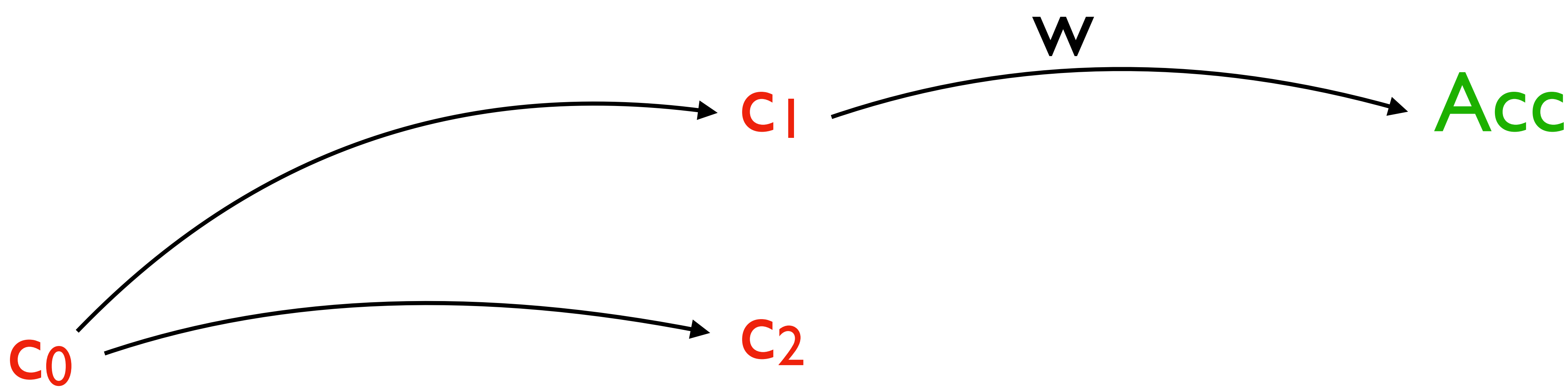

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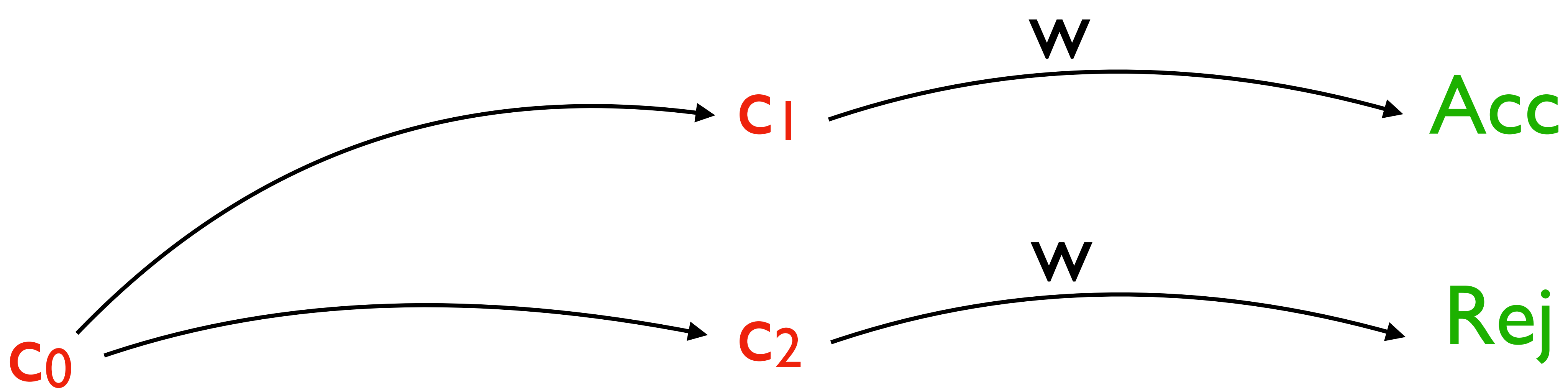

