Techniques for Unambiguous Systems

Wojciech Czerwiński

GandALF 2022

Plan

Plan

- unambiguity

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- unambiguity
- equivalence of FA

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- equivalence of FA
  - weighted automata (Schutzenberger `61)

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- equivalence of VASSes

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  - lookahead technique (Cz., Hofman `22)

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- unambiguity
- equivalence of FA
  - weighted automata (Schutzenberger `61)
- equivalence of VASSes
  - lookahead technique (Cz., Hofman `22)
- connections to other fields

Unambiguity

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for each word there is at most one accepting run

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more expressive than deterministic

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many problems become simpler

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mathematically interesting

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recently a lot of research

Simplicity

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many problems become simpler:

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universality for UFA (PTime vs PSpace)

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universality for UFA (PTime vs PSpace)

equivalence for UFA (PTime vs PSpace)

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equivalence for URA (ExpTime vs undec)

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universality for UFA (PTime vs PSpace)

equivalence for UFA (PTime vs PSpace)

equivalence for URA (ExpTime vs undec)

equivalence for UVASS (Ackermann vs undec)

UFA equivalence

UFA equivalence

reduction to weighted automata

UFA equivalence

reduction to weighted automata

reduction to zeroness problem

UFA equivalence

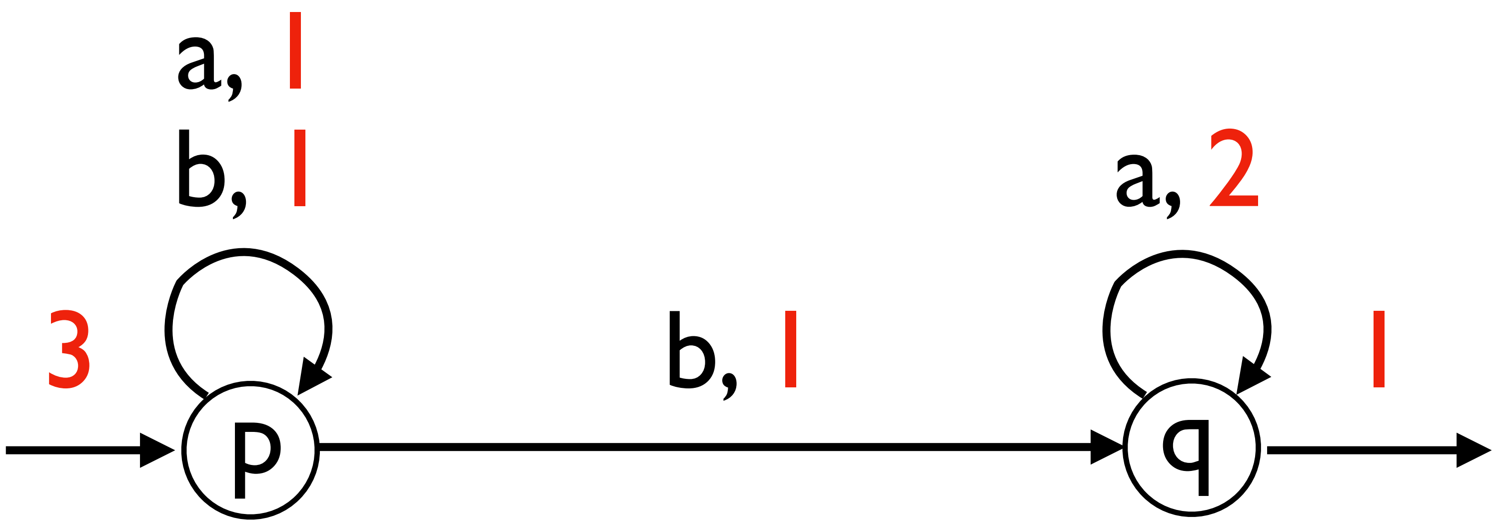
reduction to weighted automata

reduction to zeroness problem

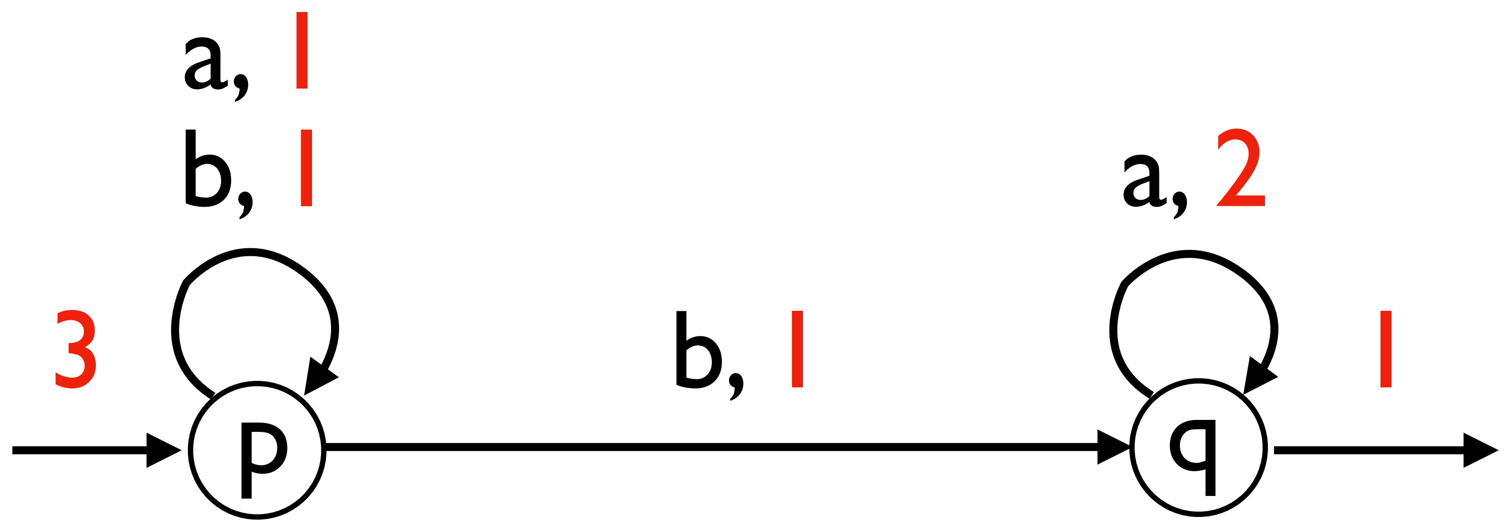
solving zeroness

Weighted automata

Weighted automata

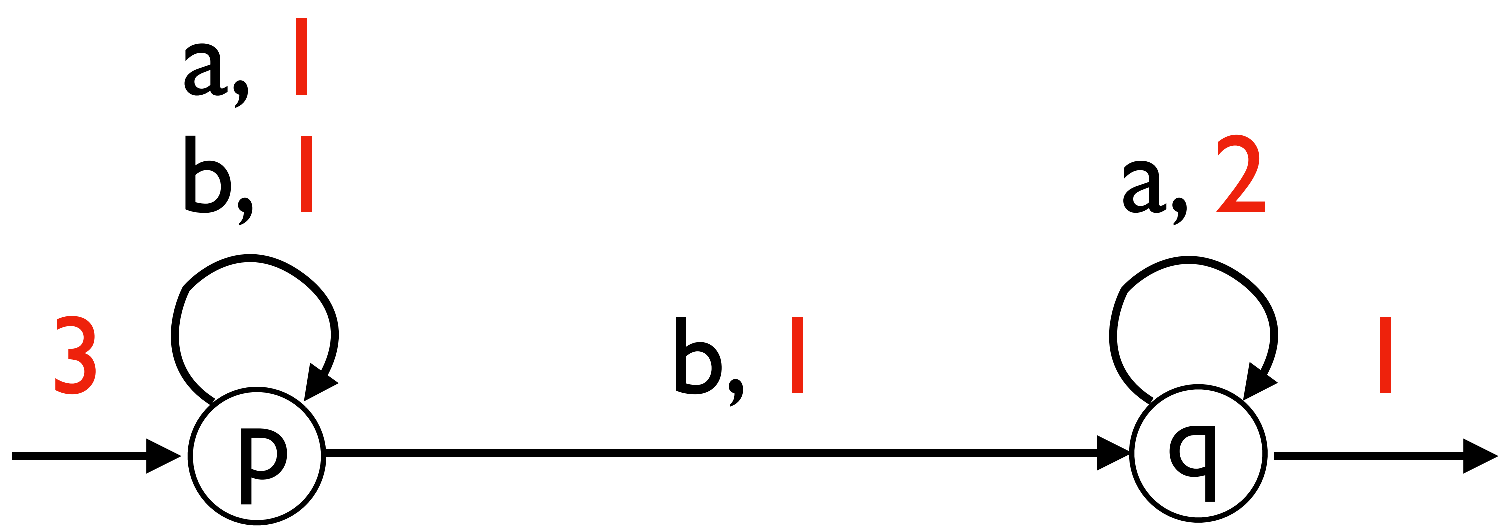


Weighted automata



weight of a run = product of transition weights

