Deciding language equivalence for unambiguous VASSes

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Plan

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- basic notions

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- equivalence for deterministic VASSes

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- regular-separability for VASSes

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- further work

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detVASS = each reachable configuration is deterministic

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Decidable also for extended VASSes
(downward closed set prohibited)

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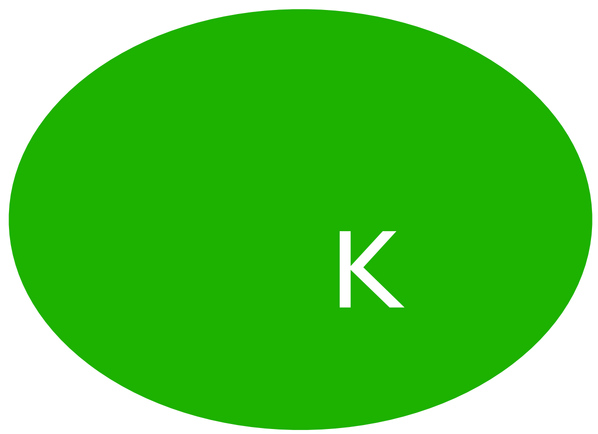

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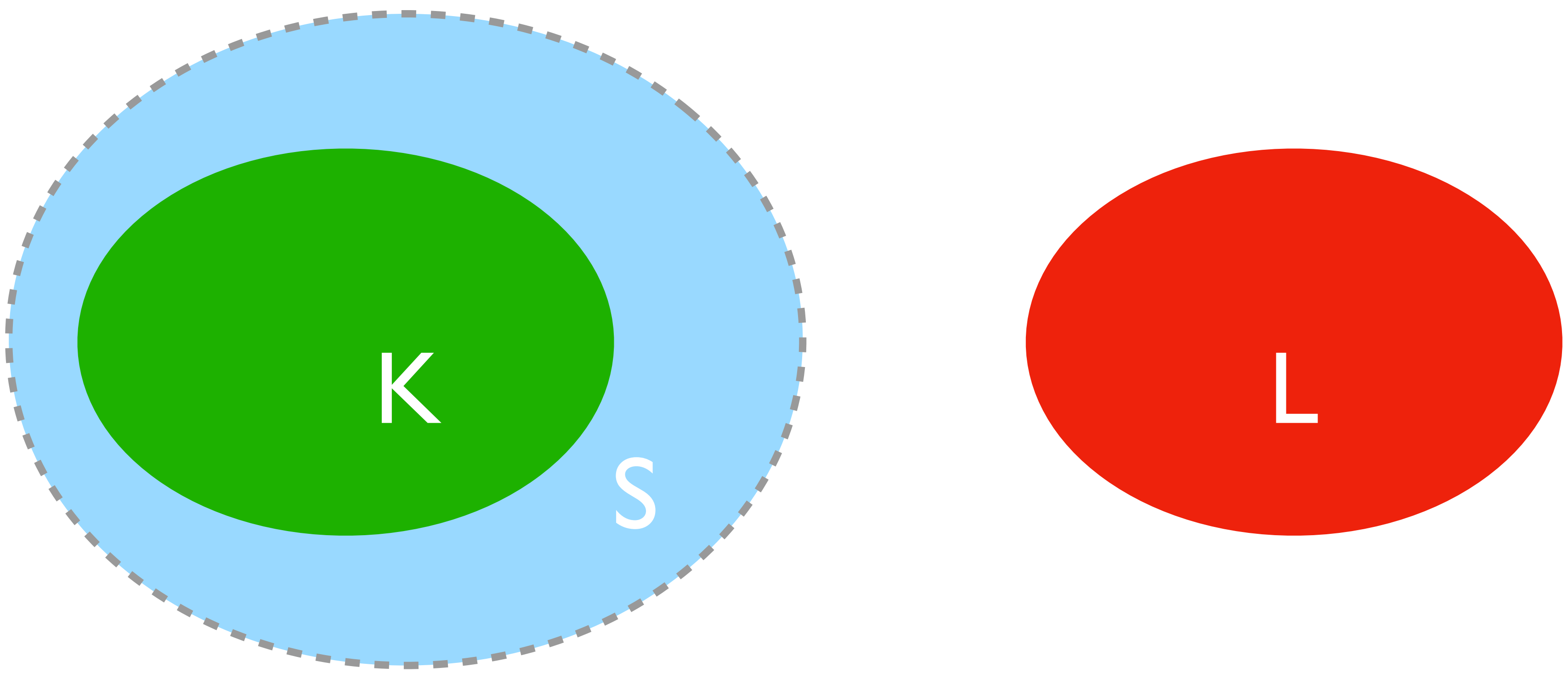

