The Reachability Problem for Computation Models

Wojciech Czerwiński

CLA 2023

Plan

Plan

- basic notions

Plan

- basic notions
- guided tour through computation models

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- basic notions
- guided tour through computation models
- hard examples for simple models

Plan

- basic notions
- guided tour through computation models
- hard examples for simple models
- open problems and message

Computation model

Computation model

Turing machine = automaton with infinite tape

Computation model

Turing machine = automaton with infinite tape

finite automaton

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Turing machine = automaton with infinite tape

finite automaton

pushdown automaton

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Turing machine = automaton with infinite tape

finite automaton

pushdown automaton

automaton with counters

Computation model

Turing machine = automaton with infinite tape

finite automaton

pushdown automaton

automaton with counters

automaton with some structure

Reachability problem

Reachability problem

Given: a model, two its configurations s and t

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Given: a model, two its configurations s and t

Question: is there a run from s to t?

Reachability problem

Given: a model, two its configurations s and t

Question: is there a run from s to t?

Why this problem?

Reachability problem

Given: a model, two its configurations s and t

Question: is there a run from s to t?

Why this problem?

Central one for a computation model

Halting problem for TM

Halting problem for TM

undecidable

Halting problem for TM

undecidable

the same as reachability problem

Halting problem for TM

undecidable

the same as reachability problem

what for other models?

Two-counter automaton

Two-counter automaton

Theorem

The reachability problem for two-counter automaton is undecidable.

Two-counter automaton

Theorem Minsky machine

The reachability problem for two-counter automaton is undecidable.

Two-counter automaton

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Two-counter automaton

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configuration = state + two nonnegative counters

Two-counter automaton

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configuration = state + two nonnegative counters

transition: increments / decrements counters

Two-counter automaton

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The reachability problem for two-counter automaton is undecidable.

configuration = state + two nonnegative counters

transition: increments / decrements counters

zero-tests possible

Proof

Proof

infinite tape = two pushdowns

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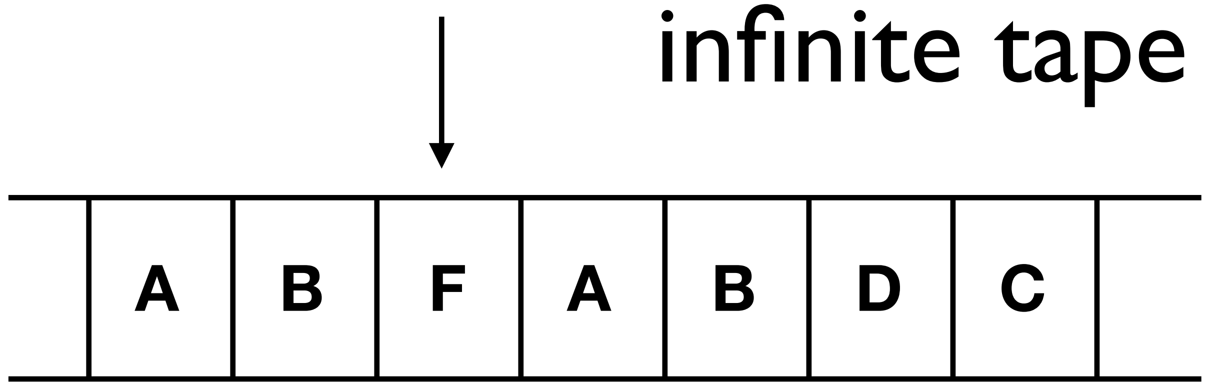
infinite tape = two pushdowns



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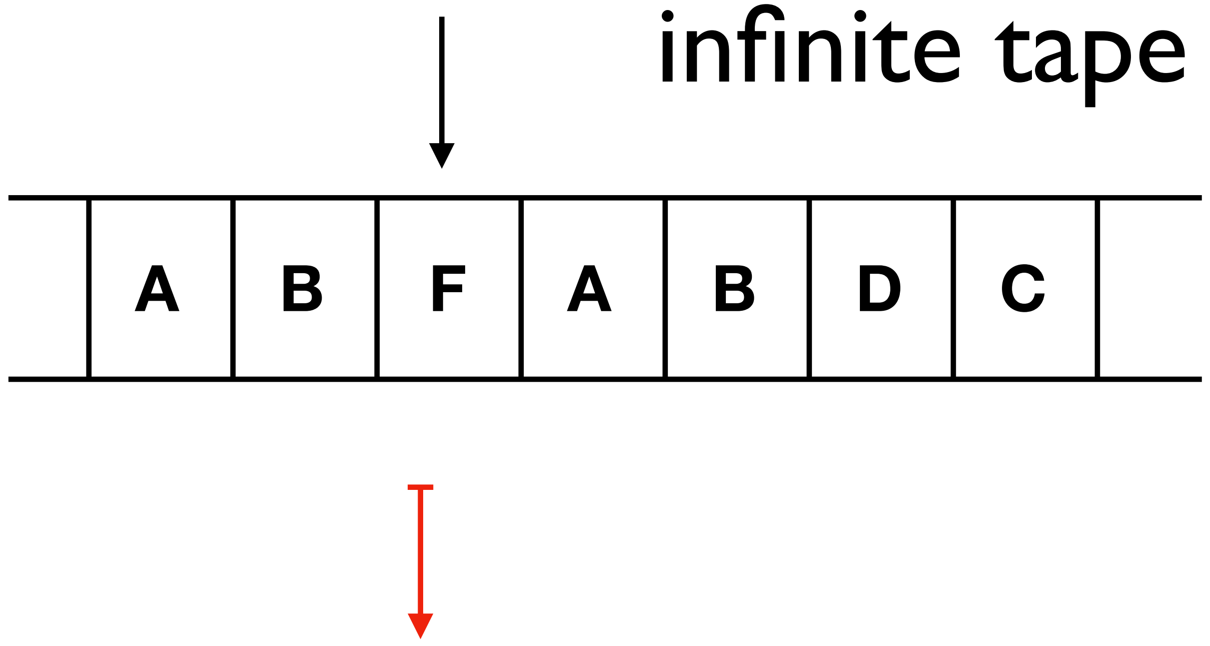
infinite tape = two pushdowns



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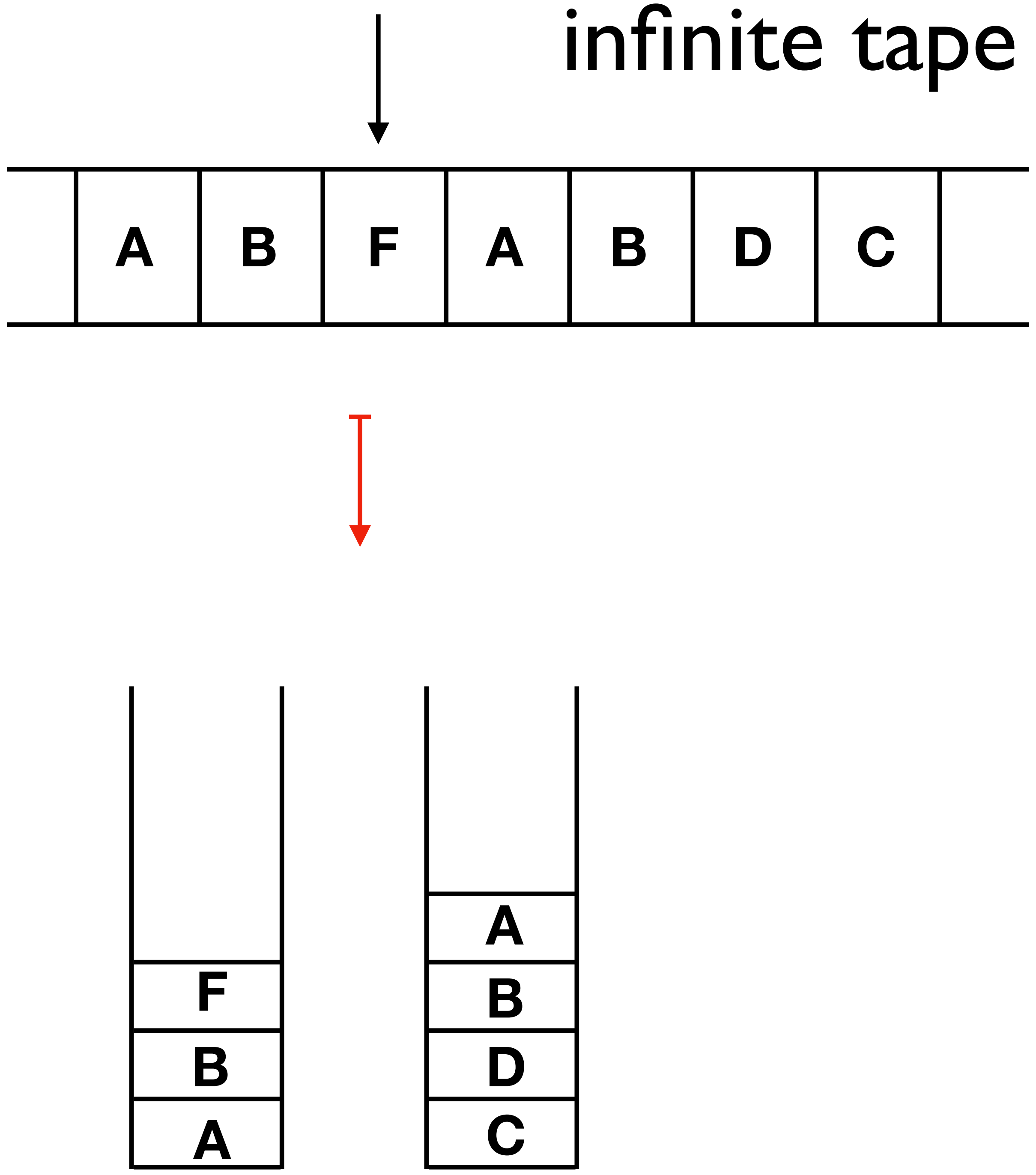

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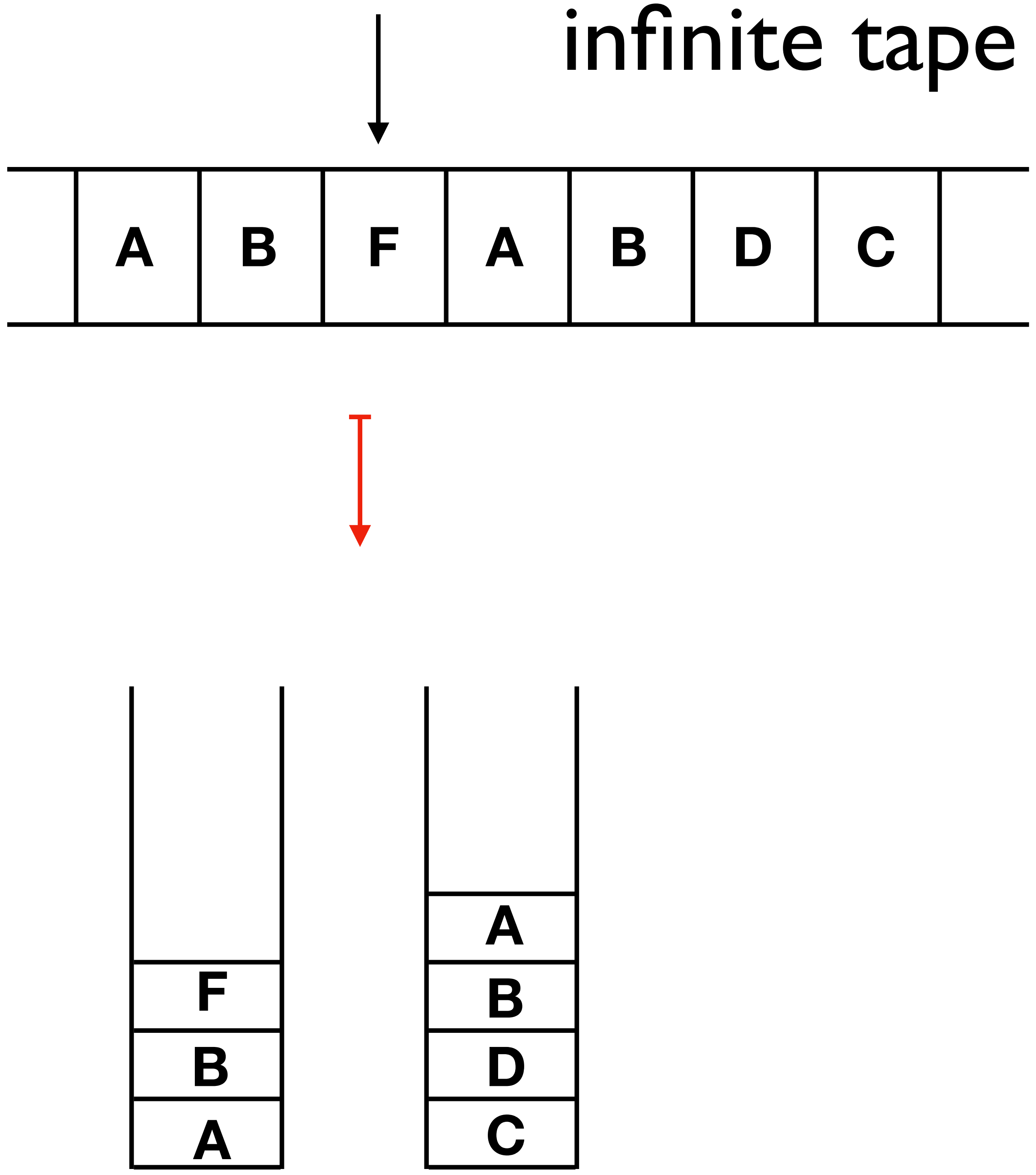

reachability problem
for
automaton with two pushdowns
is
undecidable

Proof continuation

Proof continuation

pushdown can be simulated by two counters

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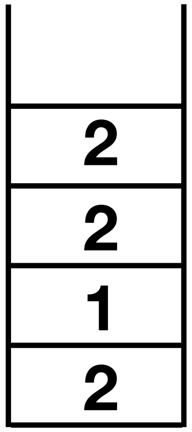

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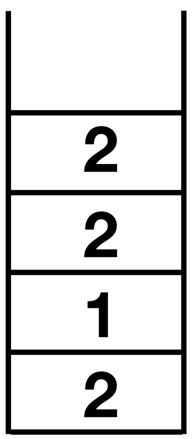
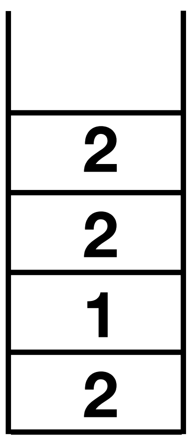


<2122> in ternary = 71

Proof continuation

pushdown can be simulated by two counters



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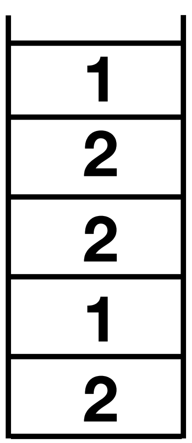

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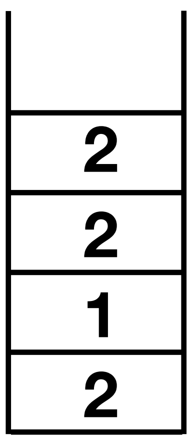

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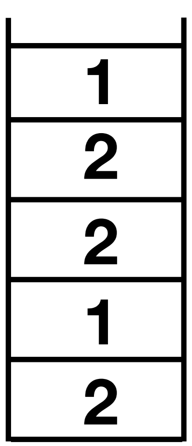
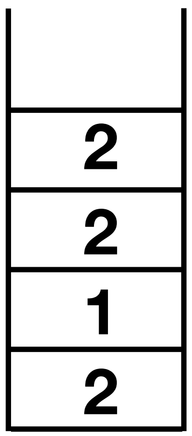


<21221> in ternary = $3 \cdot 71 + 1$

Proof continuation

pushdown can be simulated by two counters



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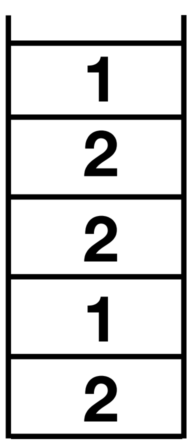
(71,0)



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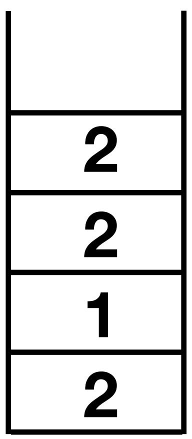
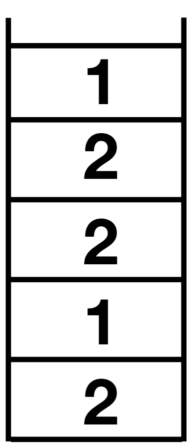


<2122> in ternary = 71

(71,0)

(-1,+1)



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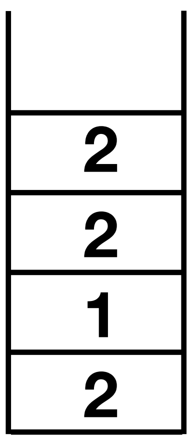
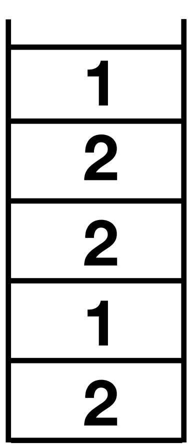


<2122> in ternary = 71

(71,0)

(-1,+1) zero-test(x)



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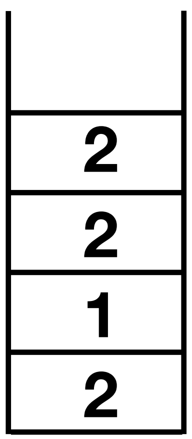

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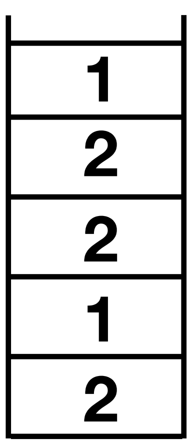

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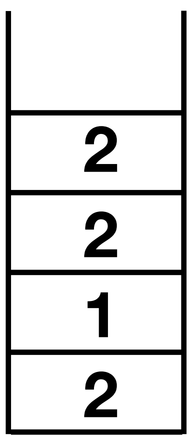

<2122> in ternary = 71

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(-1,+1) zero-test(x)

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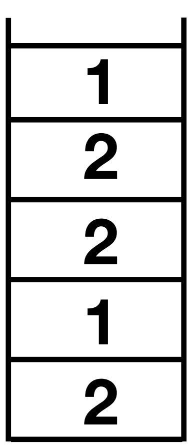
(+3,-1)



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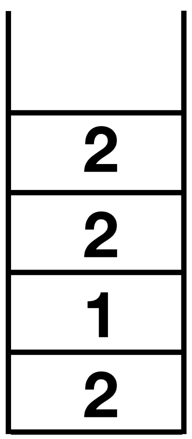


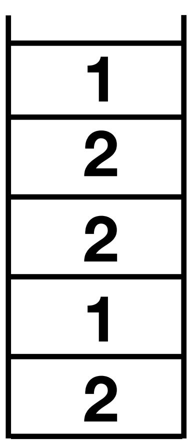
<2122> in ternary = 71

(71,0)

(-1,+1)   zero-test(x)

(0,71)

(+3,-1)   zero-test(y)



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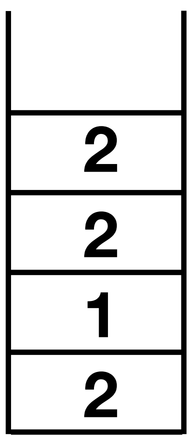

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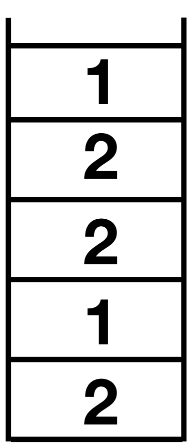
$(3\cdot 71,0)$



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the auxiliary counter can be the same

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The reachability problem for
automaton with three counters is undecidable

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(x, y, z)

Proof continuation

the auxiliary counter can be the same

The reachability problem for automaton with three counters is undecidable

(x, y, z)   encoded as

Proof continuation

the auxiliary counter can be the same

The reachability problem for
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(x, y, z) encoded as $(2^x \cdot 3^y \cdot 5^z, 0)$

Proof continuation

the auxiliary counter can be the same

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(x, y, z) encoded as $(2^x \cdot 3^y \cdot 5^z, 0)$

The reachability problem for
automaton with two counters is undecidable

Hardness

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Theorem

Turing machine with space M can be simulated by:

- three-counter automaton with counters up to exp(M)
- two-counter automaton with counters up to 2-exp(M)

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Turing machine with space M can be simulated by:

- three-counter automaton with counters up to exp(M)
- two-counter automaton with counters up to 2-exp(M)

Reachability for three-counter automaton with counters bounded by 2-exp with ExpSpace-complete

How to simplify?

How to simplify?

Counters without zero-tests

How to simplify?

Counters without zero-tests

Just one zero-tested counter

How to simplify?

Counters without zero-tests

Just one zero-tested counter

Just one pushdown

How to simplify?