Weighted automata



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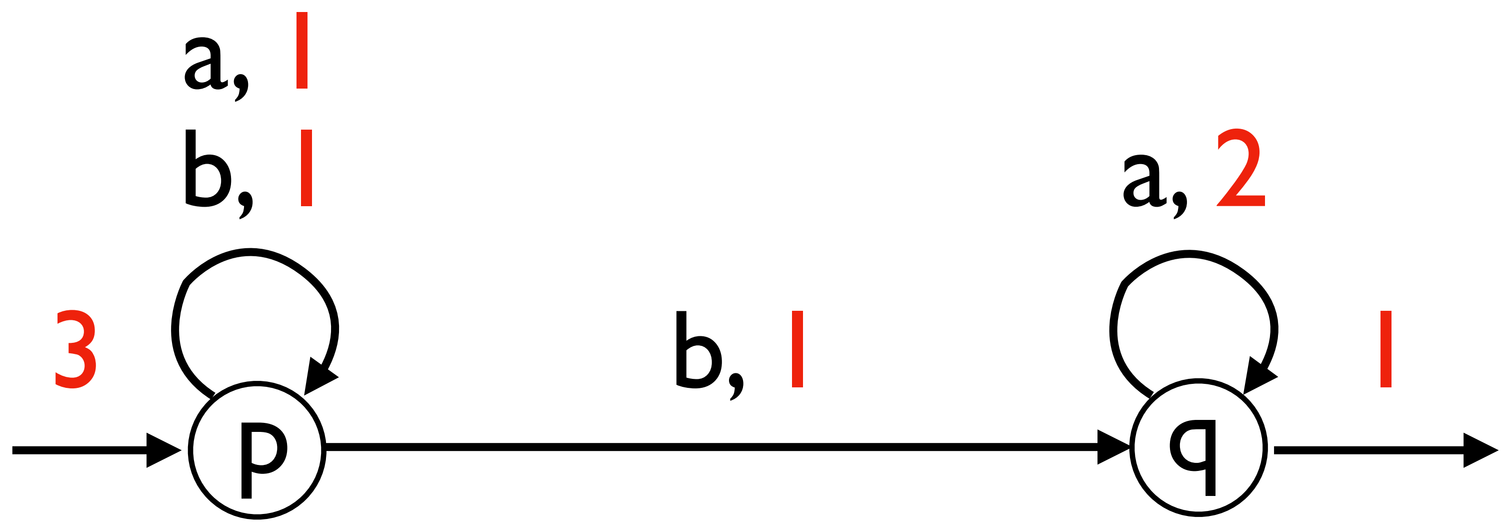
Weighted automata



weight of a run = product of transition weights

weight of a word = sum of run weights

w(abbaabaaa) = 24

UFA to WA

UFA to WA

L(A) = L(B) ⇔ A' and B' equivalent

UFA to WA

$L(A) = L(B) \Leftrightarrow A'$ and B' equivalent

each transition has weight 1

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input weights of initial states are 1

output weights of final states are 1

Zeroness

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A’ and B’ equivalent $\Leftrightarrow$ A’(w)-B’(w) is constantly zero

Zeroness

A’ and B’ equivalent ⇔ A’(w)-B’(w) is constantly zero

construct C such that C(w) = A’(w)-B’(w)

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enough to check zeroness for WA!

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solves also multiplicity equivalence!

Solving zeroness

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Fix A with n states

Solving zeroness

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Consider vec(w) in $\mathbb{Z}^n$

Solving zeroness

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Consider $\mathrm{vec}(w)$ in $\mathbb{Z}^n$

Let V_k be spanned by $\mathrm{vec}(w)$ for words of length at most k

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$$\mathrm{weight}(w) = \mathrm{out}(\mathrm{vec}(w))$$

Solving zeroness

Fix A with n states

Consider $\mathrm{vec}(w)$ in $\mathbb{Z}^n$

Let V_k be spanned by $\mathrm{vec}(w)$
for words of length at most k

$$\mathrm{weight}(w) = \mathrm{out}(\mathrm{vec}(w))$$

$$A \text{ is zero} \Leftrightarrow \text{all } V_k \subseteq \ker(\mathrm{out})$$

Solving zeroness

Fix A with n states

Consider vec(w) in $\mathbb{Z}^n$

Let V_k be spanned by vec(w)
for words of length at most k

weight(w) = out(vec(w))

A is zero $\Leftrightarrow$ all $V_k \subseteq$ ker(out)

Enough to compute V_k

Solving zeroness

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compute V_{k+1} from V_k

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V_{k+1} is generated by

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A is zero $\Leftrightarrow$ all $V_k \subseteq \ker(\mathrm{out})$