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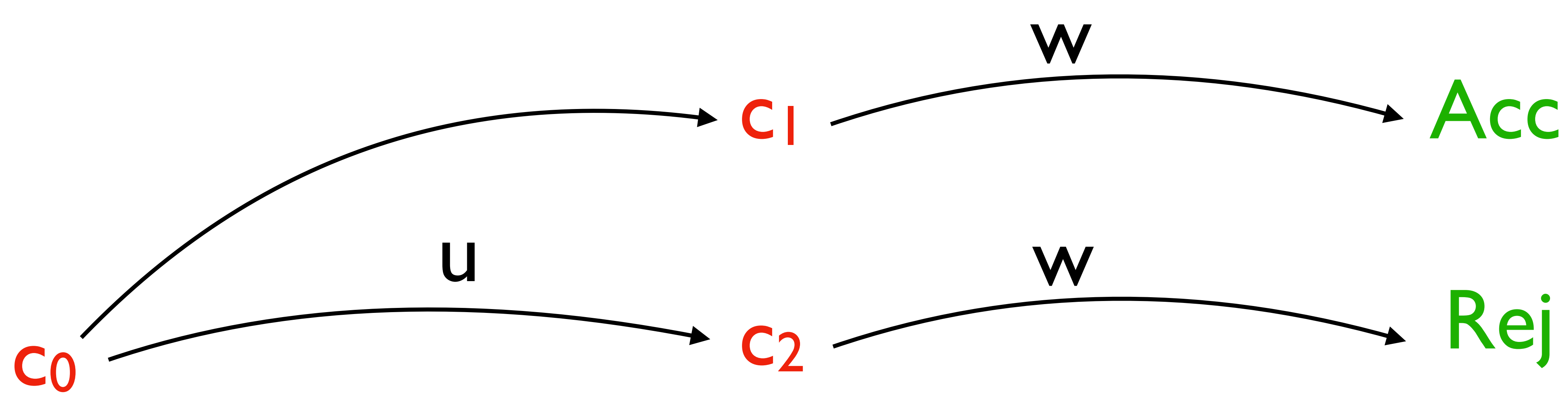

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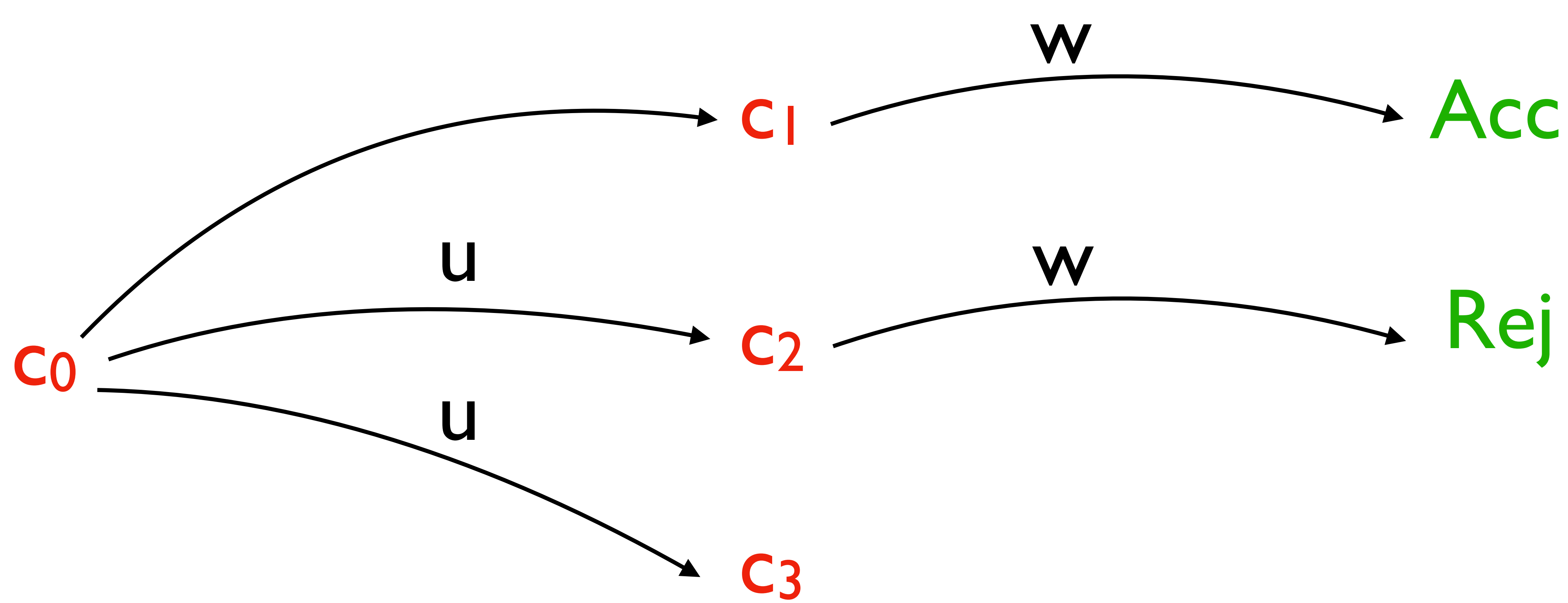

