Recent Advances on the Reachability Problem for VASSes by Examples

Wojciech Czerwiński

RP 2022

Plan

Plan

- basic notions

Plan

- basic notions
- short history

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- basic notions
- short history
- big reachability sets

Plan

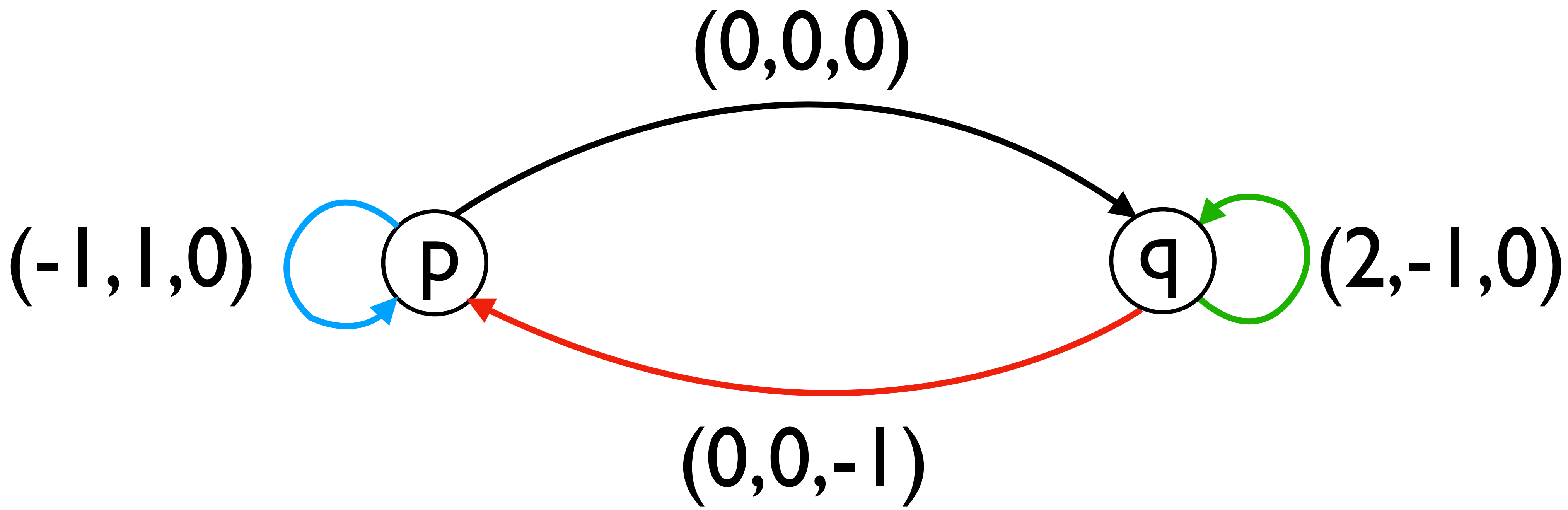
- basic notions
- short history
- big reachability sets
- zero-testing technique

Plan

- basic notions
- short history
- big reachability sets
- zero-testing technique
- pushdown VASS

Vector Addition Systems with States

Vector Addition Systems with States



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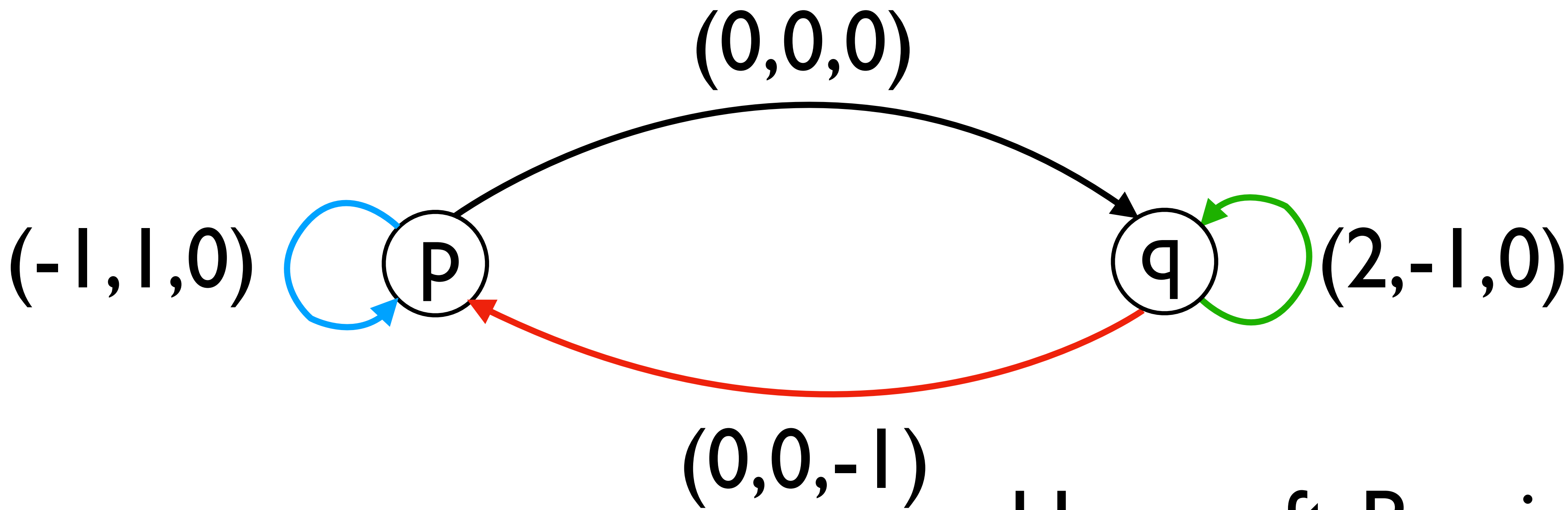
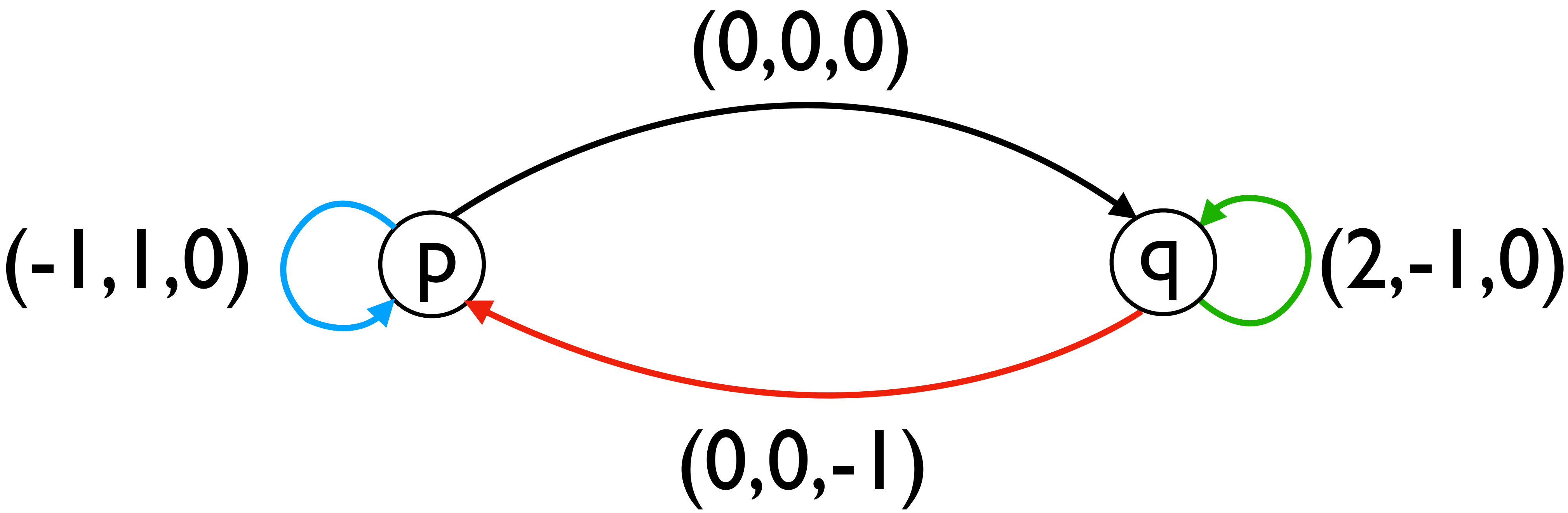


Hopcroft-Pansiot `78

Vector Addition Systems with States



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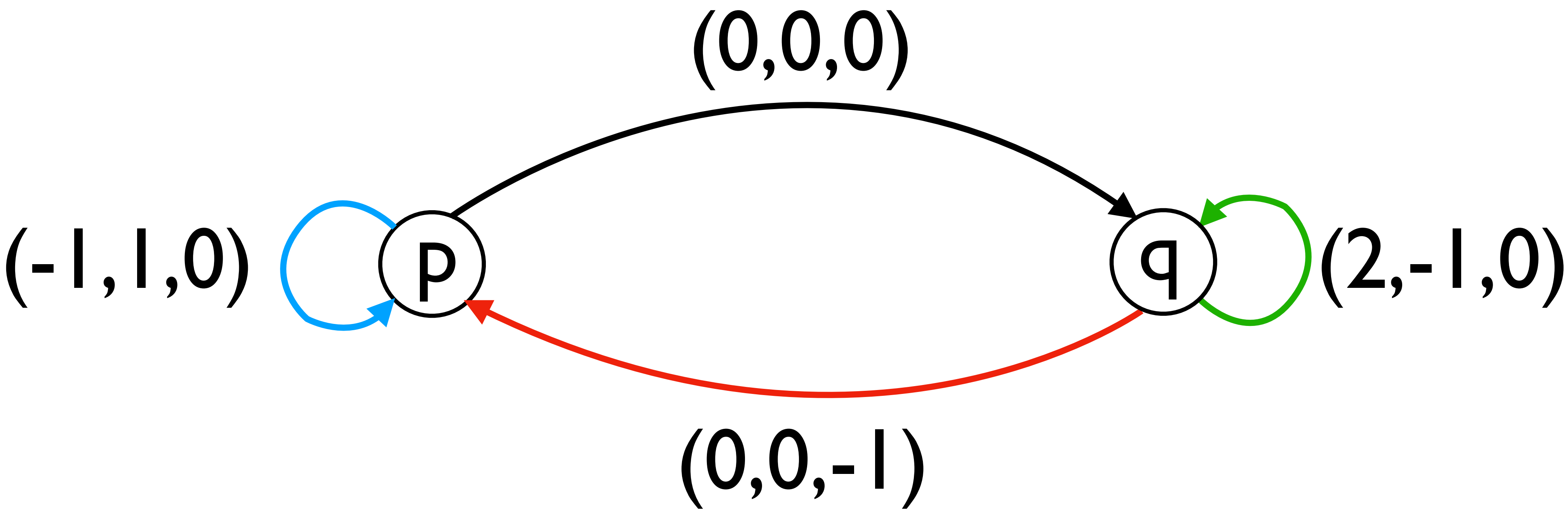
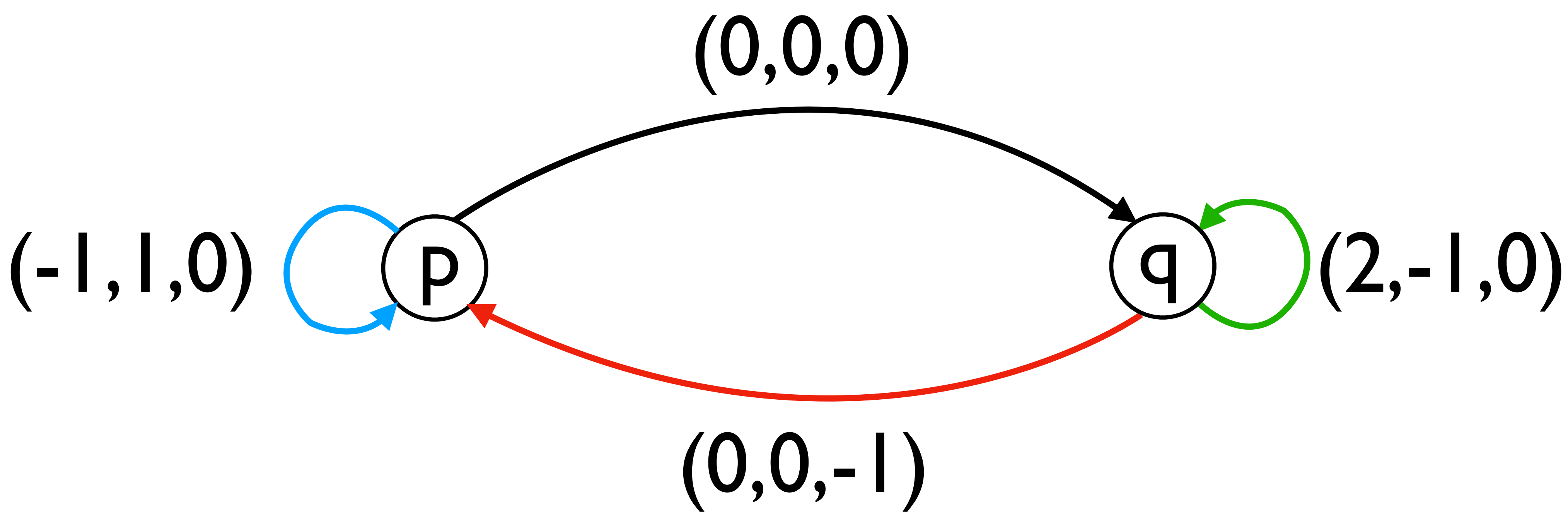


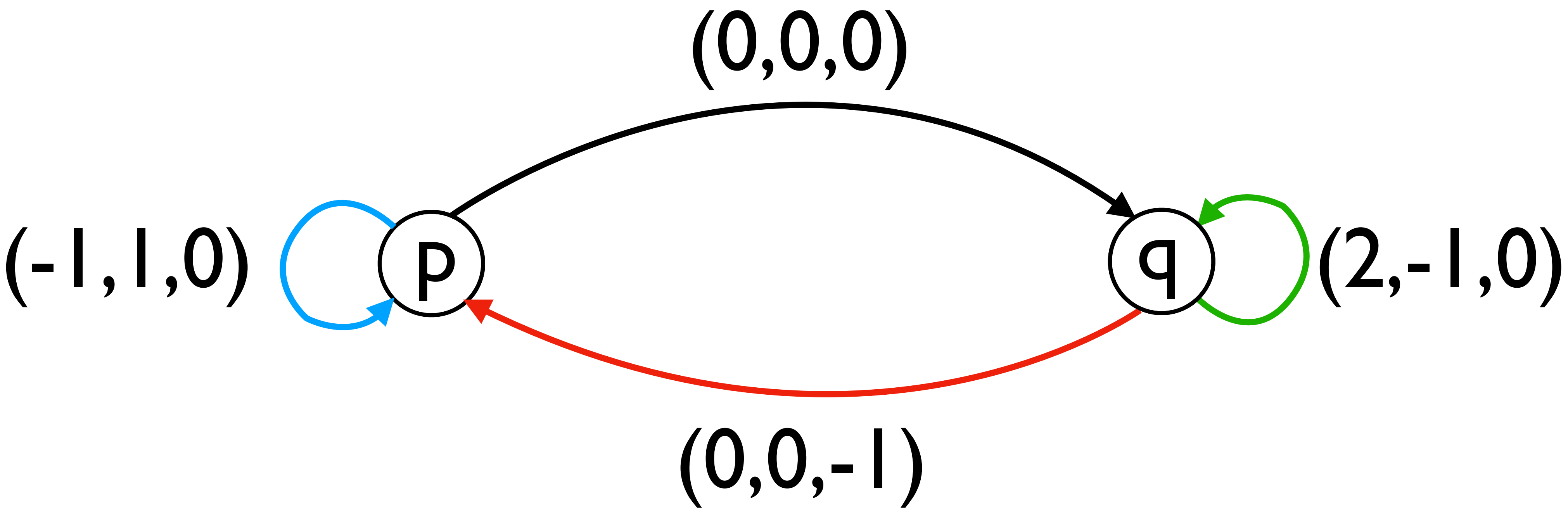
p(2,0,7)

Vector Addition Systems with States



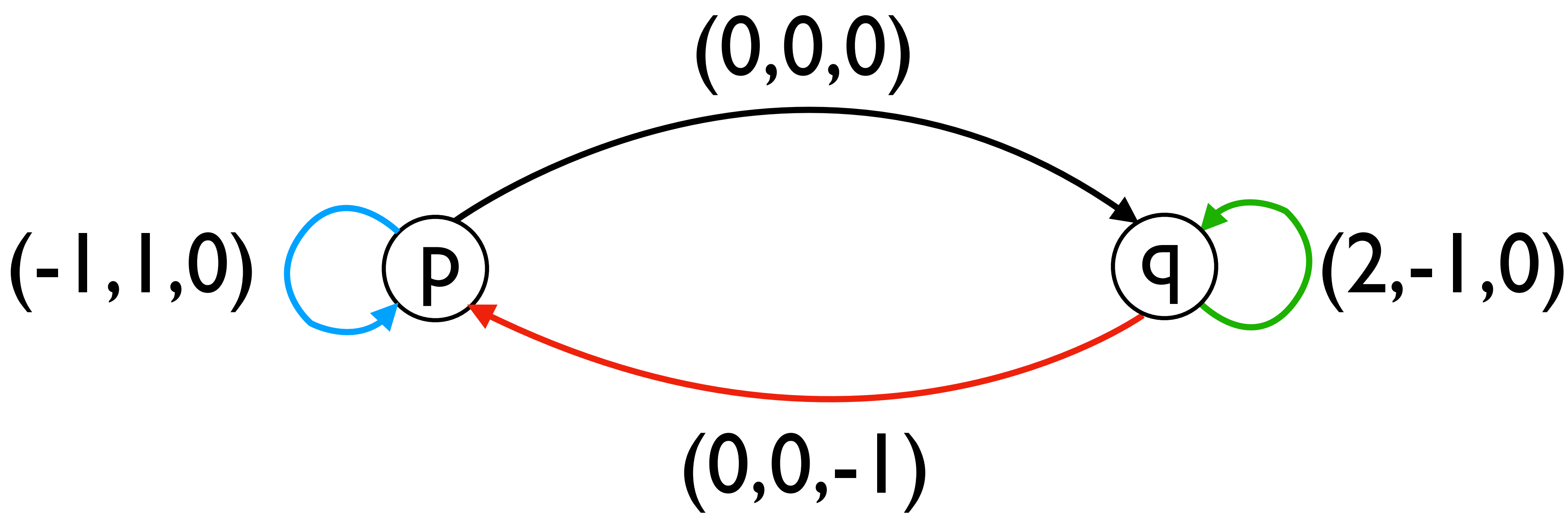
p(2,0,7) ⟶ p(1,1,7)

Vector Addition Systems with States



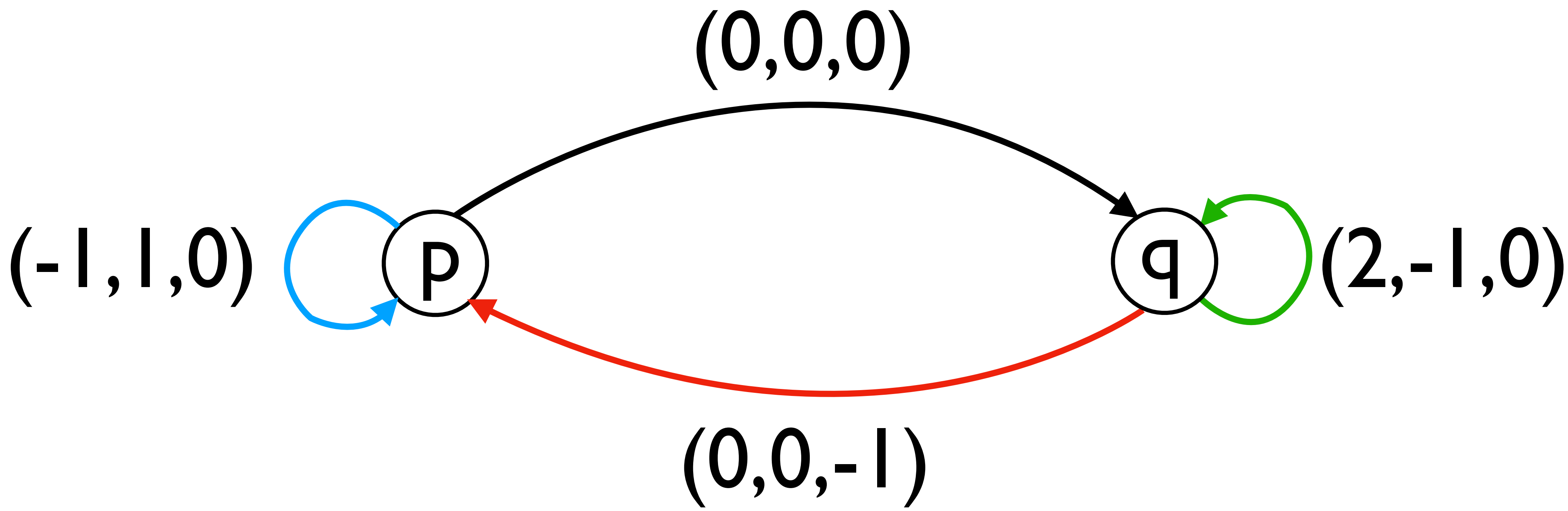
p(2,0,7) → p(1,1,7) → p(0,2,7)

Vector Addition Systems with States



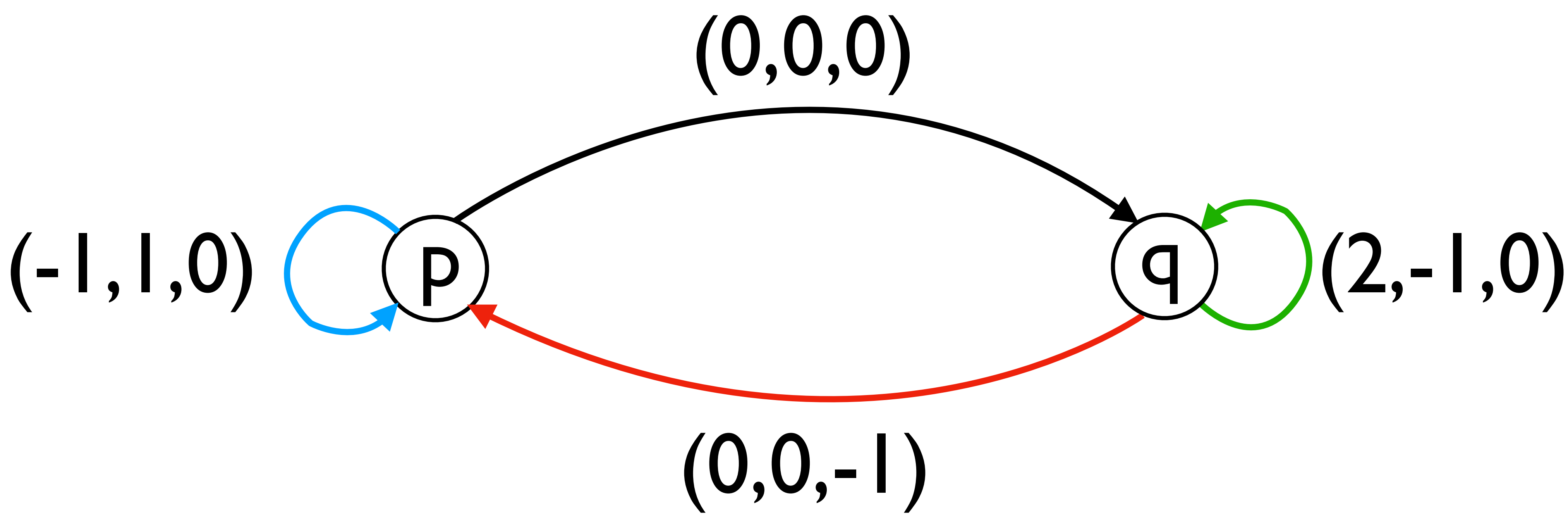
p(2,0,7) → p(1,1,7) → p(0,2,7) → q(0,2,7)