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compute V_{k+1} from V_k

V_{k+1} is generated by generators of V_k after transitions

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and generators of V_k

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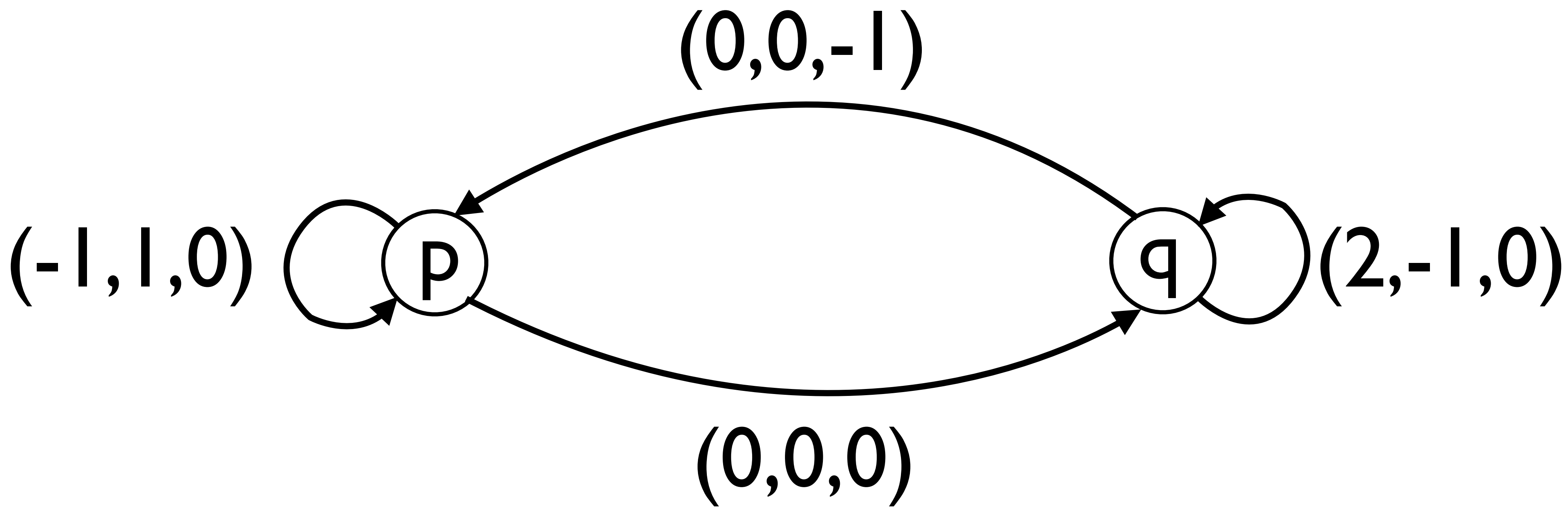
algorithm: compute V_k until stabilisation

check if included in $\ker(\text{out})$

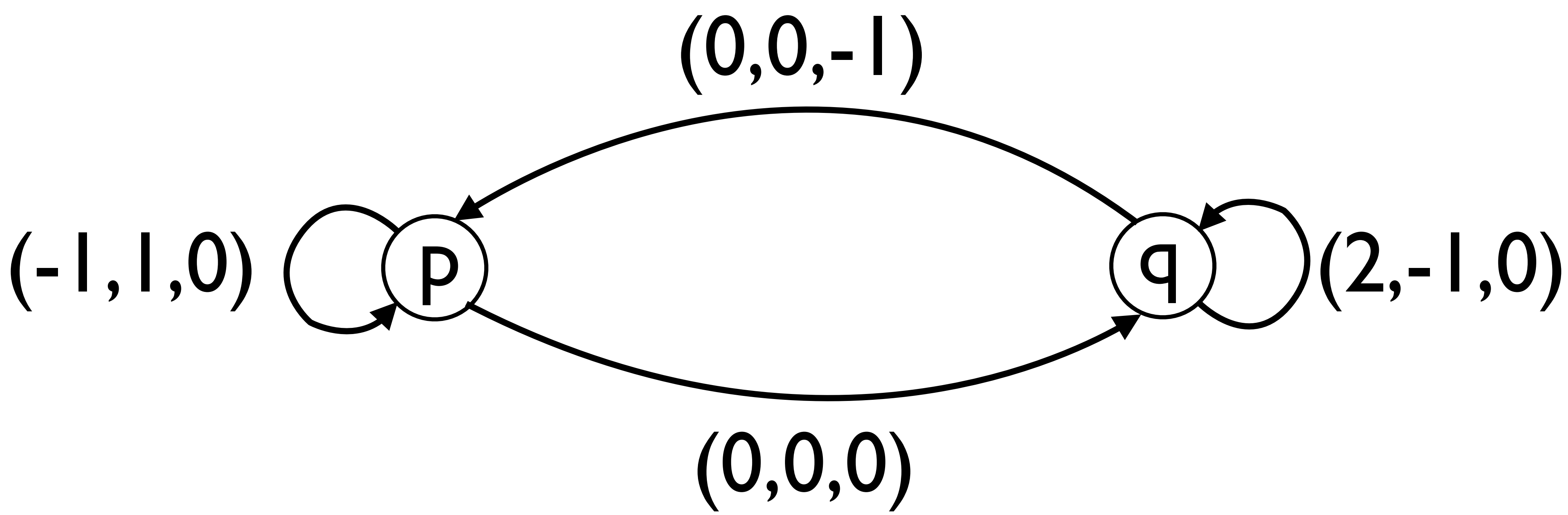
In PTime!