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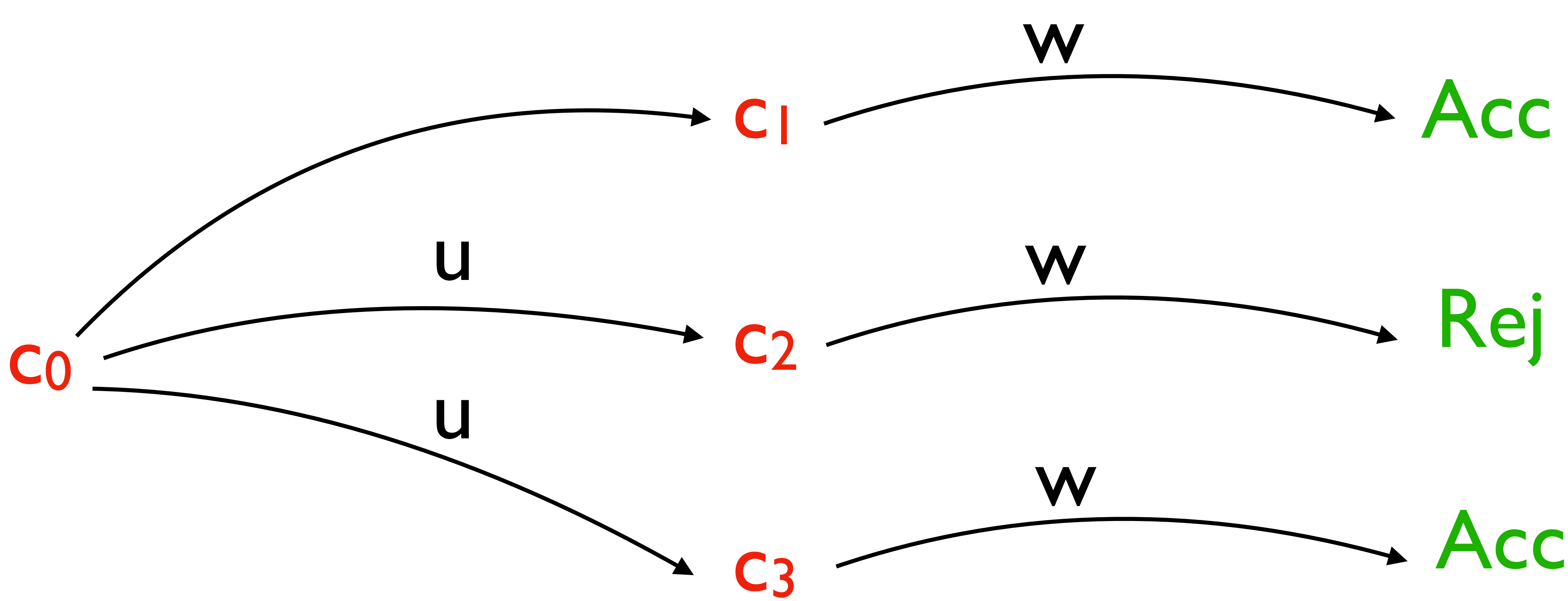