Vector Addition Systems with States



p(2,0,7) → p(1,1,7) → p(0,2,7) → q(0,2,7) → q(2,1,7)

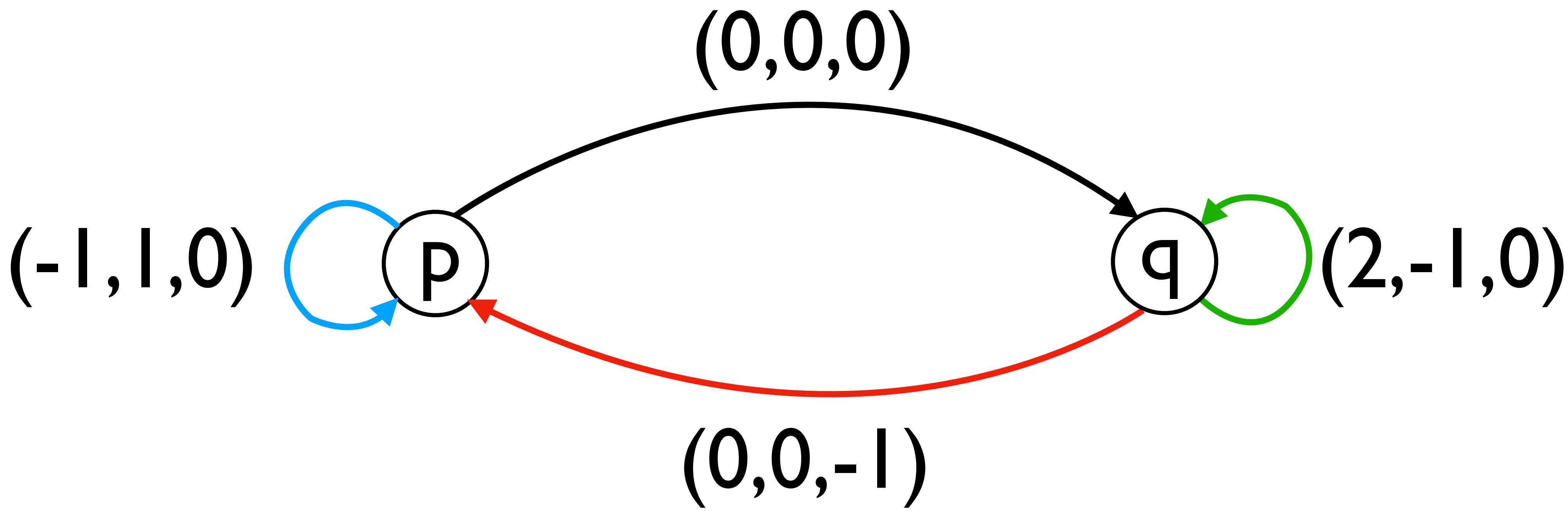
Vector Addition Systems with States



p(2,0,7) → p(1,1,7) → p(0,2,7) → q(0,2,7) → q(2,1,7)

→ q(4,0,7)

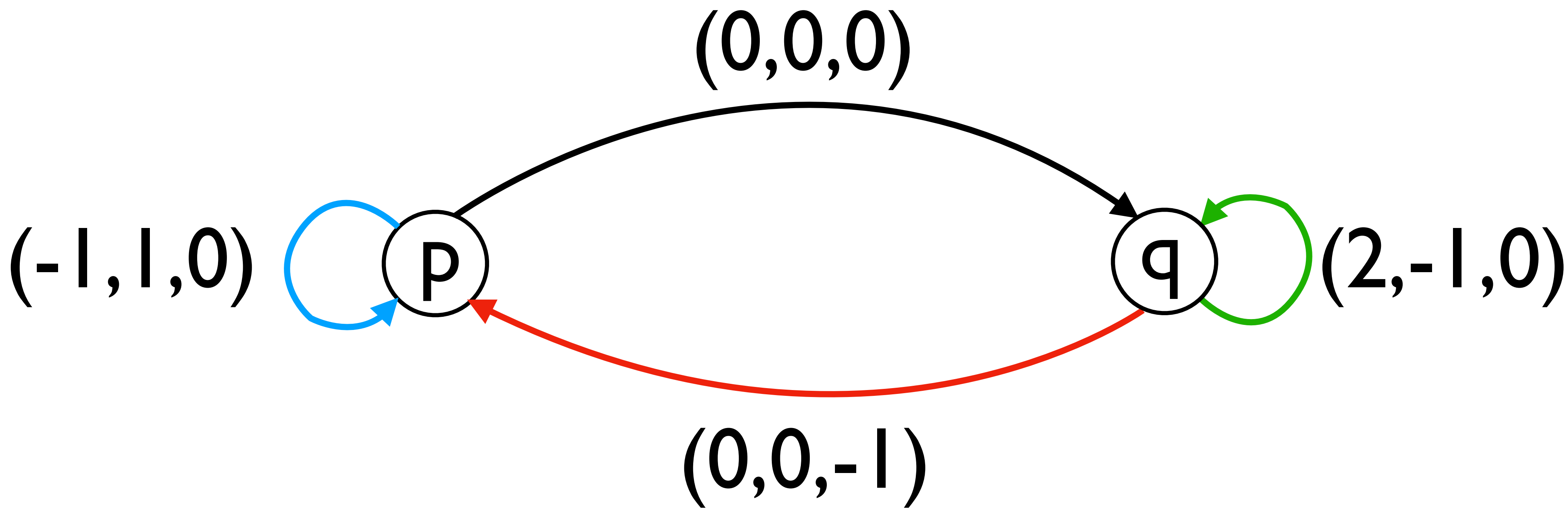
Vector Addition Systems with States



$p(2,0,7) \rightarrow p(1,1,7) \rightarrow p(0,2,7) \rightarrow q(0,2,7) \rightarrow q(2,1,7)$

$\rightarrow q(4,0,7) \rightarrow p(4,0,6)$

Vector Addition Systems with States

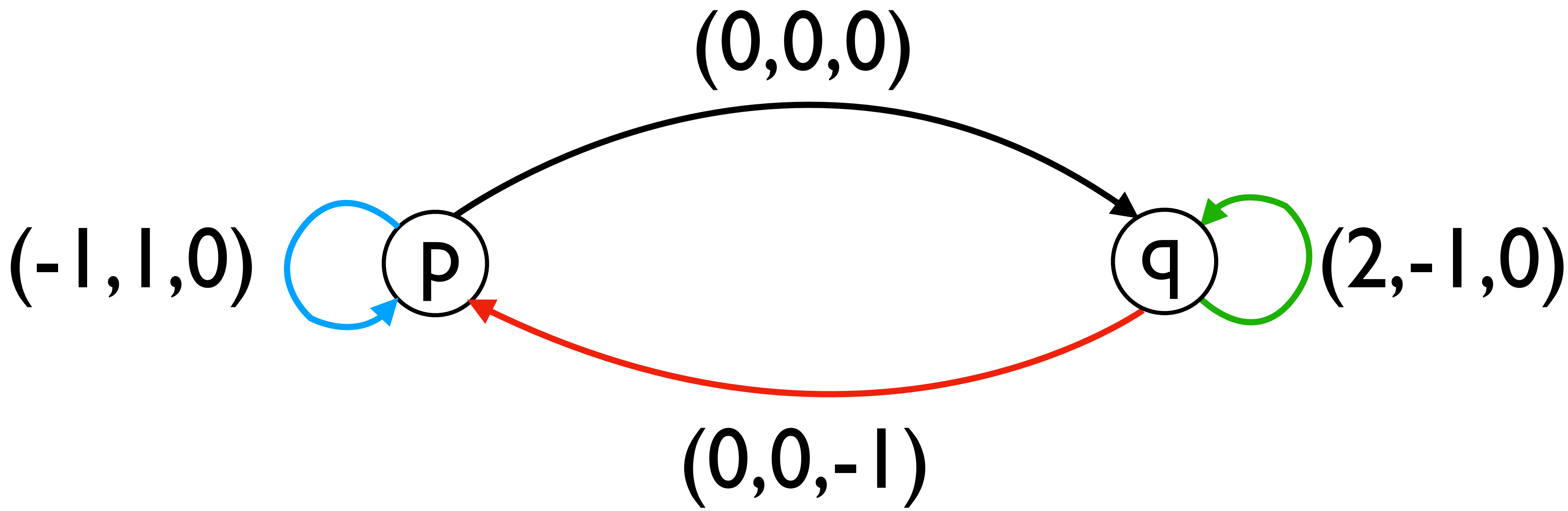


p(2,0,7) → p(1,1,7) → p(0,2,7) → q(0,2,7) → q(2,1,7)

→ q(4,0,7) → p(4,0,6)

Petri nets

Vector Addition Systems with States



p(2,0,7) → p(1,1,7) → p(0,2,7) → q(0,2,7) → q(2,1,7)

→ q(4,0,7) → p(4,0,6)

Vector Addition Systems with States

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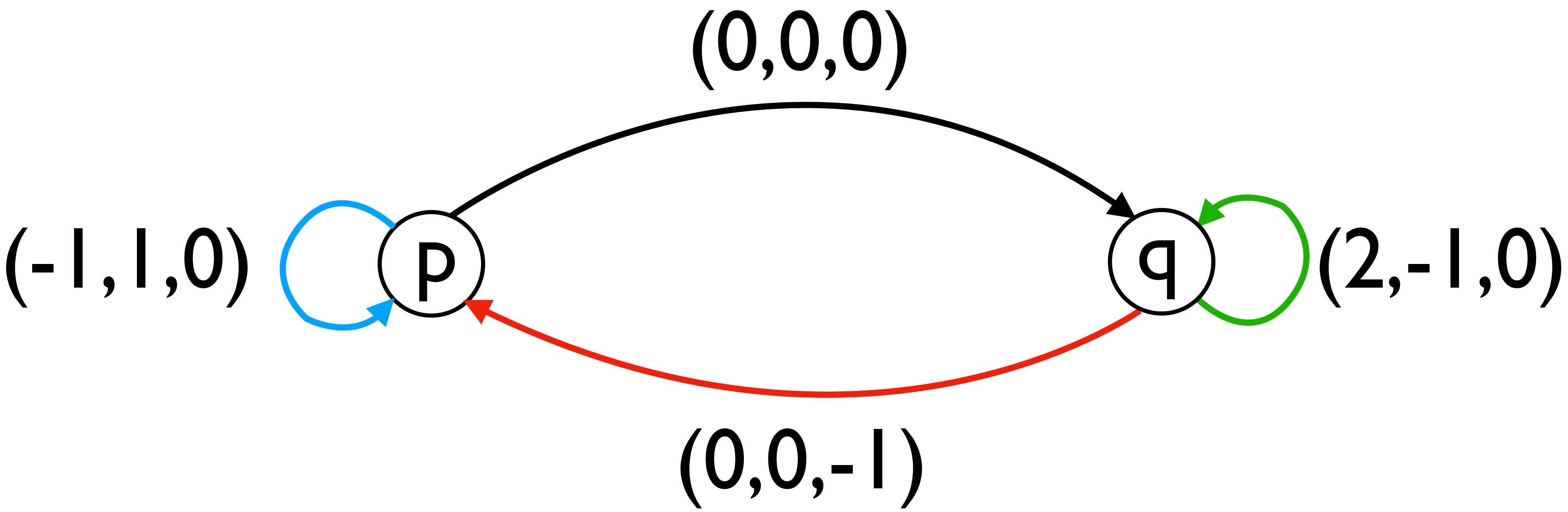

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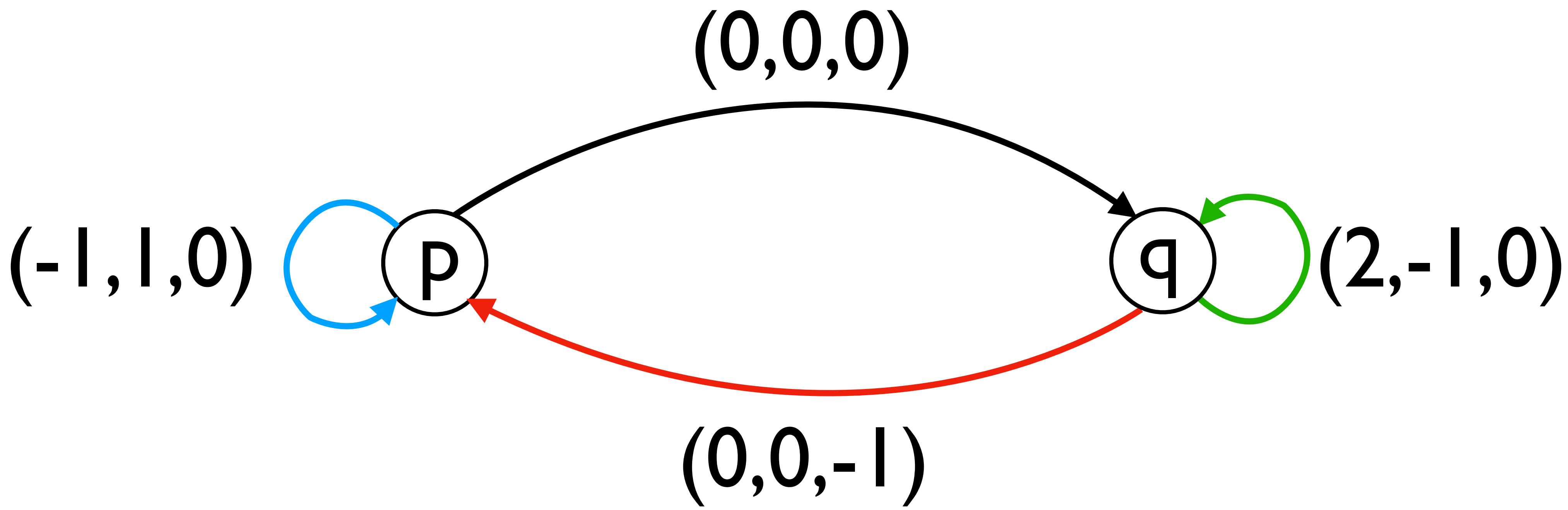
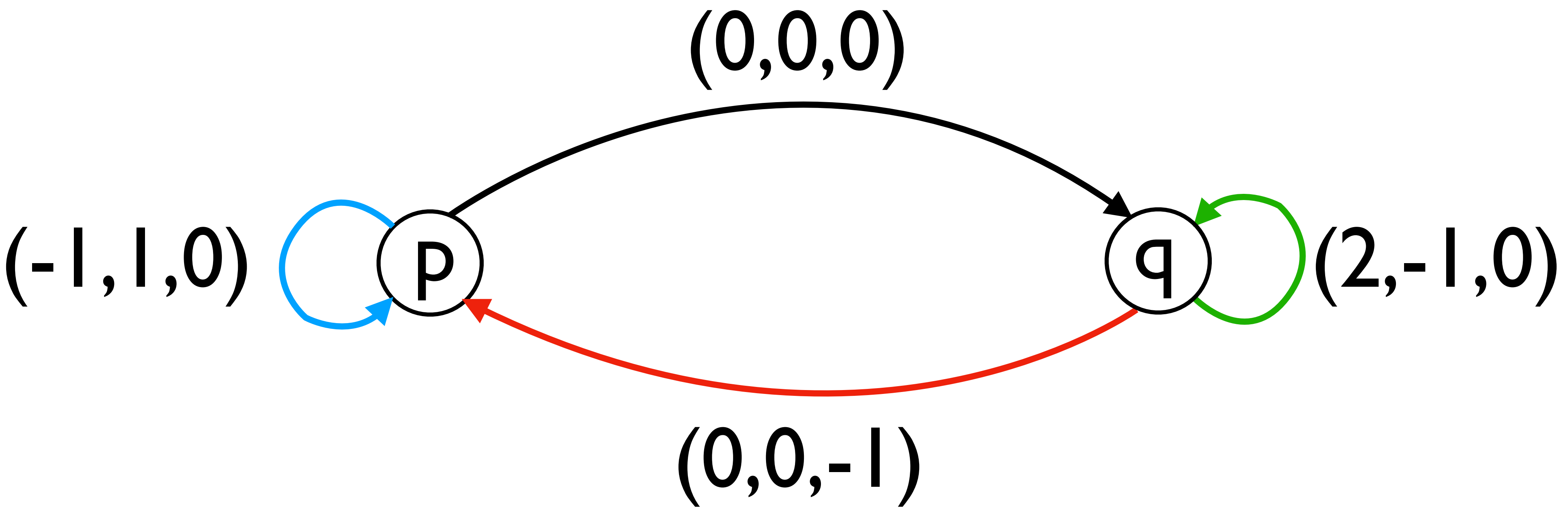


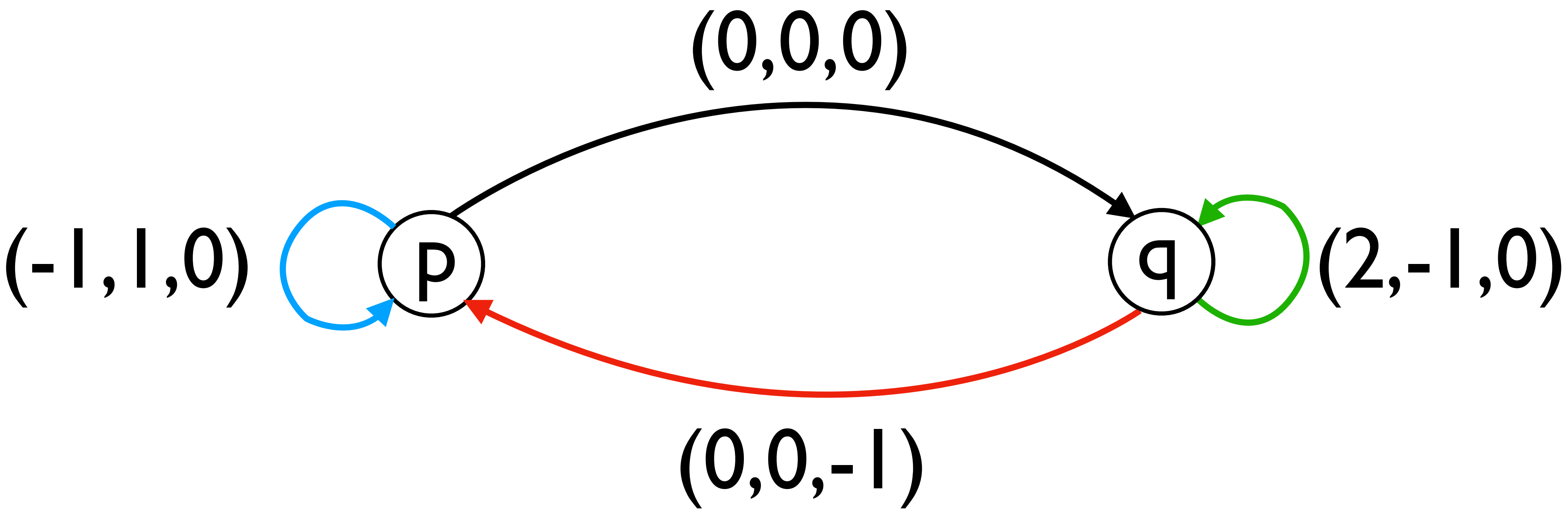
p(k,0,n)

Vector Addition Systems with States



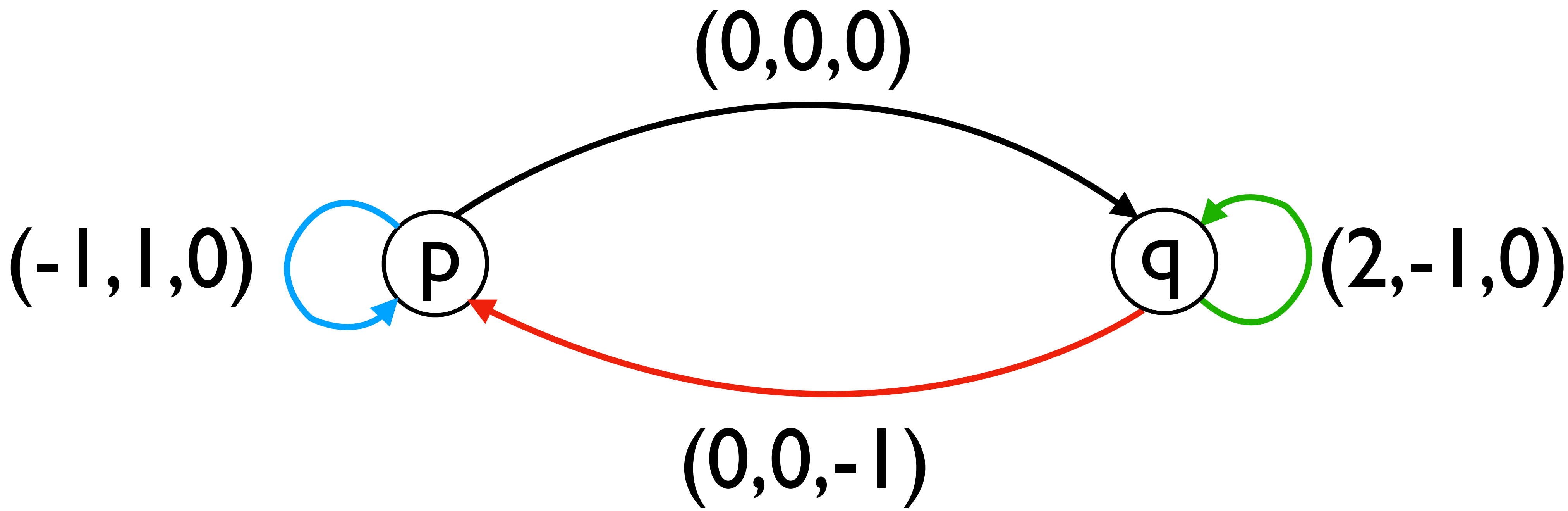
p(k,0,n) → p(0,k,n)

Vector Addition Systems with States



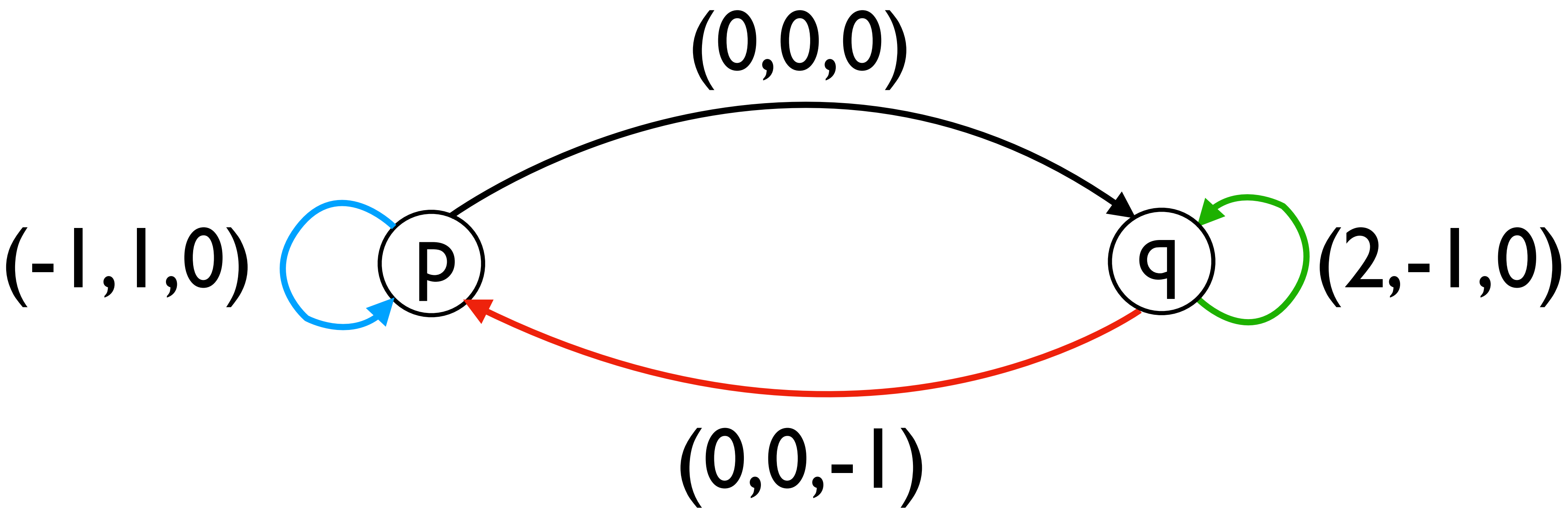
p(k,0,n) ⟶ p(0,k,n) ⟶ q(0,k,n)