Vector Addition Systems with States

Vector Addition Systems with States

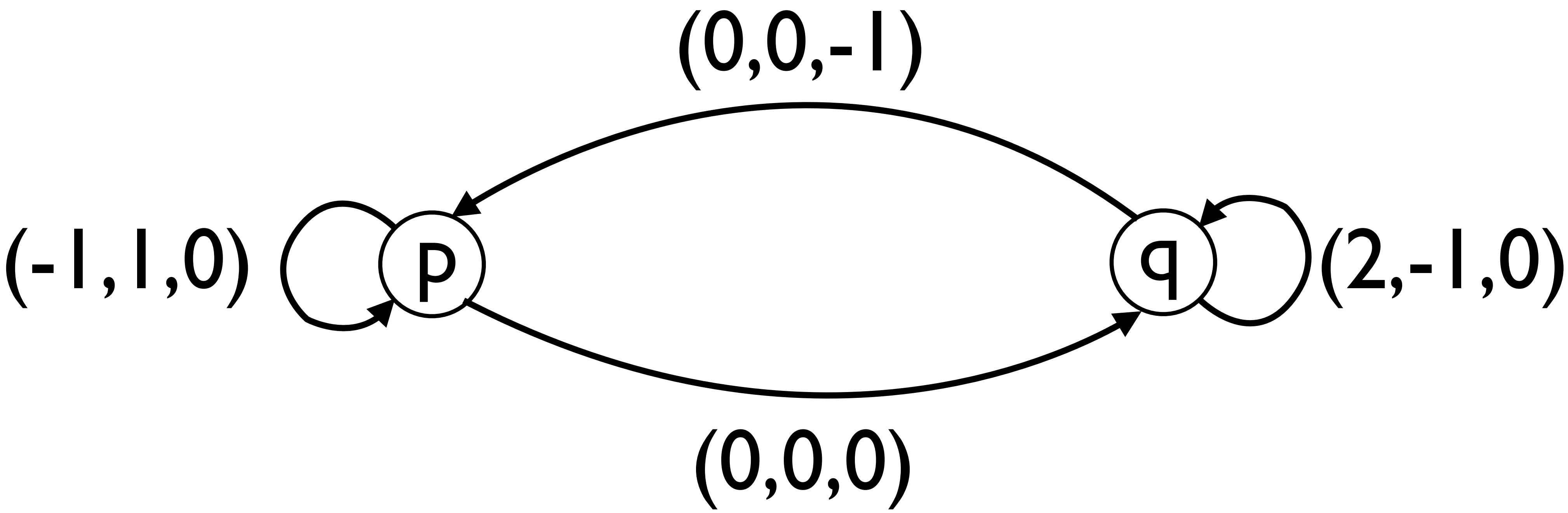


Vector Addition Systems with States



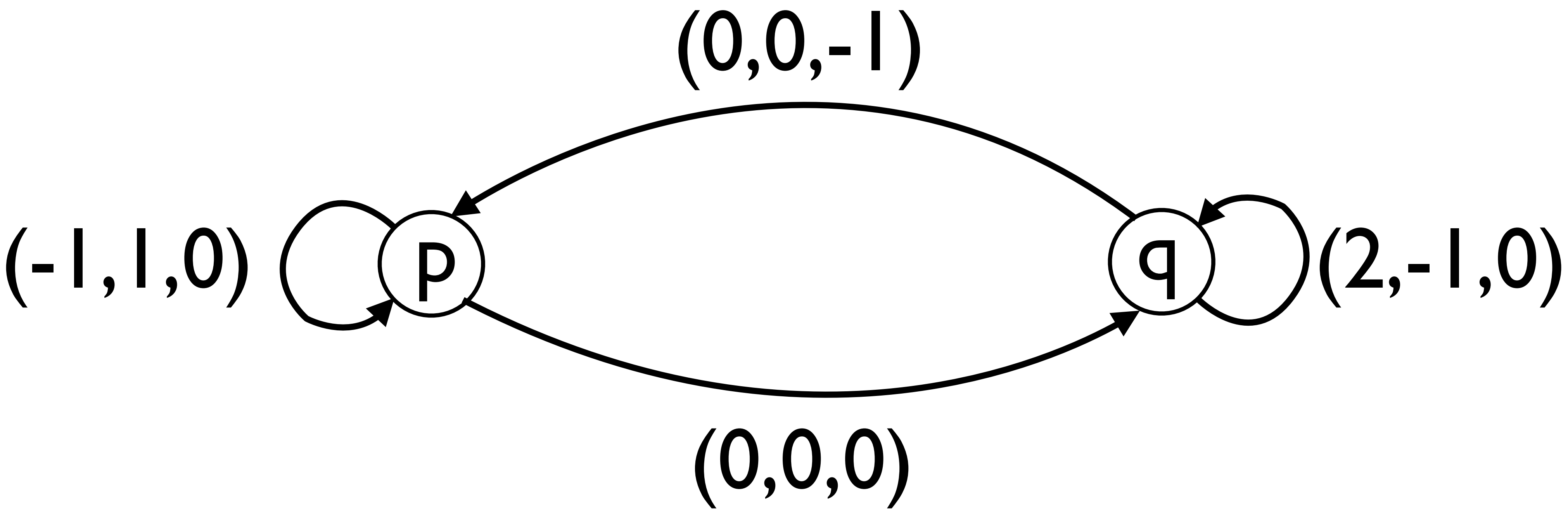
p(2,0,7)

Vector Addition Systems with States



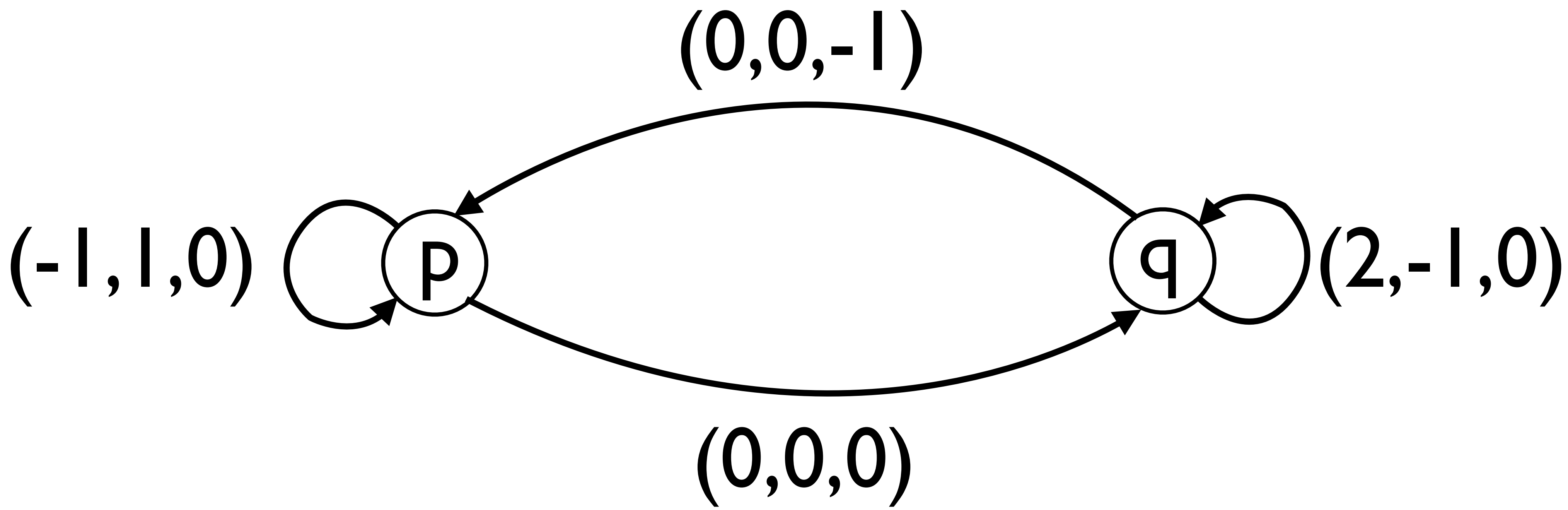
$p(2,0,7) \longrightarrow p(1,1,7)$

Vector Addition Systems with States



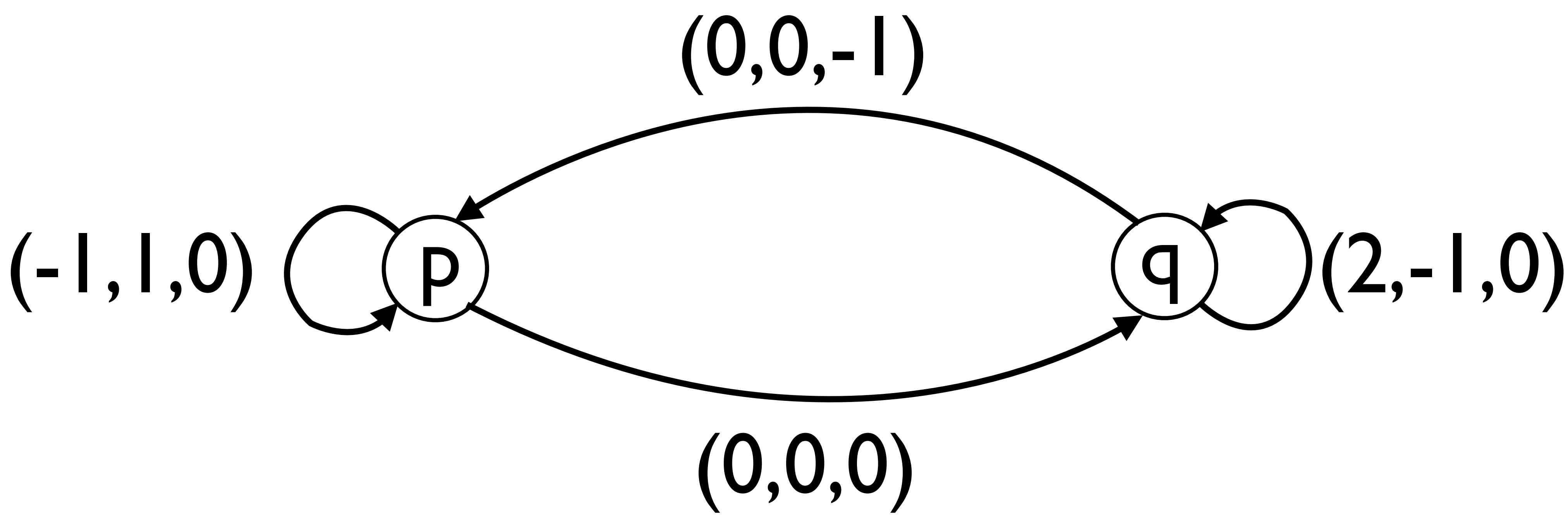
$$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7)$$