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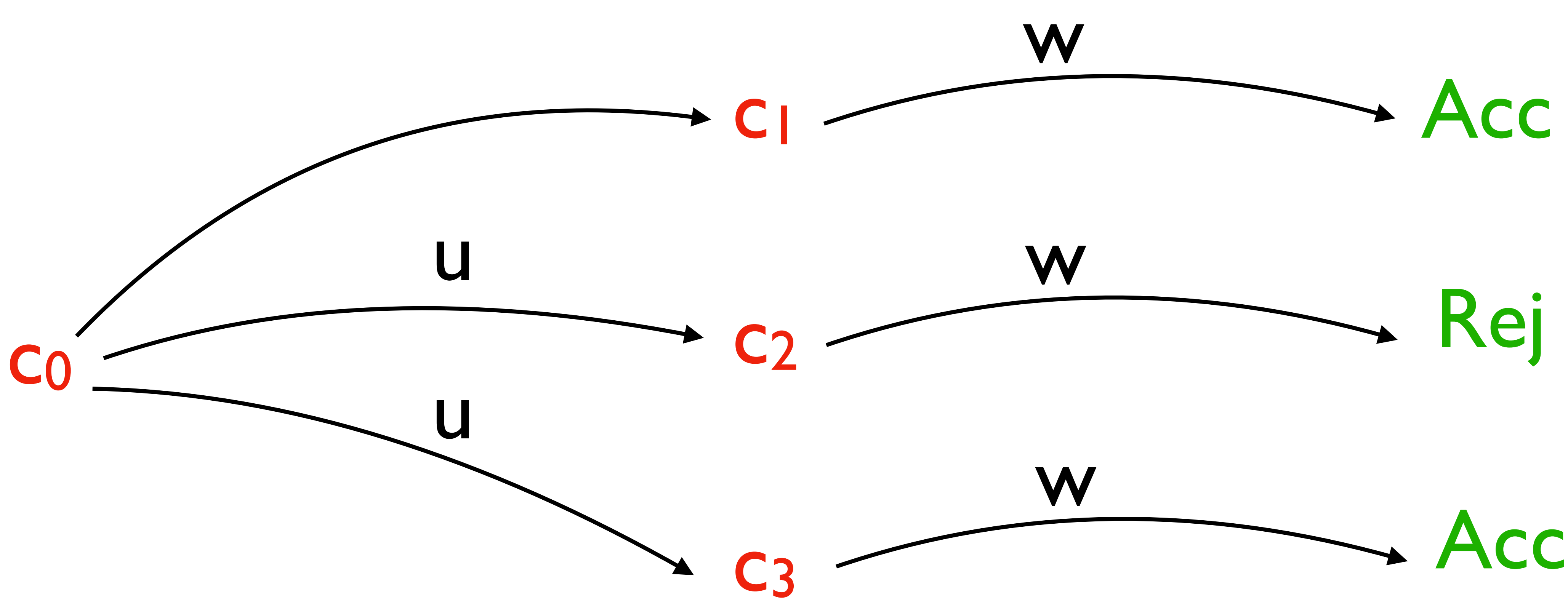