Vector Addition Systems with States



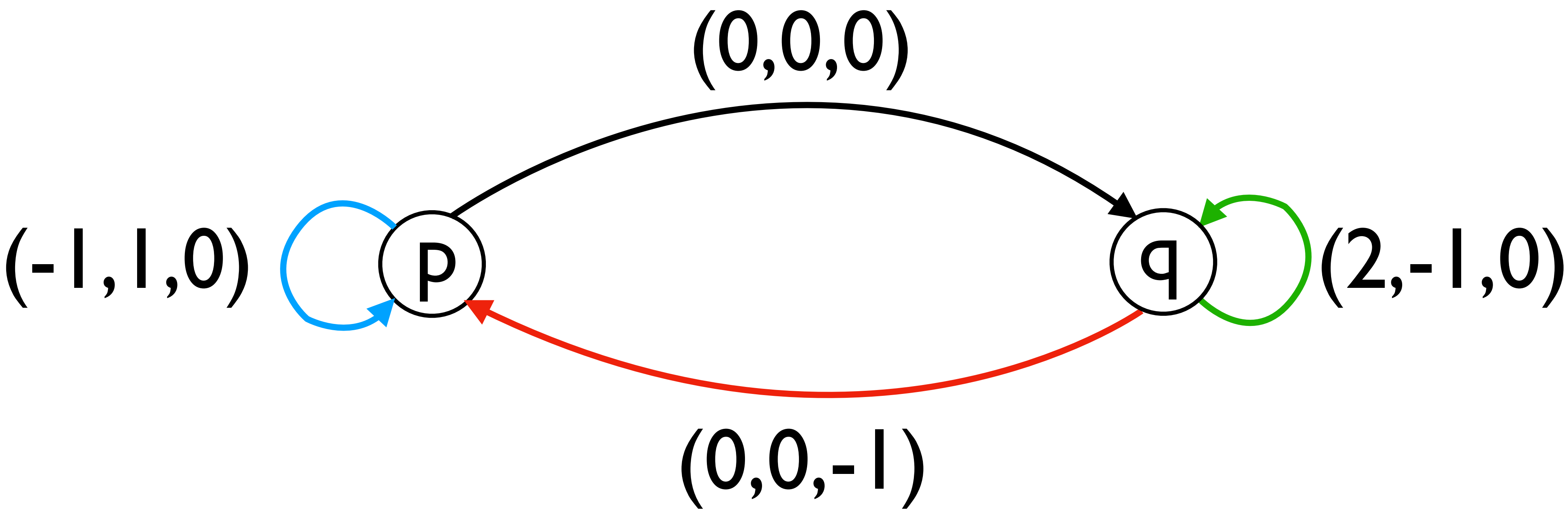
p(k,0,n) ⟶ p(0,k,n) ⟶ q(0,k,n) ⟶ q(2k,0,n)

Vector Addition Systems with States



p(k,0,n) ⟶ p(0,k,n) ⟶ q(0,k,n) ⟶ q(2k,0,n) ⟶ p(2k,0,n-1)

Vector Addition Systems with States



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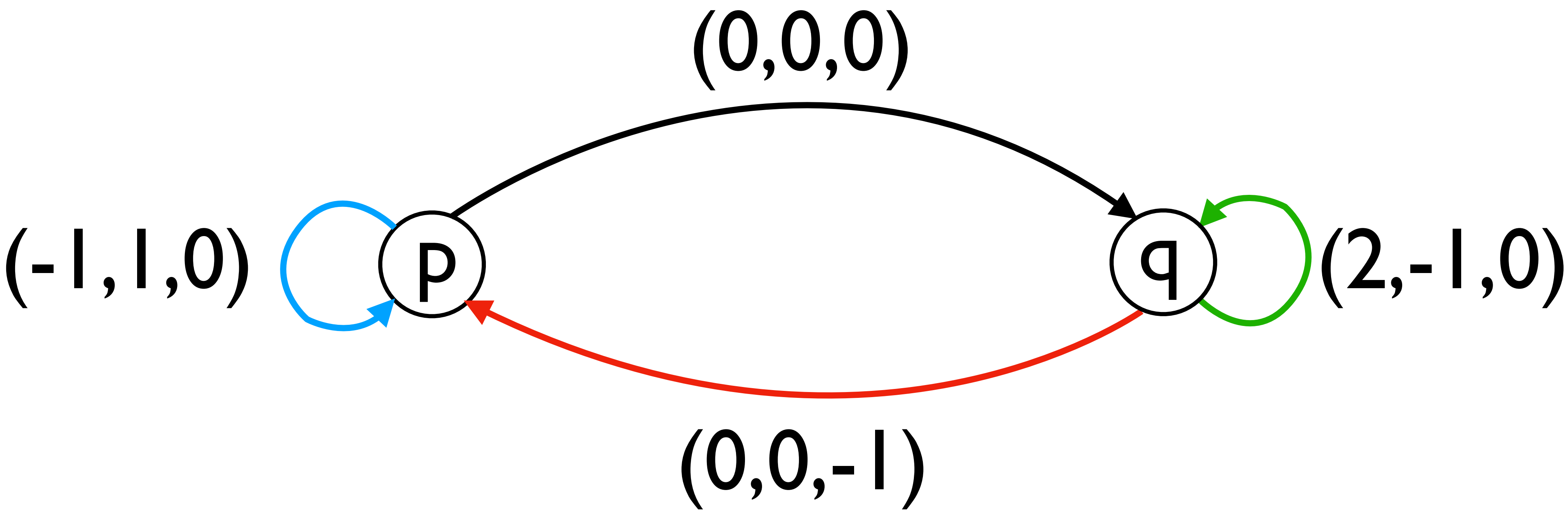
p(1,0,n)

Vector Addition Systems with States



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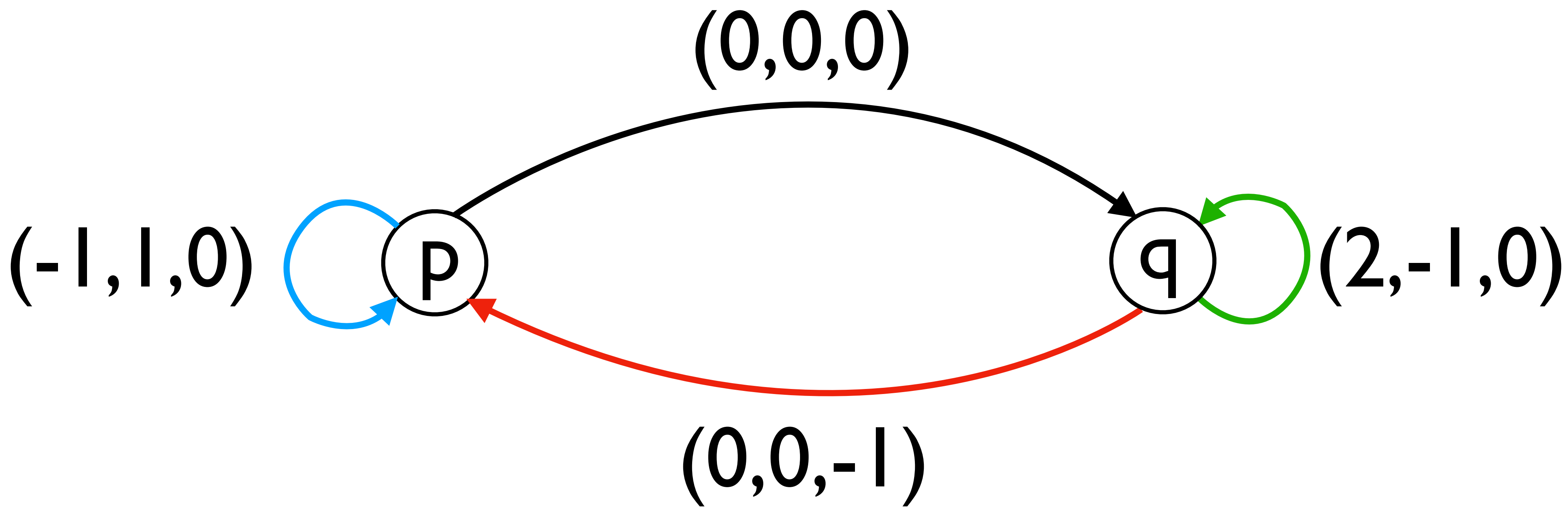
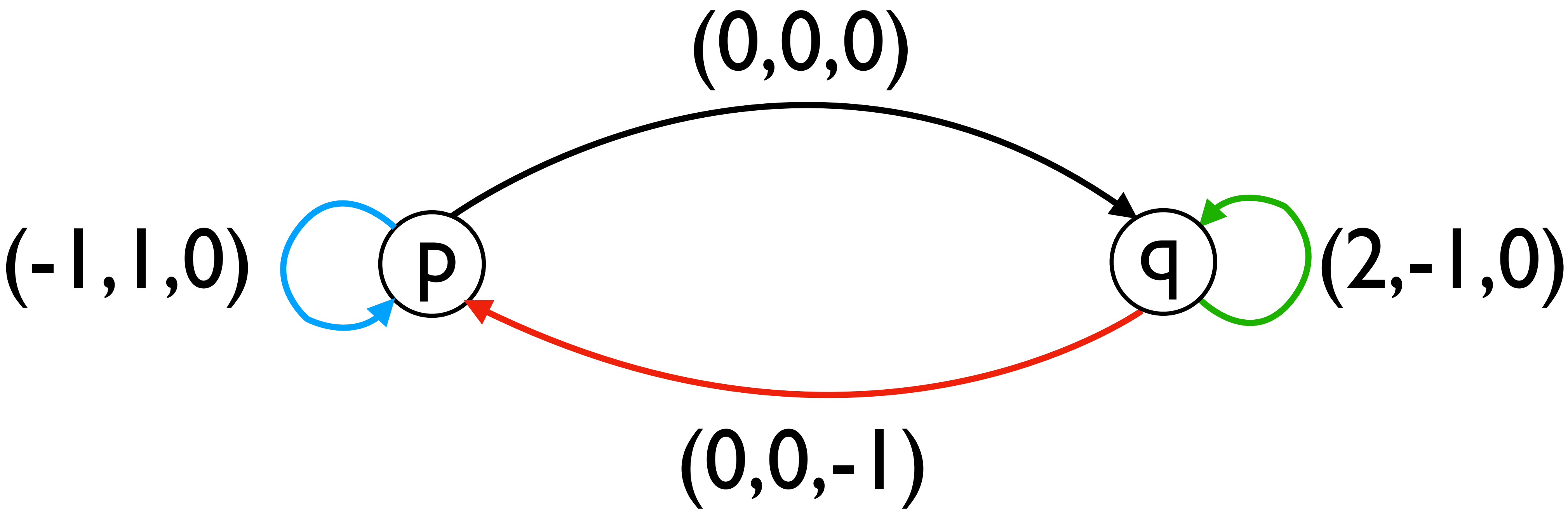
Vector Addition Systems with States



p(k,0,n) → p(0,k,n) → q(0,k,n) → q(2k,0,n) → p(2k,0,n-1)

p(1,0,n) → p(2,0,n-1) …

Vector Addition Systems with States



$$p(k,0,n) \longrightarrow p(0,k,n) \longrightarrow q(0,k,n) \longrightarrow q(2k,0,n) \longrightarrow p(2k,0,n-1)$$

$$p(1,0,n) \longrightarrow p(2,0,n-1) \ \ldots \ \longrightarrow p(2^n,0,0)$$

Vector Addition Systems with States

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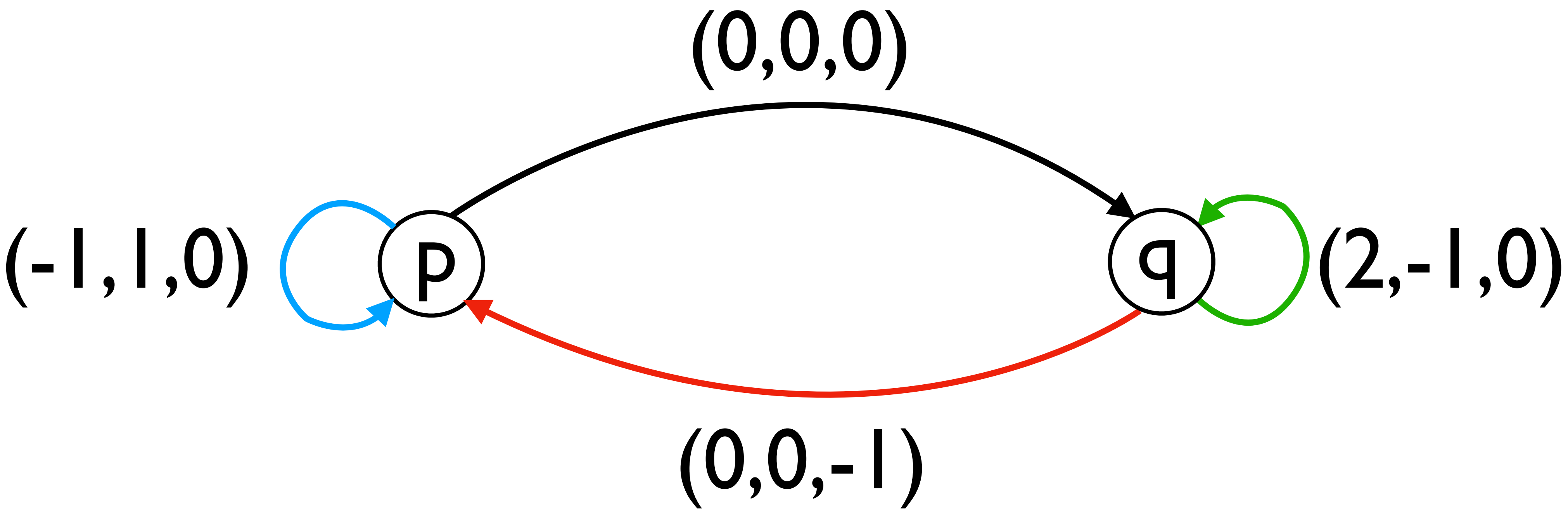

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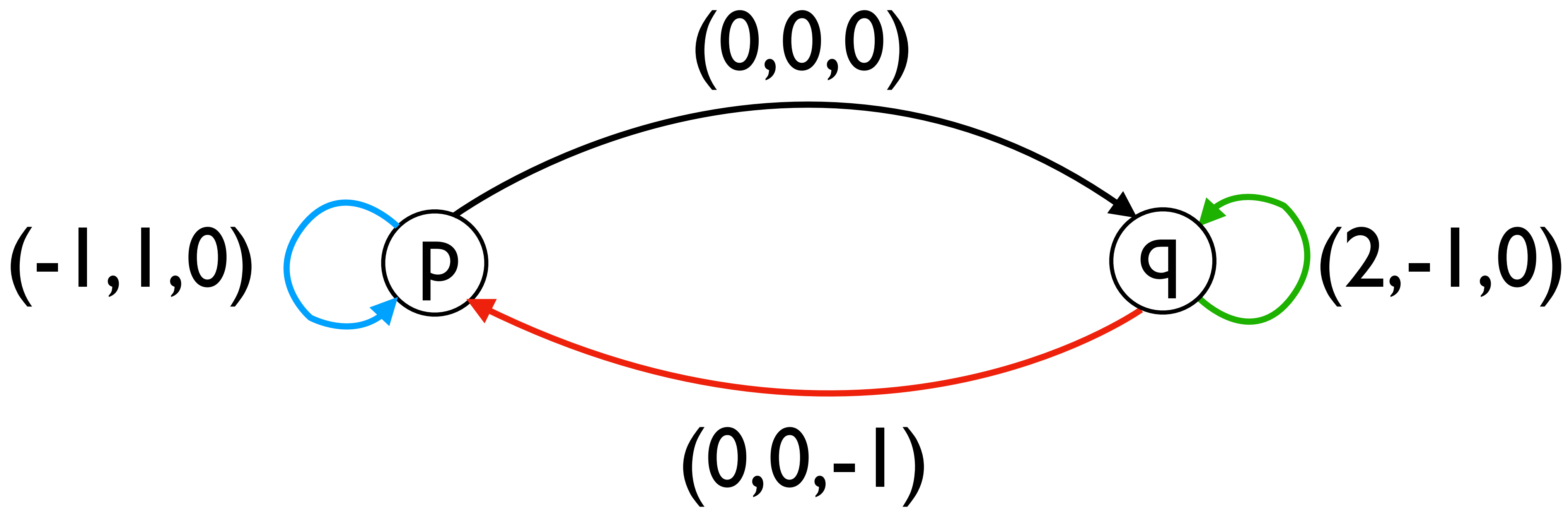
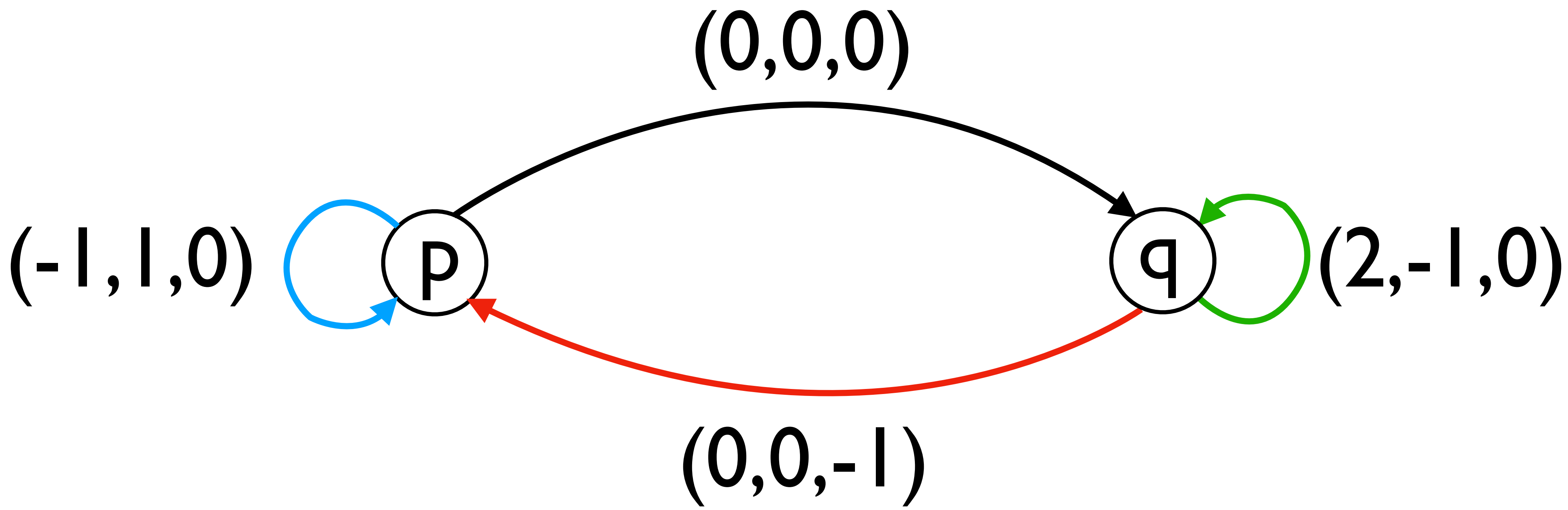


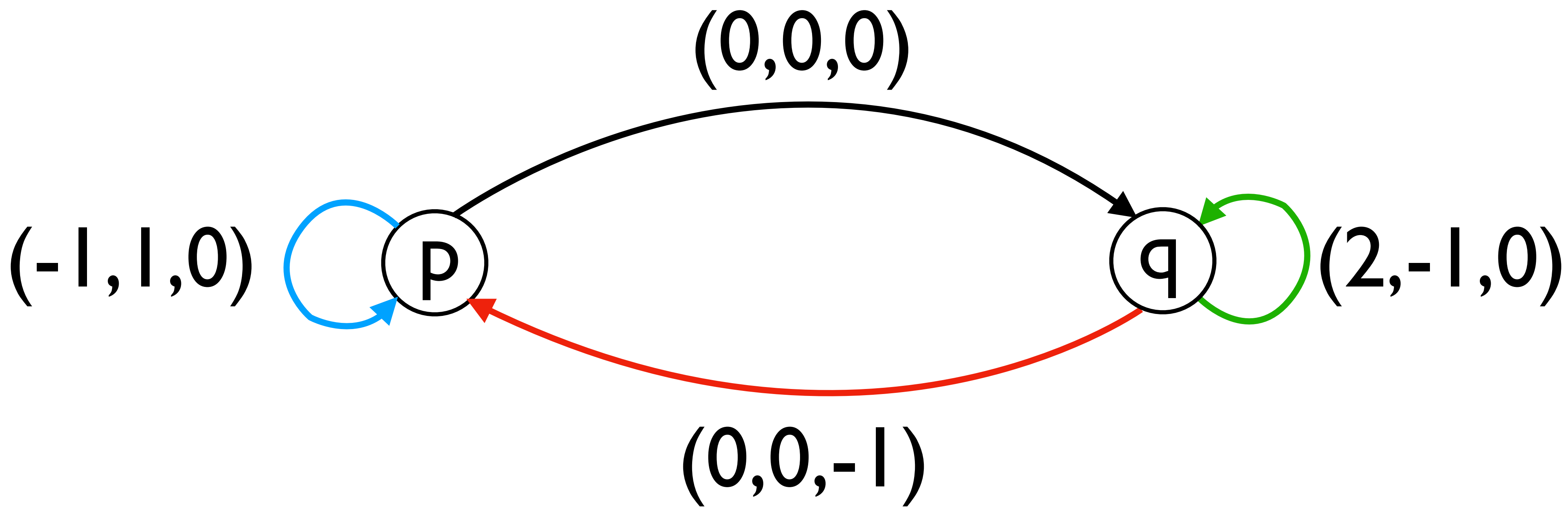
p(1,0,n)