Vector Addition Systems with States



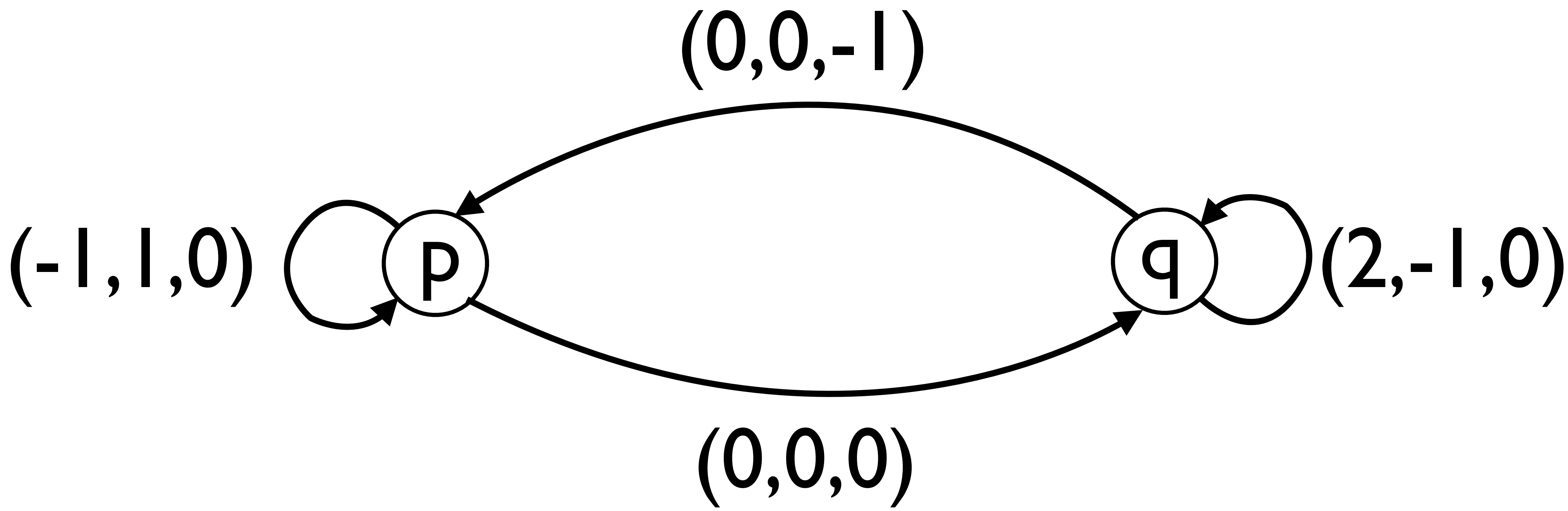
$$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7)$$

Vector Addition Systems with States



$$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7) \longrightarrow q(2,1,7)$$

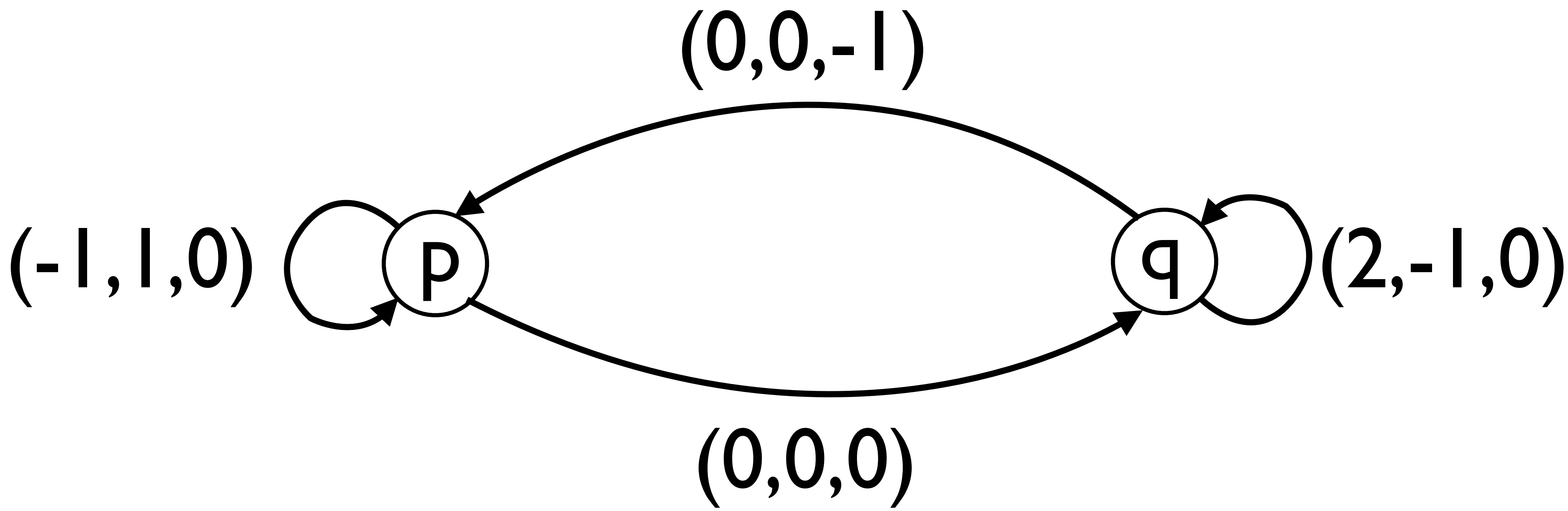
Vector Addition Systems with States



$$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7) \longrightarrow q(2,1,7)$$