Counters without zero-tests

Just one zero-tested counter

Just one pushdown

Other

Vector Addition Systems with States (VASS)

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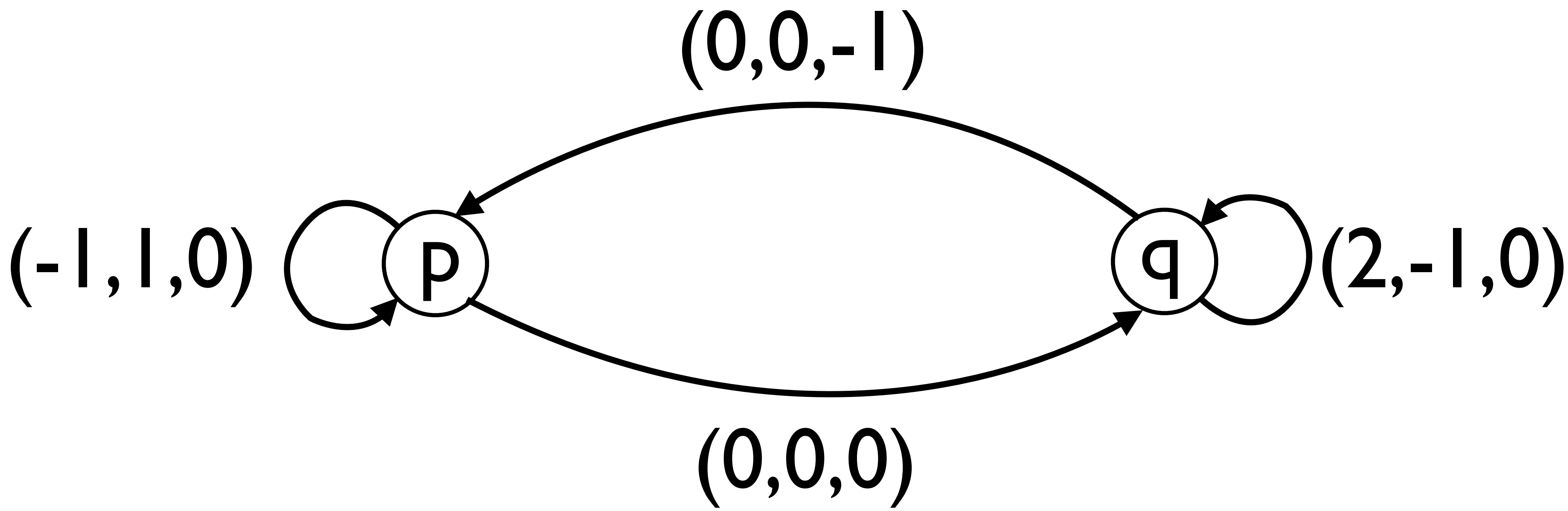
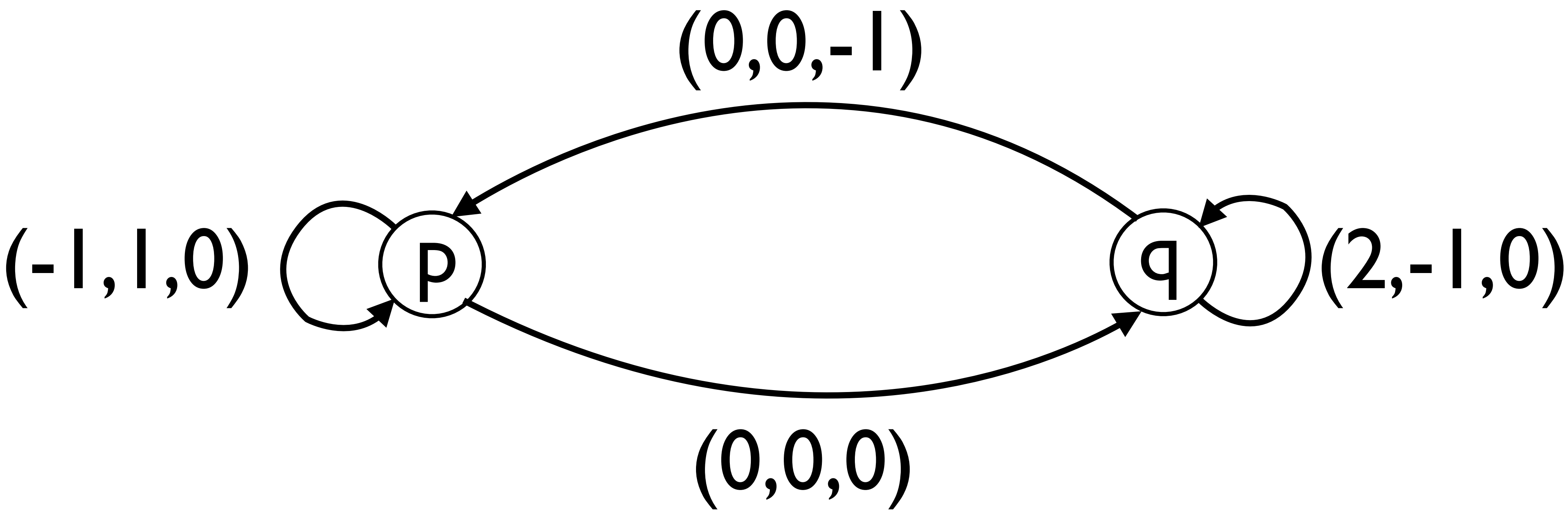
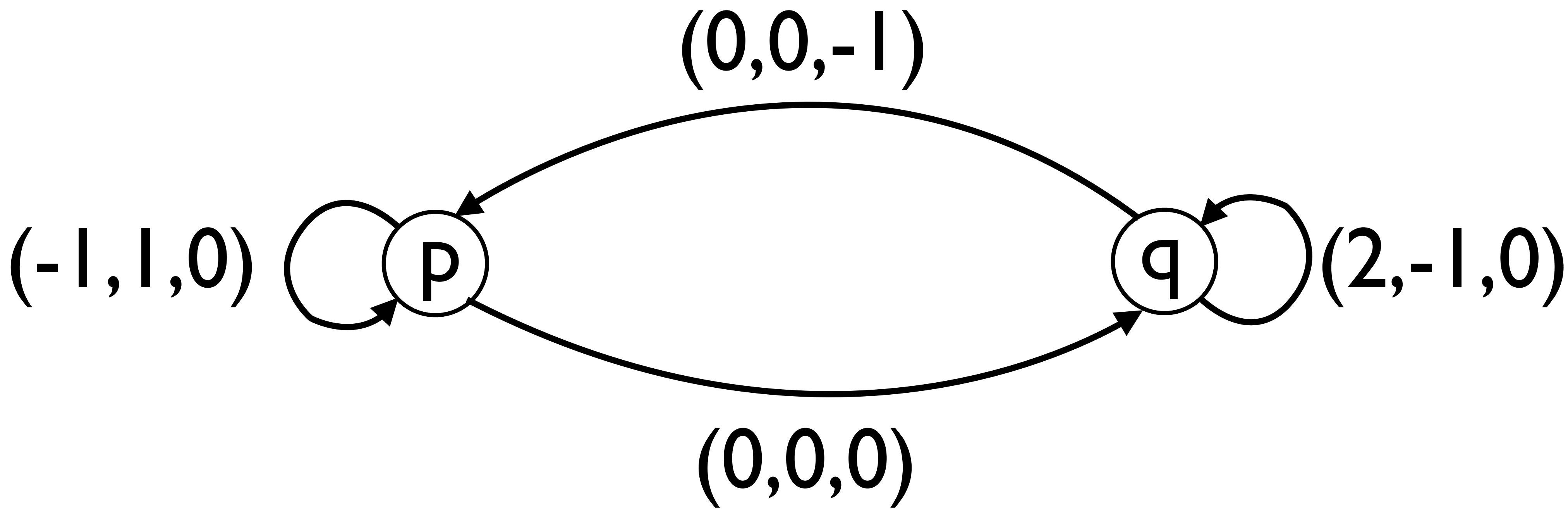


Vector Addition Systems with States (VASS)



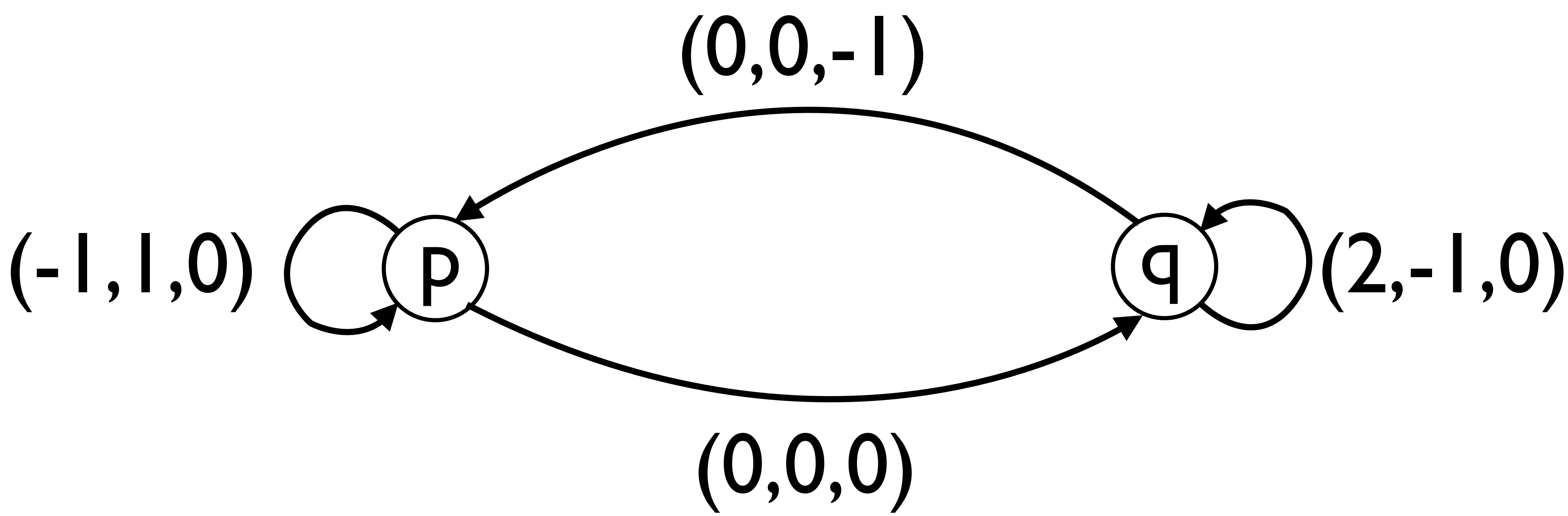
p(2,0,7)

Vector Addition Systems with States (VASS)



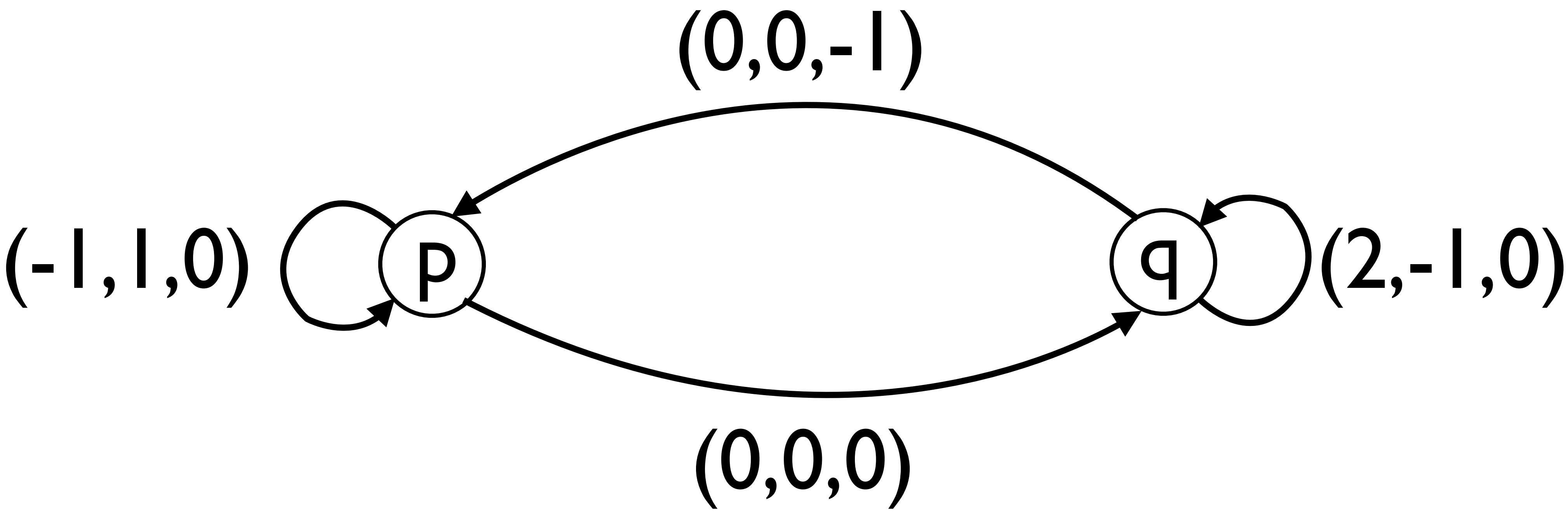
$p(2,0,7) \longrightarrow p(1,1,7)$

Vector Addition Systems with States (VASS)



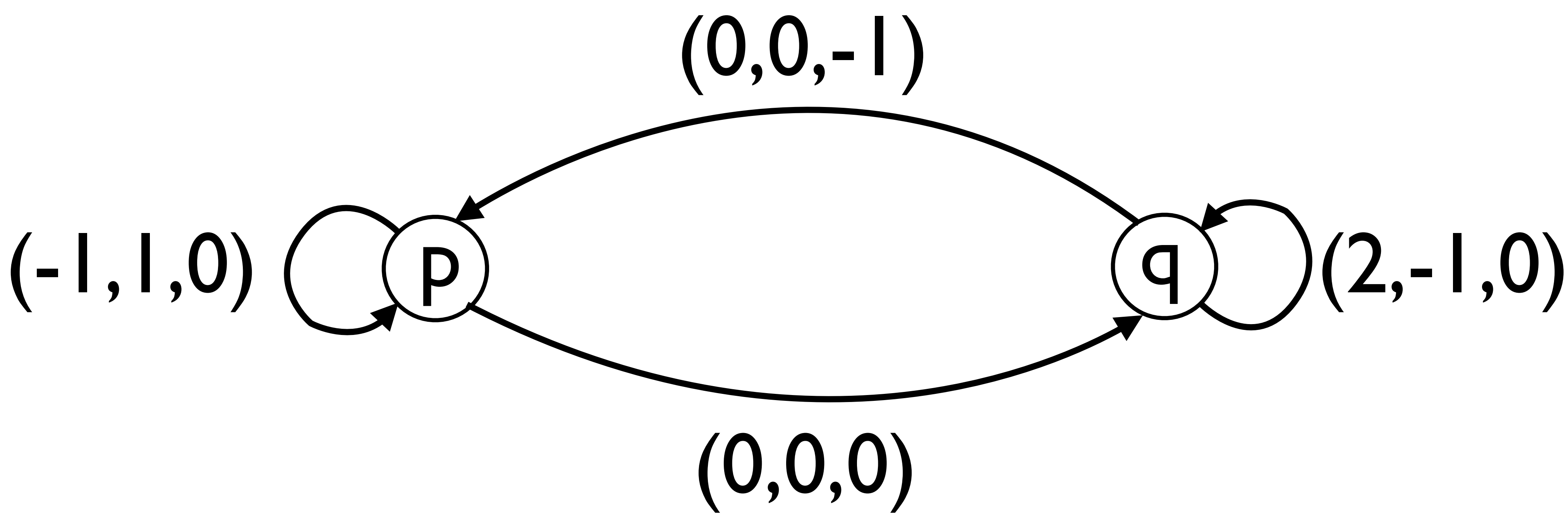
$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7)$

Vector Addition Systems with States (VASS)



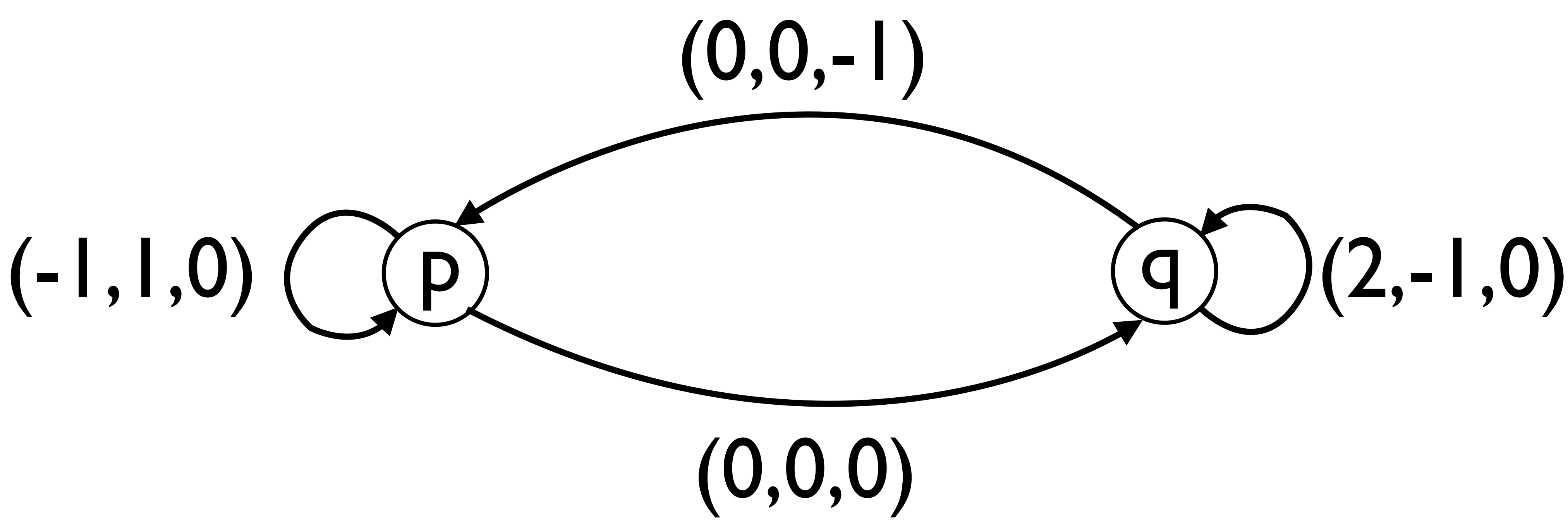
$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7)$

Vector Addition Systems with States (VASS)



p(2,0,7) ⟶ p(1,1,7) ⟶ p(0,2,7) ⟶ q(0,2,7) ⟶ q(2,1,7)

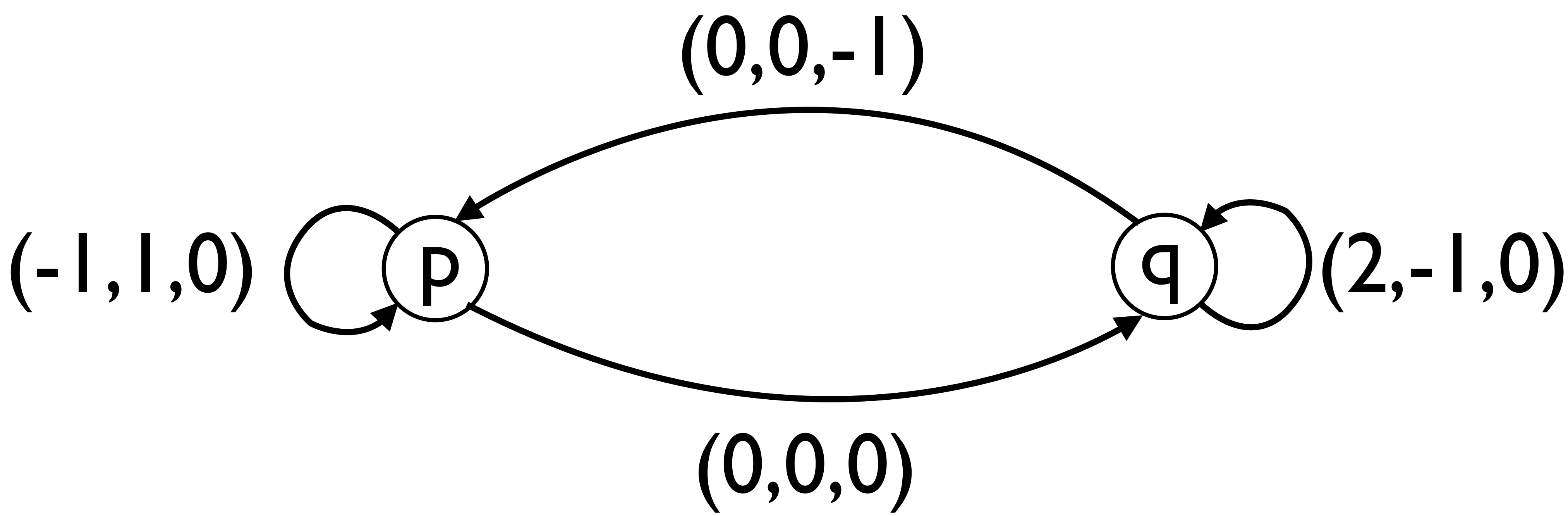
Vector Addition Systems with States (VASS)



$p(2,0,7) \longrightarrow p(1,1,7) \longrightarrow p(0,2,7) \longrightarrow q(0,2,7) \longrightarrow q(2,1,7)$

$\longrightarrow q(4,0,7)$

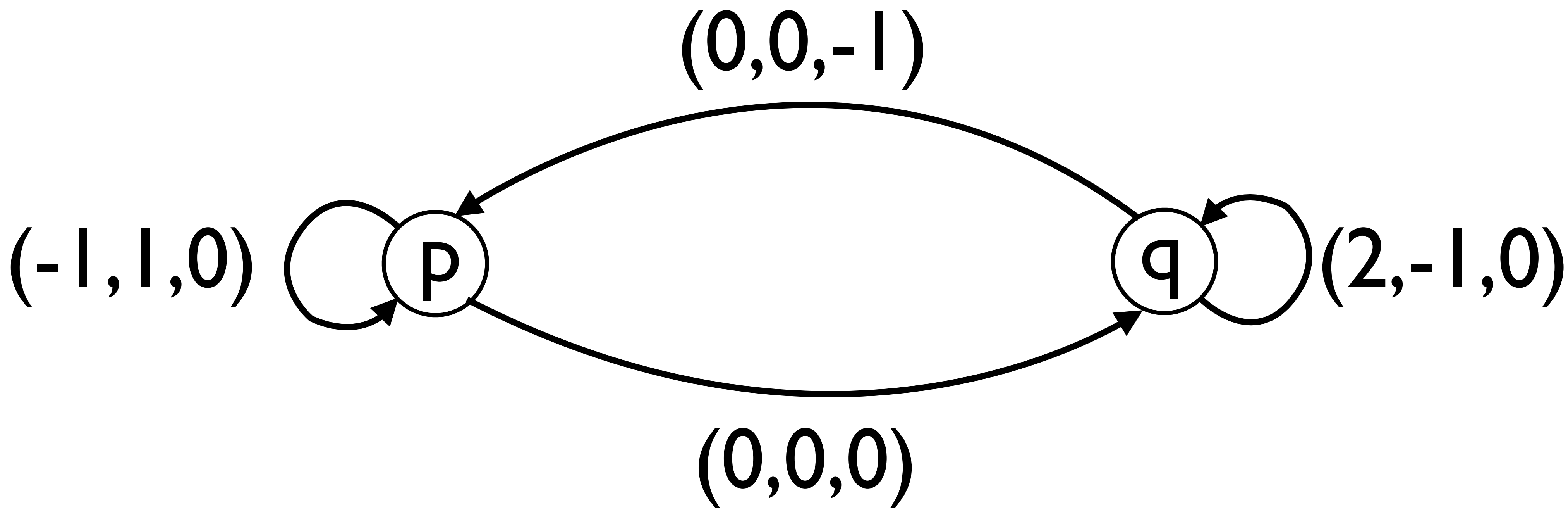
Vector Addition Systems with States (VASS)



p(2,0,7) ⟶ p(1,1,7) ⟶ p(0,2,7) ⟶ q(0,2,7) ⟶ q(2,1,7)

⟶ q(4,0,7) ⟶ p(4,0,6)

Vector Addition Systems with States (VASS)



p(2,0,7) ⟶ p(1,1,7) ⟶ p(0,2,7) ⟶ q(0,2,7) ⟶ q(2,1,7)

⟶ q(4,0,7) ⟶ p(4,0,6)

Petri nets

Short history of reachability

Short history of reachability

Lipton `76: ExpSpace-hardness

Short history of reachability

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Mayr `81: decidability of reachability

Short history of reachability

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Leroux, Schmitz `15: triple-Ackermann

Short history of reachability

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Cz., Lasota, Lazic, Leroux, Mazowiecki `19:
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Short history of reachability

Lipton `76: ExpSpace-hardness

Mayr `81: decidability of reachability

Leroux, Schmitz `15: triple-Ackermann

Leroux, Schmitz `19: Ackermann

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Tower-hardness

Leroux & Cz., Orlikowski`21: Ackermann-hardness

Functions F_k

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$F_1(n) = 2n$

Functions F_k

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$F_{k+1}(n) = F_k \circ \ldots \circ F_k(1)$

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composed n times

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$F_2(n) = 2^n$

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Functions F_k

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$F_2(n) = 2^n$

$F_3(n) = \text{Tower}(n)$

$\text{Ack}(n) = F_n(n)$

Special cases of VASS

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dimension 2: NL-complete

Special cases of VASS

dimension 2: NL-complete

dim 3: NP-hard, in Tower

Special cases of VASS

dimension 2: NL-complete

dim 3: NP-hard, in Tower

dim 6: ExpSpace-hard

Special cases of VASS

dimension 2: NL-complete

dim 3: NP-hard, in Tower

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dim 8: Tower-hard

Special cases of VASS

dimension 2: NL-complete

dim 3: NP-hard, in Tower

dim 6: ExpSpace-hard

dim 8: Tower-hard

many striking open problems!