Vector Addition Systems with States



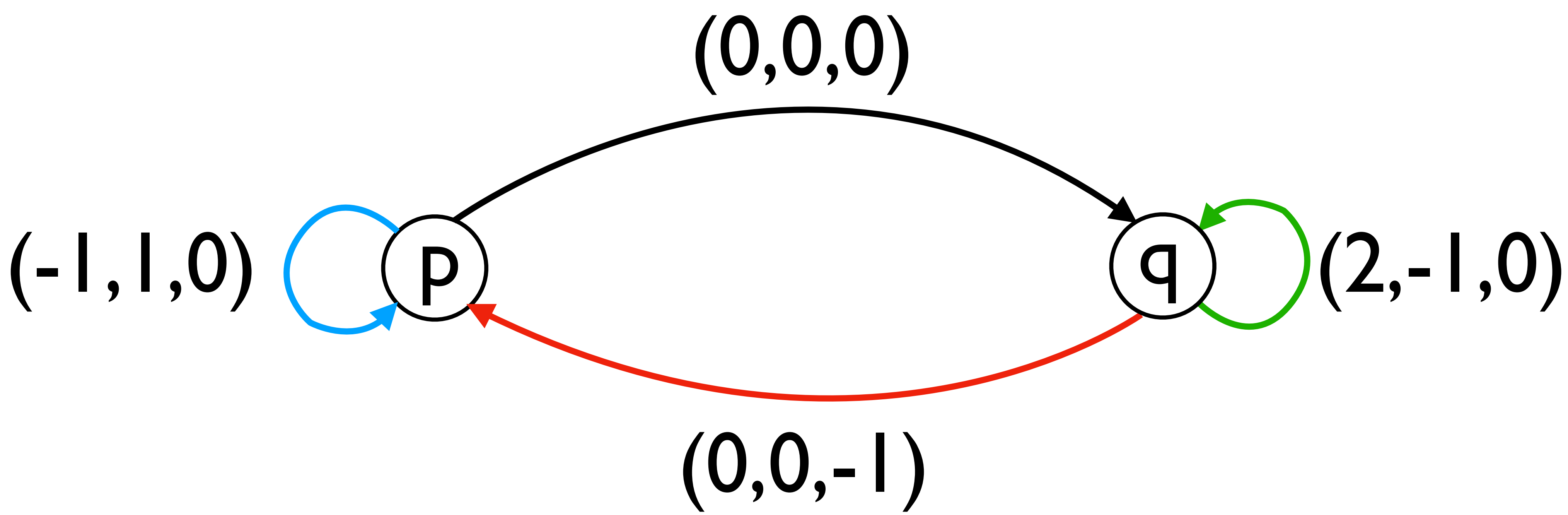
p(1,0,n) ⟶ p(2,0,n-1)

Vector Addition Systems with States



$p(1,0,n) \longrightarrow p(2,0,n-1)$ …

Vector Addition Systems with States



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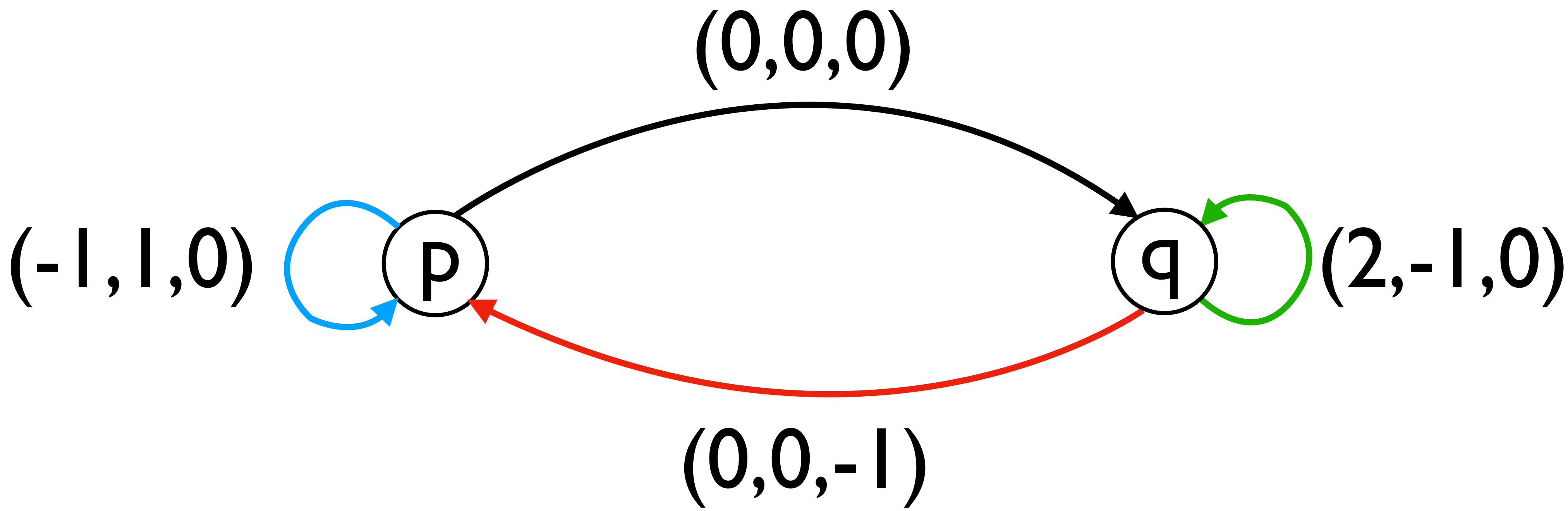
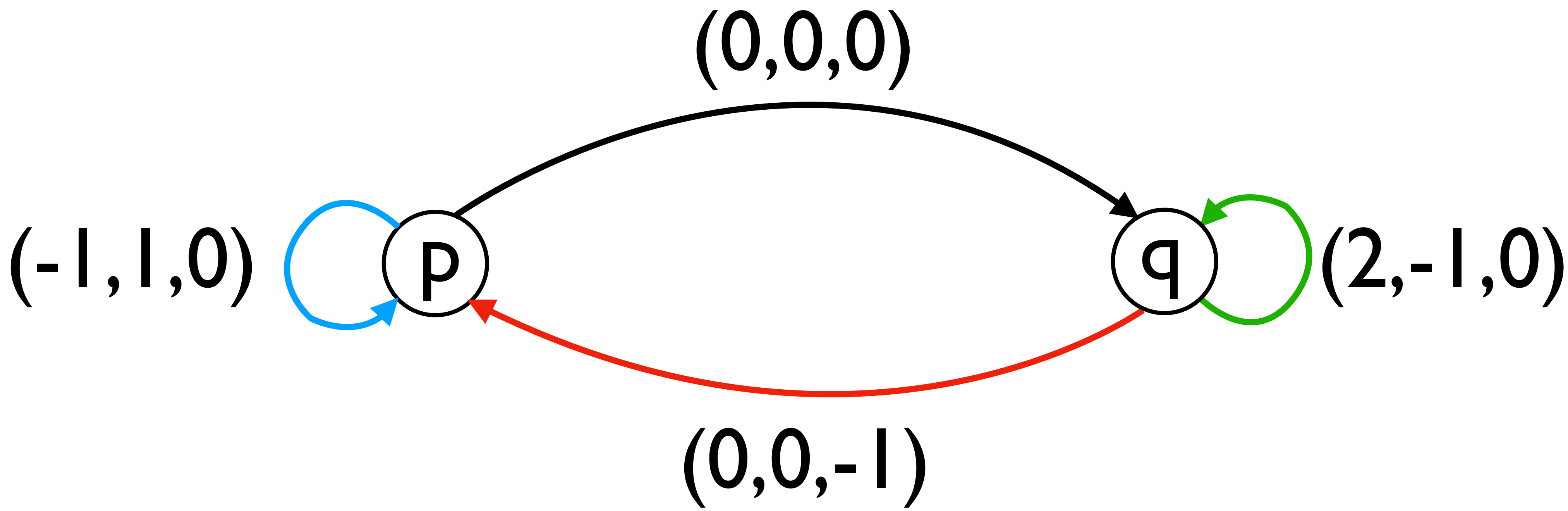


$$p(1,0,n) \longrightarrow p(2,0,n-1) \ \ldots \ \longrightarrow p(2^n,0,0) \longrightarrow p(2^n-a, a, 0)$$

Vector Addition Systems with States



$p(1,0,n) \longrightarrow p(2,0,n-1) \ \ldots \ \longrightarrow p(2^n,0,0) \longrightarrow p(2^n\text{-}a, a, 0)$

finite exponential reachability set

Reachability problem

Reachability problem

Given: a VASS, two its configurations s and t

Reachability problem

Given: a VASS, two its configurations s and t

Question: is there a run from s to t?

Short history

Short history

Lipton `76: ExpSpace-hardness of reachability

Short history

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Mayr `81: decidability of reachability

Short history

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Kosaraju `82, Lambert `92: simplifications

Short history

Lipton `76: ExpSpace-hardness of reachability

Mayr `81: decidability of reachability

Kosaraju `82, Lambert `92: simplifications

Blondin at el. `15: reachability PSpace-complete for 2-VASSes

Short history

Short history

Leroux, Schmitz `15: cubic-Ackermann

Short history

Leroux, Schmitz `15: cubic-Ackermann

Leroux, Schmitz `19: Ackermann

Short history

Leroux, Schmitz `15: cubic-Ackermann

Leroux, Schmitz `19: Ackermann

Cz., Lasota, Lazic, Leroux, Mazowiecki `19:
Tower-hardness

Short history

Leroux, Schmitz `15: cubic-Ackermann

Leroux, Schmitz `19: Ackermann

Cz., Lasota, Lazic, Leroux, Mazowiecki `19:
Tower-hardness

Leroux & Cz., Orlikowski`21: Ackermann-hardness

Functions F_k

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$F_1(n) = 2n$

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$F_{k+1}(n) = F_k \circ \ldots \circ F_k(1)$

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$F_2(n) = 2^n$

Functions F_k

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$F_{k+1}(n) = F_k \circ \ldots \circ F_k(1)$

composed n times

$F_2(n) = 2^n$ $F_3(n) = \text{Tower}(n)$

Functions F_k

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$F_{k+1}(n) = F_k \circ \ldots \circ F_k(1)$

composed n times

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$\text{Ack}(n) = F_\omega(n) = F_n(n)$

Ackermann-hardness

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Theorem
The Reachability Problem for (2k+4)-VASSes is $\mathbb{F}_k$-hard.

Ackermann-hardness

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↑

Jerome Leroux

Ackermann-hardness

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Cz., Łukasz Orlikowski: 6k

Ackermann-hardness

Theorem

The Reachability Problem for (2k+4)-VASSes is $\mathbb{F}_k$-hard.

↑

Jerome Leroux

Cz., Łukasz Orlikowski: 6k

open: is k+C enough?

Everything solved?

Everything solved?

reachability for 3-VASSes

Everything solved?

reachability for 3-VASSes (Tower? PSpace?)

Everything solved?

reachability for 3-VASSes (Tower? PSpace?)

reachability for fixed VASSes

Everything solved?

reachability for 3-VASSes (Tower? PSpace?)

reachability for fixed VASSes (Ackermann? PSpace?)

Everything solved?

reachability for 3-VASSes (Tower? PSpace?)

reachability for fixed VASSes (Ackermann? PSpace?)

reachability for VASS extensions

Everything solved?

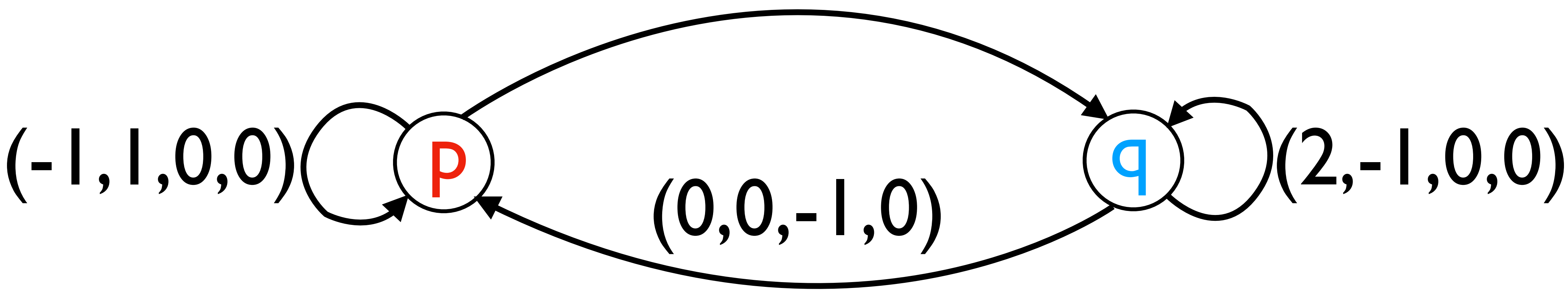
reachability for 3-VASSes (Tower? PSpace?)

reachability for fixed VASSes (Ackermann? PSpace?)

reachability for VASS extensions (decidable?)

Hard examples

Hard examples



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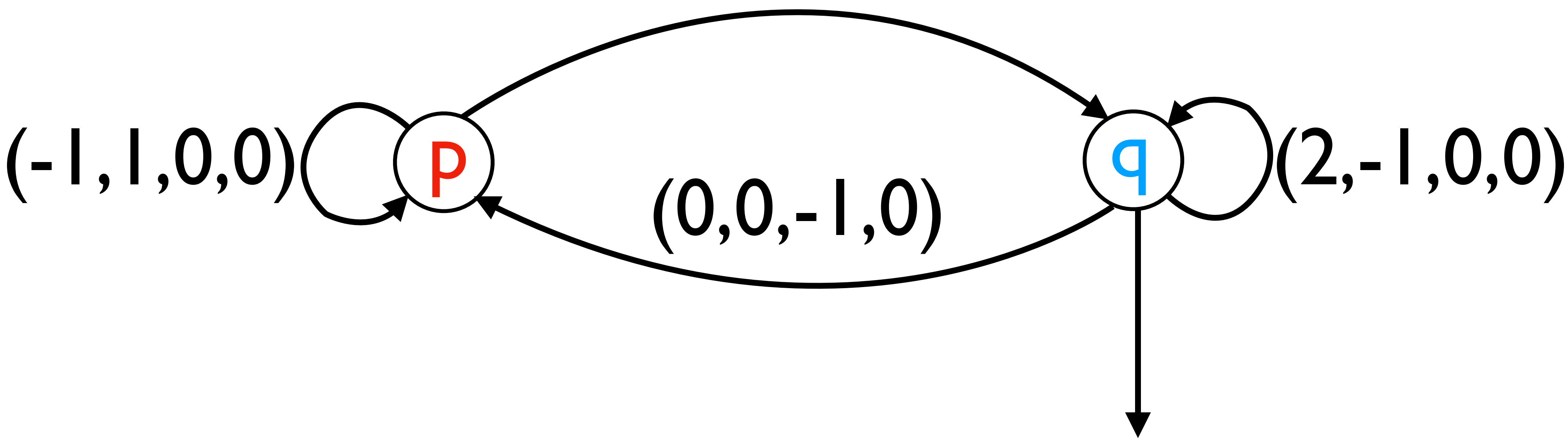

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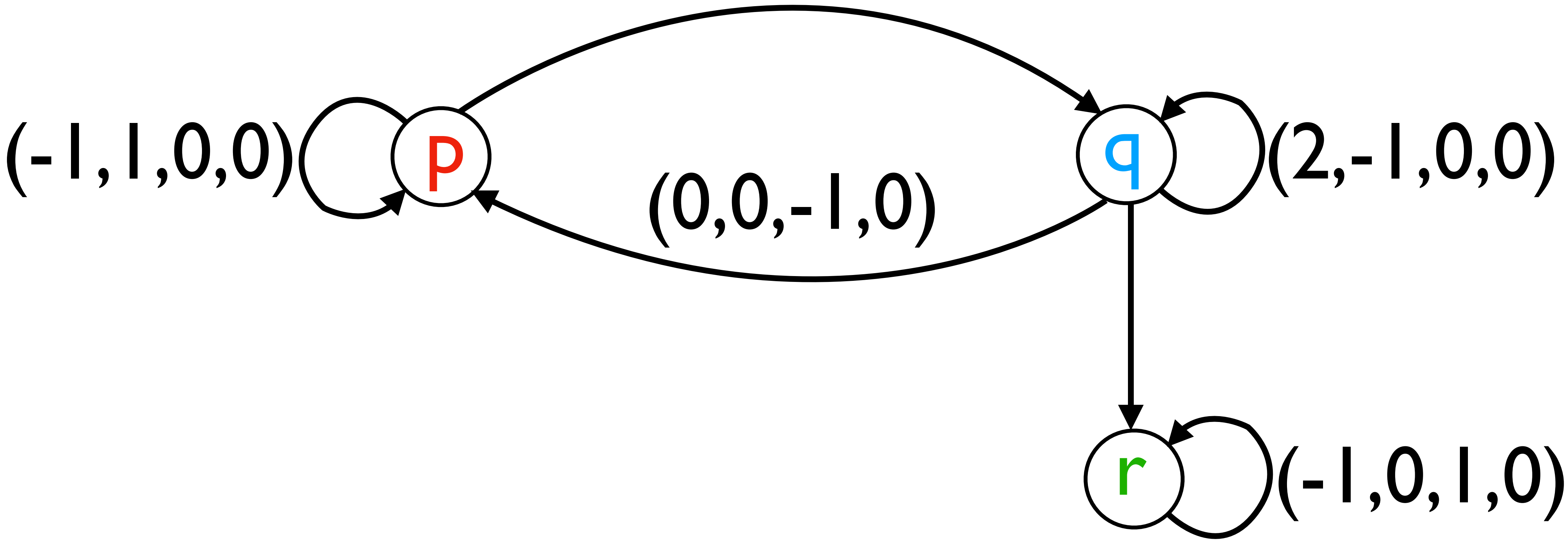

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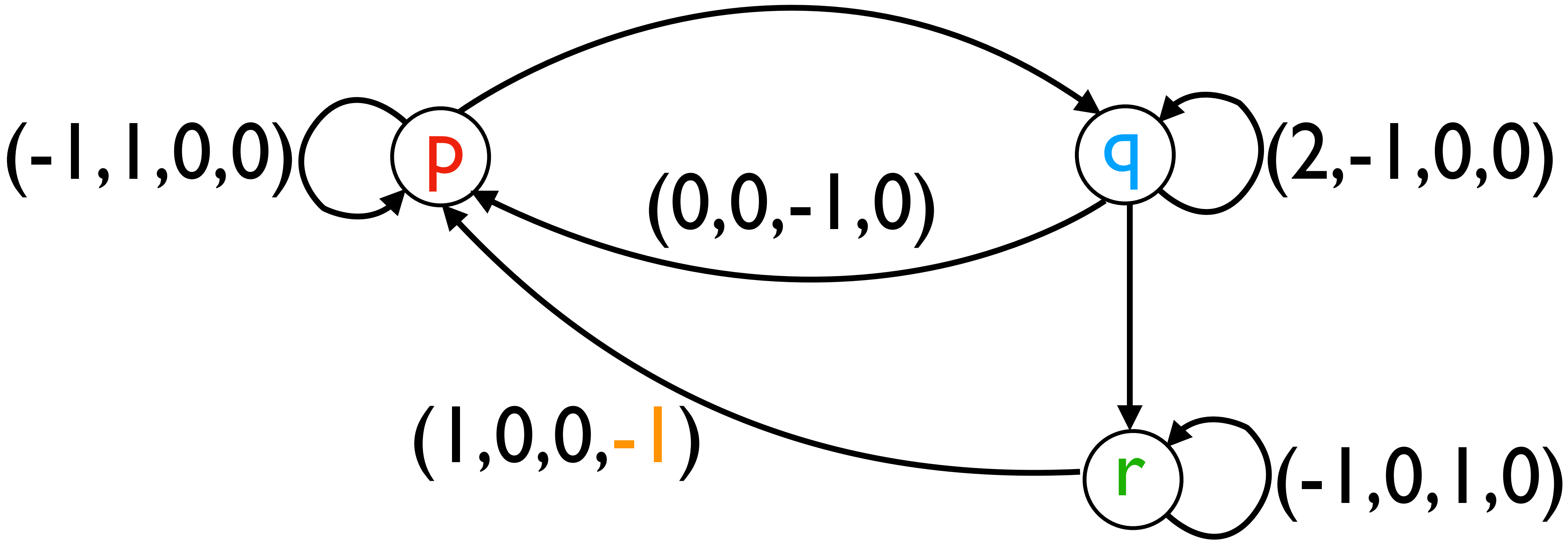

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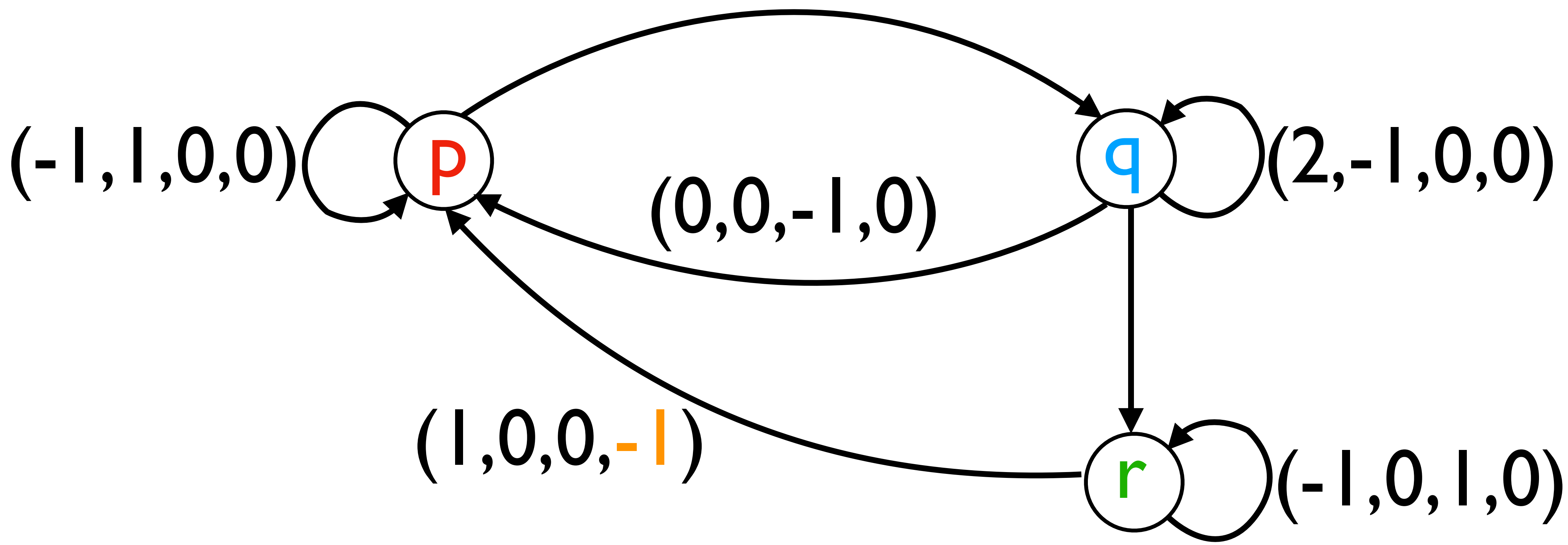
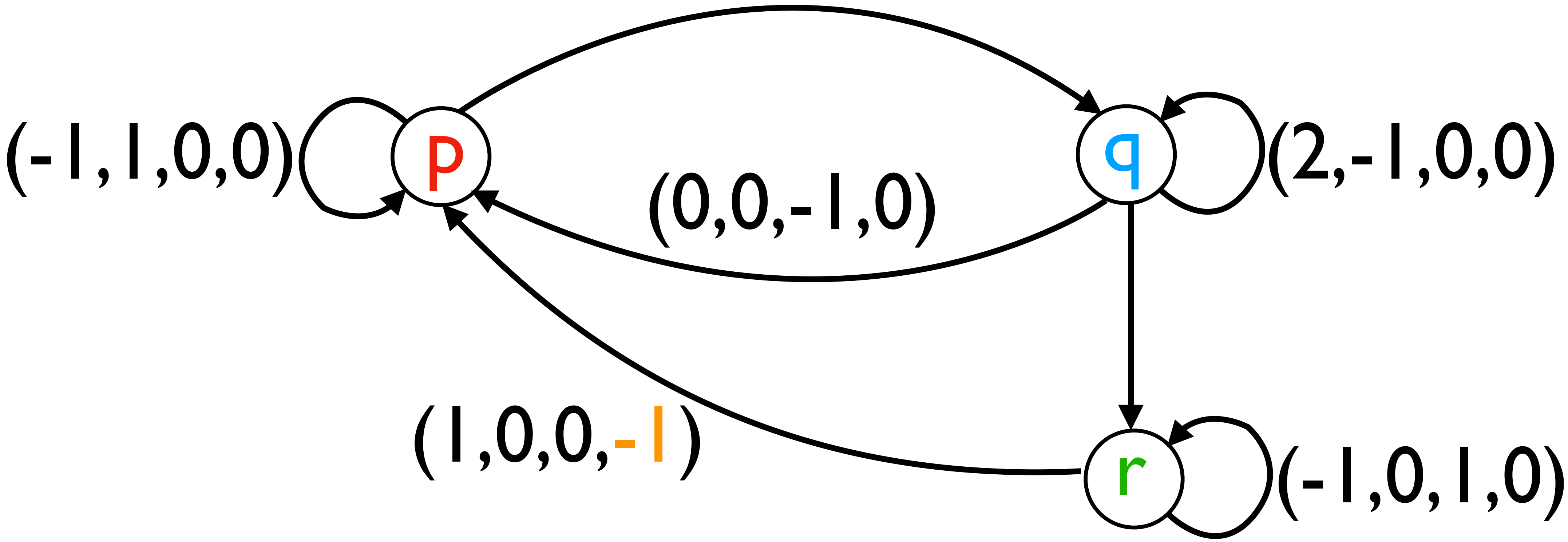