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Thus L(V_1) is regular

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Well Structured Transition Systems (WSTSes):

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- $\preccurlyeq$-bigger configurations can simulate smaller
- $\preccurlyeq$-upward closed acceptance condition

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if $m = h(a)\, m'$ or $m = \$, a = *$

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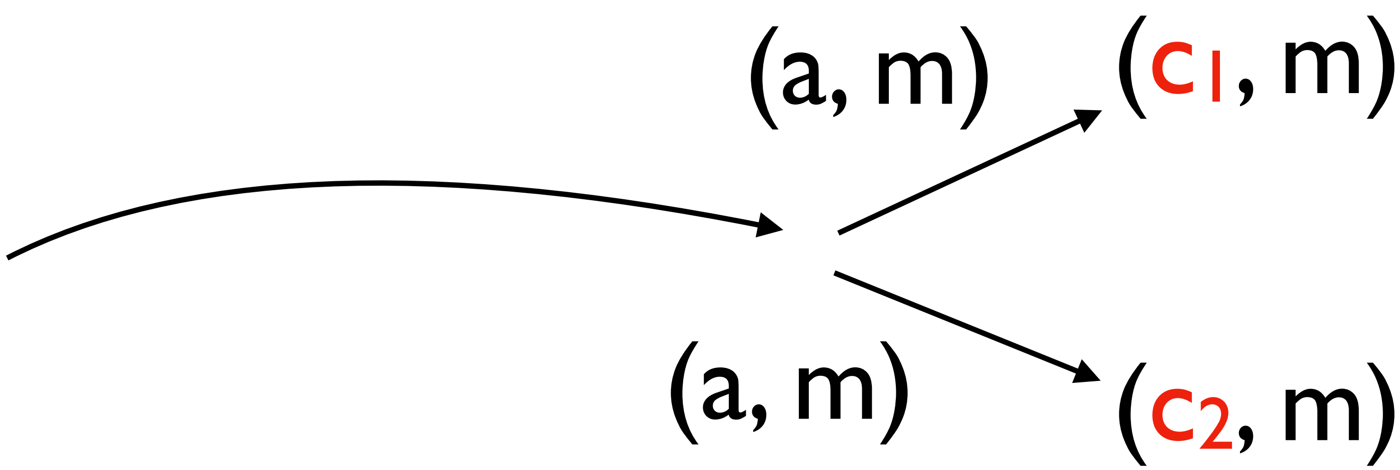

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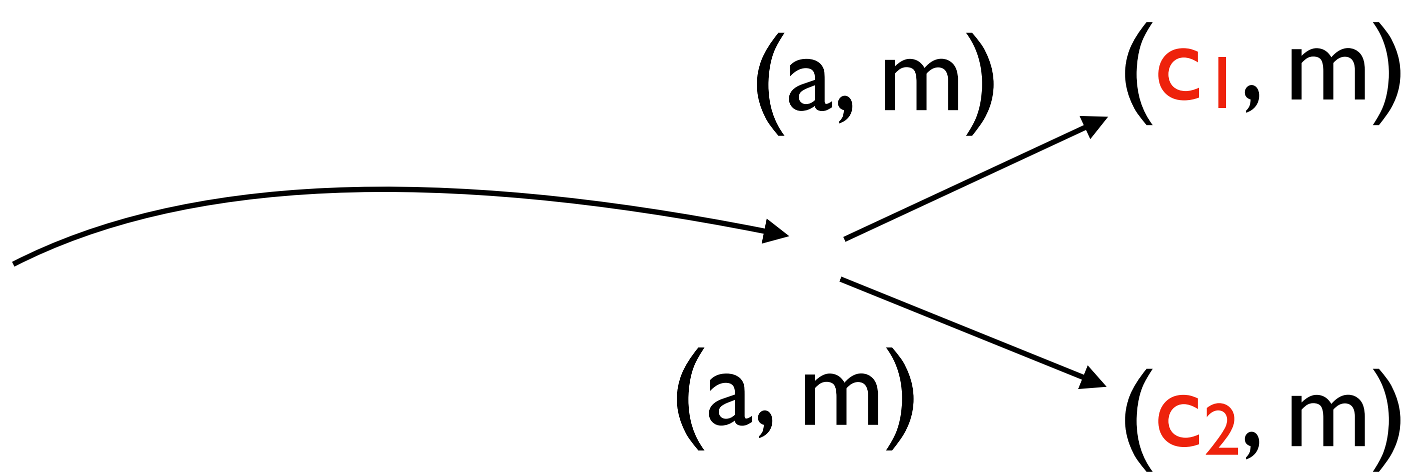