$$\longrightarrow q(4,0,7)$$

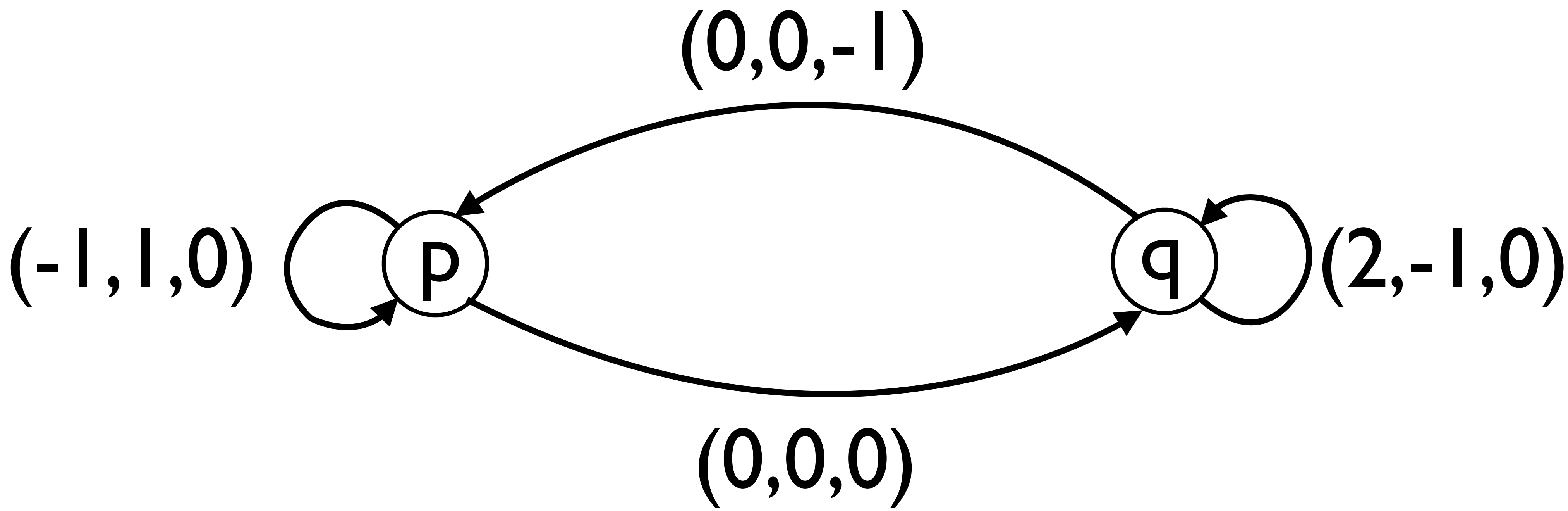
Vector Addition Systems with States



$$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7) \longrightarrow q(2,1,7)$$

$$\longrightarrow q(4,0,7) \longrightarrow p(4,0,6)$$

Vector Addition Systems with States



$$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7) \longrightarrow q(2,1,7)$$

$$\longrightarrow q(4,0,7) \longrightarrow p(4,0,6)$$

Petri nets

Language of a VASS

Language of a VASS

VASS = vector addition system with states

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initial configuration, acceptance by states

Language of a VASS

VASS = vector addition system with states

initial configuration, acceptance by states

detVASS = each reachable configuration is deterministic

Equivalence for VASS

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for 2-dimensional VASSes - undecidable

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decidable, Ackermann-complete

Equivalence for VASS

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decidable, Ackermann-complete   CONCUR 2022

Zeroness?

Zeroness?

zeroness decidable?

Zeroness?

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multiplicity equivalence undecidable for VASSes

Zeroness?

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Petr Jancar 2001
Nonprimitive recursive complexity and undecidability for Petri net equivalences

Zeroness?

zeroness decidable?

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reduction to a deterministic case!

Idea

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add information to get determinism

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add information to get determinism lookahead

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does the suffix belong to several fixed regular languages?

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best captured by a monoid

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best captured by a monoid $h: \Sigma^* \to M$

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For each regular L over Σ
there are $h: \Sigma^* \to M$ and $F \subseteq M$
such that $L = h^{-1}(F)$.

Reduction

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Let h: $\Sigma^* \rightarrow M$

Reduction

Let h: $\Sigma^* \to M$ Then map_h: $\Sigma^* \to (\Sigma \times M)^*$

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$map_h(a\ b\ c) = (*, h(abc))\ (a, h(bc))\ (b, h(c))\ (c, h(\varepsilon))$

Reduction

Let $h\colon \Sigma^* \to M$ Then $map_h\colon \Sigma^* \to (\Sigma \times M)^*$

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Claim 1: For each $h\colon \Sigma^* \to M$ it holds

$$K \subseteq L \Leftrightarrow map_h(K) \subseteq map_h(L)$$

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Claim 2: For each $h: \Sigma^* \to M$ and each VASS V one can construct another VASS V_h accepting $map_h(L(V))$

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Goal: choose h such that both V_h are deterministic

Reduction

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$Q \times M \cup \{(init, \$)\}$

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States are pairs in
$Q \times M \cup \{(\mathrm{init}, \$)\}$

initial is $(\mathrm{init}, \$)$

final are $F \times \{h(\varepsilon)\}$

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For a transition

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For a transition $p \xrightarrow[v]{a} q$

We create $(p,m) \xrightarrow[v]{(a,m')} (q,m')$

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For a transition $p \xrightarrow[v]{a} q$

We create $(p,m) \xrightarrow[v]{(a,m')} (q,m')$

if $m = h(a)\ m'$ or $m = \$, a = *$

Separability

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Languages K and L are regular-separable
if there is a regular S such that
$K \subseteq S$ and $L \cap S = \varnothing$.

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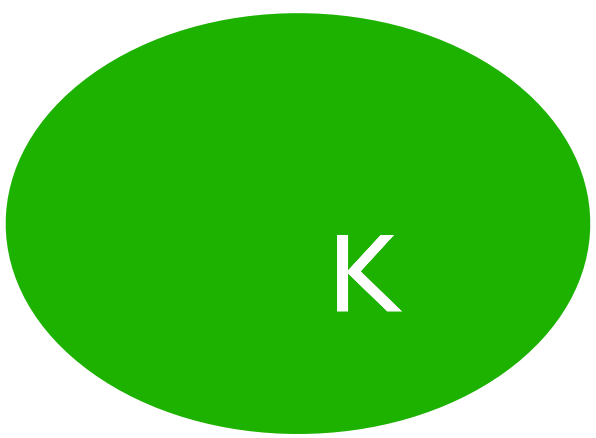

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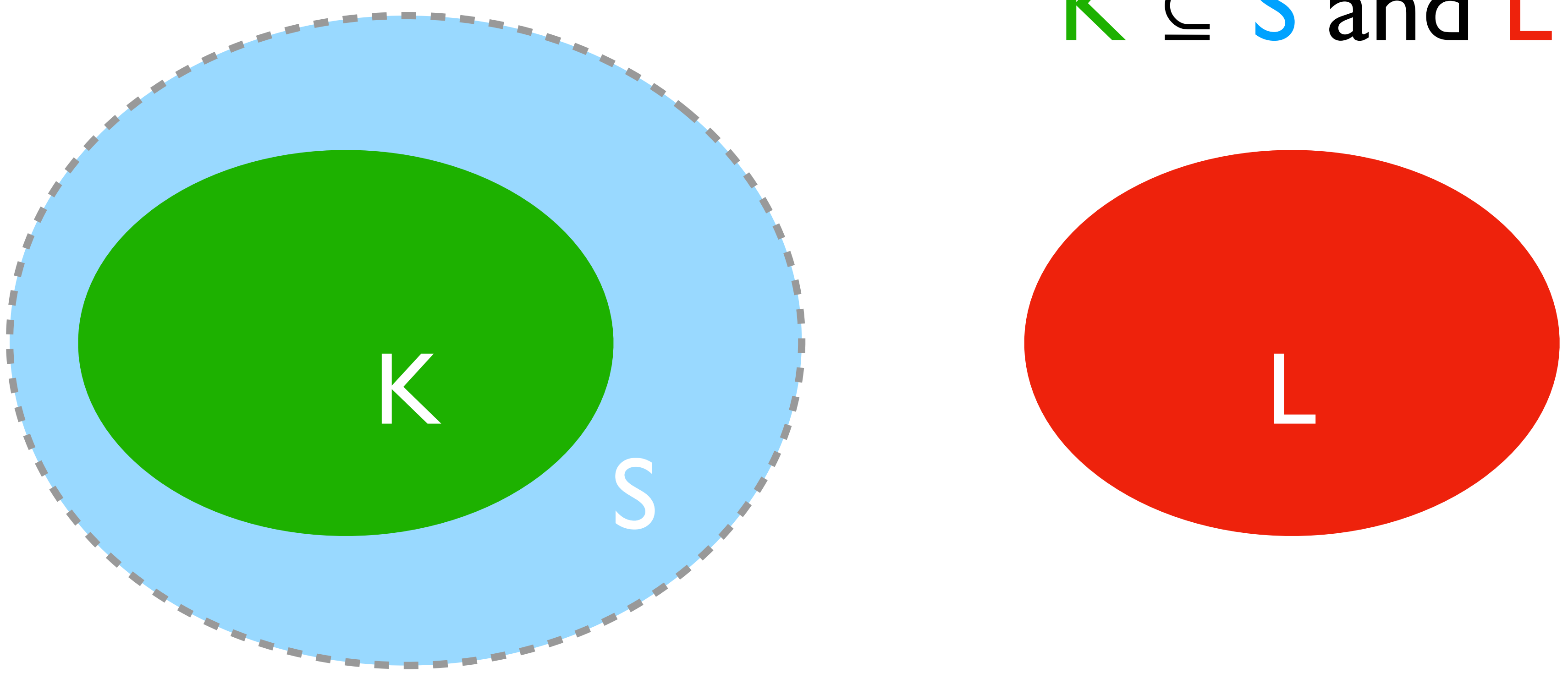