One zero-test

One zero-test

one zero-tested counter: NL-complete

One zero-test

one zero-tested counter: NL-complete

VASS + one zero-tested counter: decidable

One zero-test

one zero-tested counter: NL-complete

VASS + one zero-tested counter: decidable

Klaus Reinhardt 2008

One zero-test

one zero-tested counter: NL-complete

VASS + one zero-tested counter: decidable

Klaus Reinhardt 2008

in fact nested zero-tests

One pushdown

One pushdown

pushdown automaton: in PTime

One pushdown

pushdown automaton: in PTime

CYK algorithm for context-free grammars

One pushdown

pushdown automaton: in PTime

CYK algorithm for context-free grammars

VASS with pushdown: open

One pushdown

pushdown automaton: in PTime

CYK algorithm for context-free grammars

VASS with pushdown: open

one counter with pushdown: open

Other

Other

automaton with $\mathbb{Z}$-counters: NP-complete

Other

automaton with $\mathbb{Z}$-counters: NP-complete

more exotic combinations

Other

automaton with $\mathbb{Z}$-counters: NP-complete

more exotic combinations

reachability for very simple models is hard

Other

automaton with $\mathbb{Z}$-counters: NP-complete

more exotic combinations

reachability for very simple models is hard

might be decidable for all simplifications

Hard examples

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big reachability sets for VASS, 1-PVASS

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finite, up to Ackermann size

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big reachability sets for VASS, 1-PVASS

finite, up to Ackermann size

does not prove Ackermann-hardness

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big reachability sets for VASS, 1-PVASS

finite, up to Ackermann size

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provide intuition for hardness

3-dim. VASS (3-VASS)

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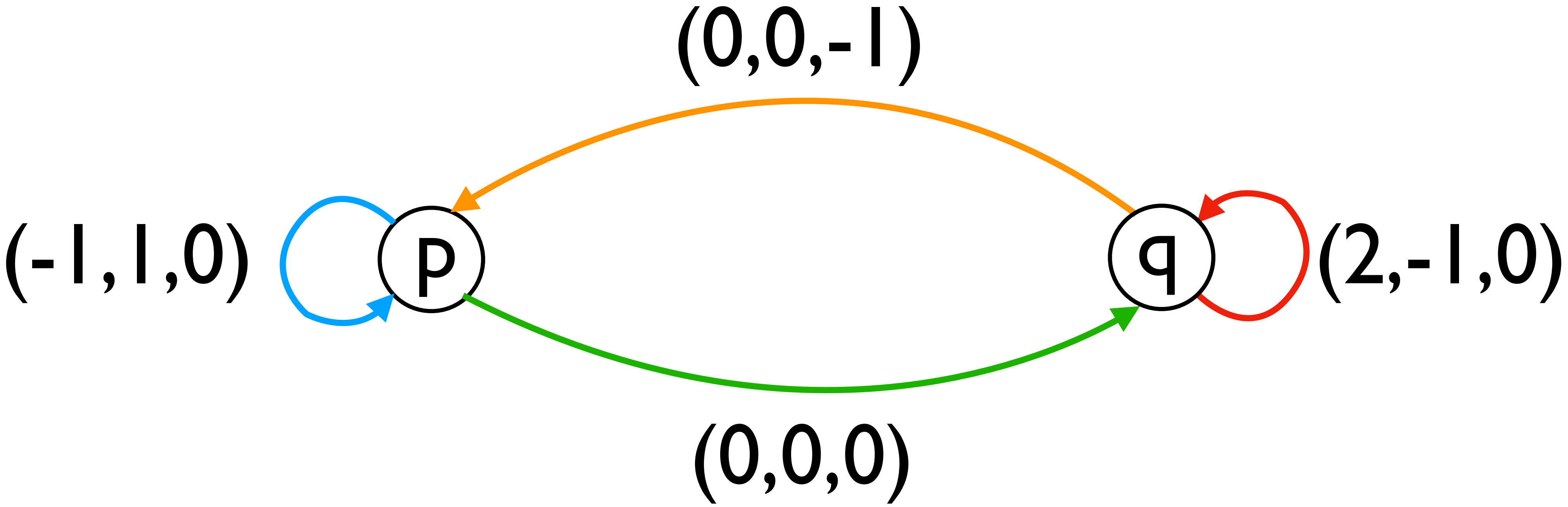
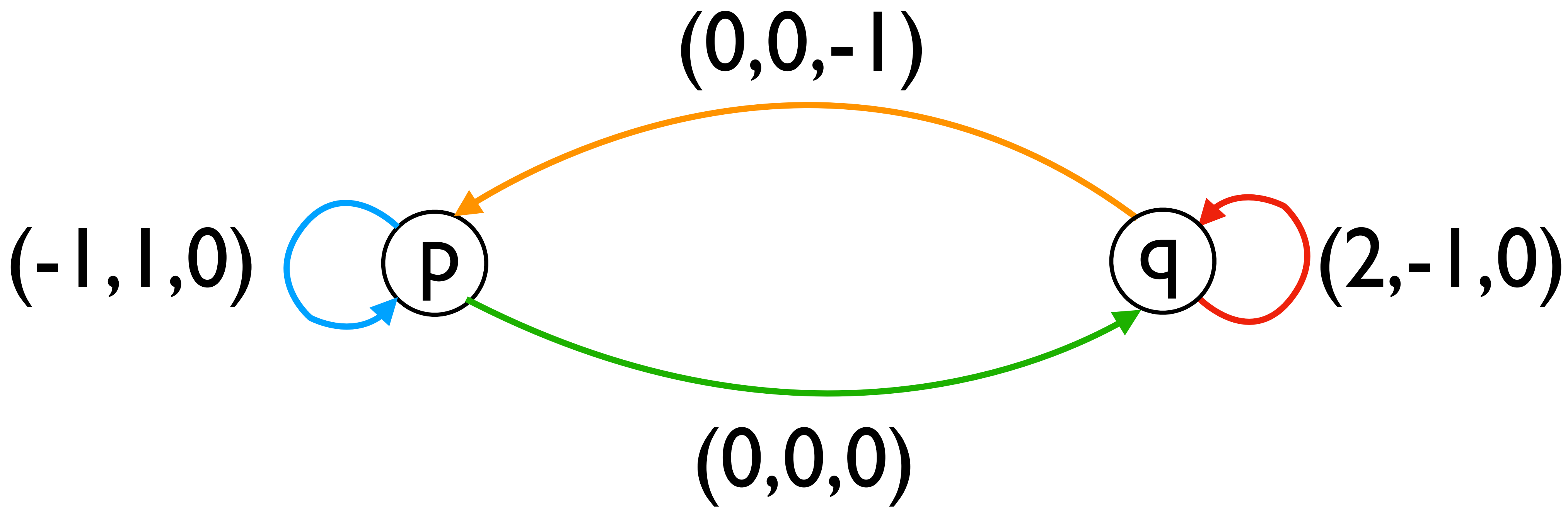
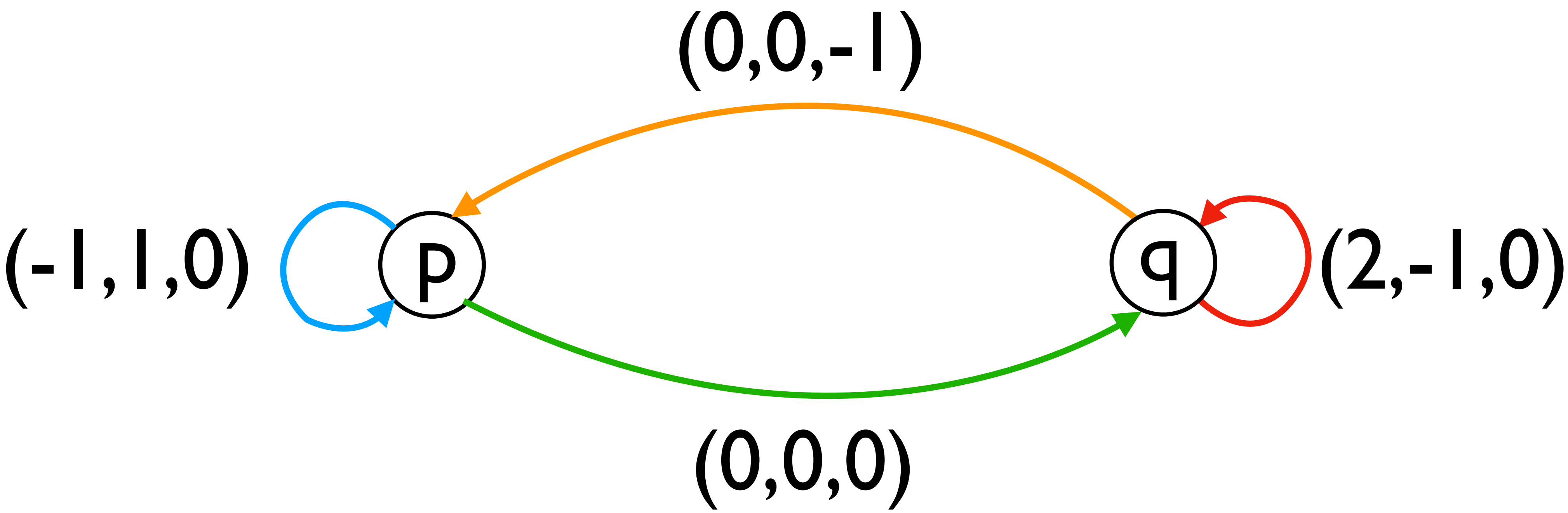


3-dim. VASS (3-VASS)



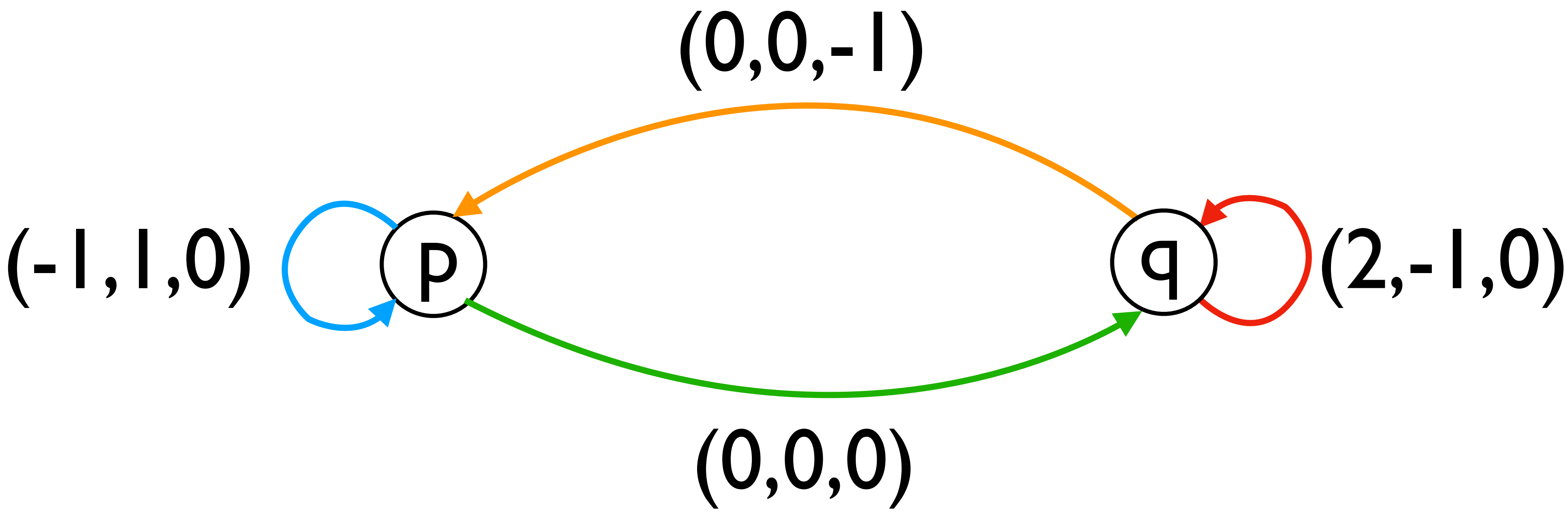
p(k,0,n)

3-dim. VASS (3-VASS)



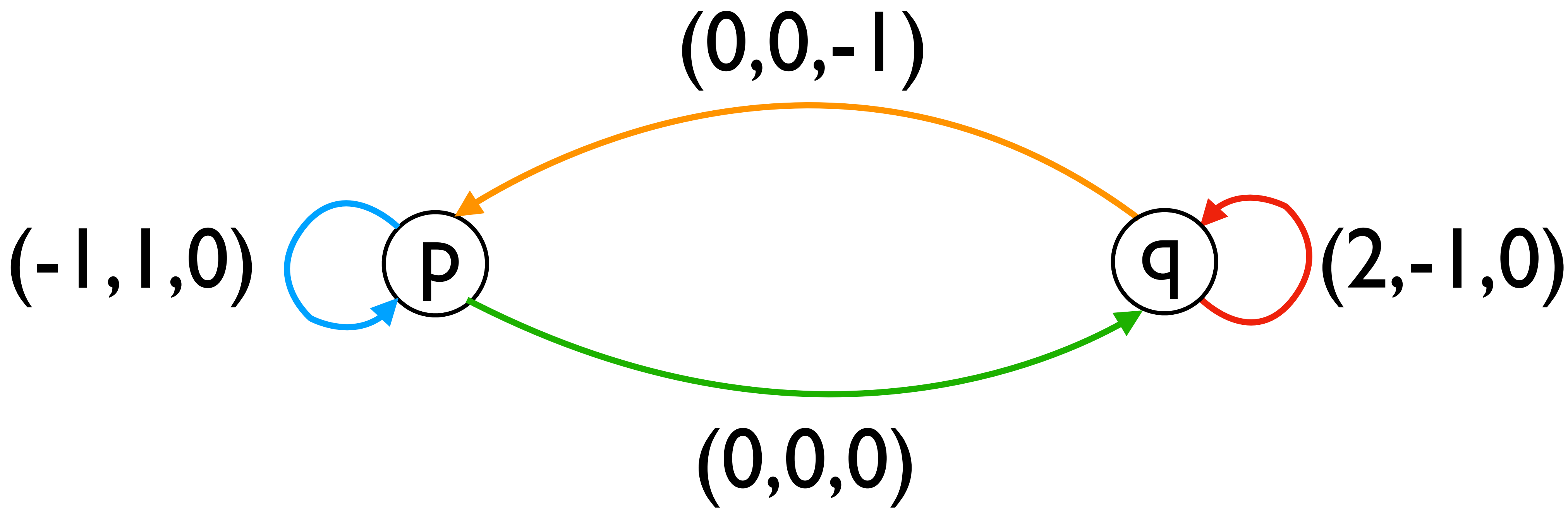
p(k,0,n) ⟶ p(0,k,n)

3-dim. VASS (3-VASS)



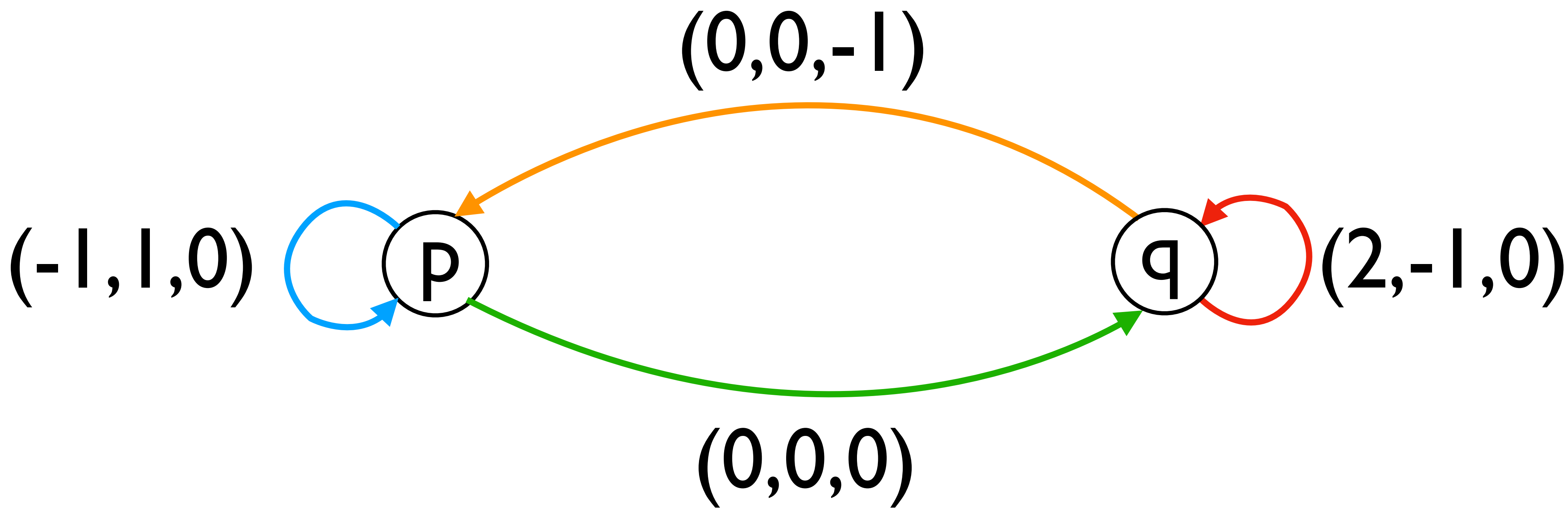
p(k,0,n) ⟶ p(0,k,n) ⟶ q(0,k,n)

3-dim. VASS (3-VASS)



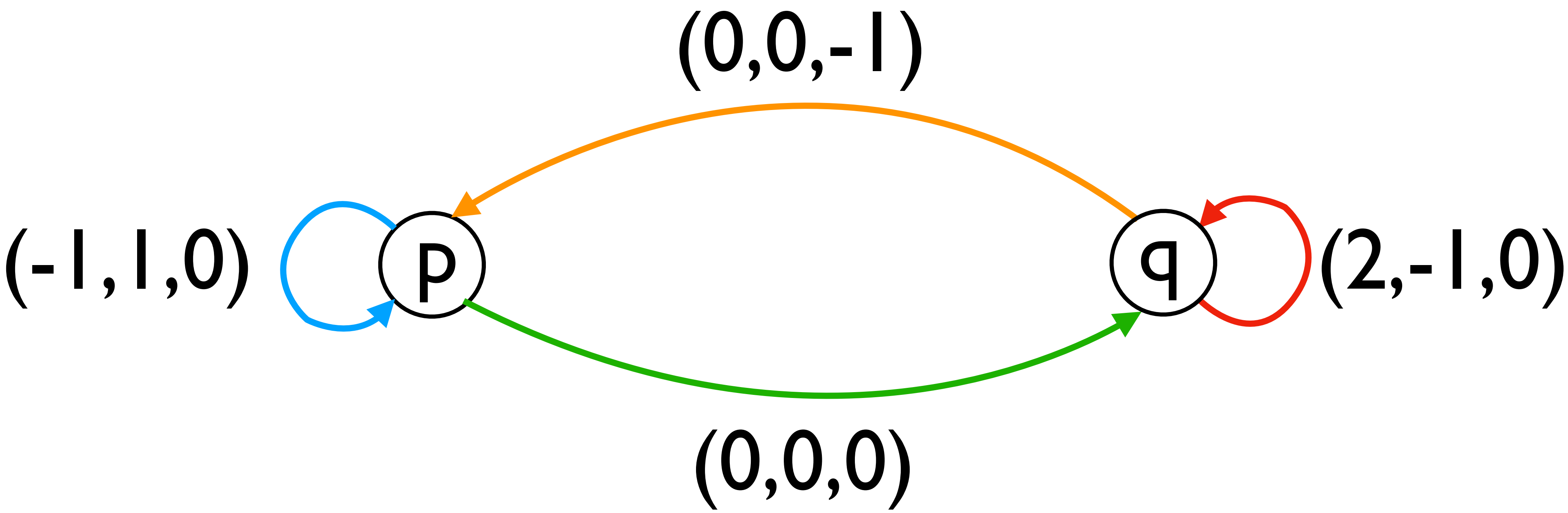
p(k,0,n) $\longrightarrow$ p(0,k,n) $\longrightarrow$ q(0,k,n) $\longrightarrow$ q(2k,0,n)

3-dim. VASS (3-VASS)



$p(k,0,n) \rightarrow p(0,k,n) \rightarrow q(0,k,n) \rightarrow q(2k,0,n) \rightarrow p(2k,0,n-1)$

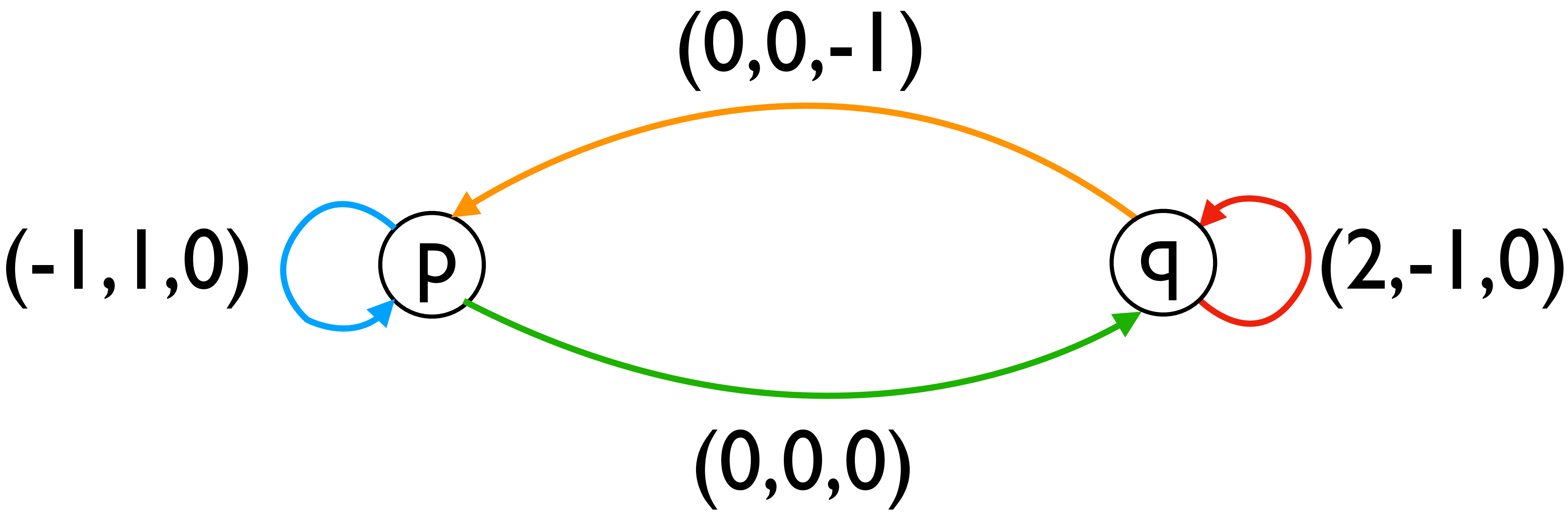
3-dim. VASS (3-VASS)



p(k,0,n) → p(0,k,n) → q(0,k,n) → q(2k,0,n) → p(2k,0,n-1)

p(1,0,n)