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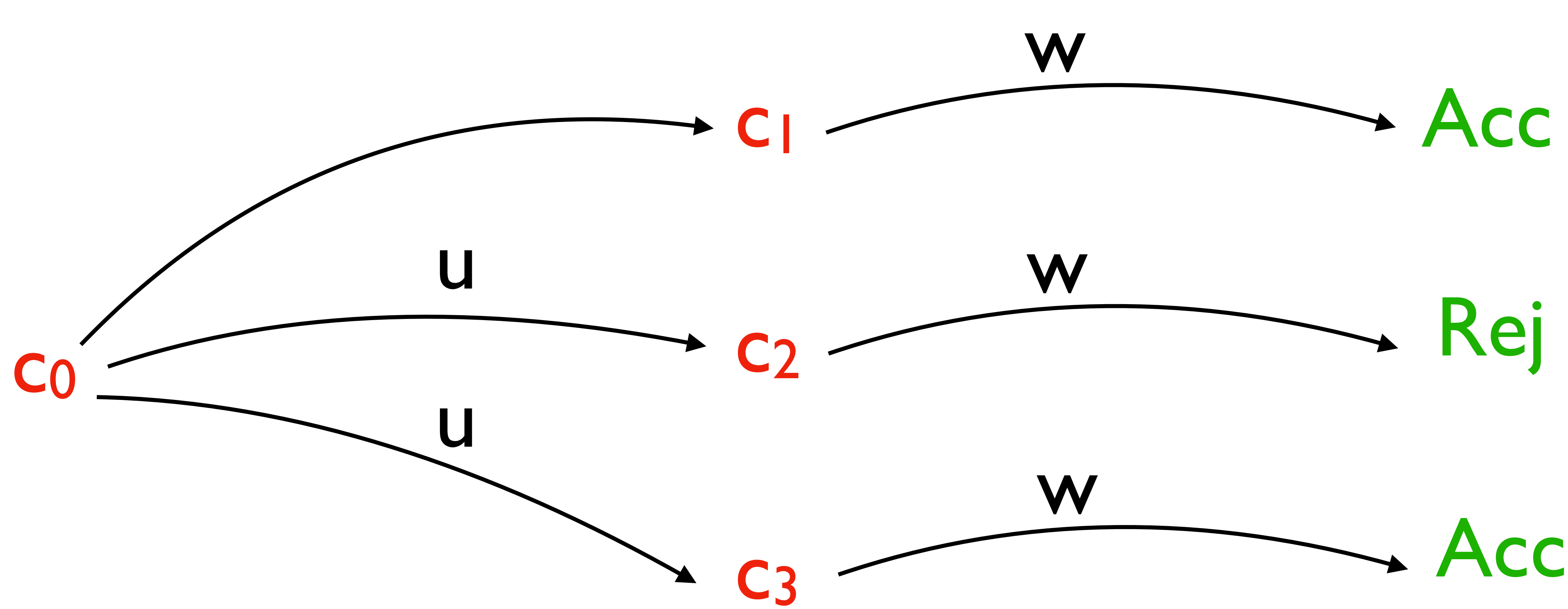

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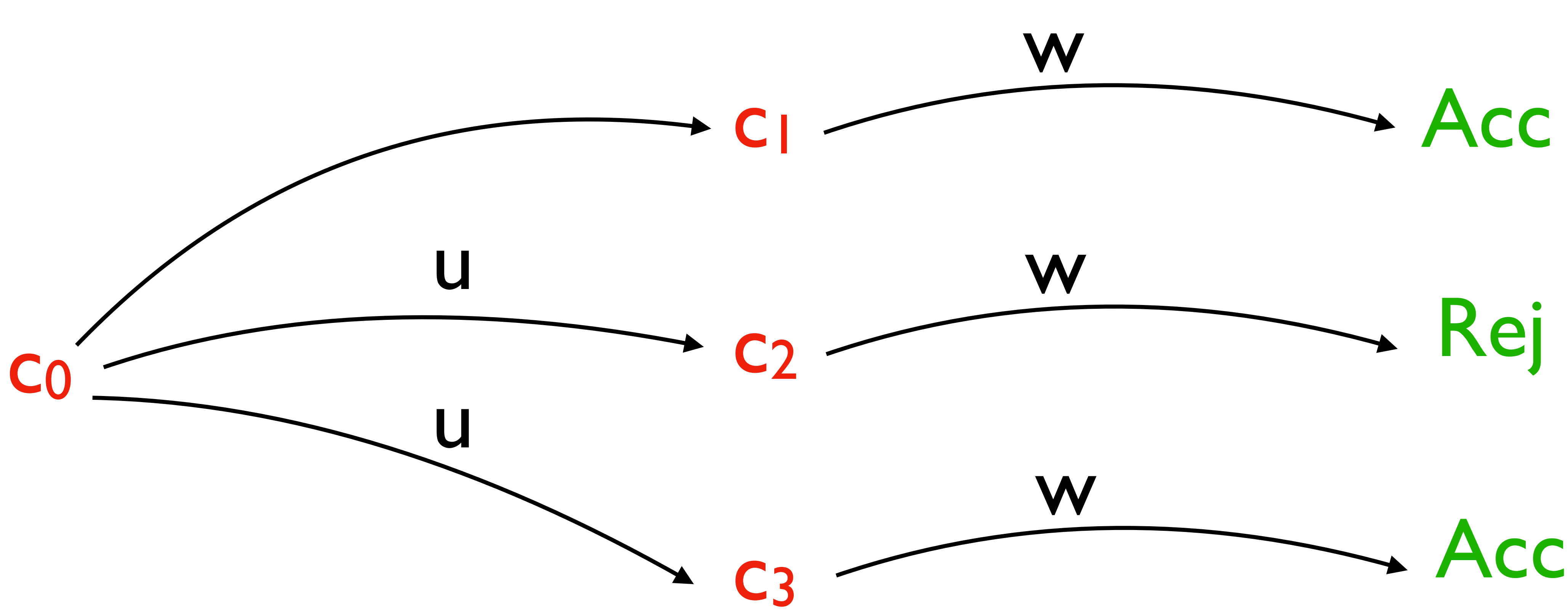