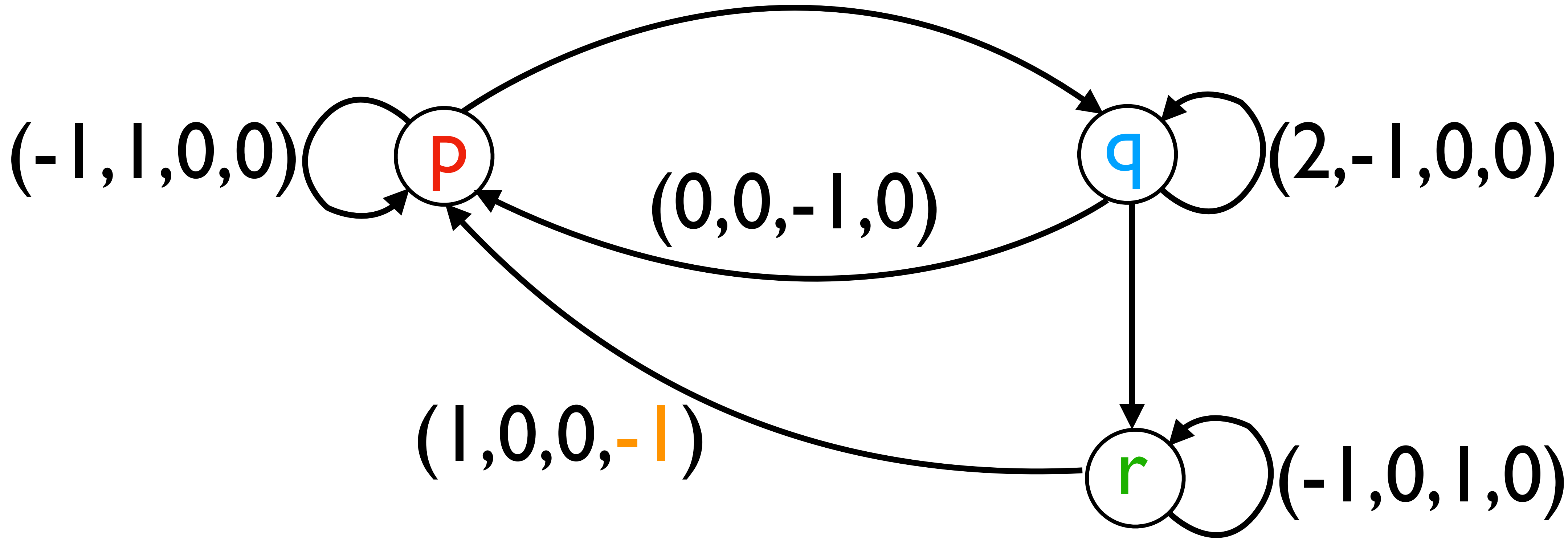
p(1,0,k,m)

Hard examples



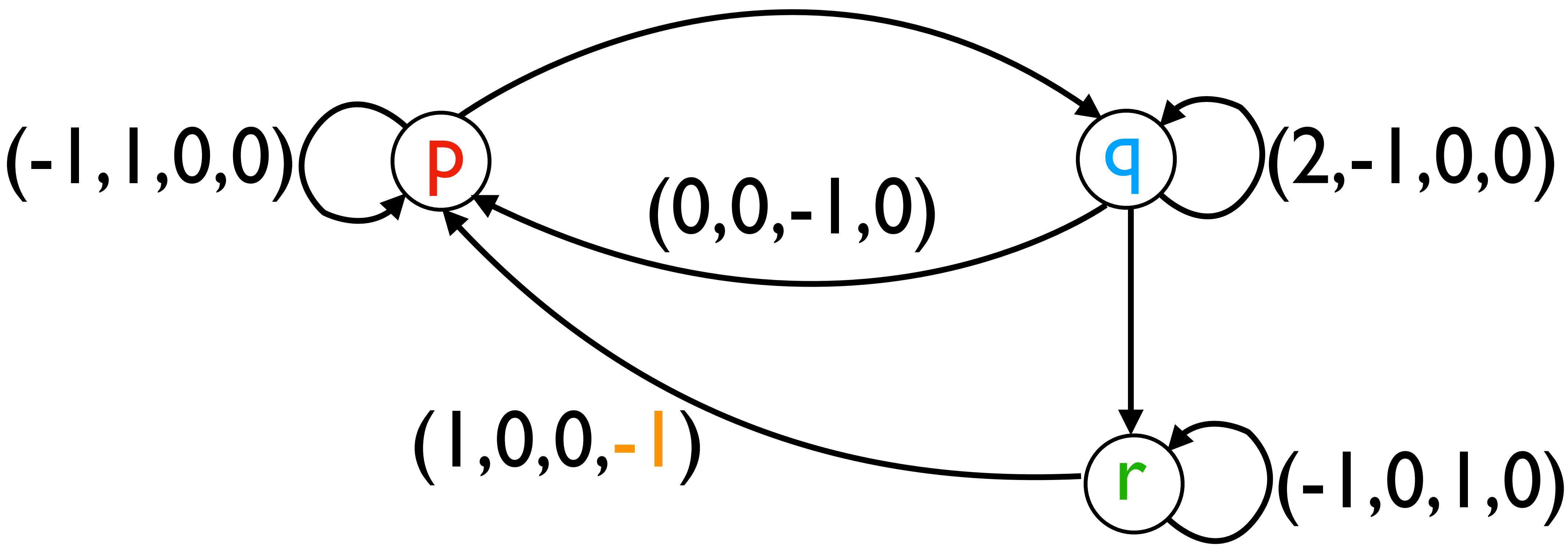
$$p(1,0,k,m) \longrightarrow p(2^k,0,0,m)$$

Hard examples



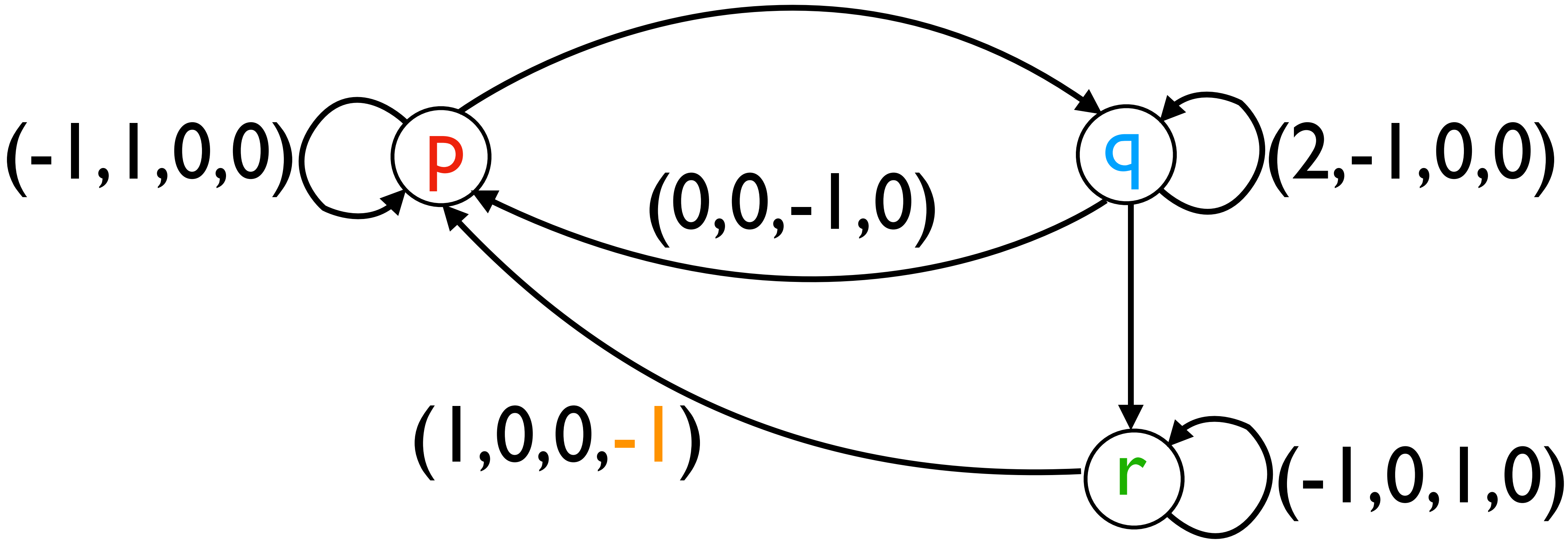
$$p(1,0,k,m) \longrightarrow p(2^k,0,0,m) \longrightarrow r(0,0,2^k,m)$$

Hard examples



p(1,0,k,m) ⟶ p(2^k,0,0,m) ⟶ r(0,0,2^k,m) ⟶ r(1,0,2^k,m-1)

Hard examples



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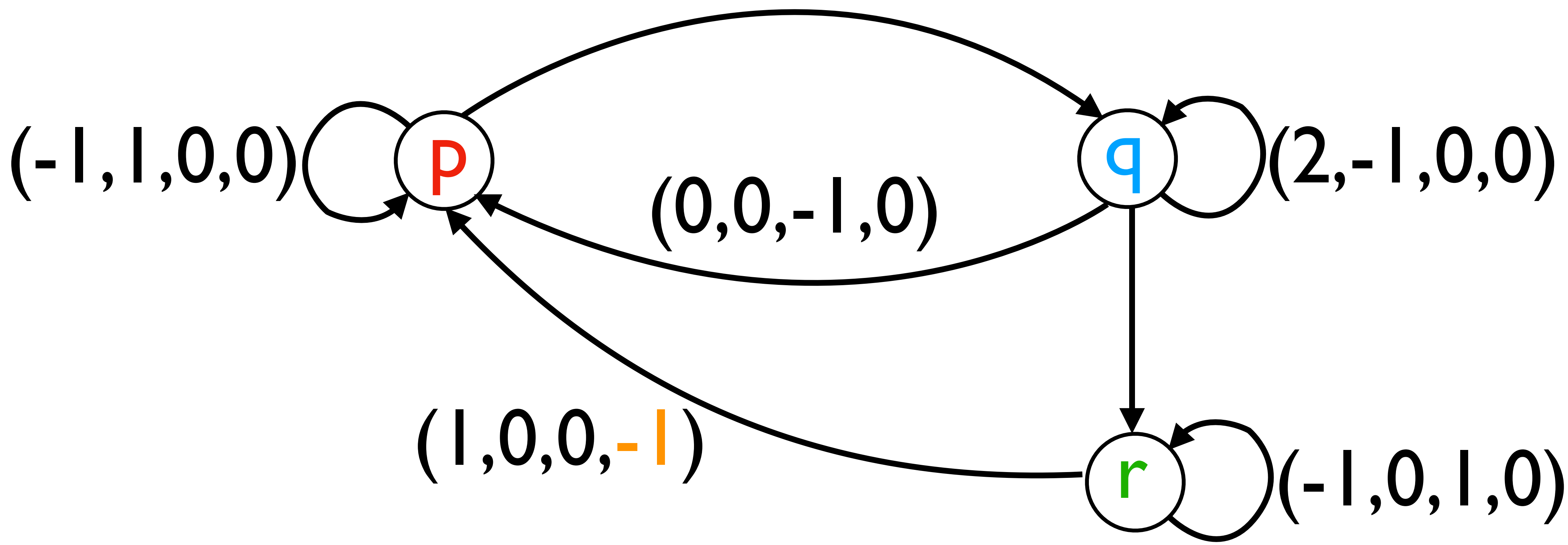
$p(1,0,1,n)$

Hard examples



$$p(1,0,k,m) \longrightarrow p(2^k,0,0,m) \longrightarrow r(0,0,2^k,m) \longrightarrow r(1,0,2^k,m-1)$$

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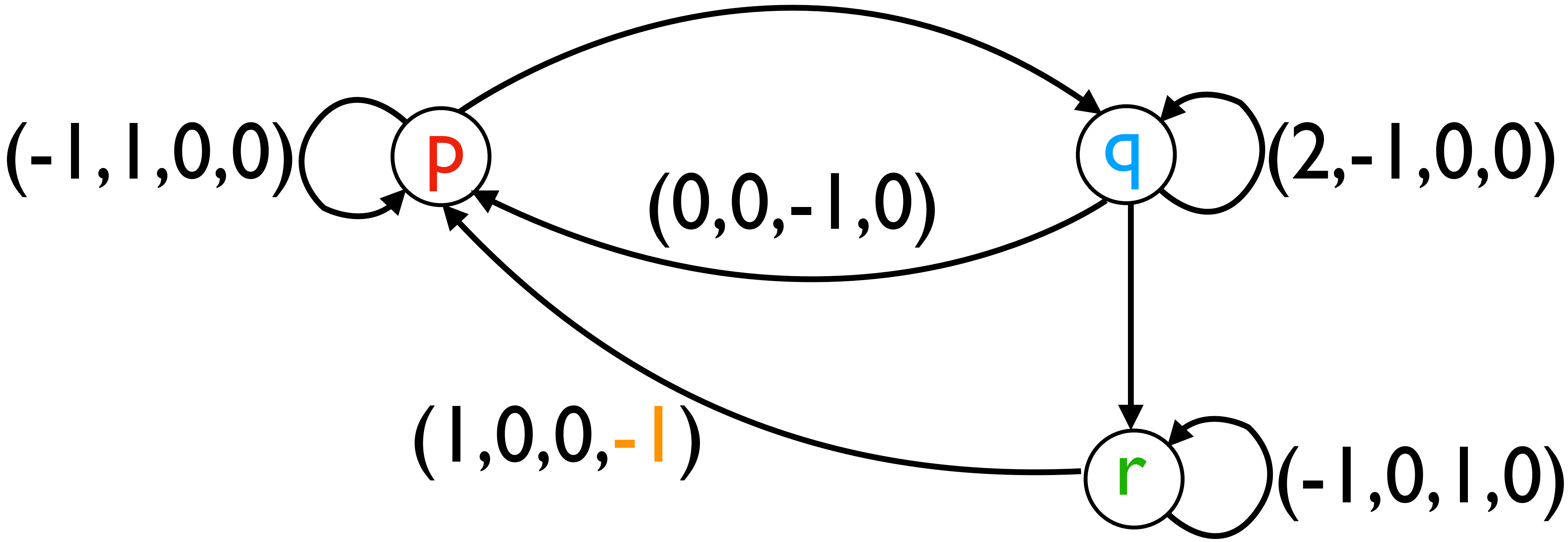
Hard examples



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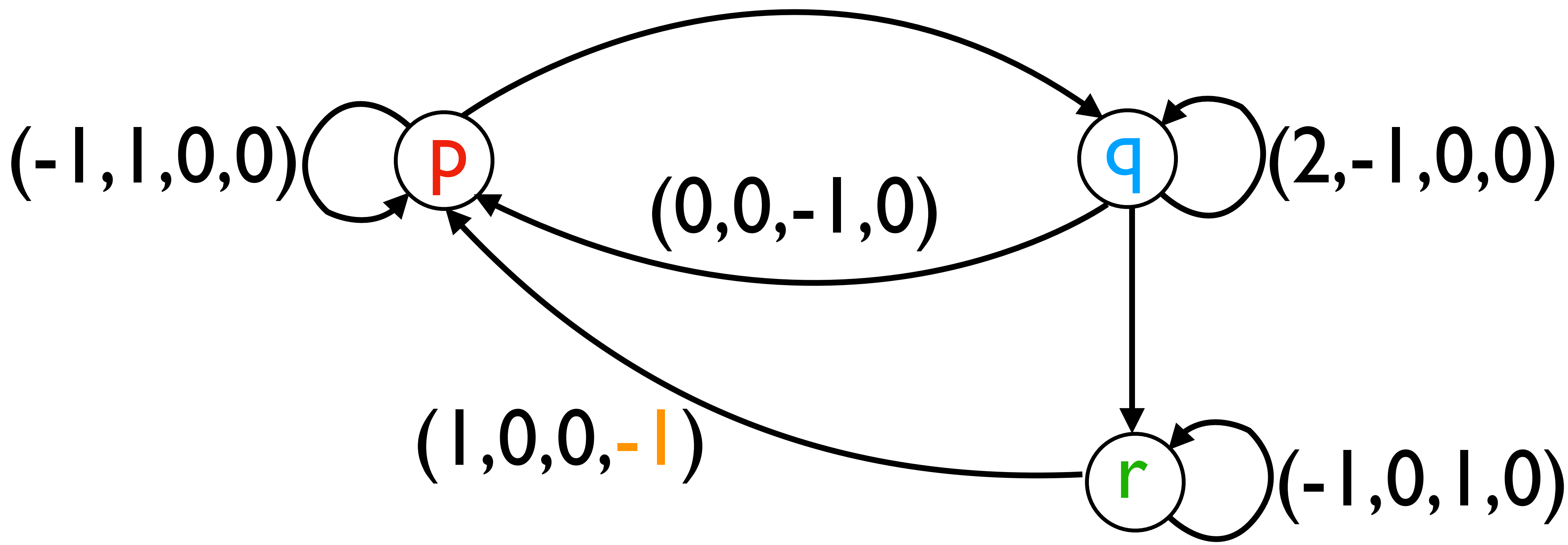
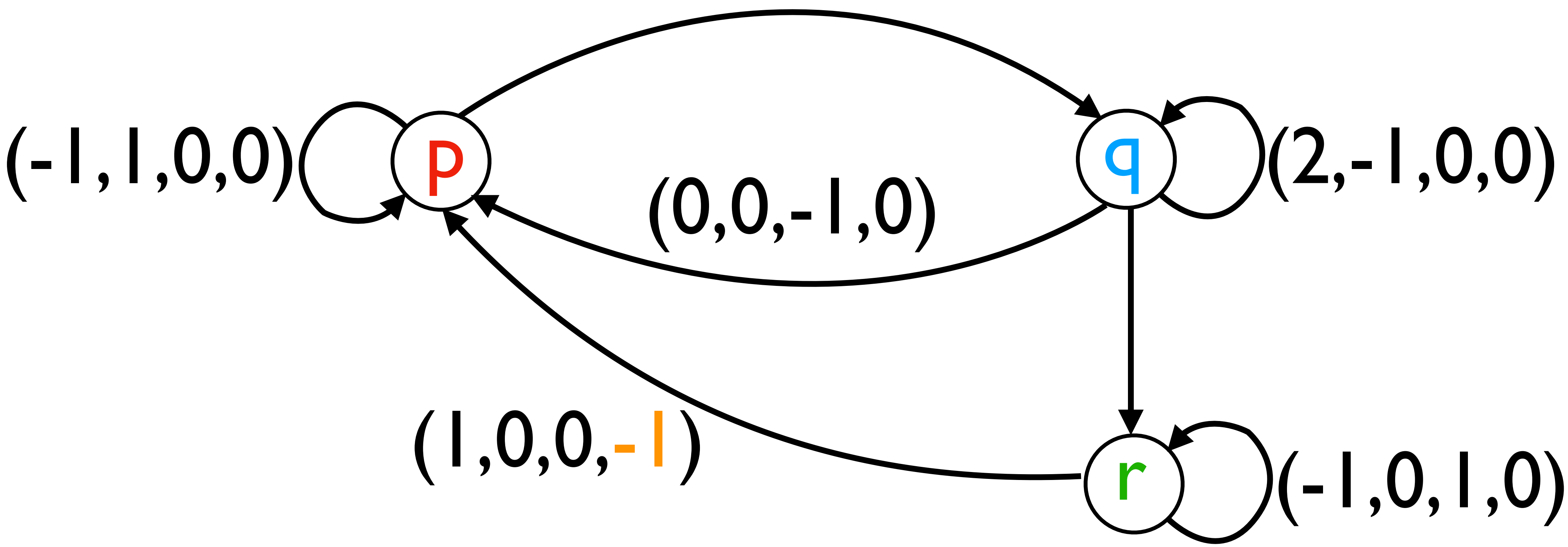


$p(1,0,k,m) \longrightarrow p(2^k,0,0,m) \longrightarrow r(0,0,2^k,m) \longrightarrow r(1,0,2^k,m-1)$

$p(1,0,1,n) \longrightarrow p(1,0,2^1,n-1) \ldots \longrightarrow p(1,0,\mathrm{Tower}(n),0)$

Hard examples



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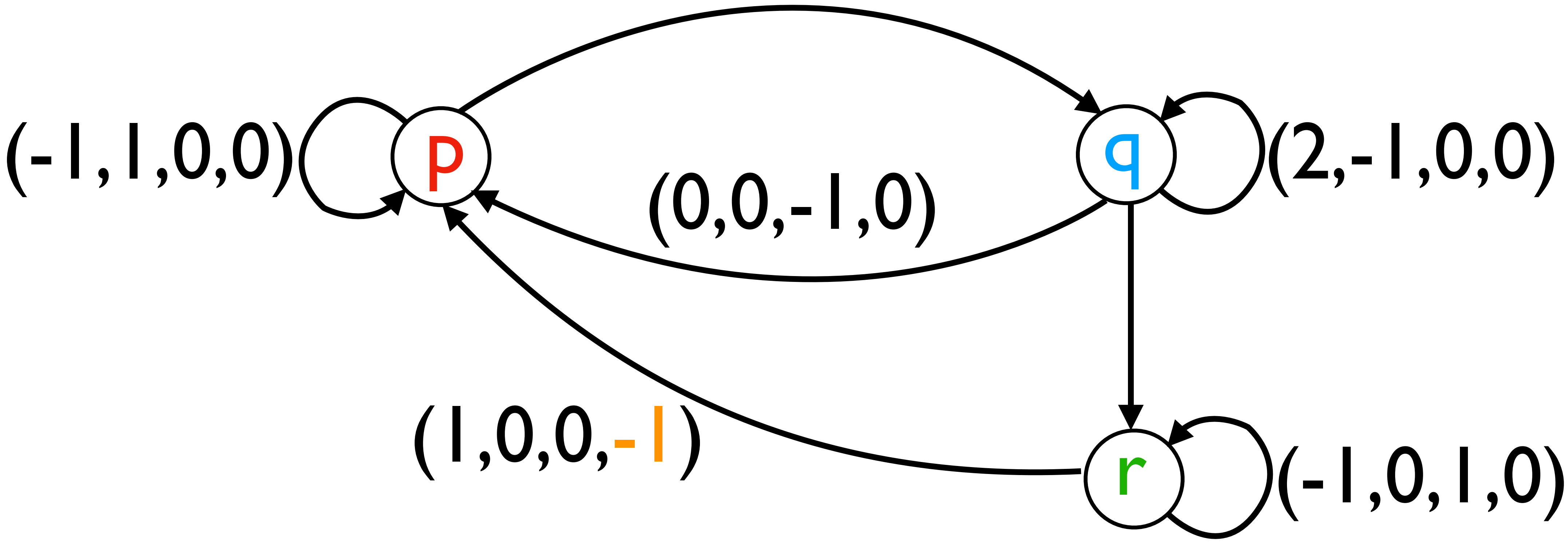
finite tower-size reachability set