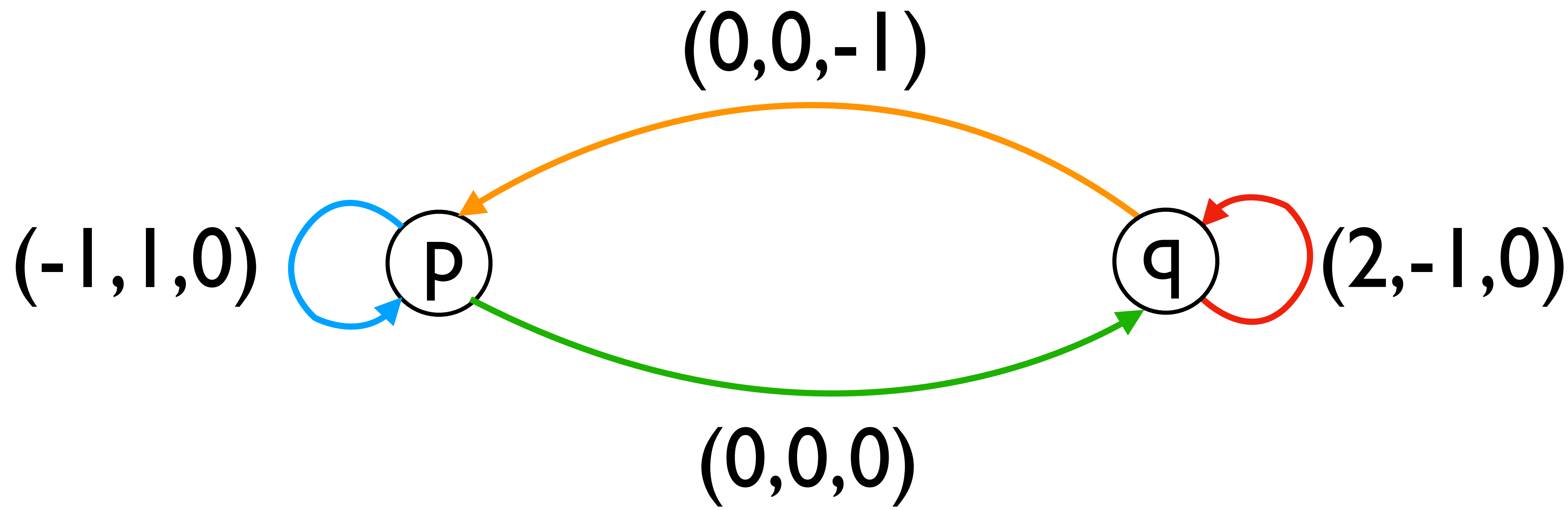
3-dim. VASS (3-VASS)



$p(k,0,n) \rightarrow p(0,k,n) \rightarrow q(0,k,n) \rightarrow q(2k,0,n) \rightarrow p(2k,0,n-1)$

$p(1,0,n) \longrightarrow p(2,0,n-1)$

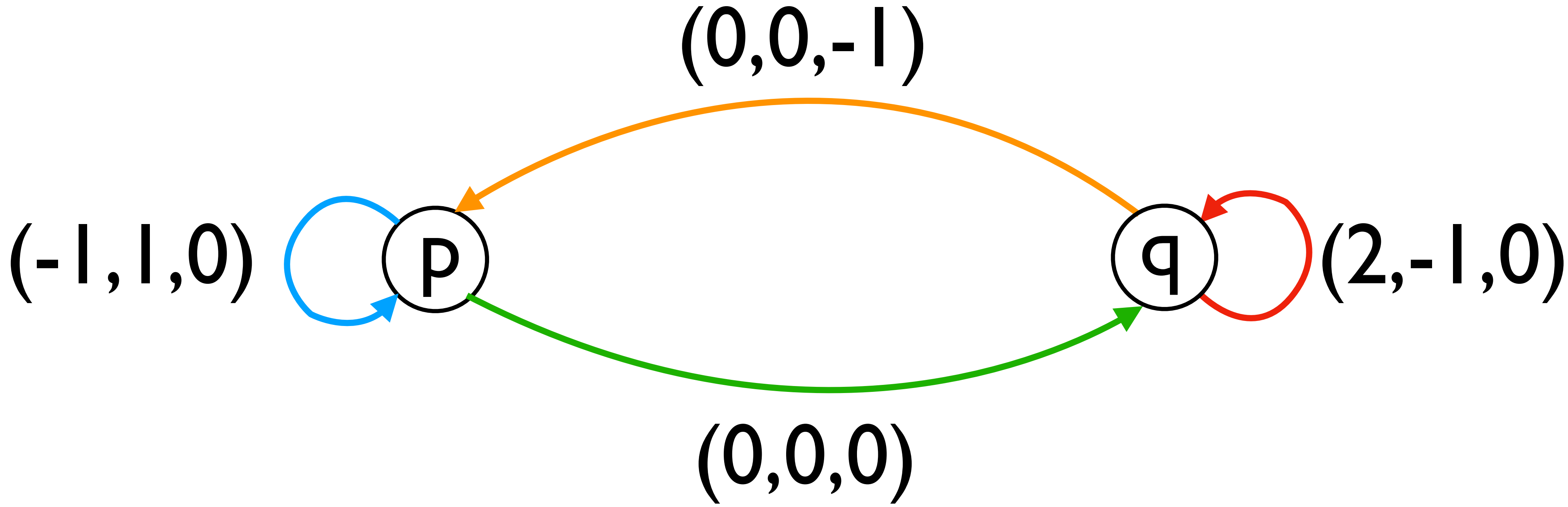
3-dim. VASS (3-VASS)



$p(k,0,n) \rightarrow p(0,k,n) \rightarrow q(0,k,n) \rightarrow q(2k,0,n) \rightarrow p(2k,0,n-1)$

$p(1,0,n) \longrightarrow p(2,0,n-1) \ \ldots$

3-dim. VASS (3-VASS)

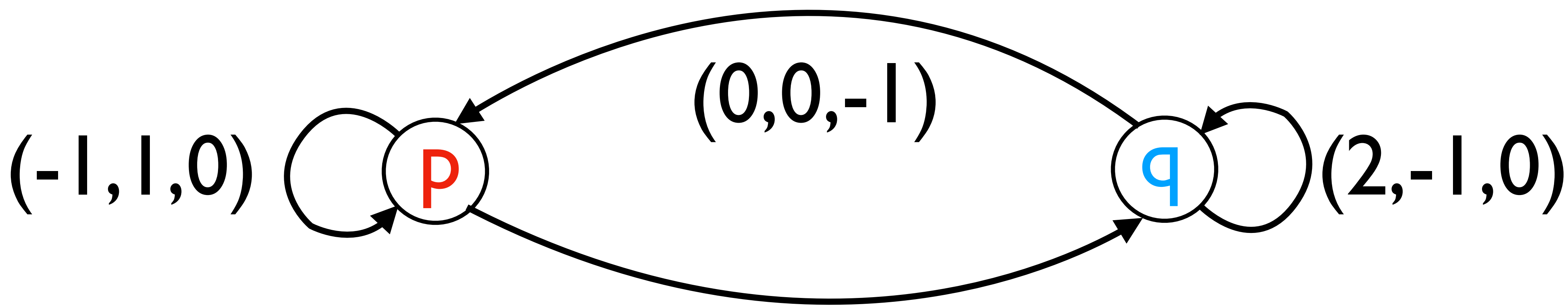


$$p(k,0,n) \longrightarrow p(0,k,n) \longrightarrow q(0,k,n) \longrightarrow q(2k,0,n) \longrightarrow p(2k,0,n-1)$$

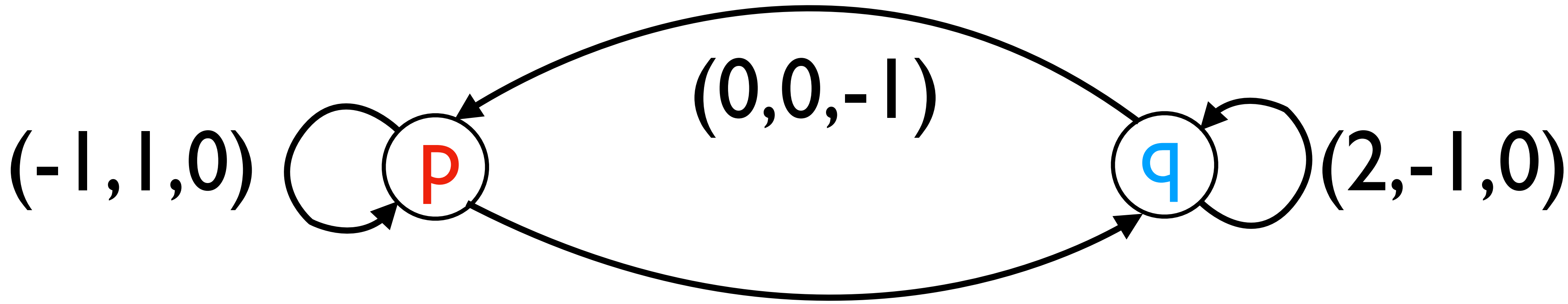
$$p(1,0,n) \longrightarrow p(2,0,n-1) \;\ldots\; \longrightarrow p(2^n,0,0)$$

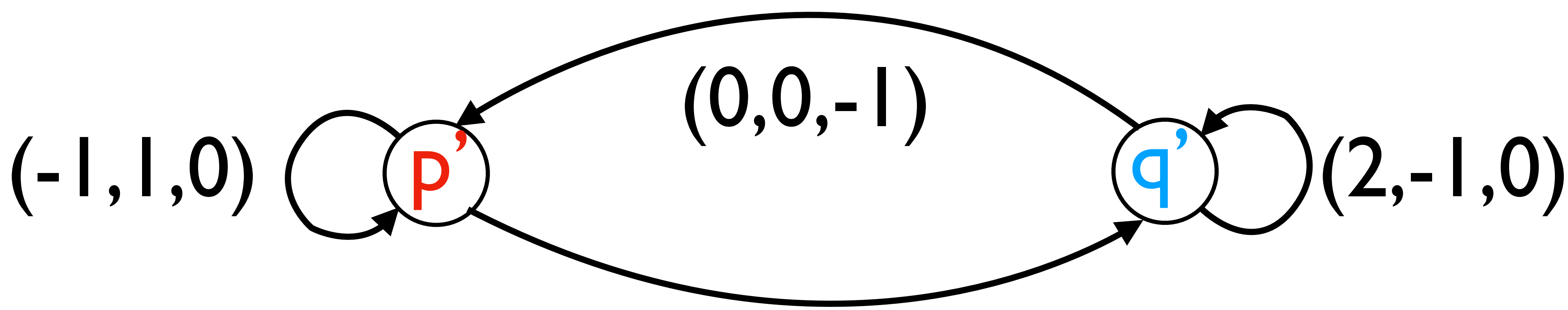
3-VASS

3-VASS



3-VASS





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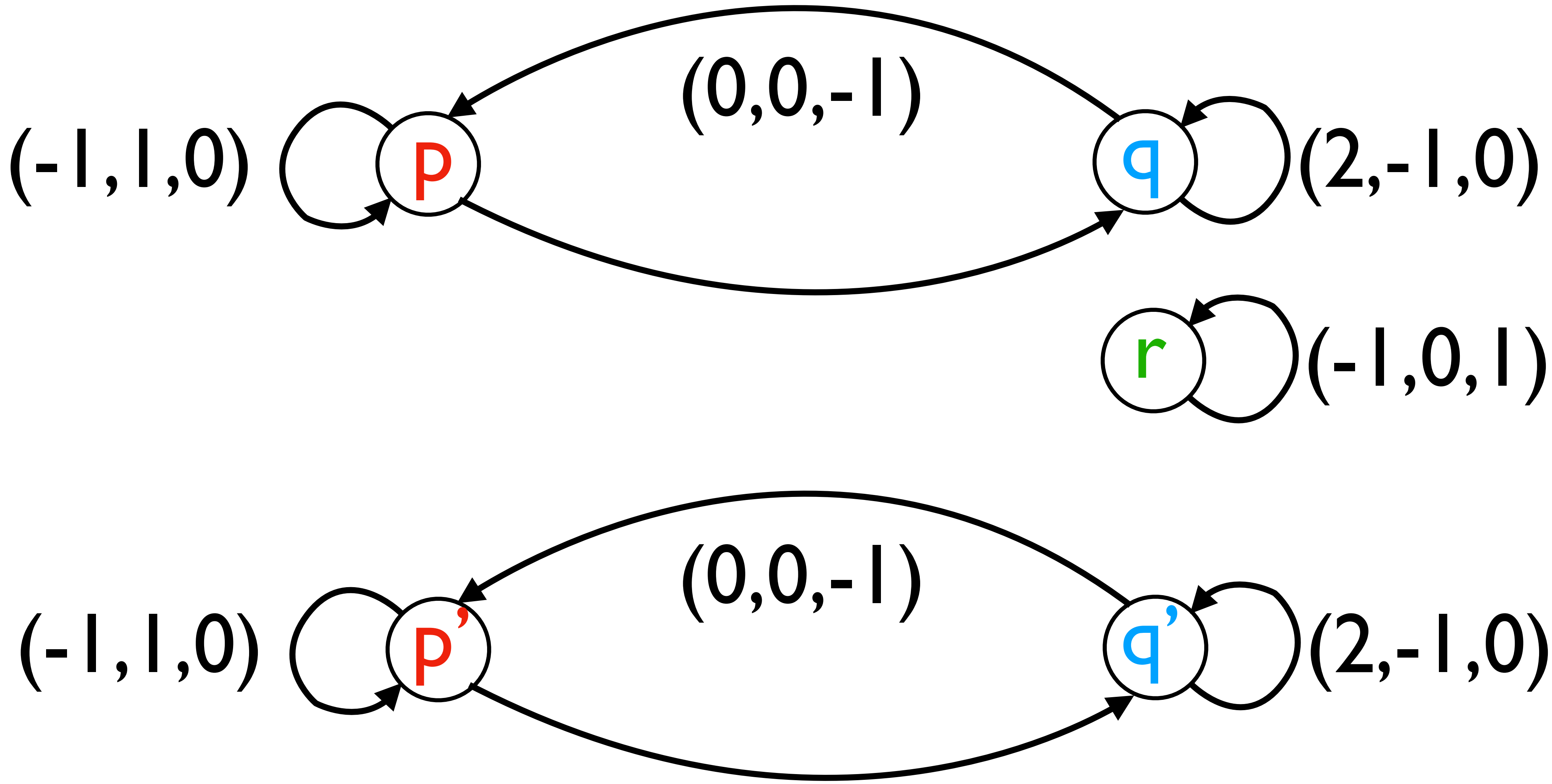

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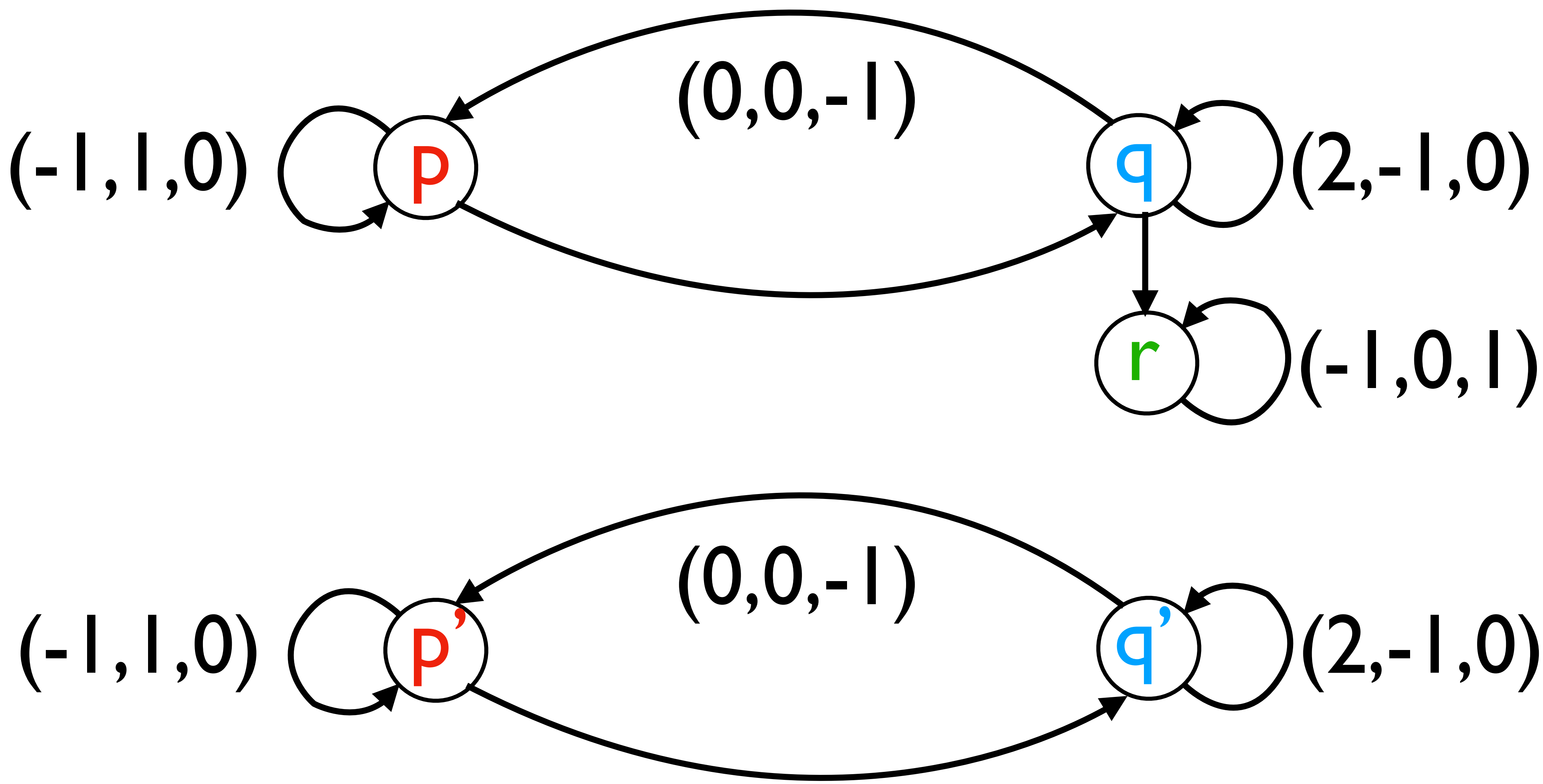
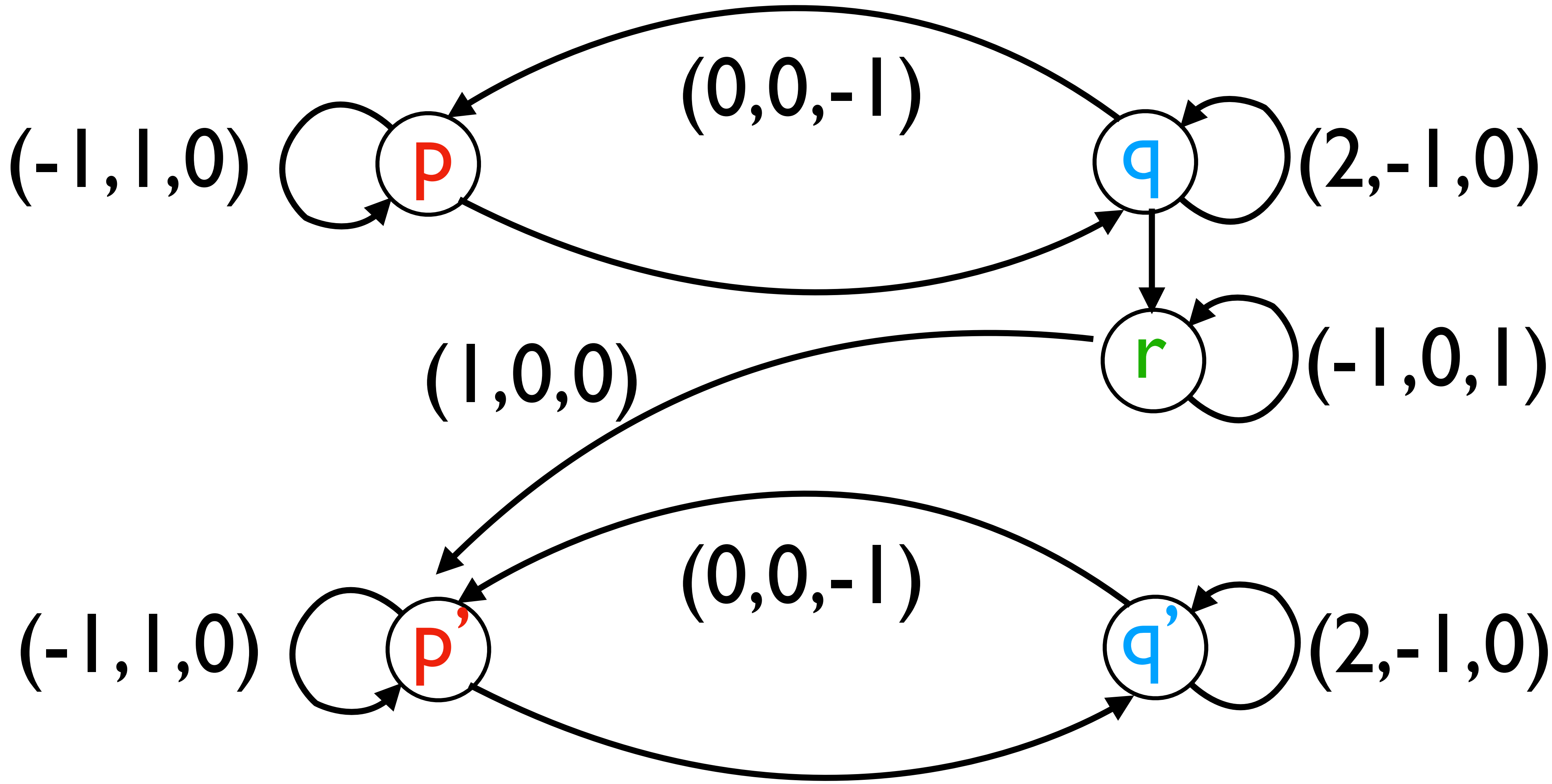
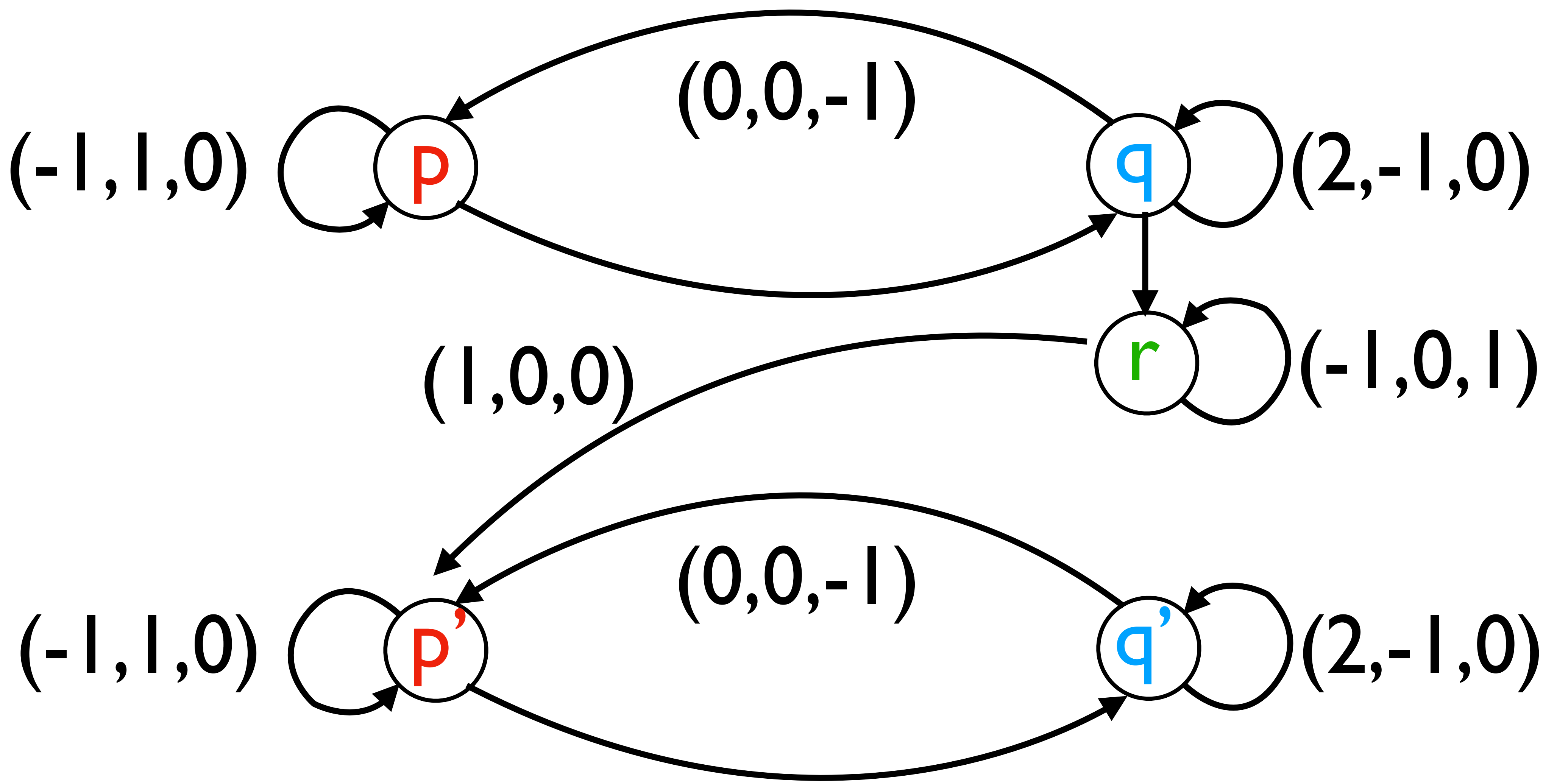