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two runs for uw'

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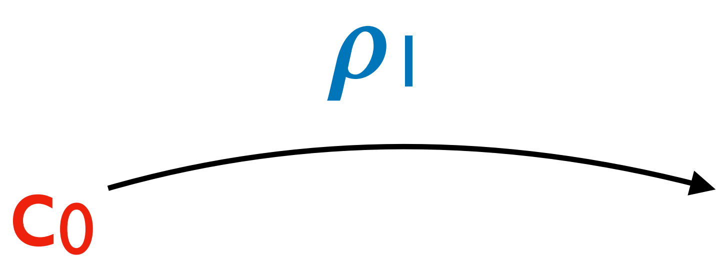

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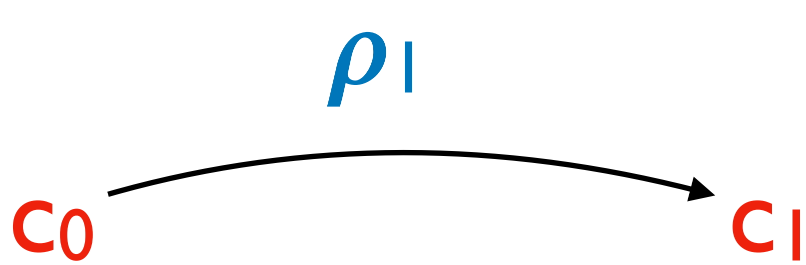

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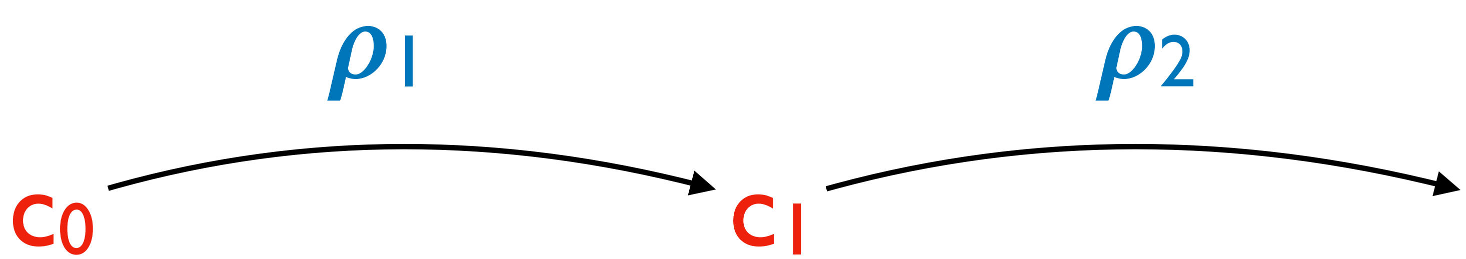

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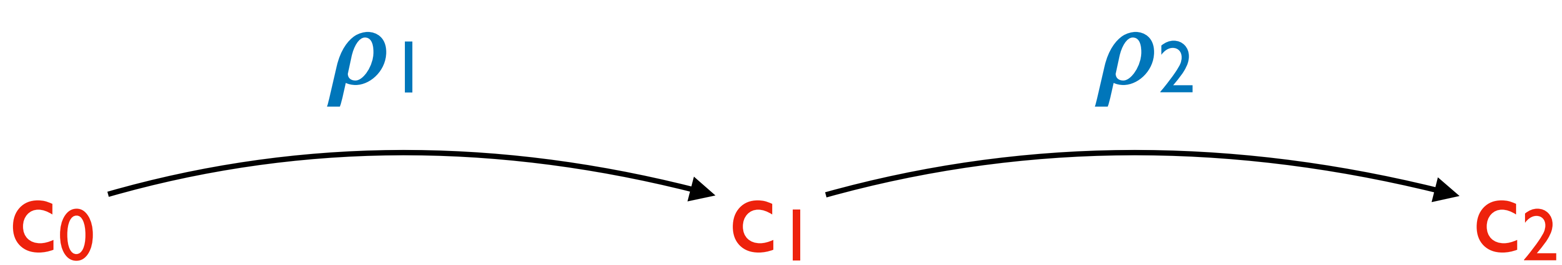