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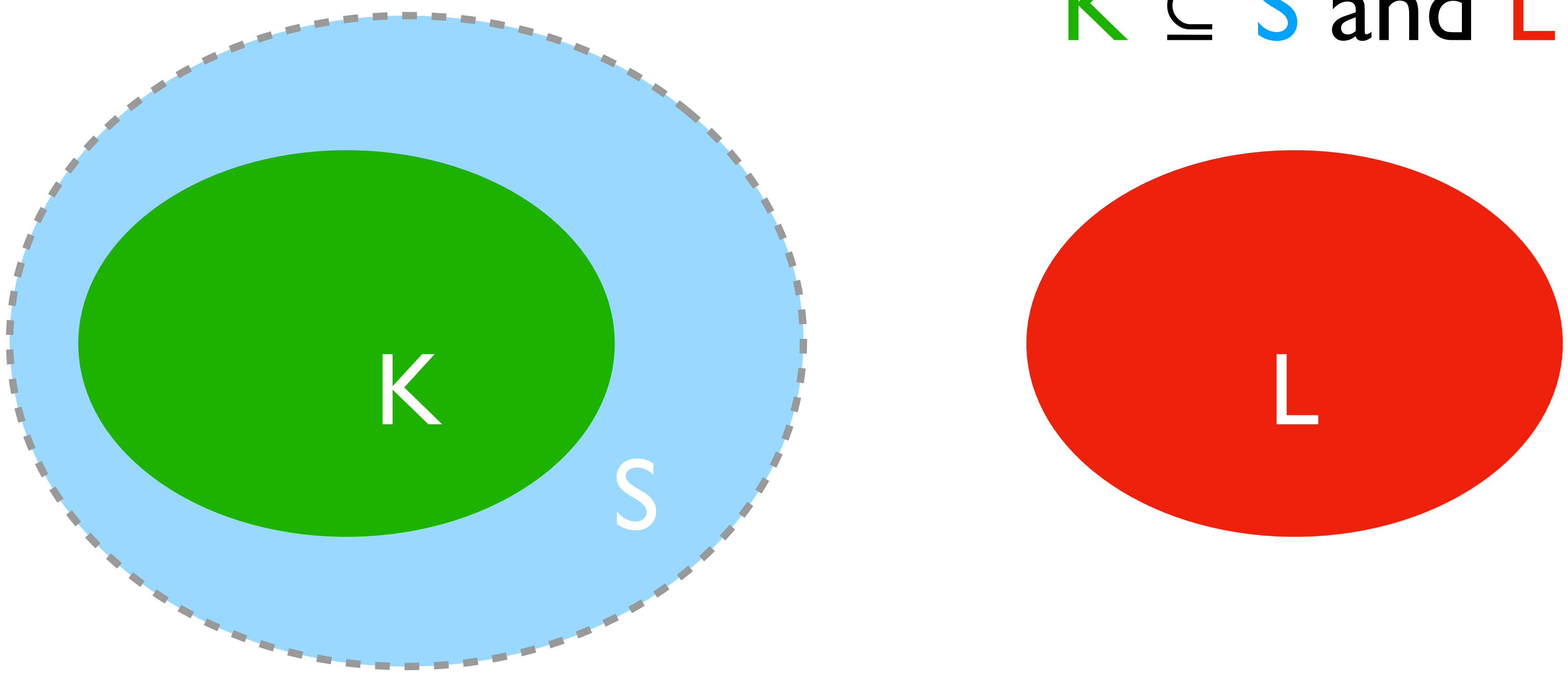

Theorem [Cz., Lasota, Meyer, Muskalla, Kumar, Saivasan]
Each two disjoint VASS languages are regular-separable.

Separability

Separability

Theorem For each VASS V there is a finite family of regular languages $\mathcal{F}_V$ such that if $L(c_1)$ and $L(c_2)$ are disjointed then they are separable by some language from $\mathcal{F}_V$.

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So language equivalence is decidable for UVASSes

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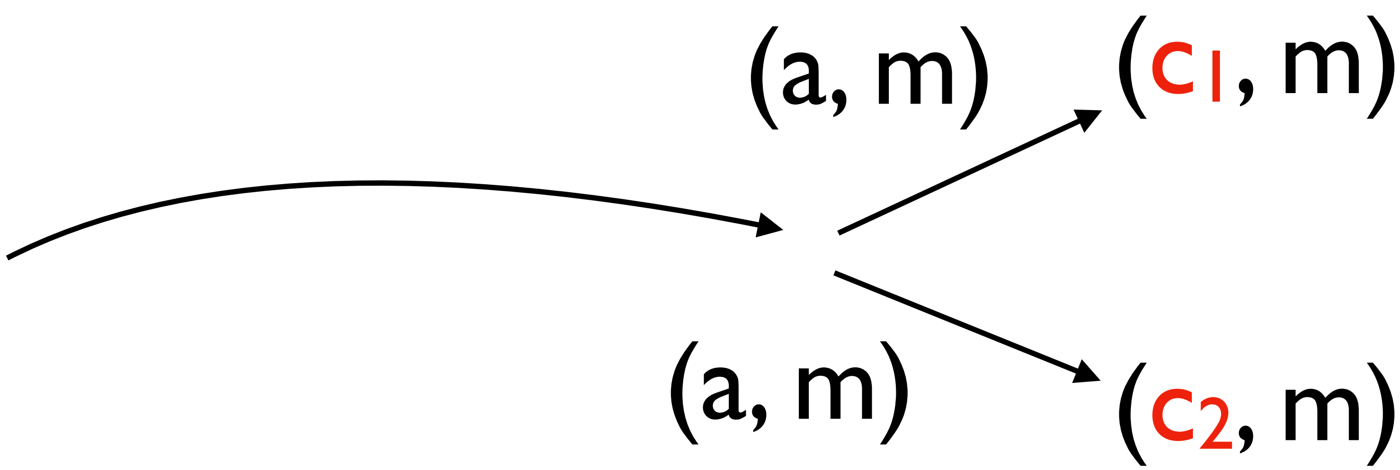