3-VASS

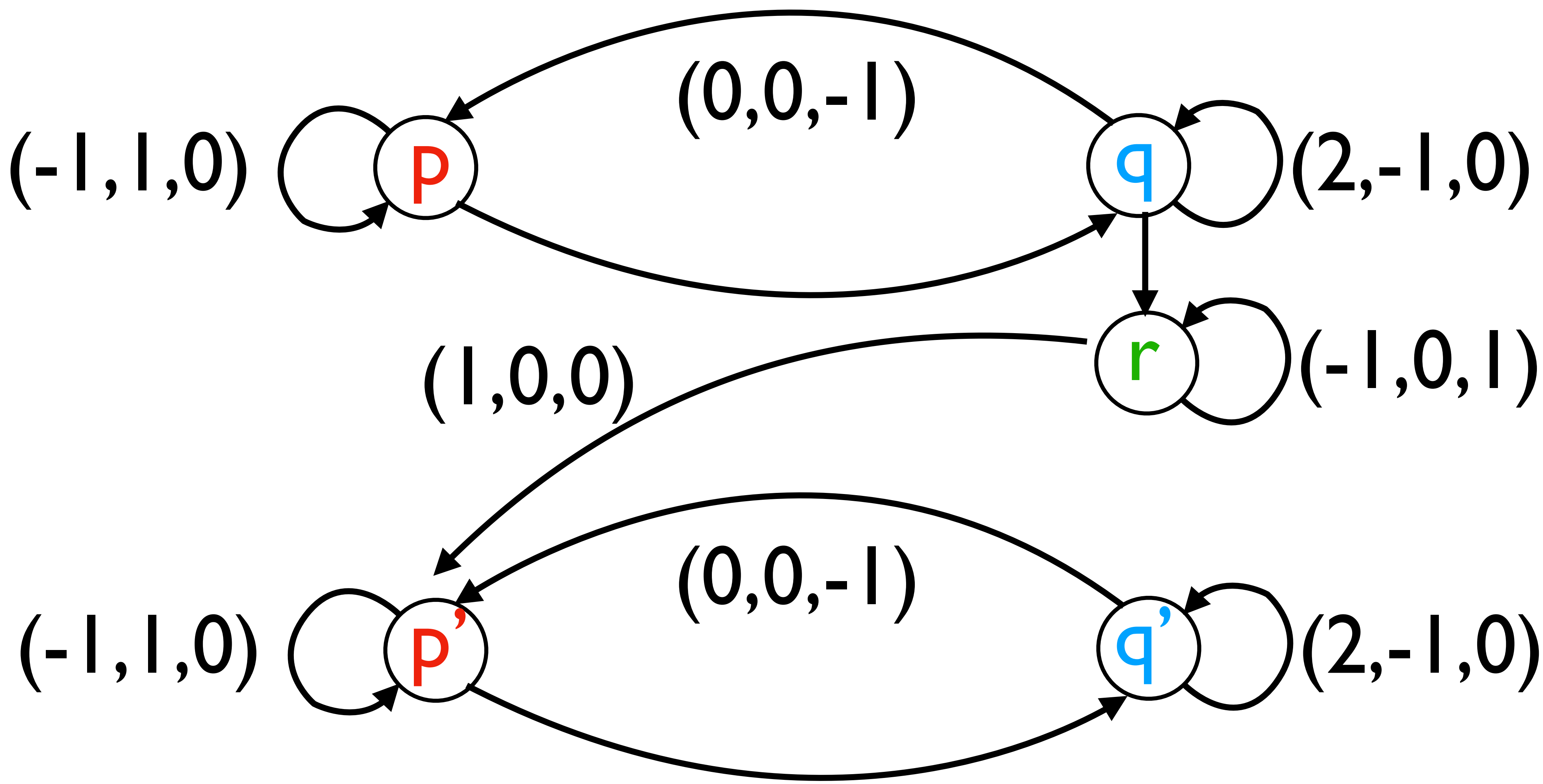


3-VASS



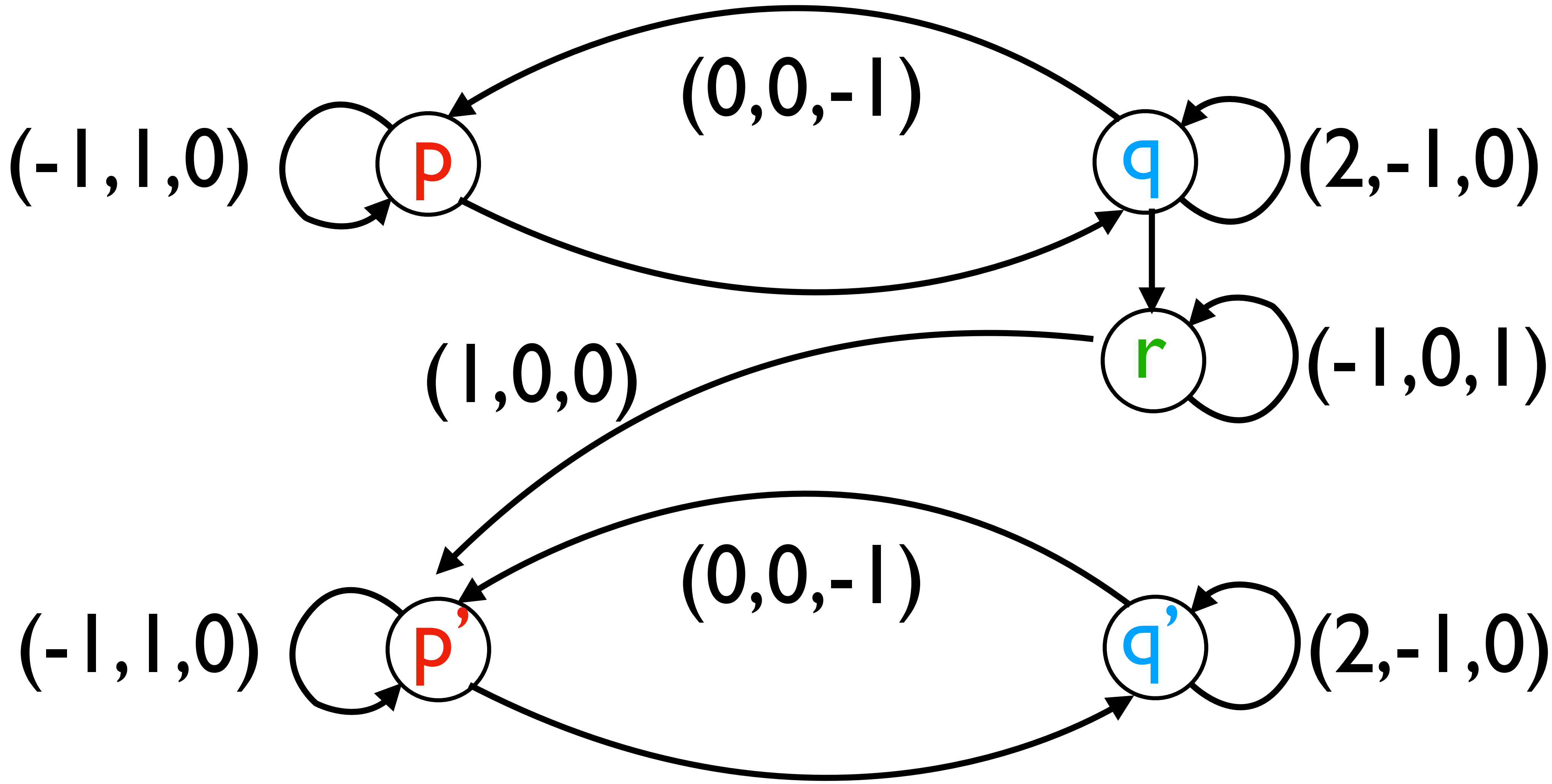
p(1,0,n)

3-VASS



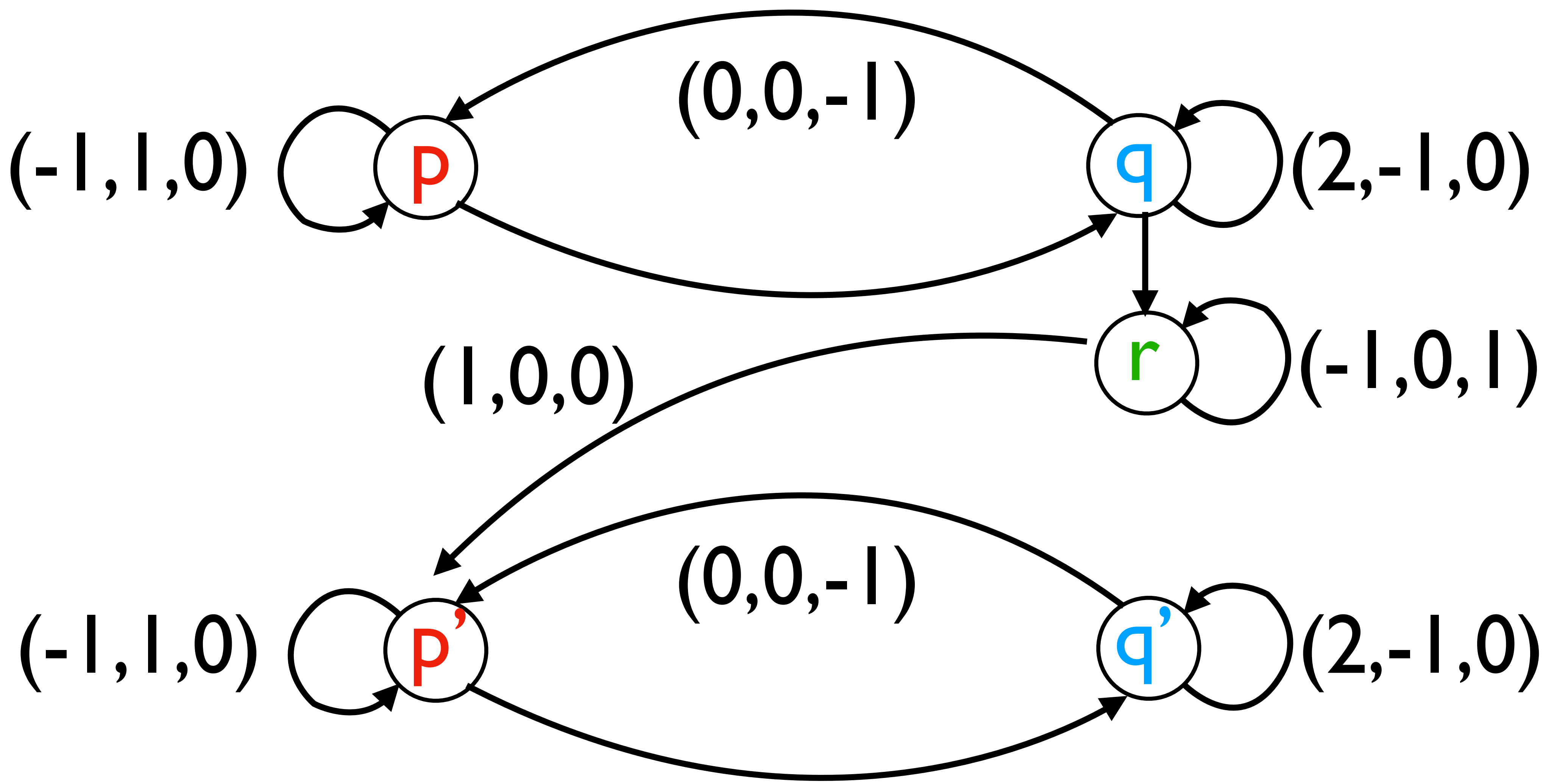
$p(1,0,n) \longrightarrow q(2^n,0,0)$

3-VASS



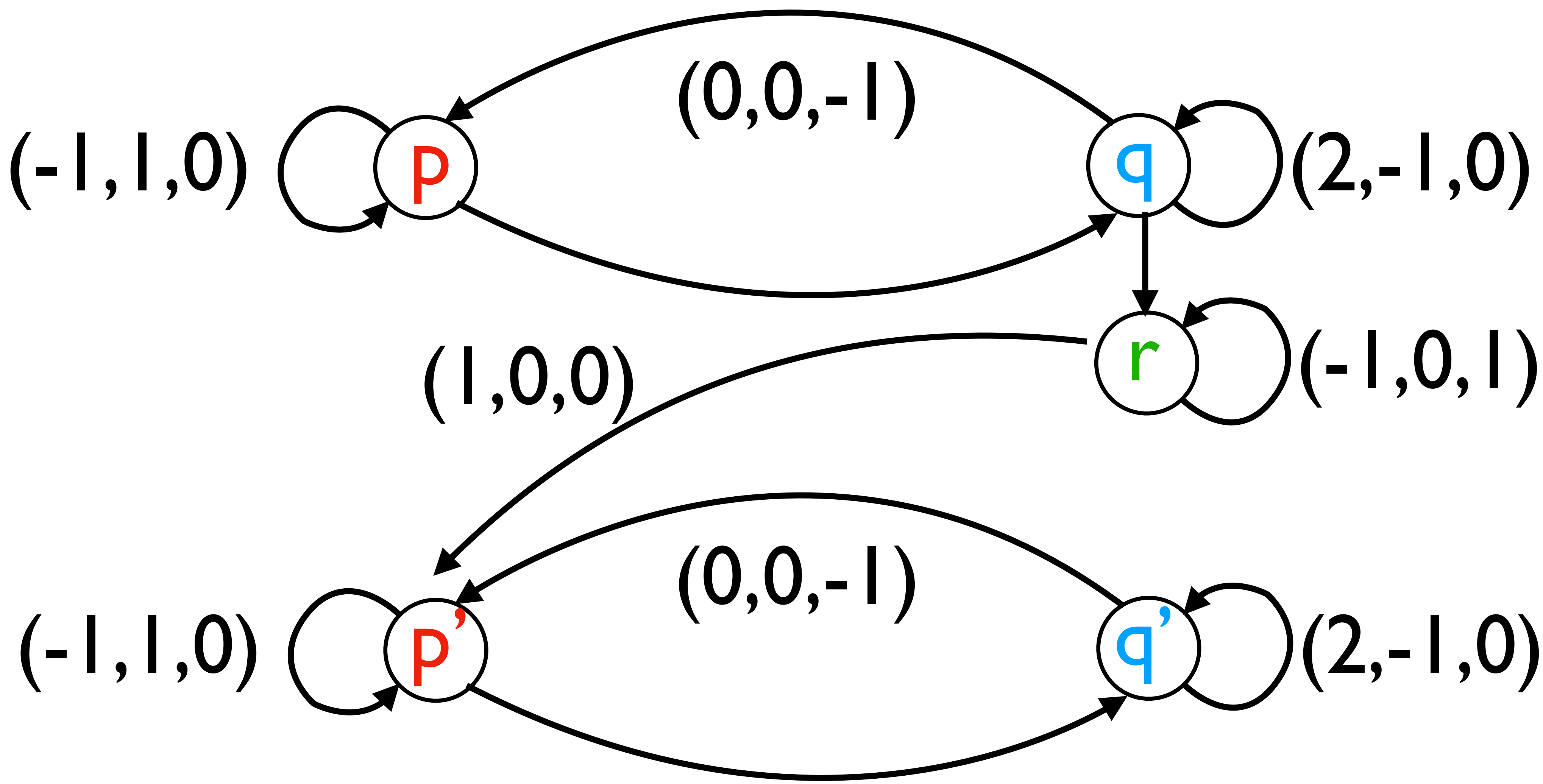
p(1,0,n) ⟶ q(2^n,0,0) ⟶ r(2^n,0,0)

3-VASS



$$p(1,0,n) \longrightarrow q(2^n,0,0) \longrightarrow r(2^n,0,0) \longrightarrow r(0,0,2^n)$$

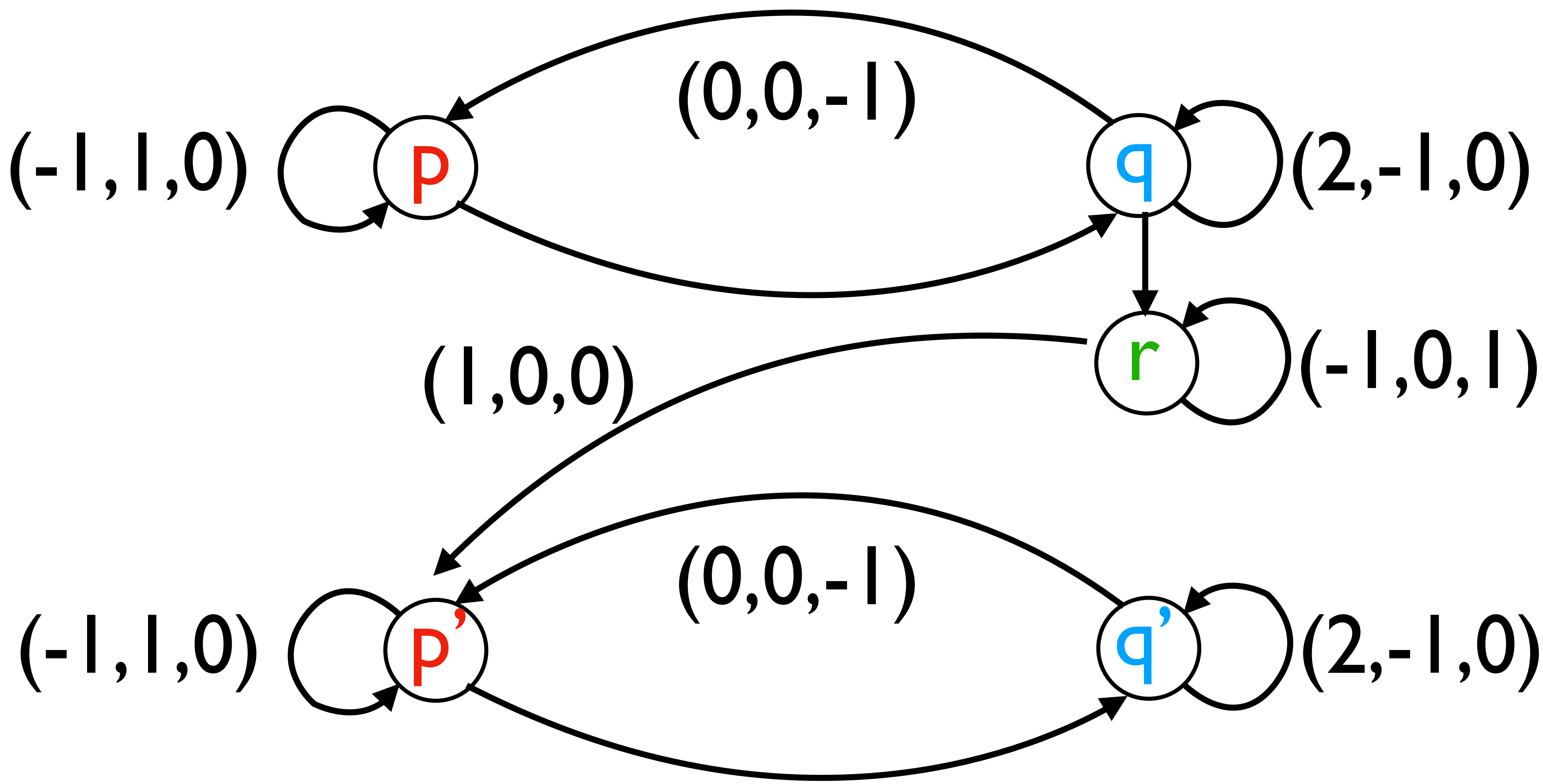
3-VASS



$p(1,0,n) \longrightarrow q(2^n,0,0) \longrightarrow r(2^n,0,0) \longrightarrow r(0,0,2^n)$