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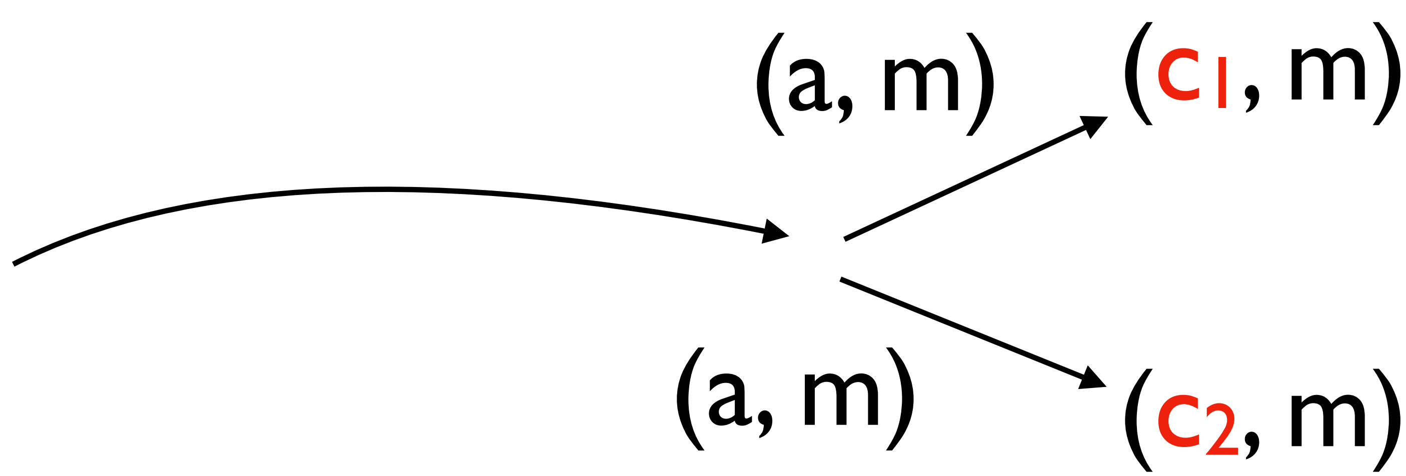

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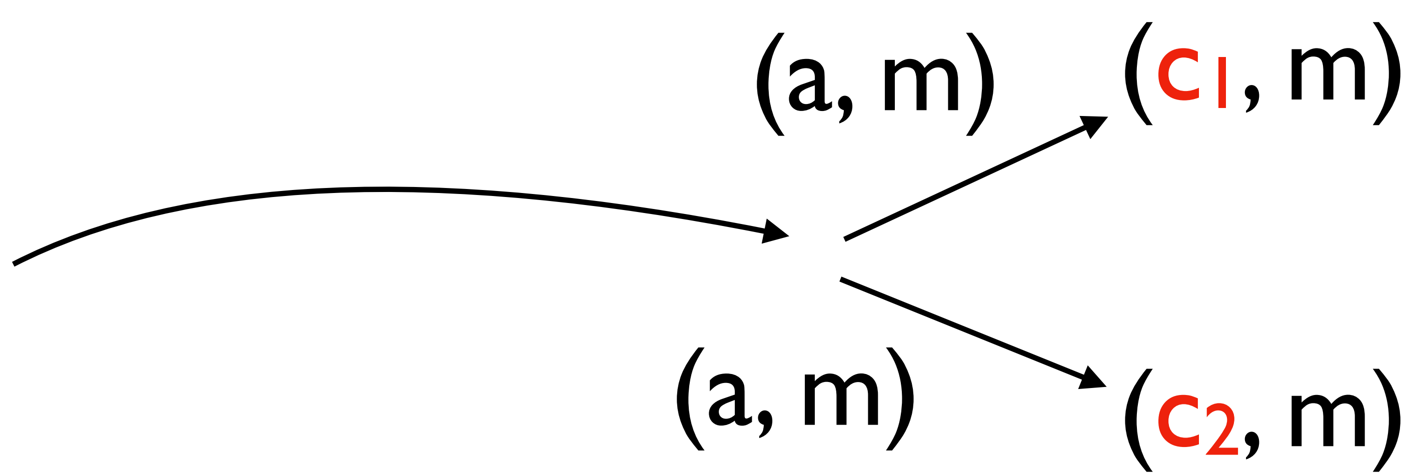

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so V_h is deterministic after removing c with $L(c) = \varnothing$

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Some deeper connection between
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Can weighted models help?

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Thank you!