Hard examples



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finite tower-size reachability set

finite F_d-size reachability set

Enforcing

Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

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Presented examples not sufficient

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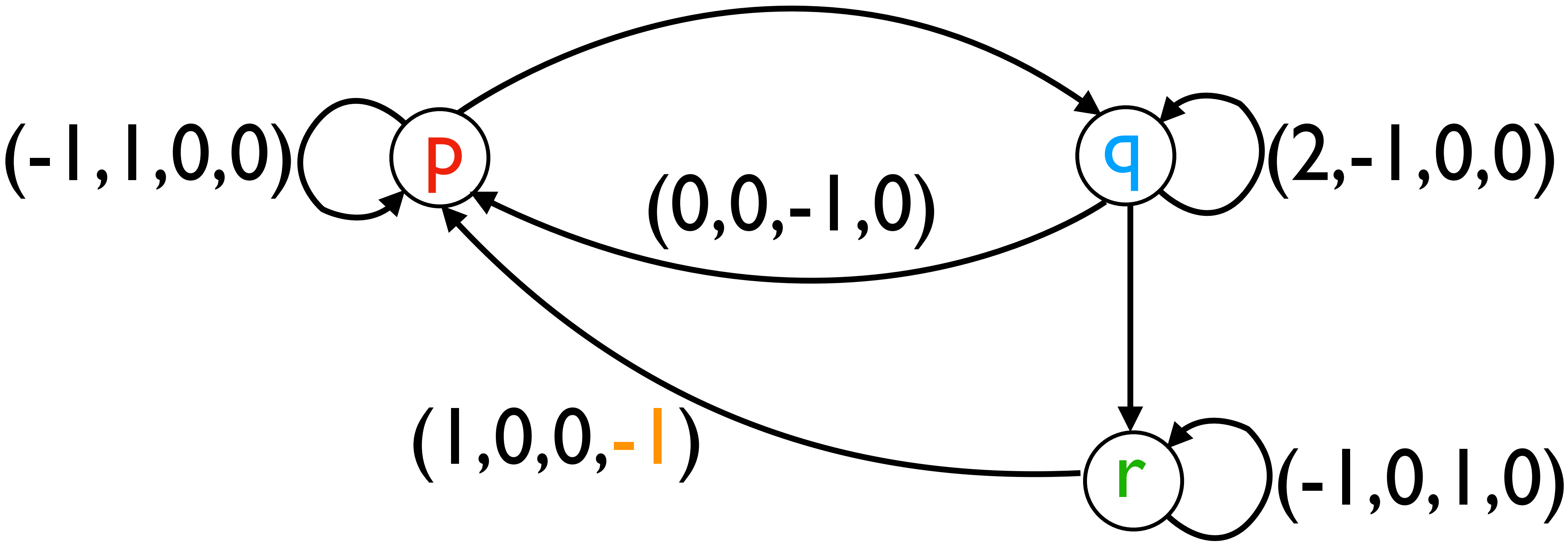
$\mathbb{F}_d$-hardness implies no F_d-short run

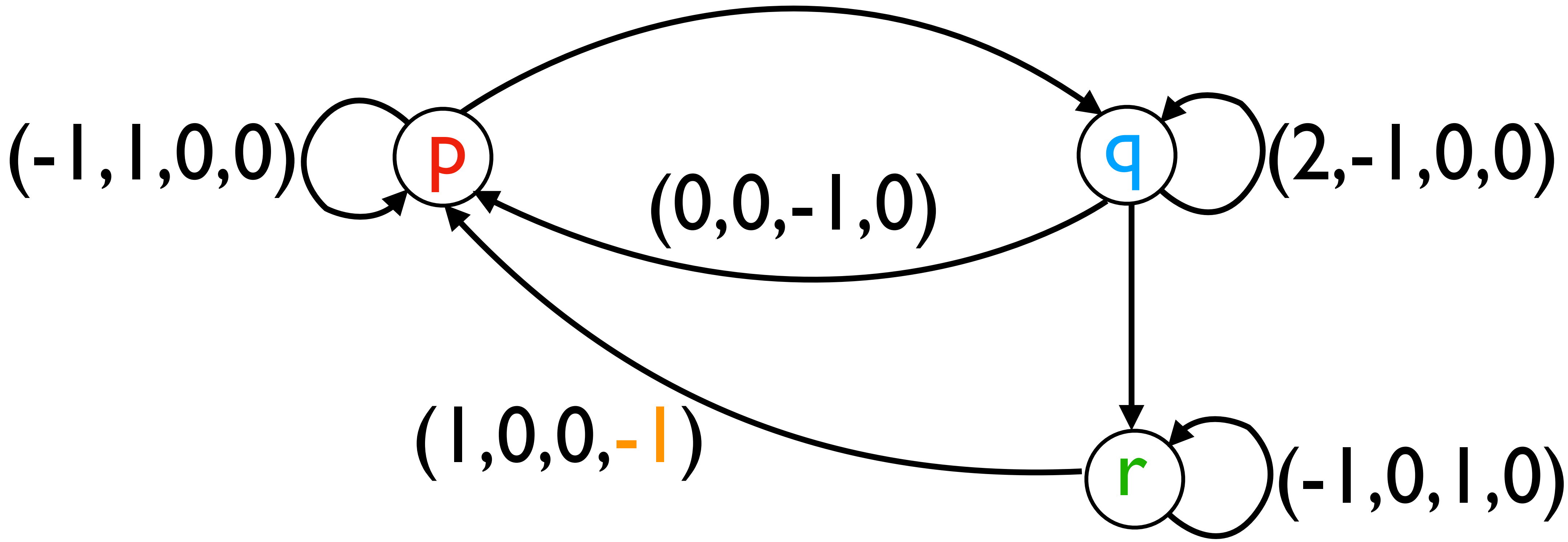
Presented examples not sufficient



Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

Presented examples not sufficient

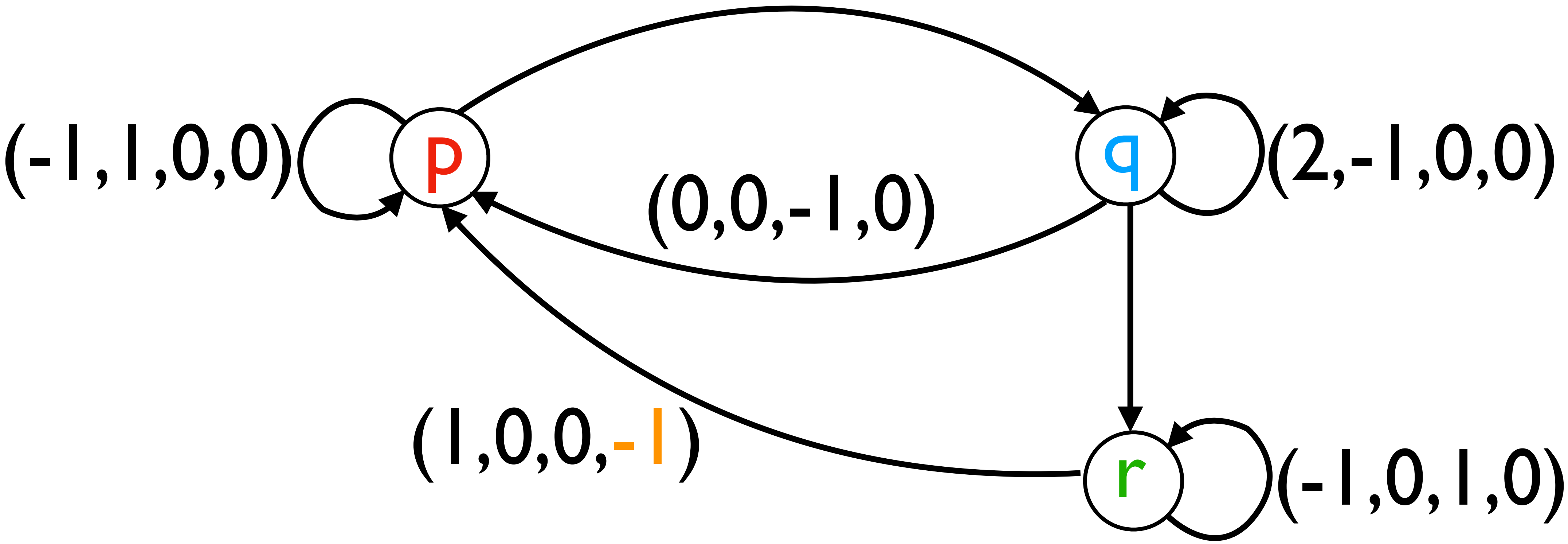


p(1,0,1,n)

Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

Presented examples not sufficient

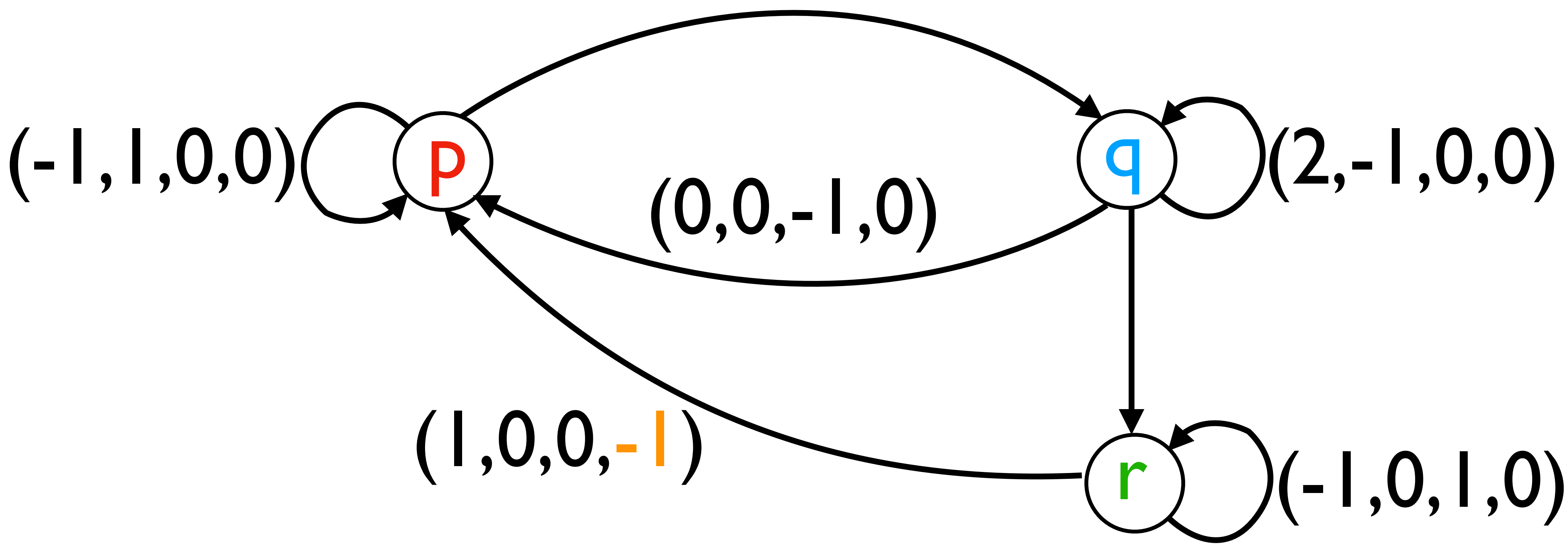


p(1,0,1,n) ⟶ p(1,0,2^1,n-1)

Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

Presented examples not sufficient

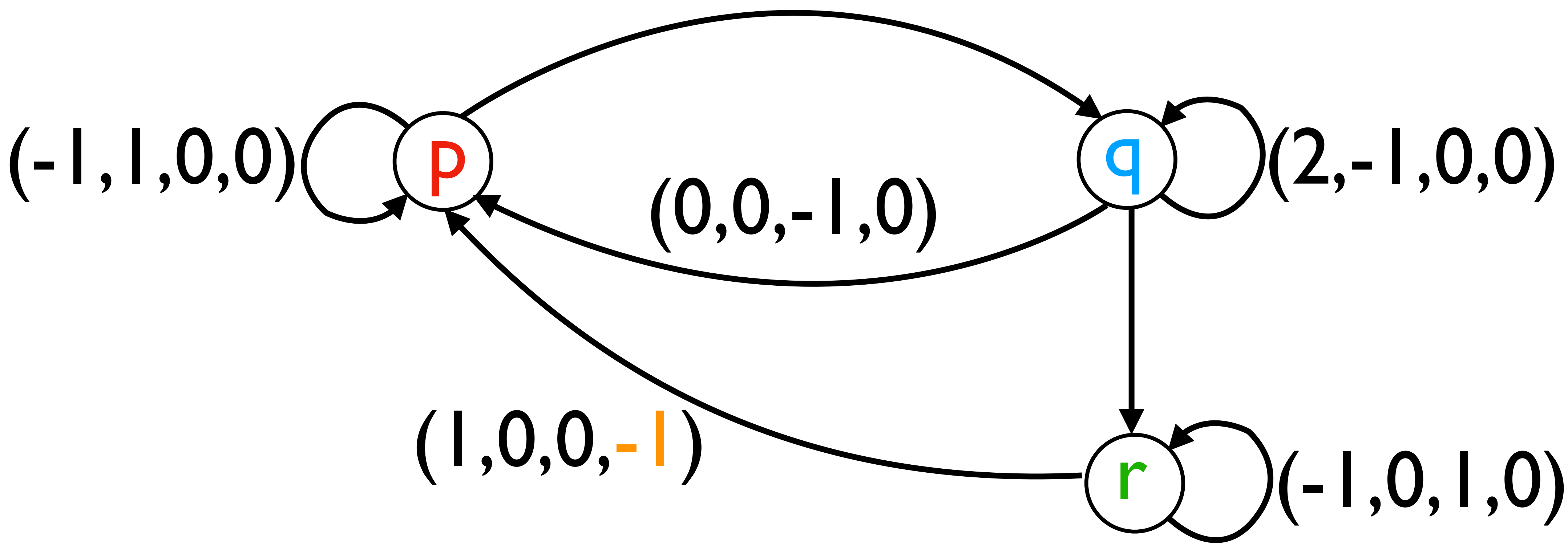


$p(1,0,1,n) \longrightarrow p(1,0,2^1,n-1) \ldots$

Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

Presented examples not sufficient

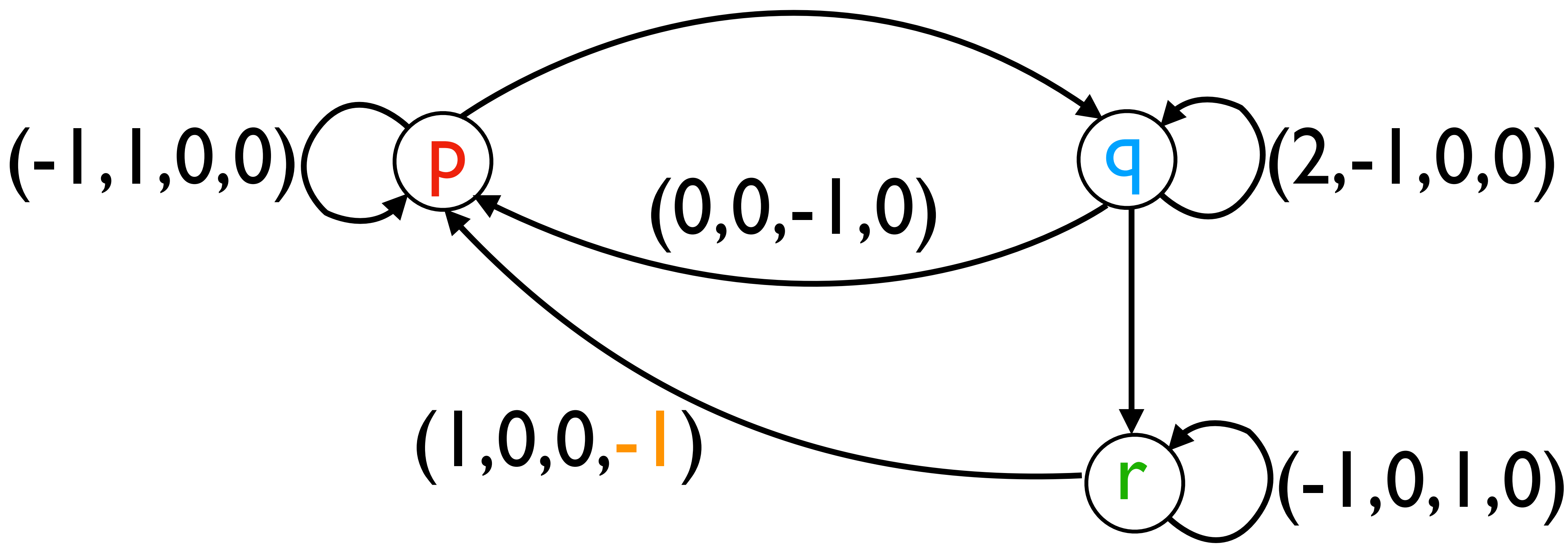


p(1,0,1,n) ⟶ p(1,0,2^1,n-1) … ⟶ p(1,0,Tower(n),0)

Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

Presented examples not sufficient



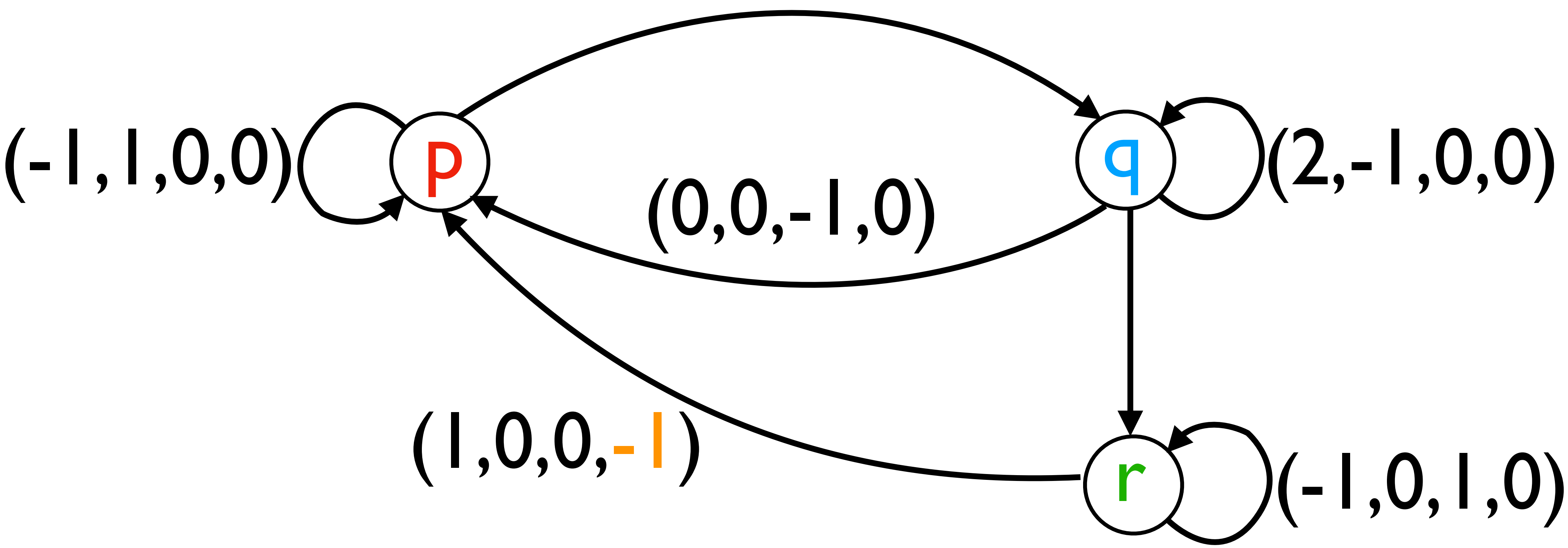
p(1,0,1,n) ⟶ p(1,0,2^1,n-1) … ⟶ p(1,0,Tower(n),0)

Need enforcing techniques!

Enforcing

$\mathbb{F}_d$-hardness implies no F_d-short run

Presented examples not sufficient

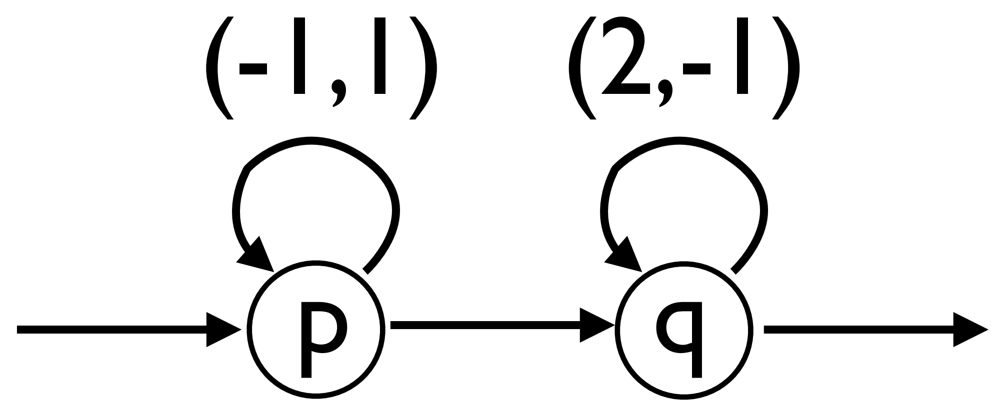


$p(1,0,1,n) \longrightarrow p(1,0,2^1,n-1) \ldots \longrightarrow p(1,0,\mathrm{Tower}(n),0)$

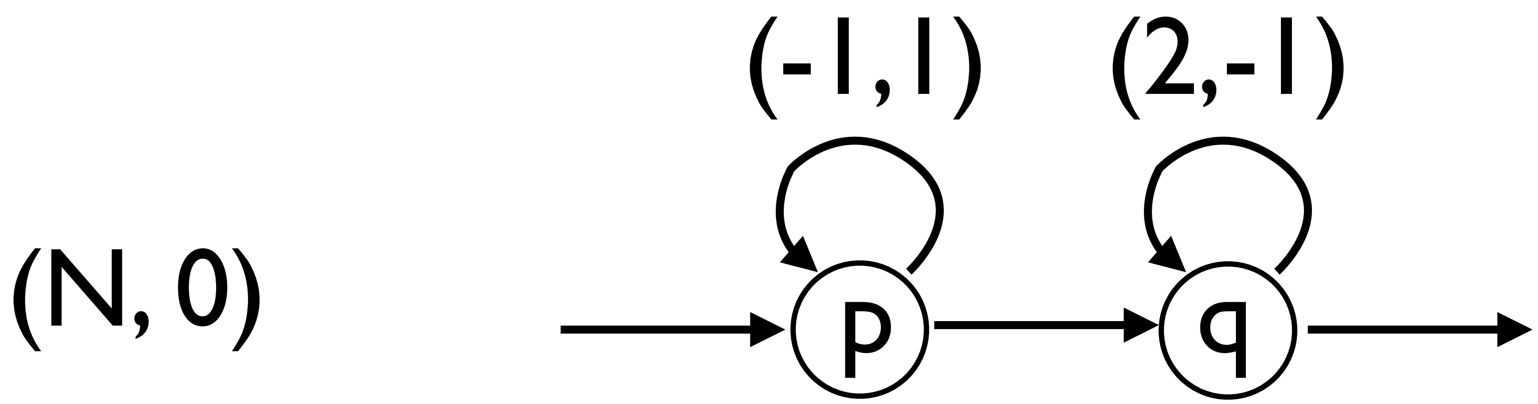
Need enforcing techniques! Consider small dimensions!