$\longrightarrow p'(1,0,2^n)$

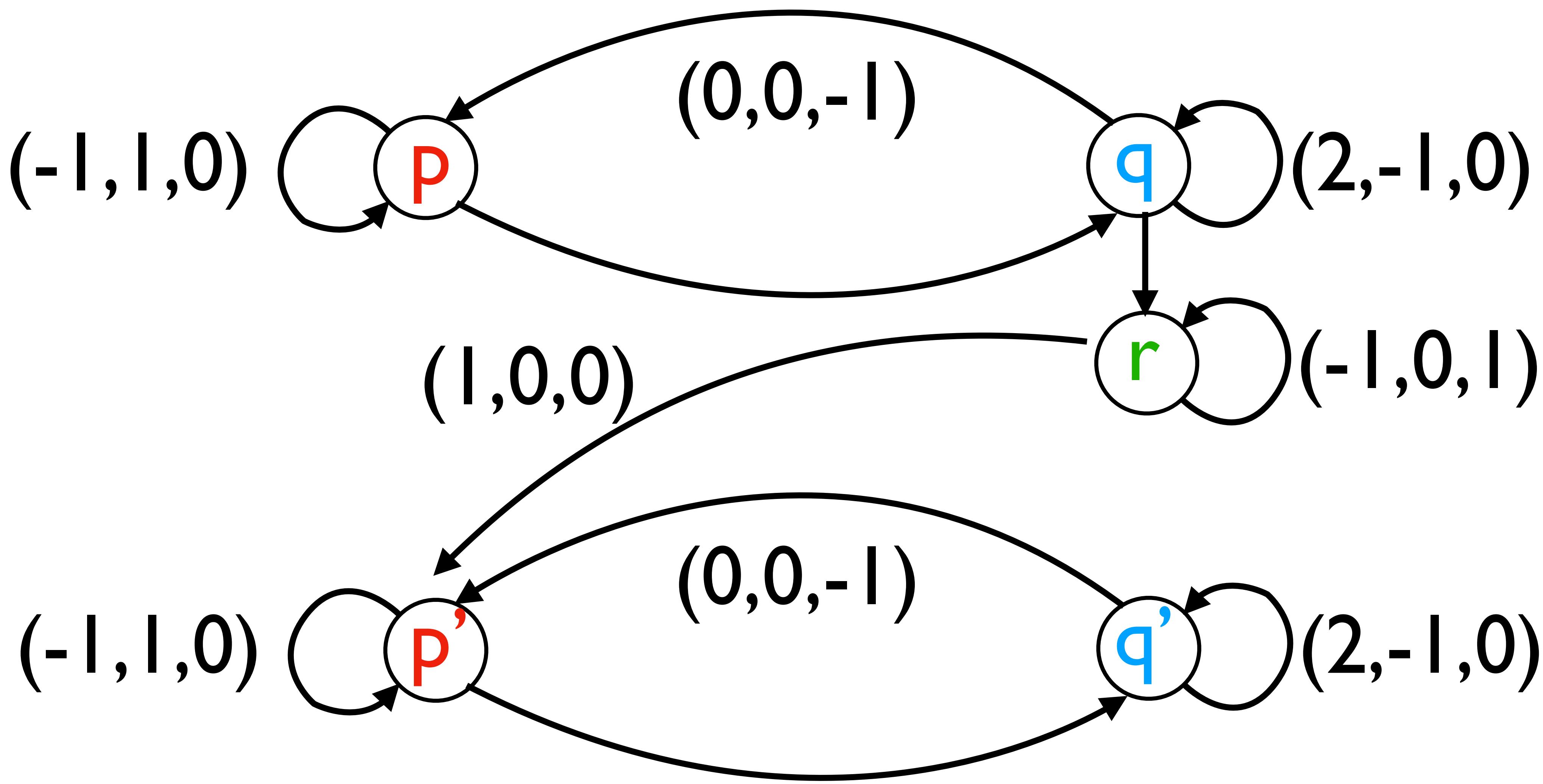
3-VASS



$$p(1,0,n) \longrightarrow q(2^n,0,0) \longrightarrow r(2^n,0,0) \longrightarrow r(0,0,2^n)$$

$$\longrightarrow p'(1,0,2^n) \longrightarrow p'(2^{2^n},0,0)$$

3-VASS



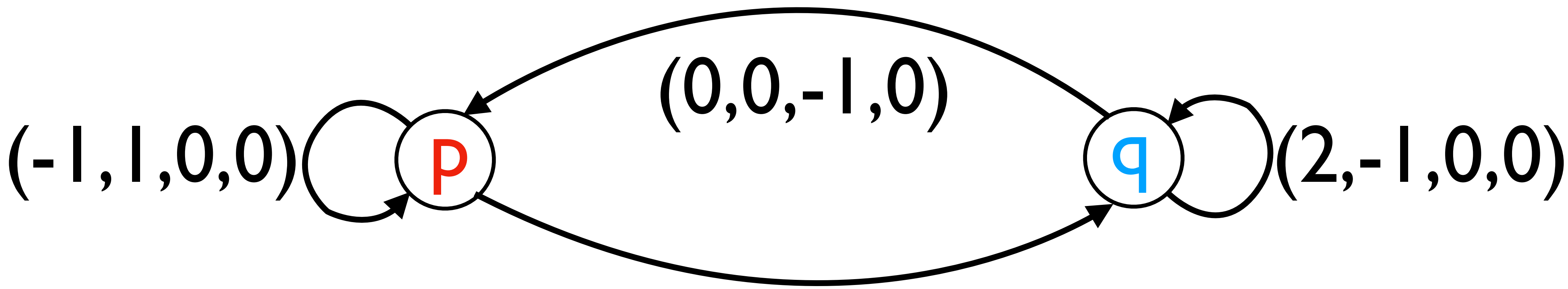
$p(1,0,n) \longrightarrow q(2^n,0,0) \longrightarrow r(2^n,0,0) \longrightarrow r(0,0,2^n)$

$\longrightarrow p'(1,0,2^n) \longrightarrow p'(2^{2^n},0,0)$

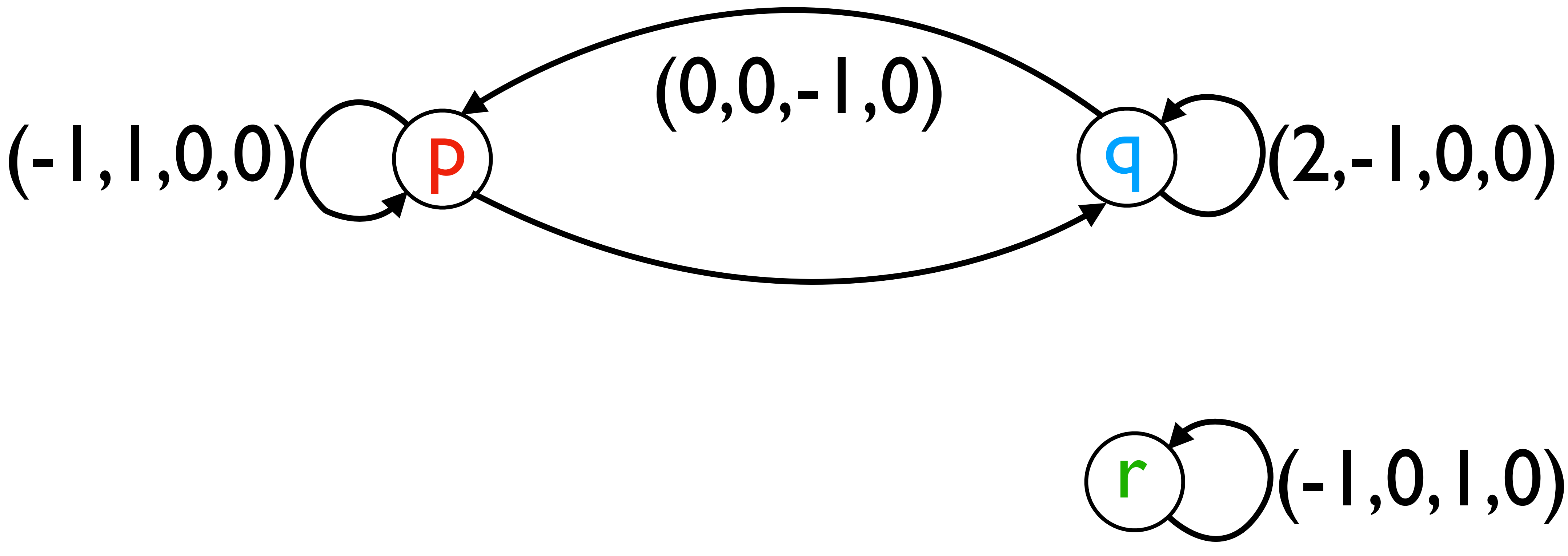
finite doubly-exponential reachability set

VASS

VASS



VASS



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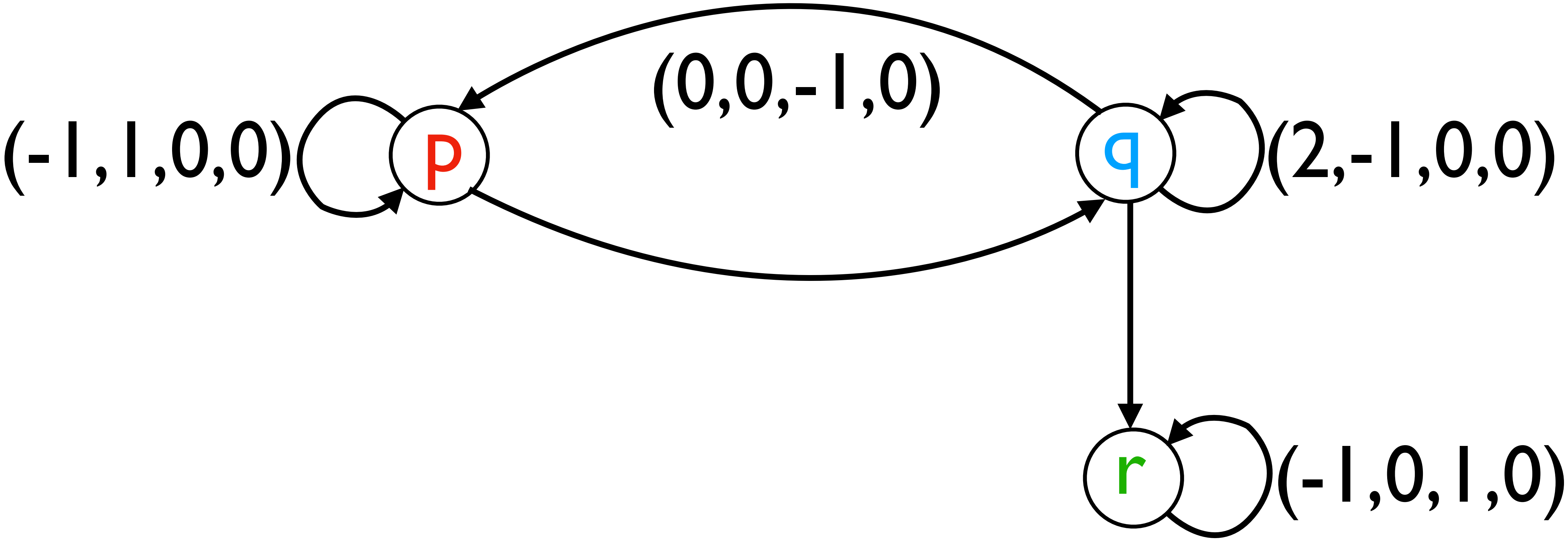

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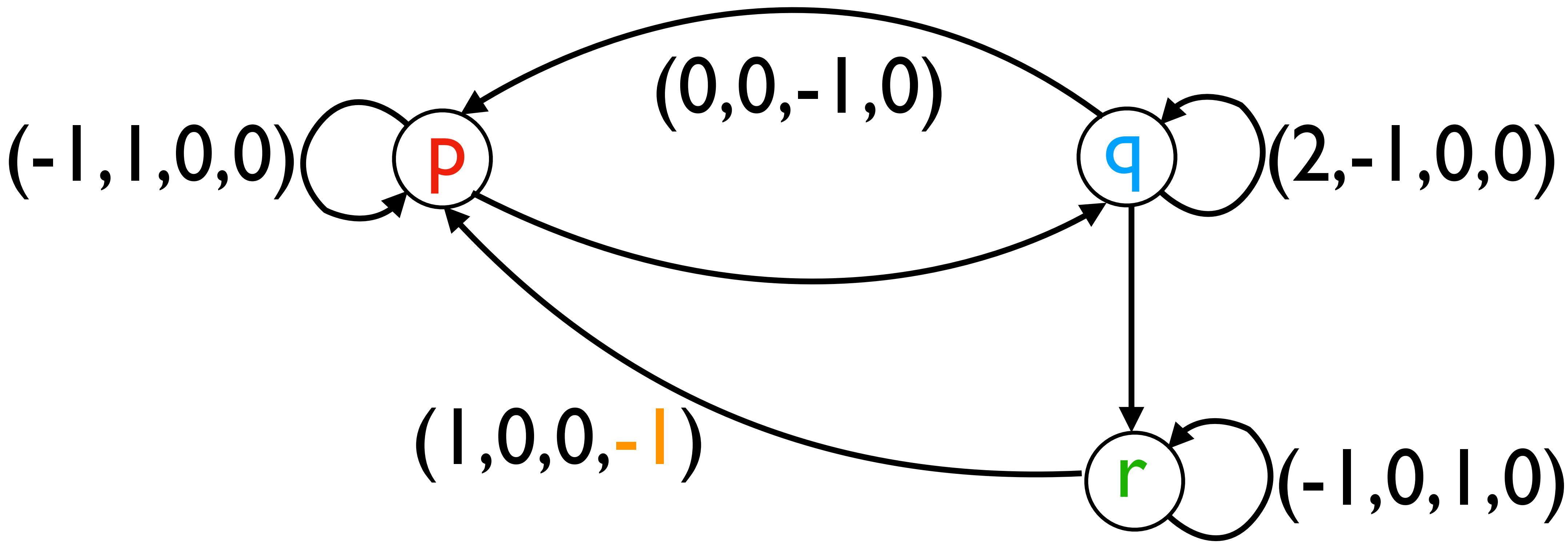

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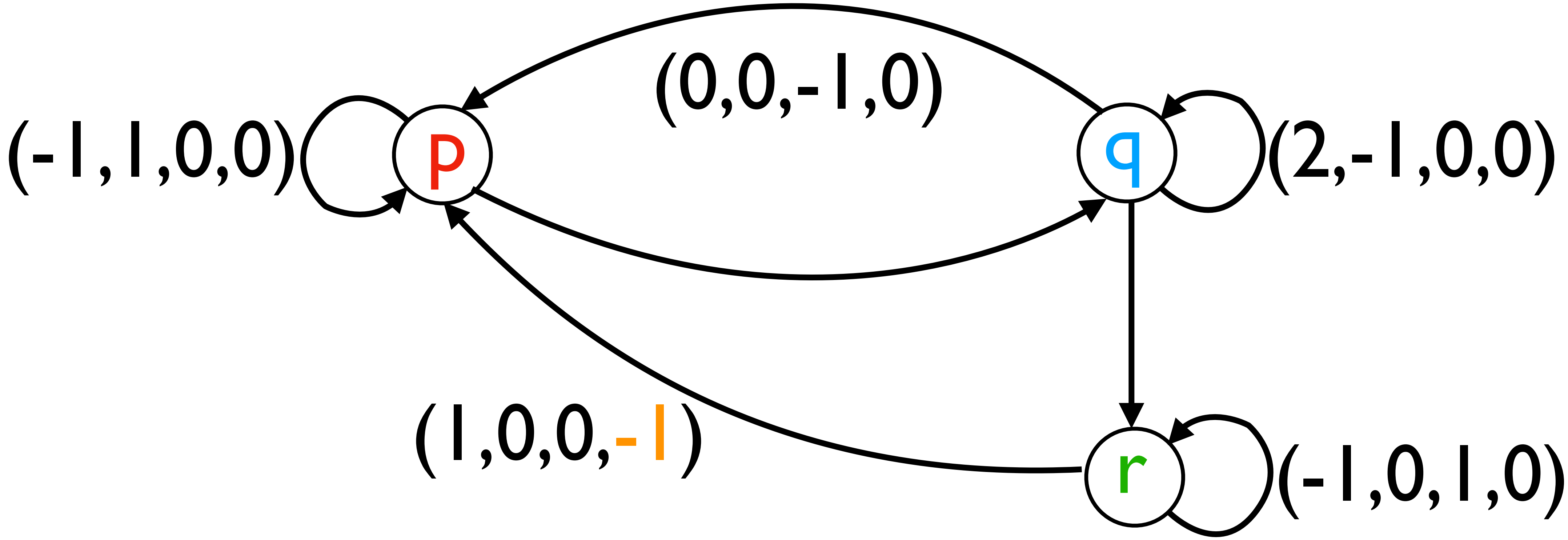
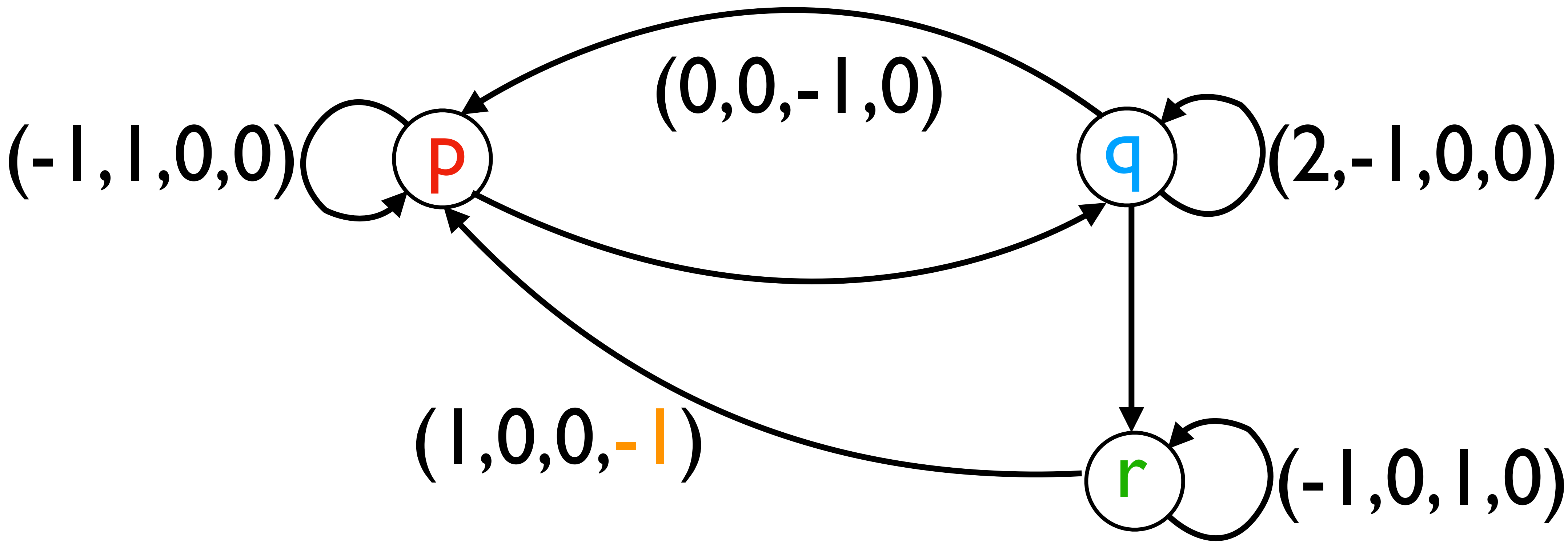


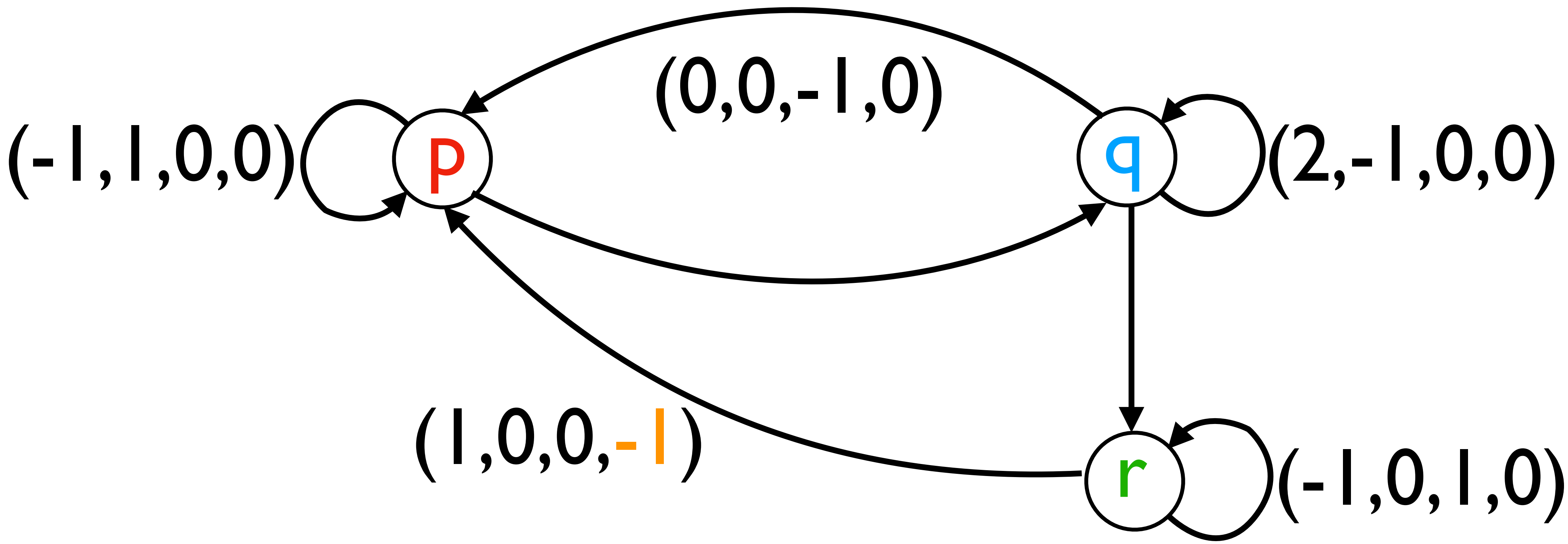
p(1,0,1,n)