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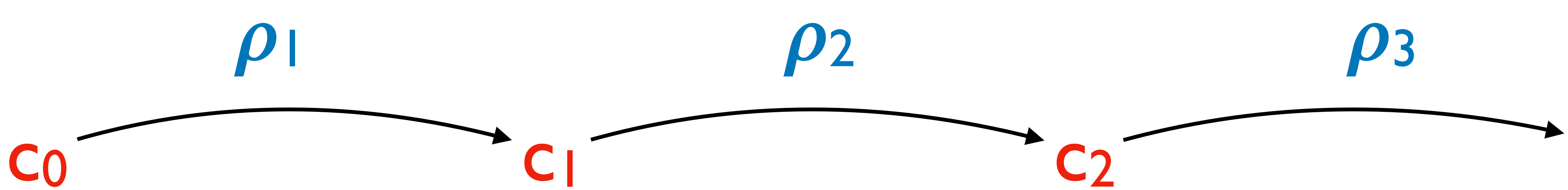

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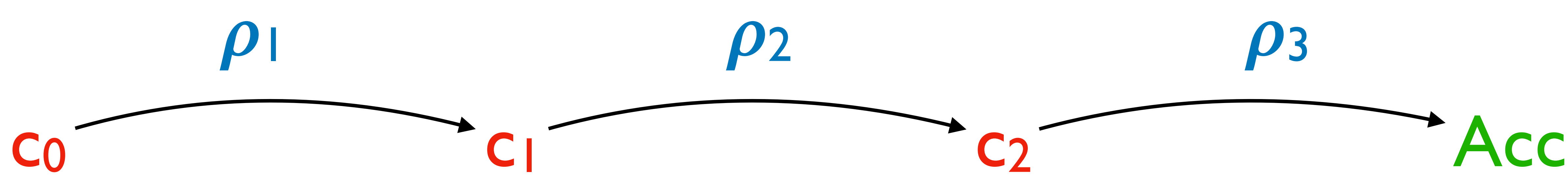

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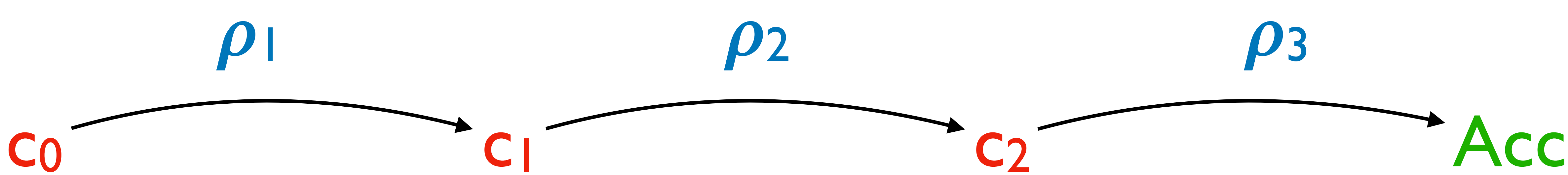

$(\rho_2)^n\rho_3$ is accepting from c_1 but not from c_2 for some n

Finite automaton

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A is universal $\iff$ A_N is universal

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Assume L(A) is universal.

Take any w in L(A). Corresponding run of A_N is invalid only if it first reaches N and then 0. Such a drop contradicts Lemma 3.

ExpSpace algorithm

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Lemma 4: For any uVASS A there is a 2exp-size N such that for the UFA A_N

A is universal $\Longleftrightarrow$ A_N is universal

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Producing A_N is in ExpSpace

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Composition is in ExpSpace

Fixed dimension

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For fixed dimension d size of A_N is exponential

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If additionally encoding is unary
then size of A_N is polynomial

Open problems

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equivalence, inclusion, co-finiteness problems

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OCN, VASS, counter automata
pushdown-automata, RA

Thank you!