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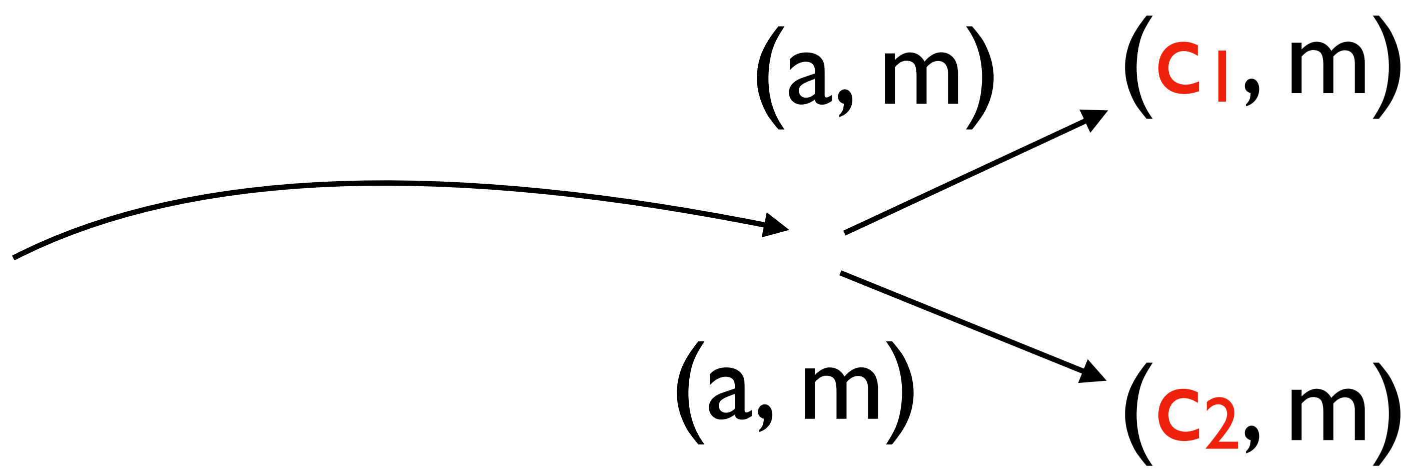

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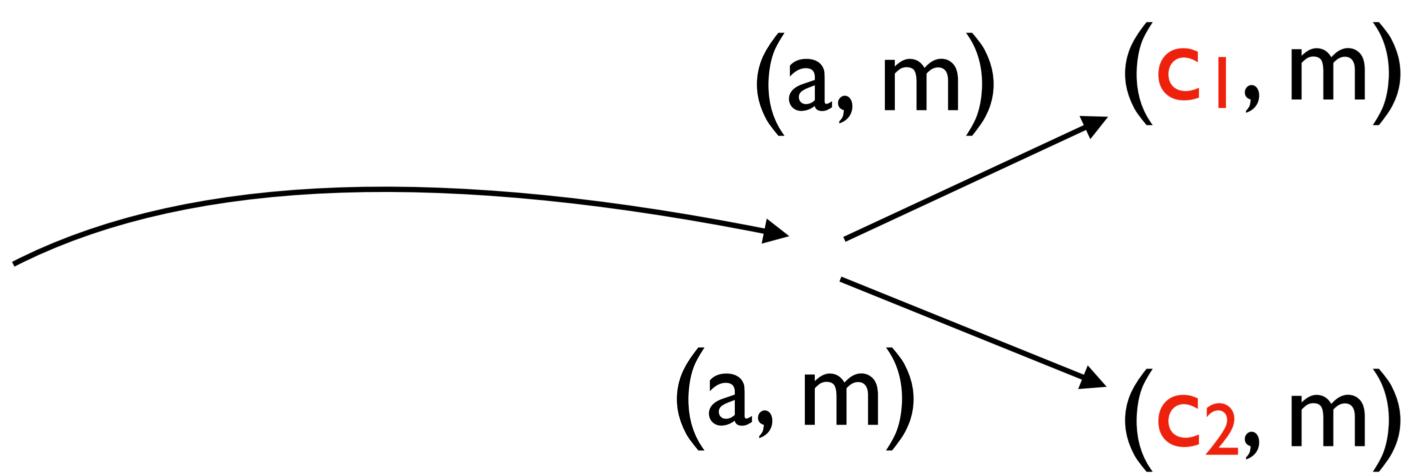

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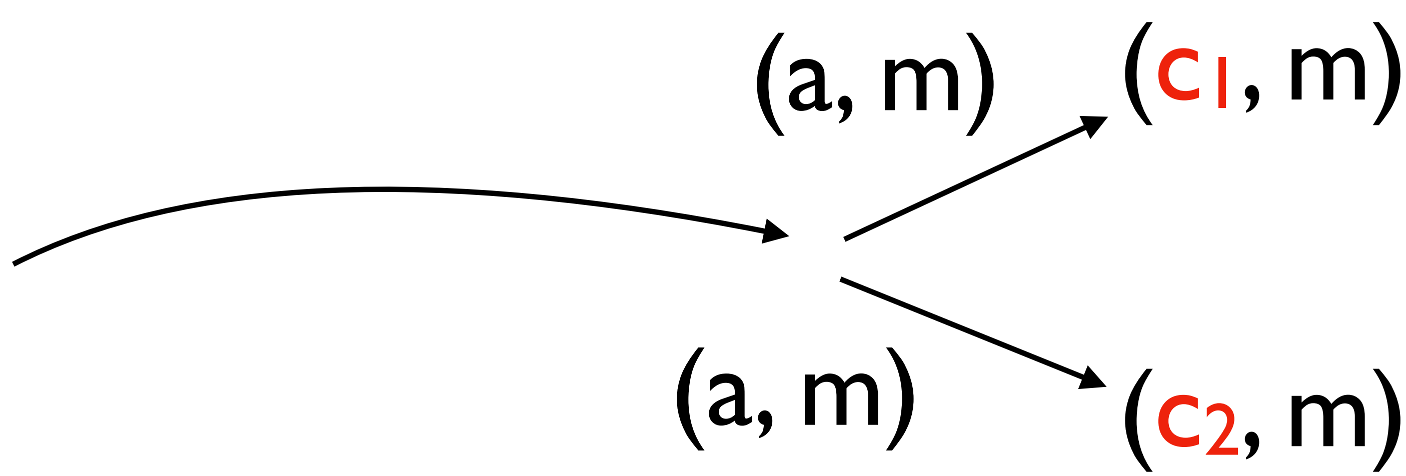

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this contradicts unambiguity!

so V_h is deterministic after removing c with $L(c) = \varnothing$

Multiplicity equivalence

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finite automata: in PTime

Multiplicity equivalence

finite automata: in PTime

context-free grammars: decidability is a big open problem

Multiplicity equivalence

finite automata: in PTime

context-free grammars: decidability is a big open problem

maybe a better semantics?

Multiplicity equivalence

finite automata: in PTime

context-free grammars: decidability is a big open problem

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(Bostan et al., ICALP `2020)

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one $\mathbb{Z}$-counter: connections to complex analysis

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Thank you!