VASS



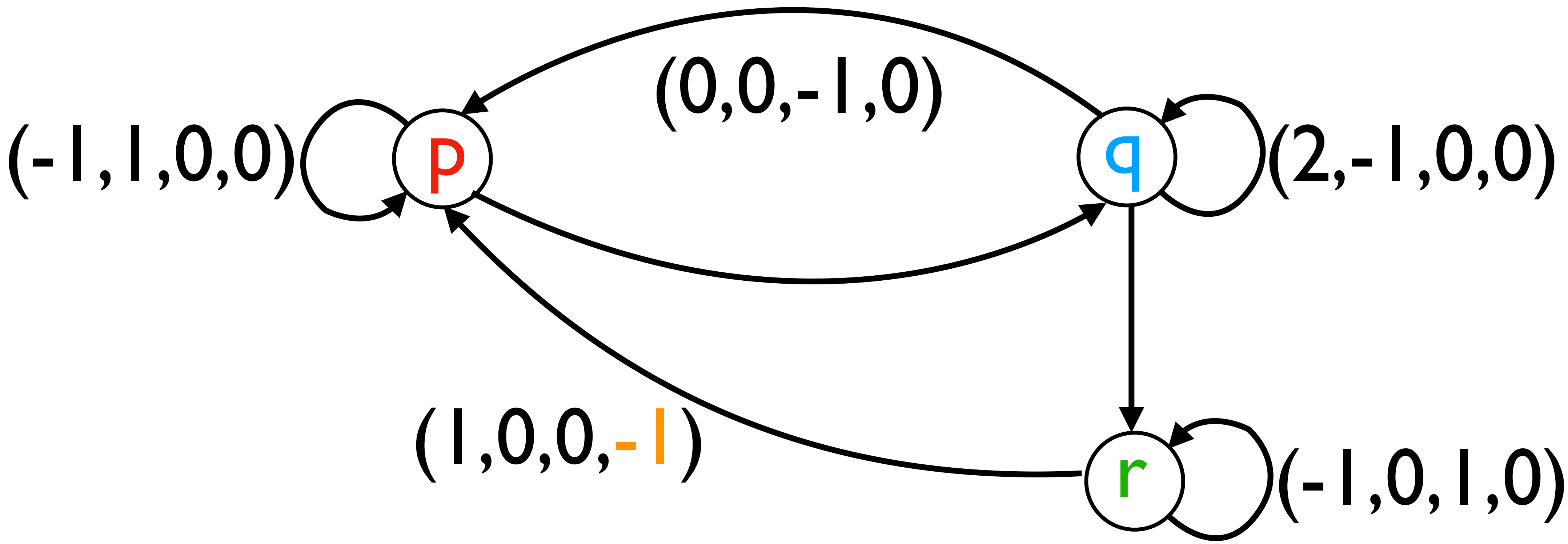
$p(1,0,1,n) \longrightarrow p(1,0,2^1,n-1)$

VASS



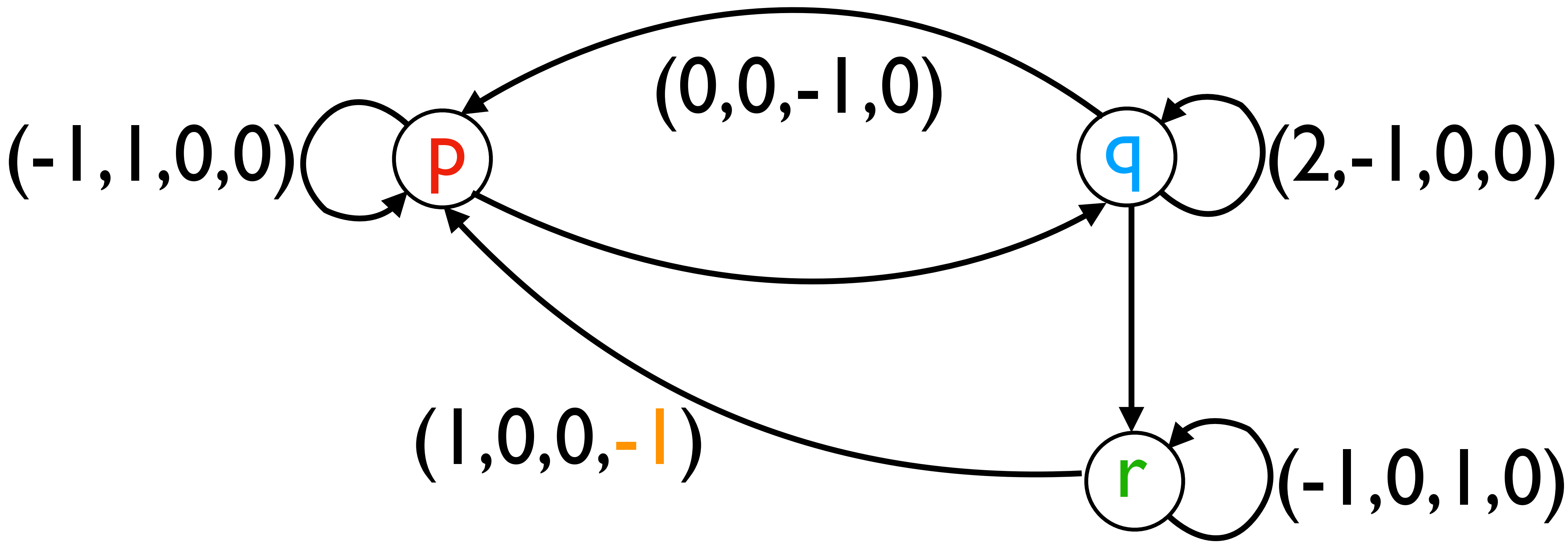
$p(1,0,1,n) \longrightarrow p(1,0,2^1,n-1) \ldots$

VASS



$p(1,0,1,n) \longrightarrow p(1,0,2^1,n-1) \ \ldots \longrightarrow p(1,0,\text{Tower}(n),0)$

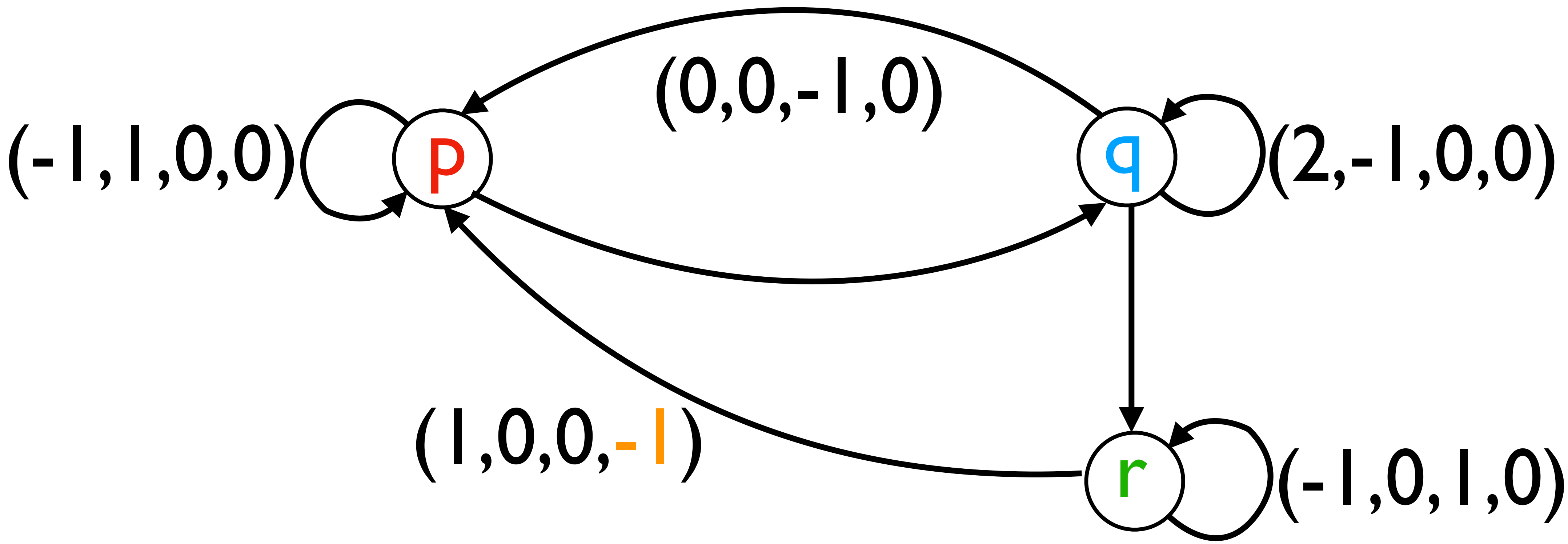
VASS



p(1,0,1,n) ⟶ p(1,0,2^1,n-1) … ⟶ p(1,0,Tower(n),0)

finite tower-size reachability set

VASS



p(1,0,1,n) ⟶ p(1,0,2^1,n-1) … ⟶ p(1,0,Tower(n),0)