Counter programs

Counter programs

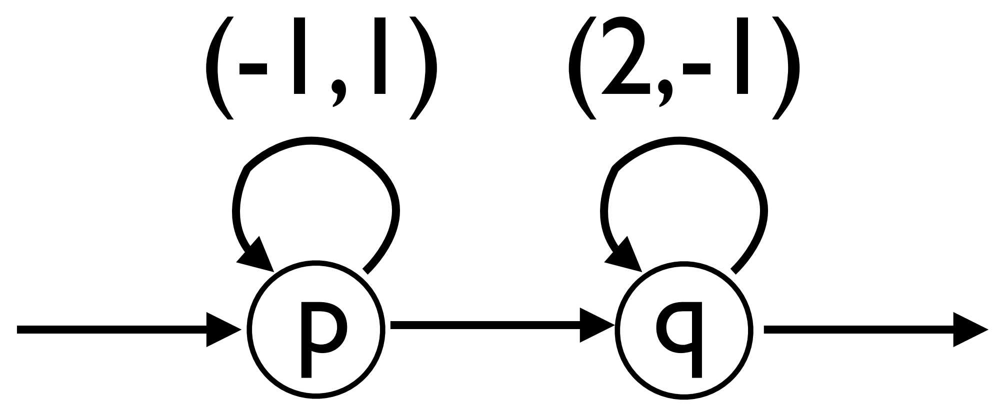


Counter programs



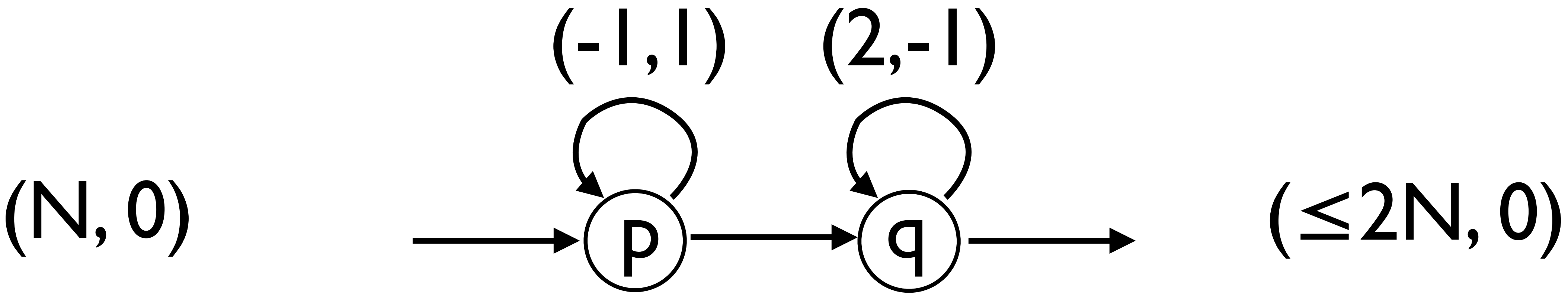
Counter programs

(N, 0)



(≤2N, 0)

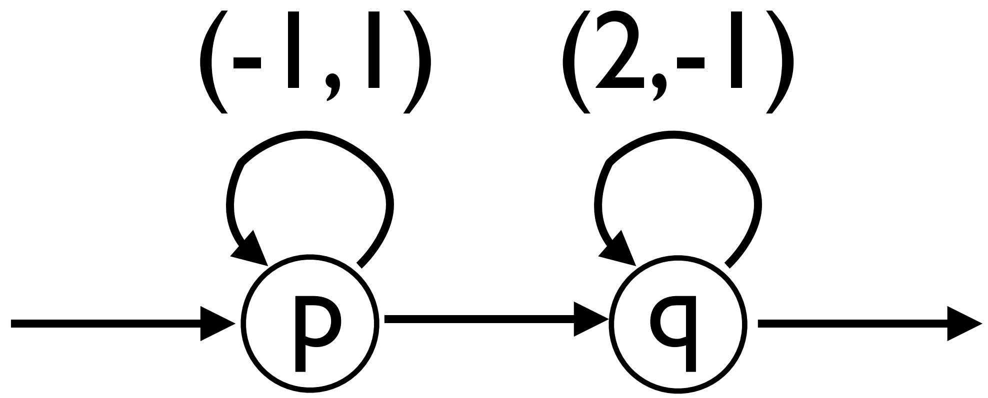
Counter programs



loop

Counter programs

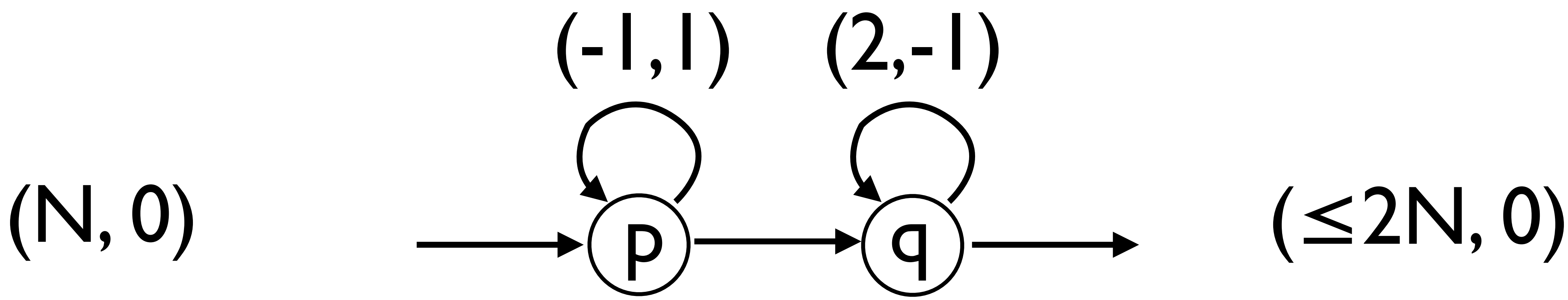
(N, 0)



(≤2N, 0)

loop   x-=1 y+=1

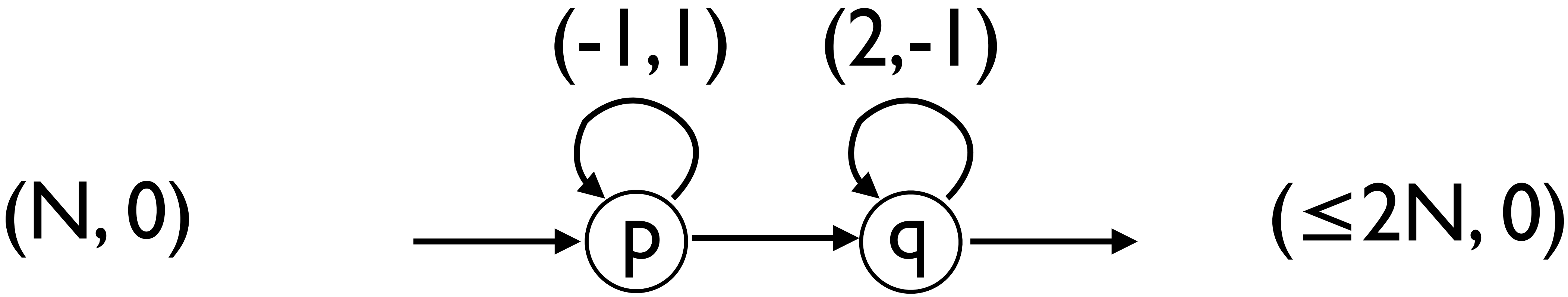
Counter programs



loop   x-=1 y+=1

loop

Counter programs



loop x-=1 y+=1

loop x+=2 y-=1

Multiplication triples

Multiplication triples

Triple (B, m, Bm) allow zero-testing

Multiplication triples

Triple (B, m, Bm) allow zero-testing

for m/2 zero-tests on B-bounded counters

Multiplication triples

Triple (B, m, Bm) allow zero-testing

for $m/2$ zero-tests on B-bounded counters

For small B easy: (B, 0, 0) + m(0, 1, B)

Multiplication triples

Triple (B, m, Bm) allow zero-testing

for m/2 zero-tests on B-bounded counters

For small B easy: (B, 0, 0) + m(0, 1, B)

For Fd size B gives $\mathbb{F}_d$-hardness

Multiplication triples

Multiplication triples

Assume (x, y, z) = (B, m, Bm)

Multiplication triples

Assume $(x, y, z) = (B, m, Bm)$

Let $x' = 0$

Multiplication triples

Assume $(x, y, z) = (B, m, Bm)$

keep $x+x' = B$

Let $x' = 0$

Multiplication triples

Assume $(x, y, z) = (B, m, Bm)$

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zero-test(x'):

Multiplication triples

keep $x+x' = B$

Assume $(x, y, z) = (B, m, Bm)$

Let $x' = 0$

zero-test(x'):

loop {inc(x'), dec(x), dec(z)}

Multiplication triples

keep x+x' = B

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Multiplication triples

keep x+x' = B

Assume (x, y, z) = (B, m, Bm)

Let x' = 0

zero-test(x'):

loop {inc(x'), dec(x), dec(z)}

loop {dec(x'), inc(x), dec(z)}

y := y-2

Multiplication triples

keep x+x' = B

y dec by 2

Assume (x, y, z) = (B, m, Bm)

Let x' = 0

zero-test(x'):

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Multiplication triples

Assume $(x, y, z) = (B, m, Bm)$

Let $x' = 0$ — keep $x+x' = B$

zero-test(x'):

loop {inc(x'), dec(x), dec(z)}

loop {dec(x'), inc(x), dec(z)} — y dec by 2

$y := y-2$ — z dec by $\leq 2B$

Multiplication triples

Assume $(x, y, z) = (B, m, Bm)$

keep $x+x' = B$

Let $x' = 0$

zero-test(x'):

loop {inc(x'), dec(x), dec(z)}

y dec by 2

loop {dec(x'), inc(x), dec(z)}

z dec by $\leq 2B$

$y := y-2$

At the end check if $z = 0$

Testing more counters

Testing more counters

Goal: z dec by $\leq 2B$

Testing more counters

Goal: z dec by $\leq 2B$

Let $x_1 + \ldots + x_k = B$

Testing more counters

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zero-test(x_1):

Testing more counters

Goal: z dec by $\leq 2B$

Let $x_1 + \ldots + x_k = B$

zero-test(x_1):

transfer(x_2, x_1, z)

Testing more counters

Goal: z dec by $\leq 2B$

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transfer(x_2, x_1, z)

loop {dec(x_2), inc(x_1), dec(z)}

Testing more counters

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transfer(x_3, x_2, z)

loop {dec(x_2), inc(x_1), dec(z)}

Testing more counters

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...

transfer(x_2, x_3, z)

transfer(x_1, x_2, z)

y := y-2

Testing more counters

Goal: z dec by $\leq 2B$

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zero-test(x_1):

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transfer(x_{k-1}, x_k, z)

...

transfer(x_2, x_3, z)

transfer(x_1, x_2, z)

y := y-2

each token moved at most twice

Testing more counters

Goal: z dec by $\leq 2B$

Let $x_1 + \ldots + x_k = B$

loop {dec(x_2), inc(x_1), dec(z)}

zero-test(x_1):

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transfer(x_3, x_2, z)

...

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transfer(x_{k-1}, x_k, z)

...

transfer(x_2, x_3, z)

transfer(x_1, x_2, z)

y := y-2

each token moved at most twice

z dec by $\leq 2B$

PSpace-hardness

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Simple proof for binary 6-VASS

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2^n-bounded 3-counter automata are PSpace-hard

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Triple (2^n, $6q\ 4^n$, $6q\ 8^n$) sufficient for zero-tests

PSpace-hardness

Simple proof for binary 6-VASS

2^n-bounded 3-counter automata are PSpace-hard

Triple (2^n, $6q\ 4^n$, $6q\ 8^n$) sufficient for zero-tests

$x{+}{=}2^n$ $y{+}{=}6q\ 4^n$ $z{+}{=}6q\ 8^n$

PSpace-hardness

Simple proof for binary 6-VASS

2^n-bounded 3-counter automata are PSpace-hard

Triple (2^n, $6q\ 4^n$, $6q\ 8^n$) sufficient for zero-tests

x+=2^n y+=$6q\ 4^n$ z+=$6q\ 8^n$

simulate 3-cnt automaton on counters u, v, w

PSpace-hardness

Simple proof for binary 6-VASS

2^n-bounded 3-counter automata are PSpace-hard

Triple (2^n, $6q\ 4^n$, $6q\ 8^n$) sufficient for zero-tests

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simulate 3-cnt automaton on counters u, v, w

($x+u+v+w=2^n$)

PSpace-hardness

Simple proof for binary 6-VASS

2^n-bounded 3-counter automata are PSpace-hard

Triple (2^n, $6q\ 4^n$, $6q\ 8^n$) sufficient for zero-tests

$x{+}{=}2^n$ $y{+}{=}6q\ 4^n$ $z{+}{=}6q\ 8^n$

simulate 3-cnt automaton on counters u, v, w

target: $z = 0$ ($x+u+v+w=2^n$)

Applications

Applications

used in our Ackermann-hardness

Applications

used in our Ackermann-hardness

many small dimension results

Applications

used in our Ackermann-hardness

many small dimension results

incomparable with other techniques

Pushdown VASS

Pushdown VASS

general

Pushdown VASS

general decidability conjectured

Pushdown VASS

general

decidability conjectured

lower bounds from VASSes

Pushdown VASS

general	decidability conjectured
	lower bounds from VASSes

equivalent to grammars with counters

Pushdown VASS

general — decidability conjectured

lower bounds from VASSes

equivalent to grammars with counters

dimension one

Pushdown VASS

general — decidability conjectured

lower bounds from VASSes

equivalent to grammars with counters

dimension one — decidability not known

Pushdown VASS

general — decidability conjectured

lower bounds from VASSes

equivalent to grammars with counters

dimension one — decidability not known

PSpace-hardness

Pushdown VASS

general — decidability conjectured

lower bounds from VASSes

equivalent to grammars with counters

dimension one — decidability not known

PSpace-hardness

Example: d+1 nonterminals, runs of length $\geq F_d(n)$

Pushdown VASS

Pushdown VASS

$S \longrightarrow n\ X$

Pushdown VASS

$S \longrightarrow n\ X$

$X \longrightarrow -1\ X\ 2 \mid 0$

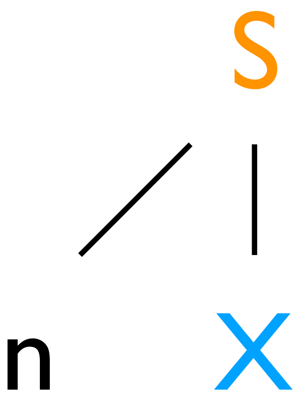
Pushdown VASS

S

$$S \longrightarrow n\ X$$

$$X \longrightarrow -1\ X\ 2 \mid 0$$

Pushdown VASS



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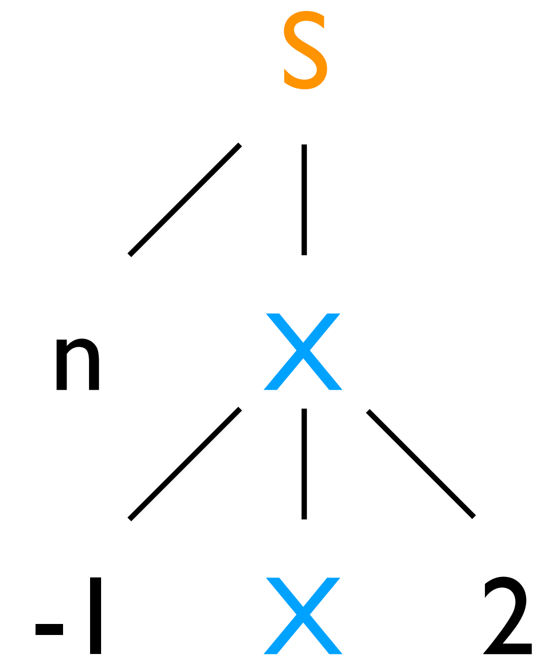

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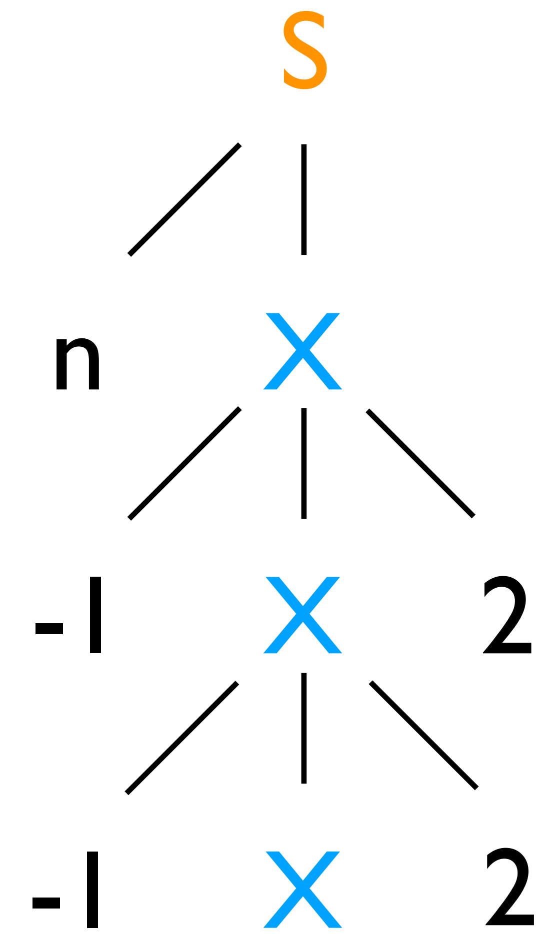

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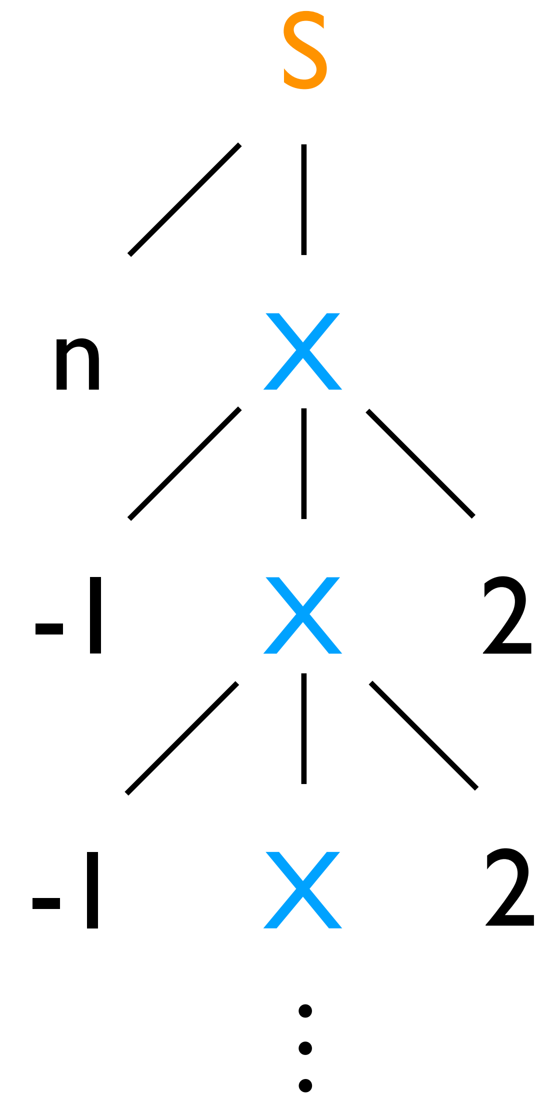

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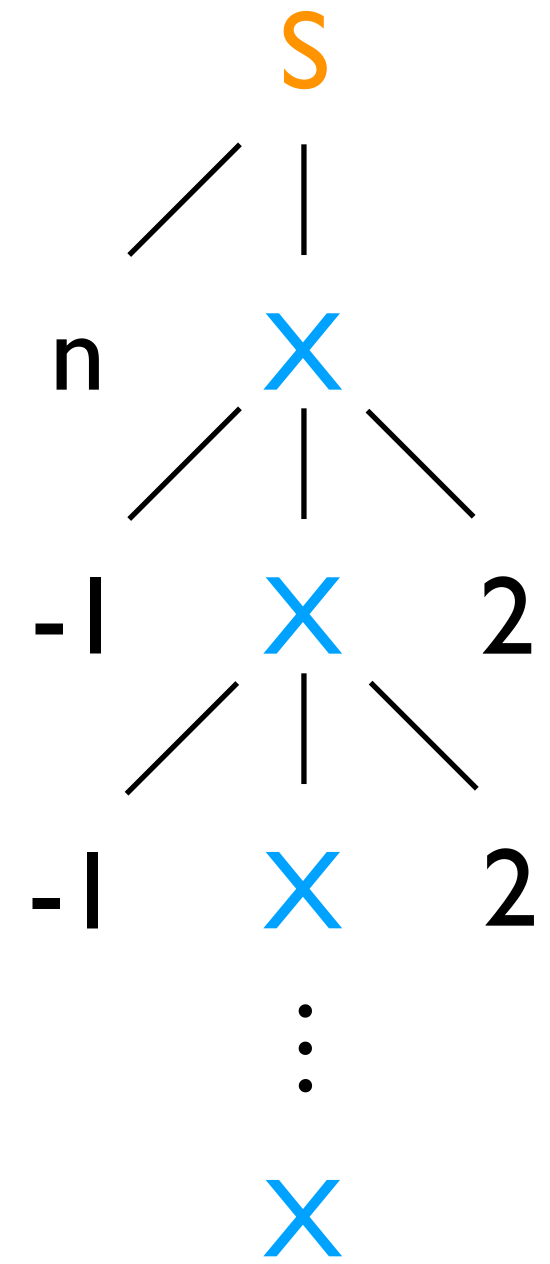