finite tower-size reachability set

finite F_d-size reachability set

1-dim Pushdown VASS

1-dim Pushdown VASS

$S \longrightarrow n\ X$

1-dim Pushdown VASS

$S \longrightarrow n\ X$

$X \longrightarrow -1\ X\ 2 \mid 0$

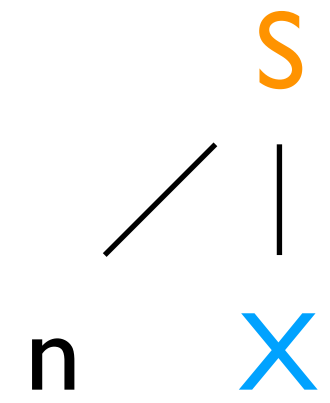
1-dim Pushdown VASS

S

S ⟶ n X

X ⟶ -1 X 2 | 0

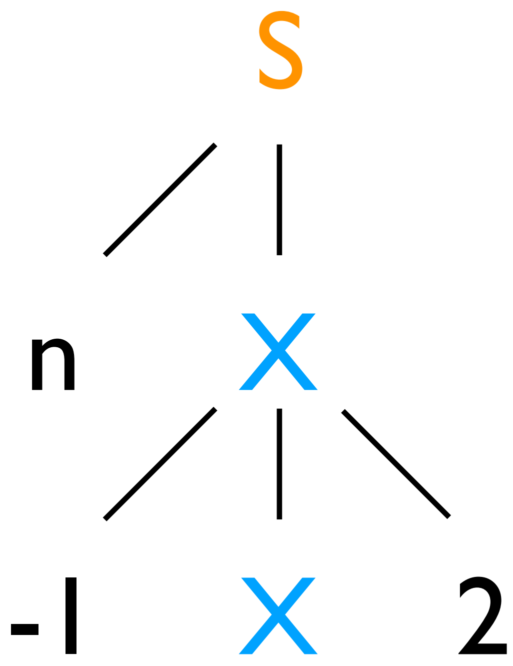
1-dim Pushdown VASS



S ⟶ n X

X ⟶ -1 X 2 | 0

1-dim Pushdown VASS



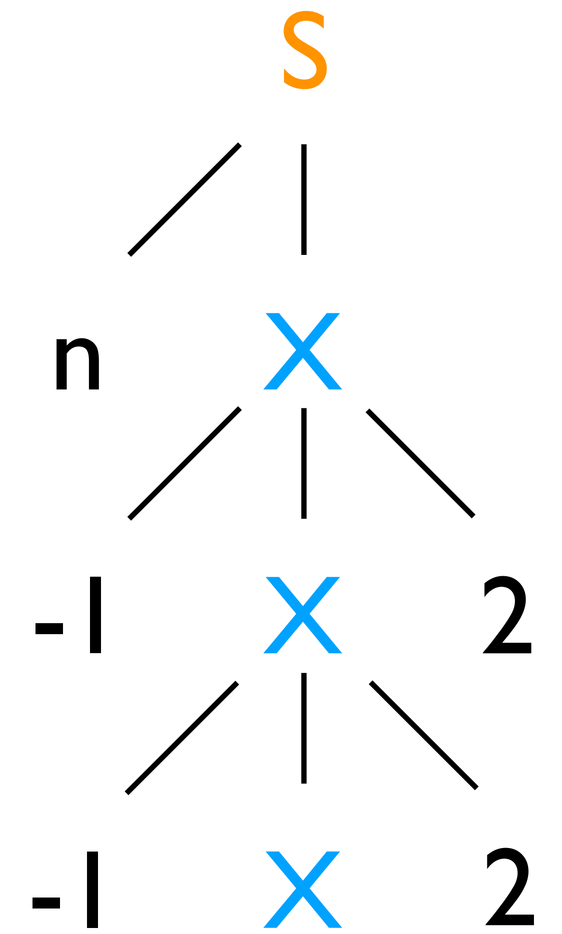
S ⟶ n X

X ⟶ -1 X 2 | 0

1-dim Pushdown VASS

S ⟶ n X

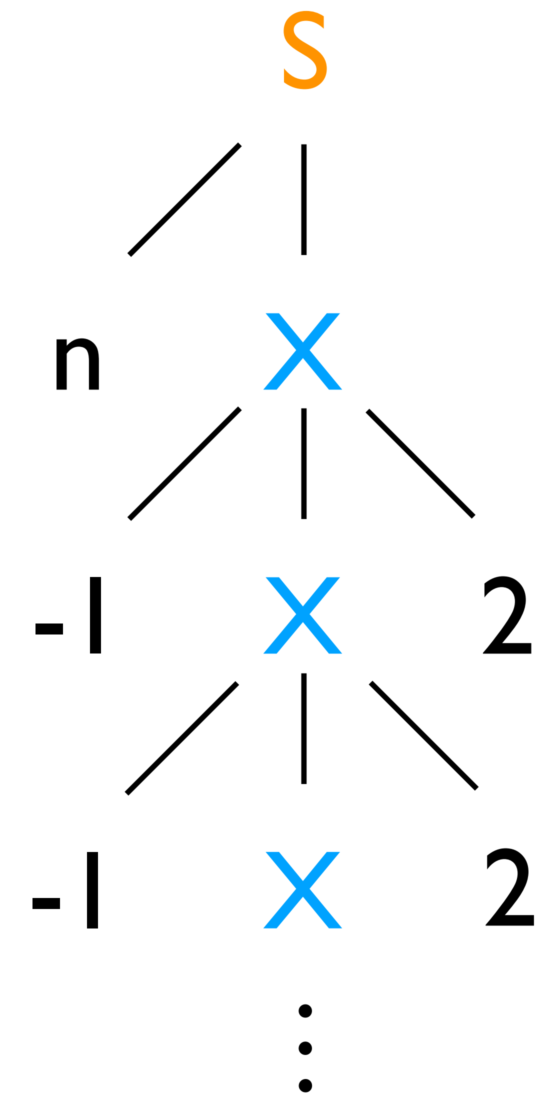
X ⟶ -1 X 2 | 0



1-dim Pushdown VASS

$S \longrightarrow n\ X$

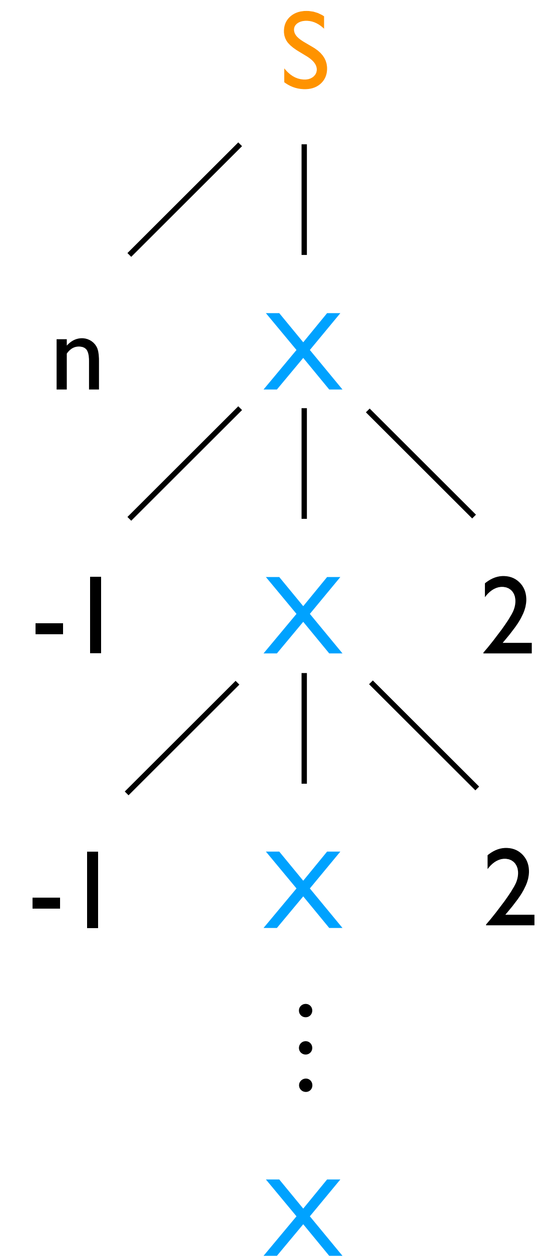
$X \longrightarrow -1\ X\ 2 \mid 0$



1-dim Pushdown VASS

S ⟶ n X

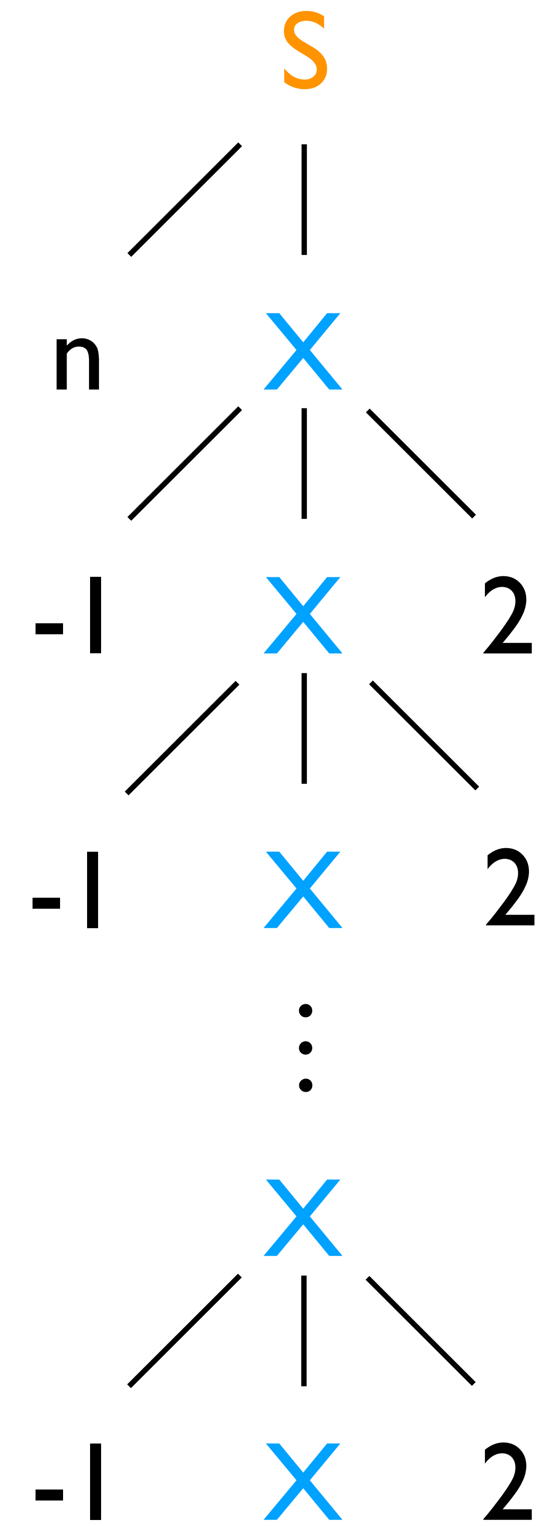
X ⟶ -1 X 2 | 0



1-dim Pushdown VASS

S ⟶ n X

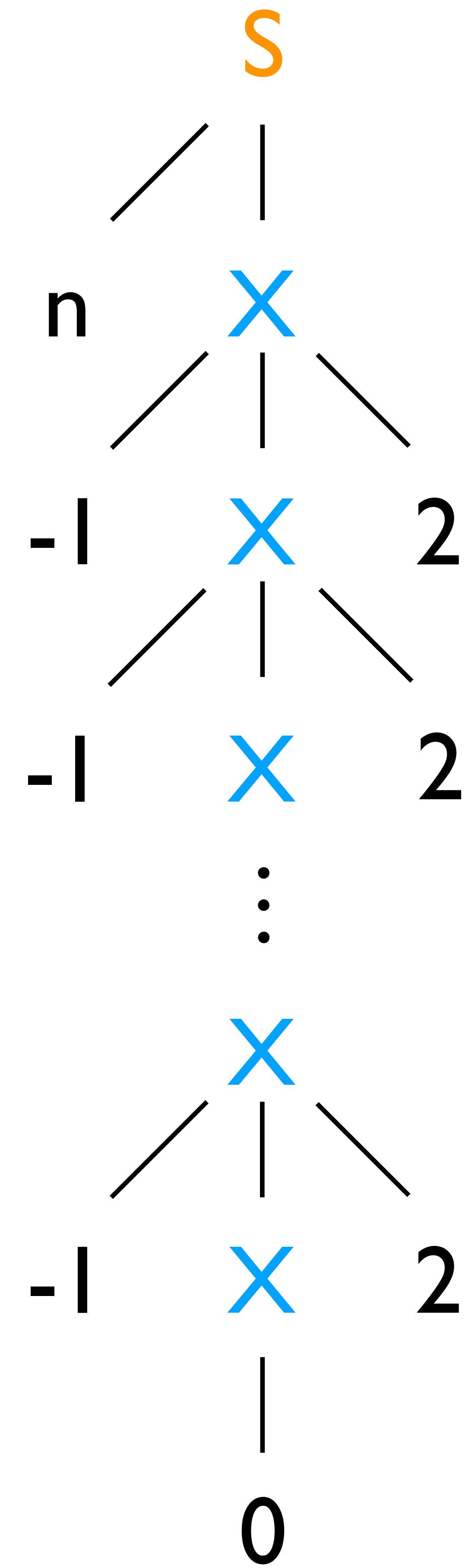
X ⟶ -1 X 2 | 0



1-dim Pushdown VASS

S ⟶ n X

X ⟶ -1 X 2 | 0

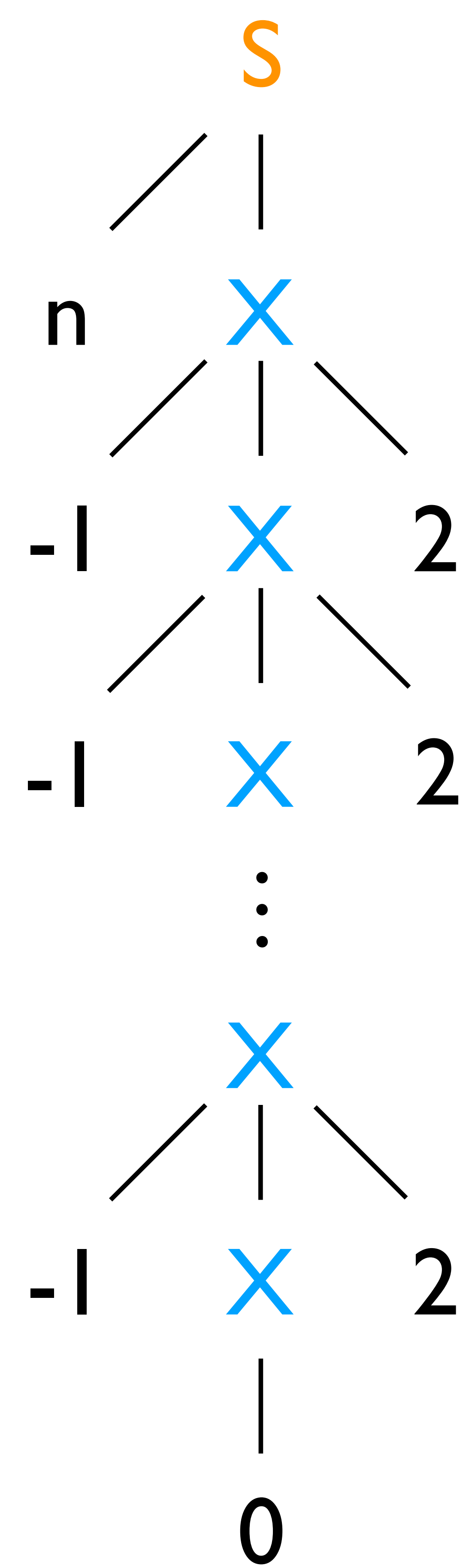


1-dim Pushdown VASS

$S \longrightarrow n\ X$

$X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$



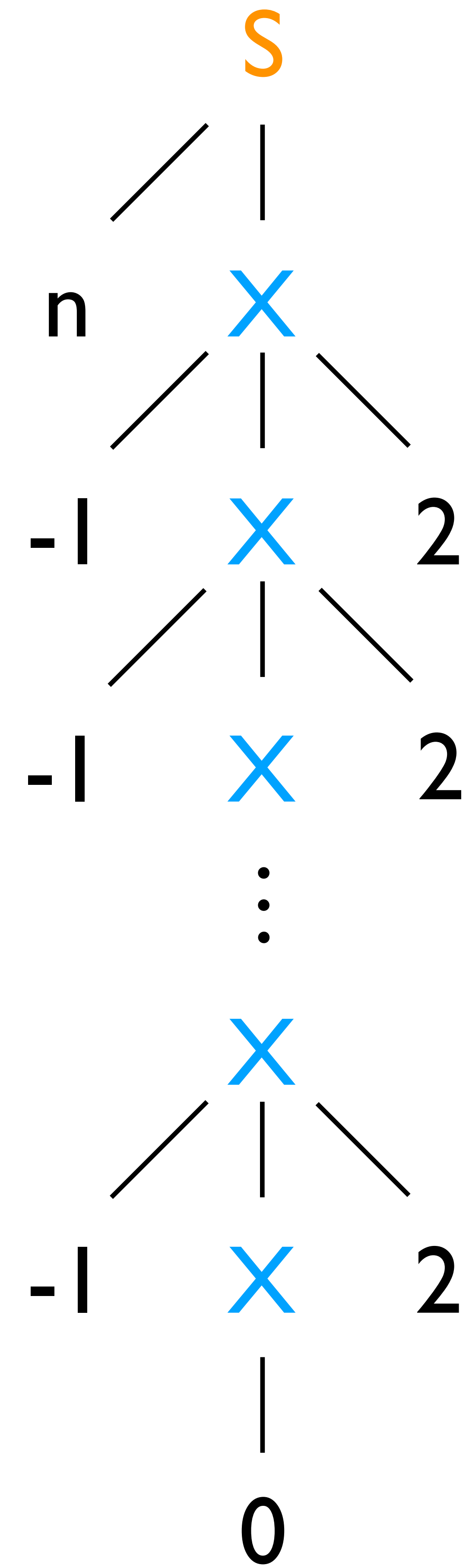
1-dim Pushdown VASS

$S \longrightarrow n\ X$

$X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$

maximally



1-PVASS

1-PVASS

$S \longrightarrow nY$

1-PVASS

$$S \longrightarrow n\,Y$$

$$Y \longrightarrow -1\,Y\,X \mid 1$$

1-PVASS

$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

$X \longrightarrow -1\;X\;2 \mid 0$

1-PVASS

$S \longrightarrow nY$

$Y \longrightarrow -1\ Y\ X \mid 1$

$X \longrightarrow -1\ X\ 2 \mid 0$

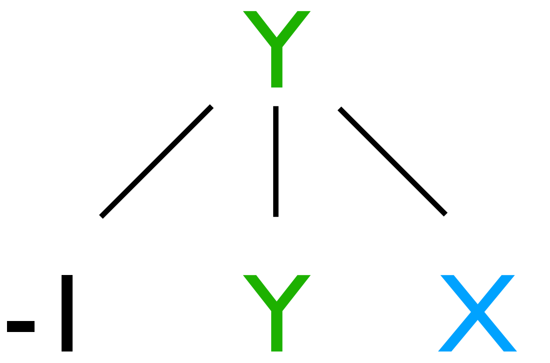
Y

1-PVASS

$S \longrightarrow n\, Y$

$Y \longrightarrow -1\, Y\, X \mid 1$

$X \longrightarrow -1\, X\, 2 \mid 0$

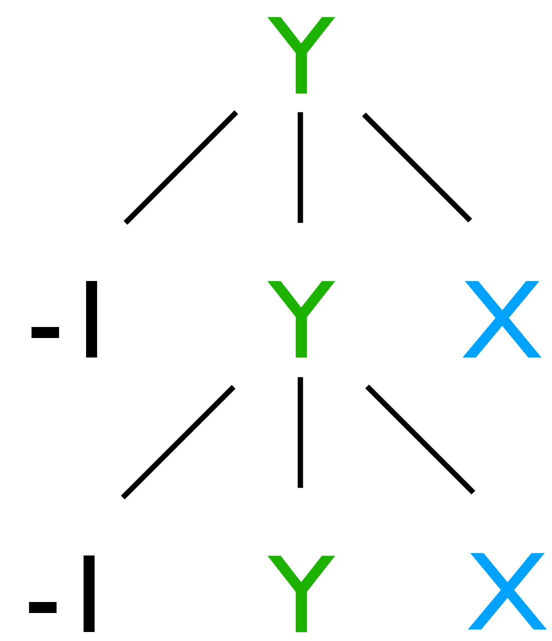


1-PVASS

$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

$X \longrightarrow -1\;X\;2 \mid 0$

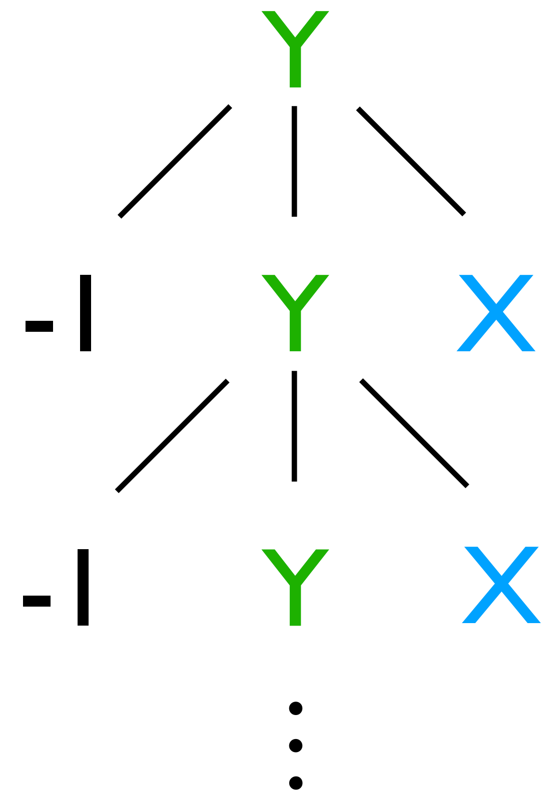


1-PVASS

$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

$X \longrightarrow -1\;X\;2 \mid 0$

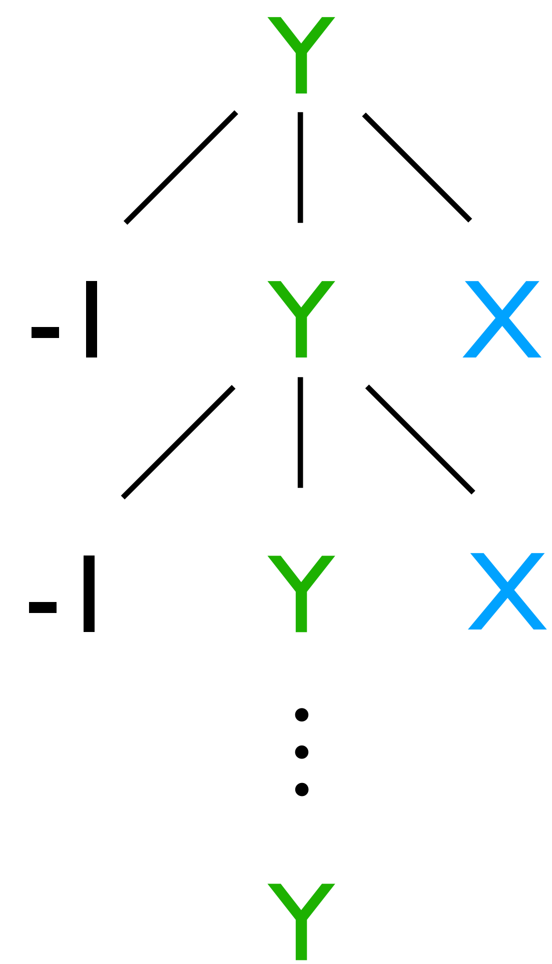


1-PVASS

$S \longrightarrow n\, Y$

$Y \longrightarrow -1\, Y\, X \mid 1$

$X \longrightarrow -1\, X\, 2 \mid 0$

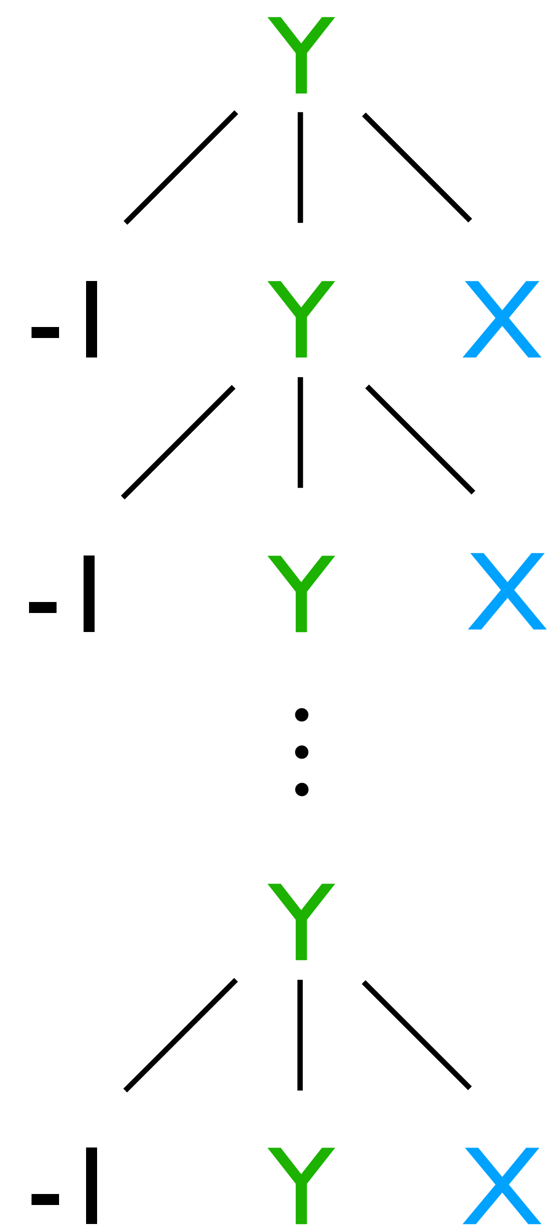


1-PVASS

S ⟶ n Y

Y ⟶ -1 Y X | 1

X ⟶ -1 X 2 | 0

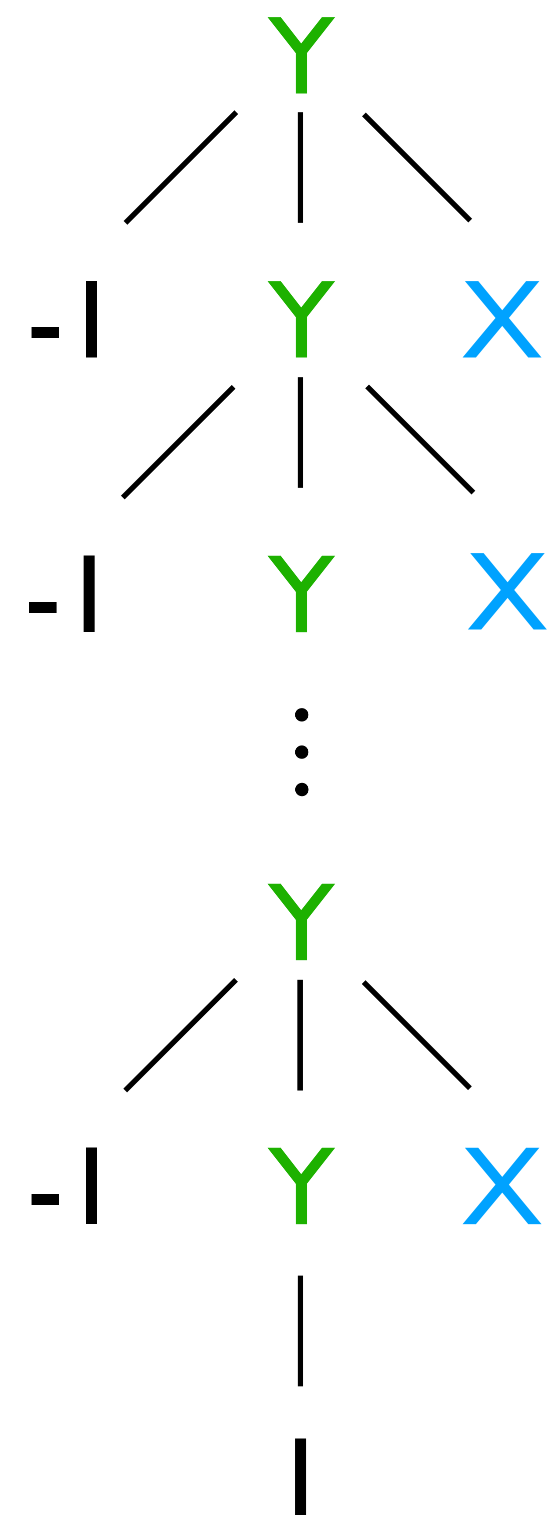


1-PVASS

S ⟶ n Y

Y ⟶ -1 Y X | 1

X ⟶ -1 X 2 | 0



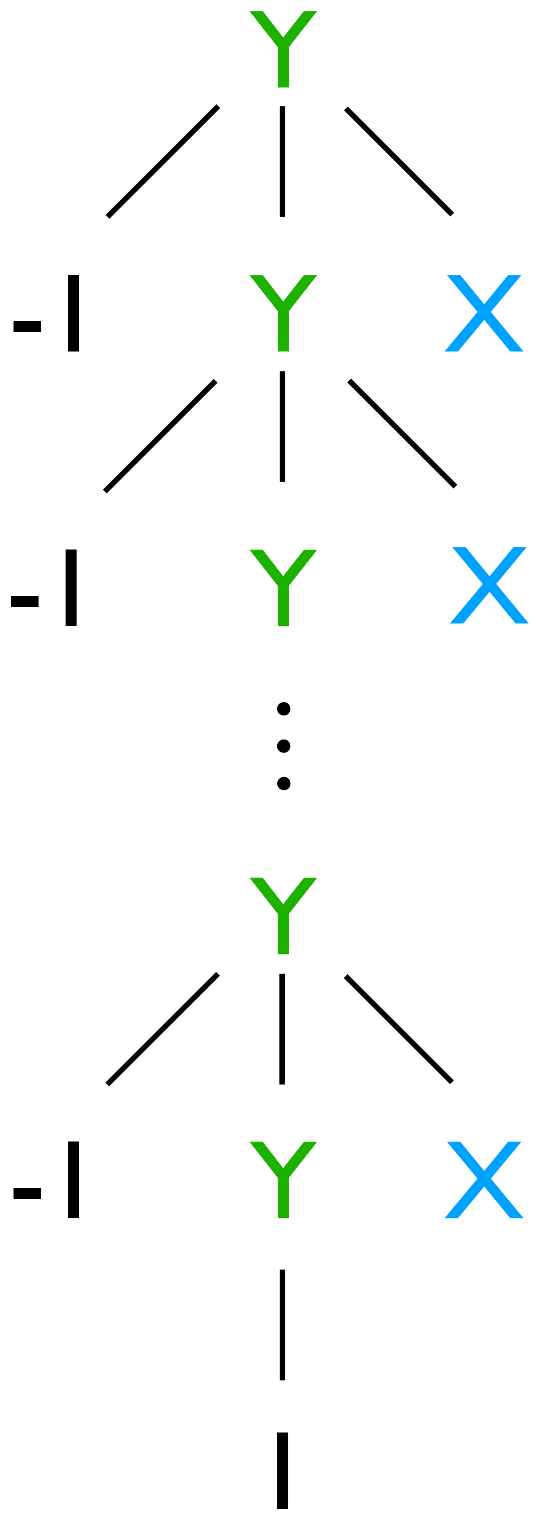
1-PVASS

$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

$X \longrightarrow -1\;X\;2 \mid 0$

$$k \xrightarrow{X} 2k$$



1-PVASS

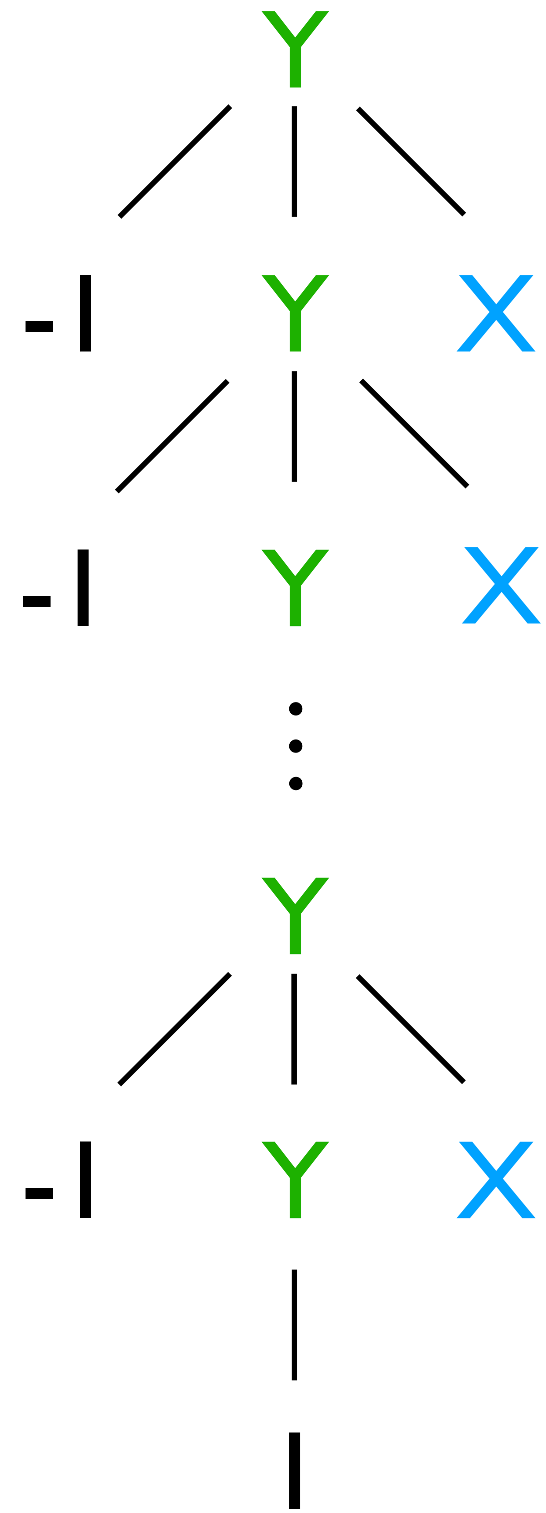
$S \longrightarrow n\,Y$

$Y \longrightarrow -1\;Y\;X \mid 1$

$X \longrightarrow -1\;X\;2 \mid 0$

$k \xrightarrow{X} 2k$

$k \xrightarrow{Y} 2^k$



1-dim Pushdown VASS

1-dim Pushdown VASS

S ⟶ n Z

1-dim Pushdown VASS

$S \longrightarrow n\ Z \qquad\qquad Z \longrightarrow -1\ Z\,Y \mid 1$

1-dim Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

1-dim Pushdown VASS

$$S \longrightarrow n\ Z \qquad\qquad Z \longrightarrow -1\ Z\ Y \mid 1$$

$$Y \longrightarrow -1\ Y\ X \mid 1 \qquad\qquad X \longrightarrow -1\ X\ 2 \mid 0$$

1-dim Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

$X \longrightarrow -1\ X\ 2 \mid 0$

$$k \xrightarrow{X} 2k$$

1-dim Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

$X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$

$k \xrightarrow{Y} 2^k$

1-dim Pushdown VASS

$S \longrightarrow n\ Z$

$Z \longrightarrow -1\ Z\ Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$

$X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$

$k \xrightarrow{Y} 2^k$

$k \xrightarrow{Z} \text{Tower}(k)$

1-dim Pushdown VASS

$S \longrightarrow n\, Z$ $\qquad$ $Z \longrightarrow -1\ Z\, Y \mid 1$

$Y \longrightarrow -1\ Y\ X \mid 1$ $\qquad$ $X \longrightarrow -1\ X\ 2 \mid 0$

$k \xrightarrow{X} 2k$ $\qquad$ $k \xrightarrow{Y} 2^k$ $\qquad$ $k \xrightarrow{Z} \mathrm{Tower}(k)$

d+1 nonterminals: reachability set of size $F_d(n)$

Message

Message

simple models are involved

Message

simple models are involved

fundamental but hard research

Message

simple models are involved

fundamental but hard research

still many open problems:

Message

simple models are involved

fundamental but hard research

still many open problems: 3-VASS

Message

simple models are involved

fundamental but hard research

still many open problems: 3-VASS   1-PVASS

Message

simple models are involved

fundamental but hard research

still many open problems: 3-VASS 1-PVASS

Thank you!