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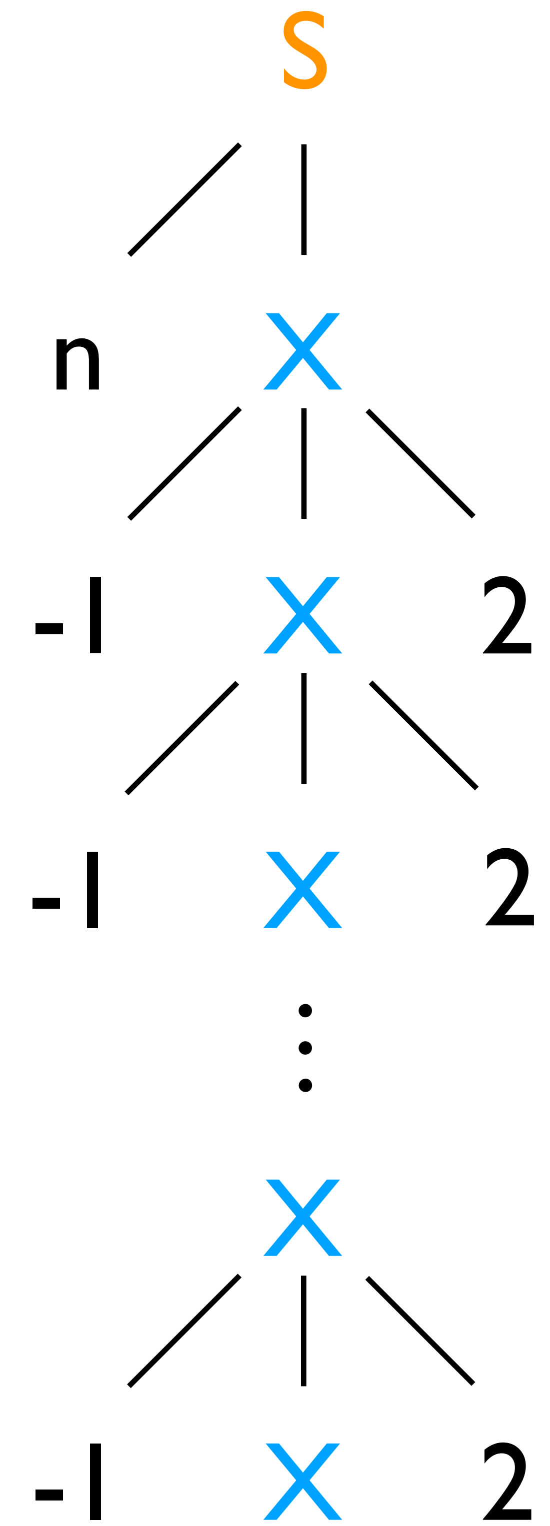
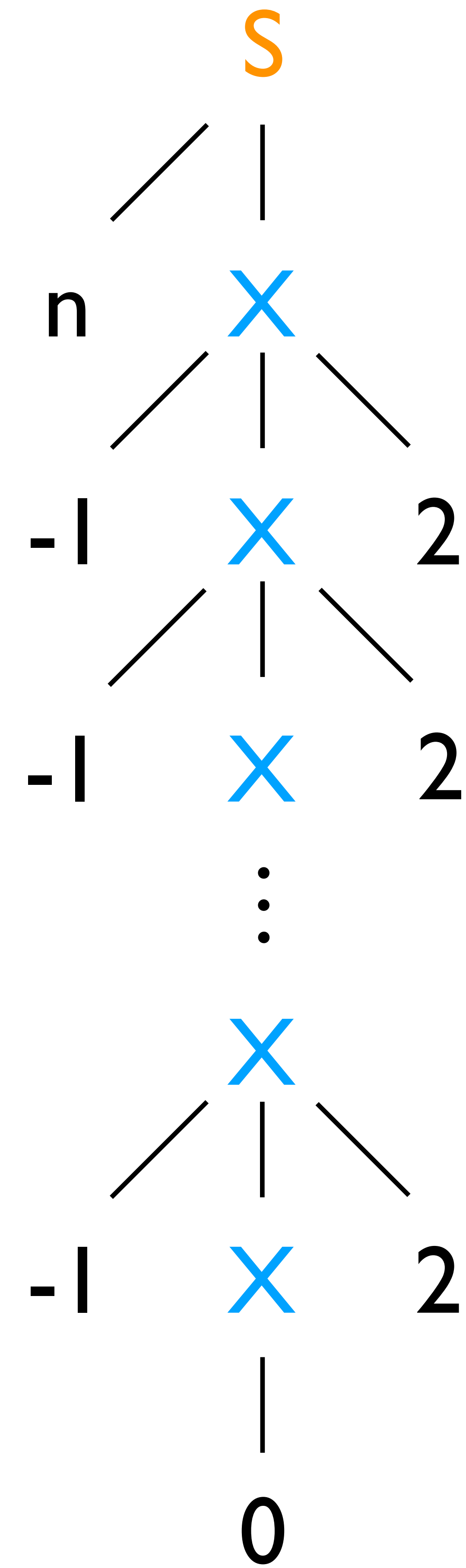


Pushdown VASS

S ⟶ n X

X ⟶ -1 X 2 | 0

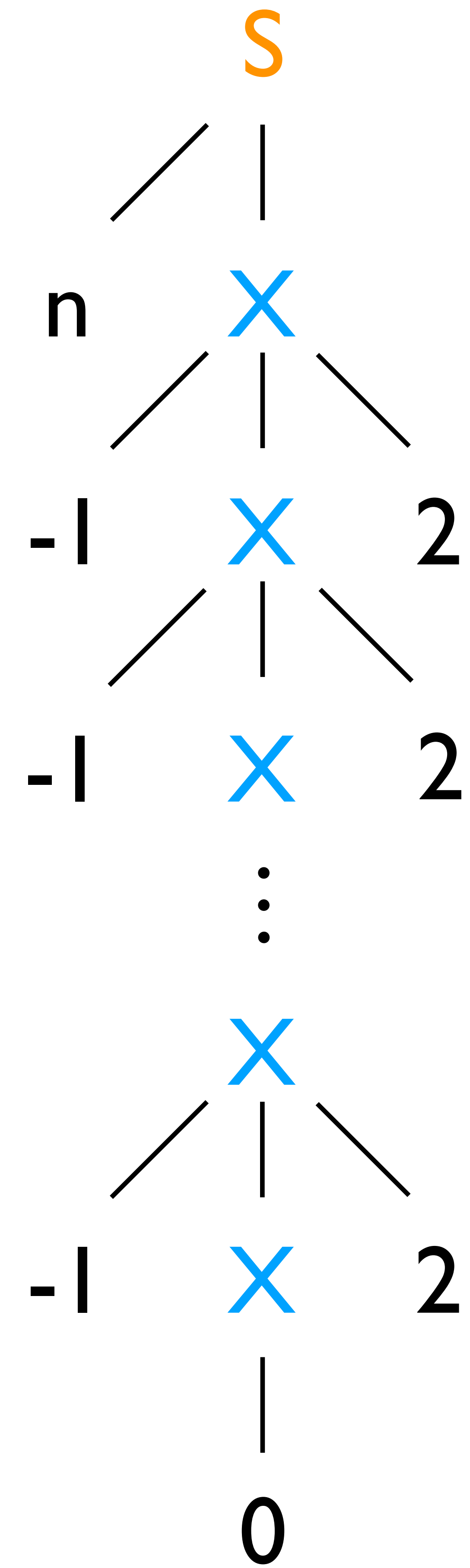


Pushdown VASS

$S \longrightarrow n\ X$

$X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$



Pushdown VASS

Pushdown VASS

$S \longrightarrow n\,Y$

Pushdown VASS

$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \;|\; 1$

Pushdown VASS

$$S \longrightarrow n\,Y$$

$$Y \longrightarrow -1\;Y\;X \mid 1$$

$$X \longrightarrow -1\;X\;2 \mid 0$$

Pushdown VASS

S ⟶ n Y

Y ⟶ -1 Y X | 1

X ⟶ -1 X 2 | 0

Y

Pushdown VASS

$$S \longrightarrow n\,Y$$

$$Y \longrightarrow -1\;Y\;X \mid 1$$

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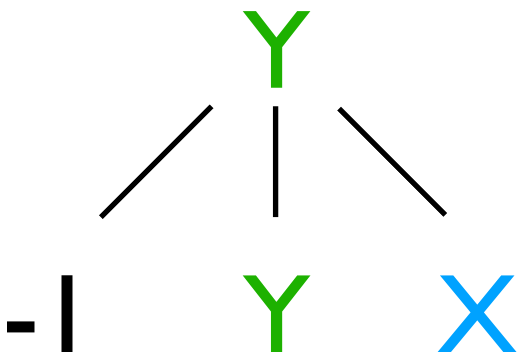

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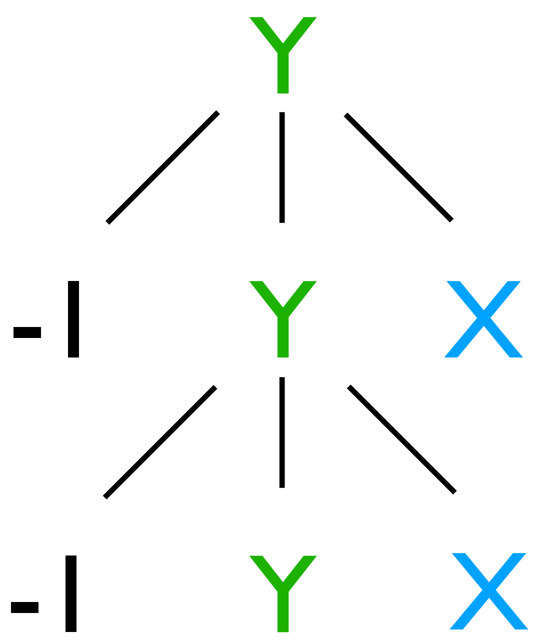

Pushdown VASS

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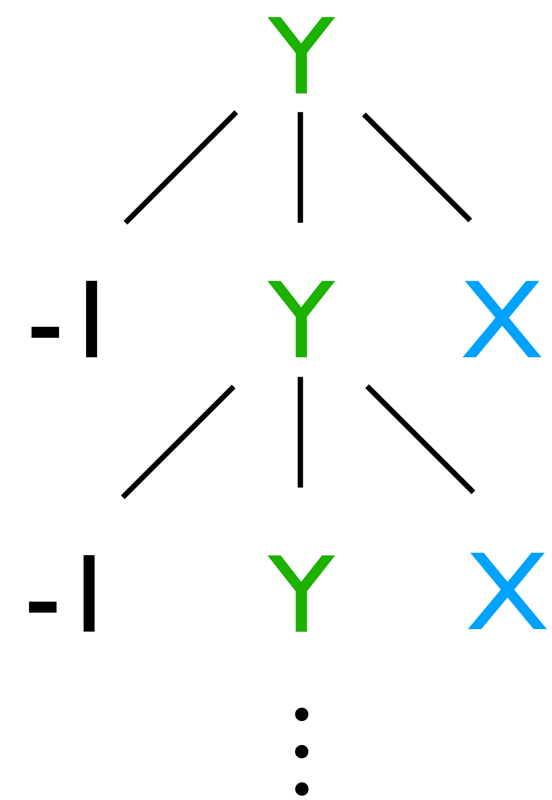
Y ⟶ -1 Y X | 1

X ⟶ -1 X 2 | 0



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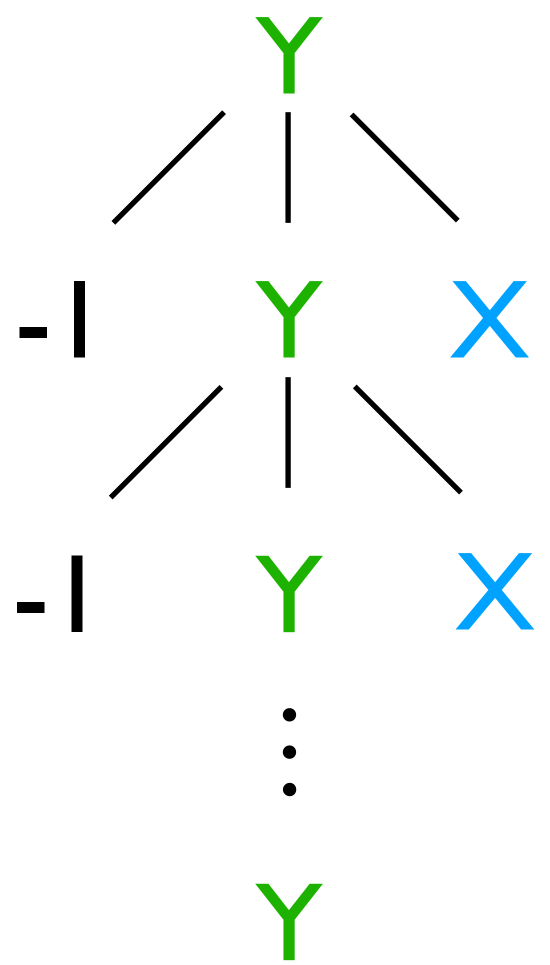

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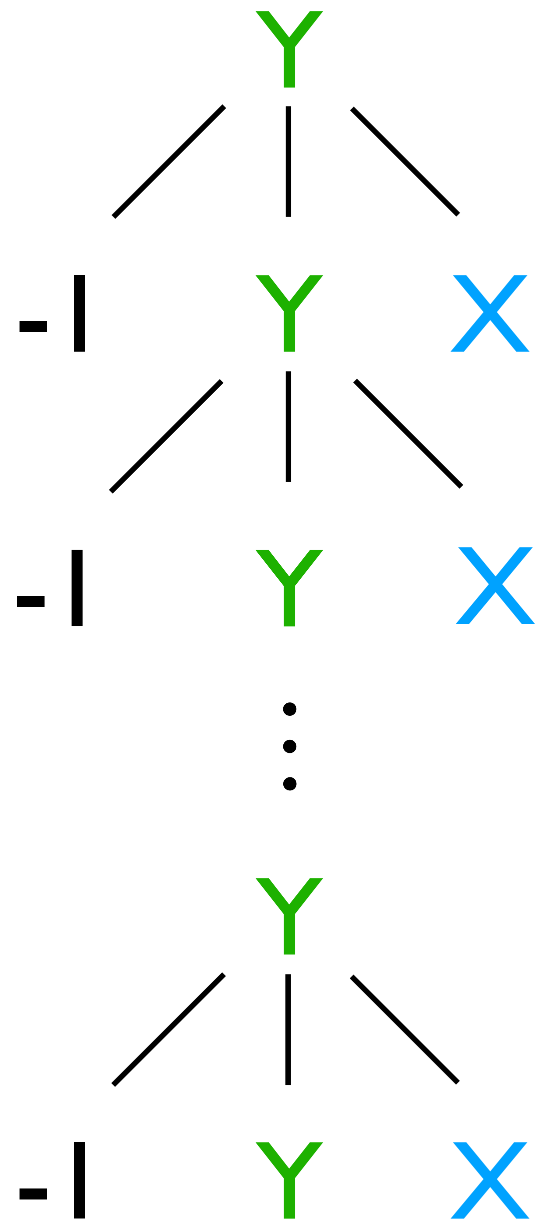

Pushdown VASS

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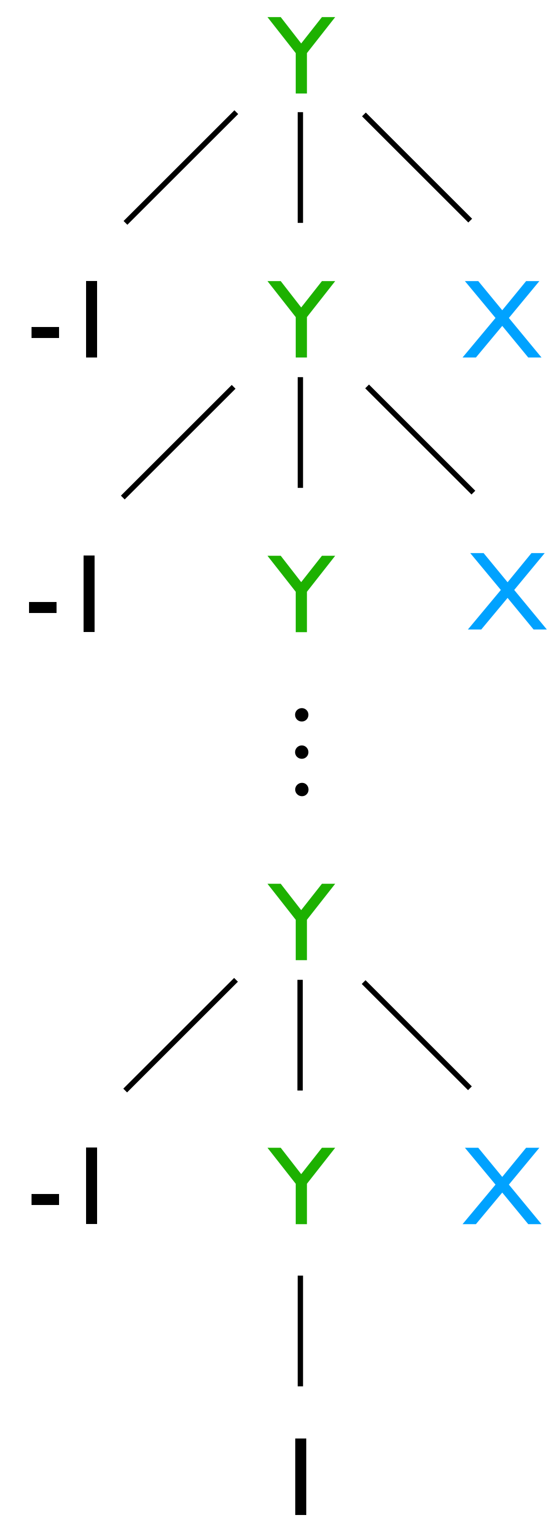

Pushdown VASS

$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

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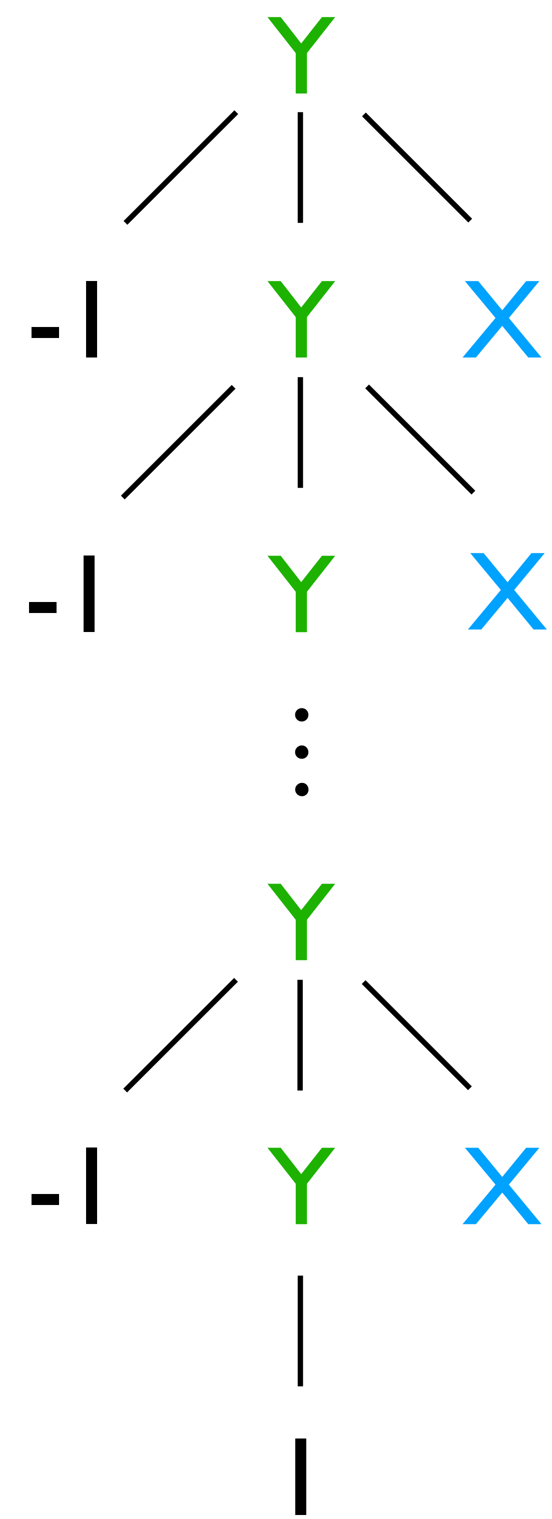
$$k \xrightarrow{X} 2k$$



Pushdown VASS

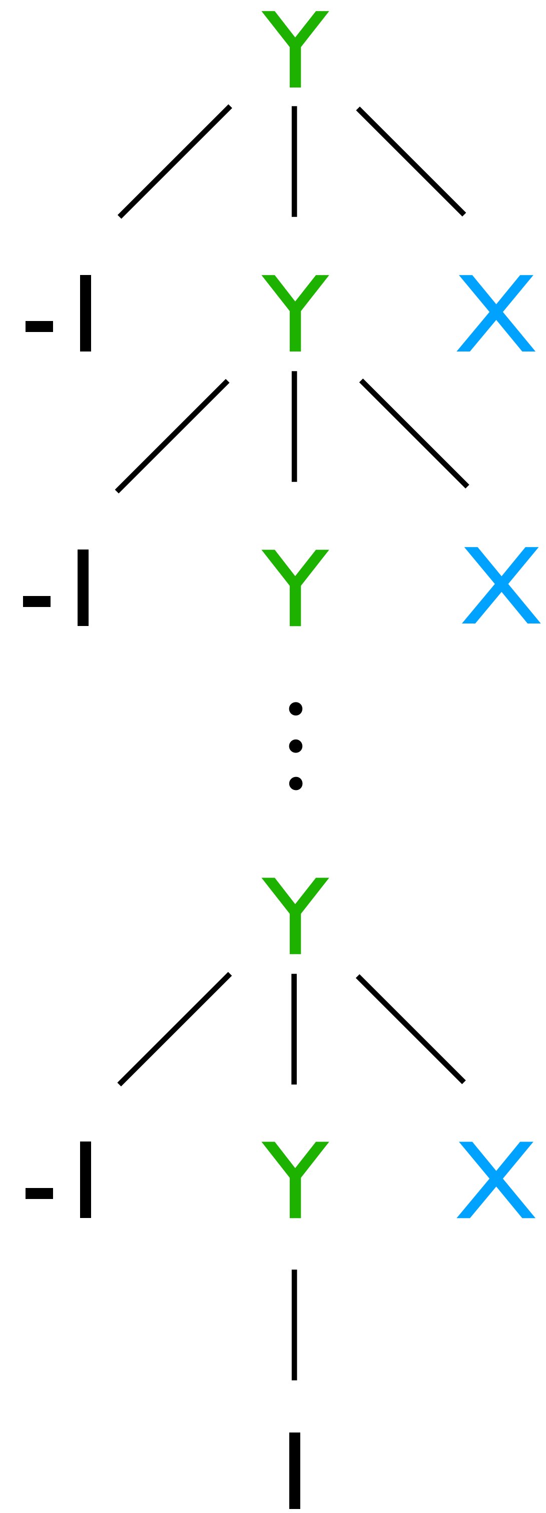
$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

$X \longrightarrow -1\;X\;2 \mid 0$

$k \xrightarrow{X} 2k$

$k \xrightarrow{Y} 2^k$



Pushdown VASS

Pushdown VASS

$S \longrightarrow n\ Z$

Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

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Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

$X \longrightarrow -1\ X\ 2 \mid 0$

$$k \xrightarrow{X} 2k$$

Pushdown VASS

$S \longrightarrow n\ Z$ $\qquad$ $Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$ $\qquad$ $X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$ $\qquad$ $k \xrightarrow{Y} 2^k$

Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

$X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$

$k \xrightarrow{Y} 2^k$

$k \xrightarrow{Z} \text{Tower}(k)$

Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

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$k \xrightarrow{Z} \text{Tower}(k)$

d+1 nonterminals: weak computation of $F_d(n)$

Pushdown VASS

$S \longrightarrow n\,Z$ $\qquad$ $Z \longrightarrow -1\,Z\,Y \mid 1$

$Y \longrightarrow -1\,Y\,X \mid 1$ $\qquad$ $X \longrightarrow -1\,X\,2 \mid 0$

$k \xrightarrow{X} 2k$ $\qquad$ $k \xrightarrow{Y} 2^k$ $\qquad$ $k \xrightarrow{Z} \text{Tower}(k)$

d+1 nonterminals: weak computation of $F_d(n)$

how to make it exact?

Message

Message

many open problems

Message

many open problems

look at small dimensions

Message

many open problems

look at small dimensions

hardness $\approx$ big reachability set + enforcing

Message

many open problems

look at small dimensions

hardness ≈ big reachability set + enforcing

Thank you!