Multi-Stack Automata

Wojciech Czerwiński

Multi-Stack Automata

(Partially-Commutive Context-Free Graphs)

Wojciech Czerwiński

Plan

Plan

- what are Multi-Stack Automata

Plan

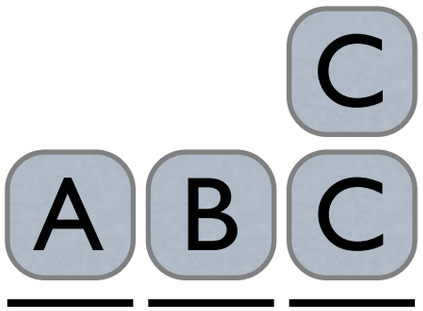
- what are Multi-Stack Automata
- why I investigate them

Plan

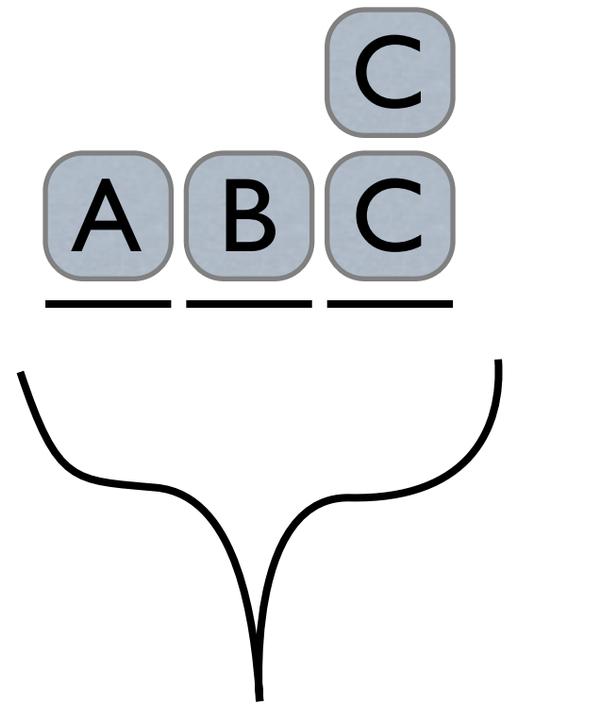
- what are Multi-Stack Automata
- why I investigate them
- what I have solved

Multi-Stack Automaton

Multi-Stack Automaton

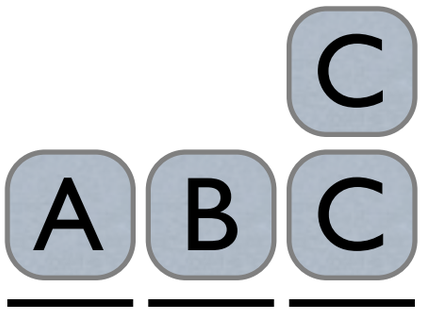


Multi-Stack Automaton



Configuration
(3 stacks)

Multi-Stack Automaton



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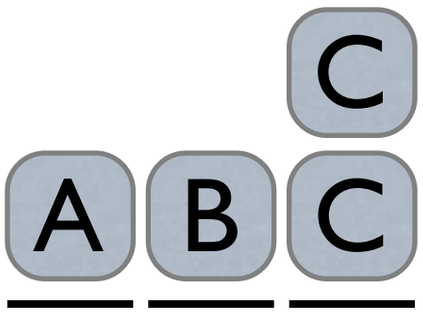
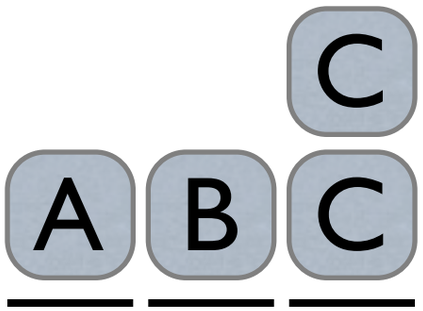


$C \longrightarrow (A, BD, C)$

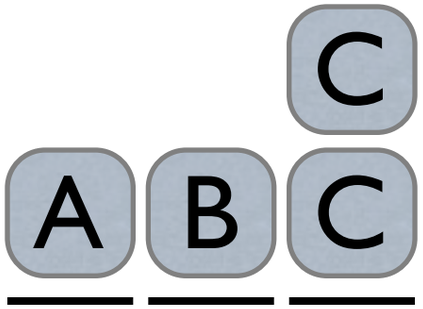
Multi-Stack Automaton



$C \longrightarrow (A, BD, C)$

$A \longrightarrow (\_, B, \_)$

Multi-Stack Automaton

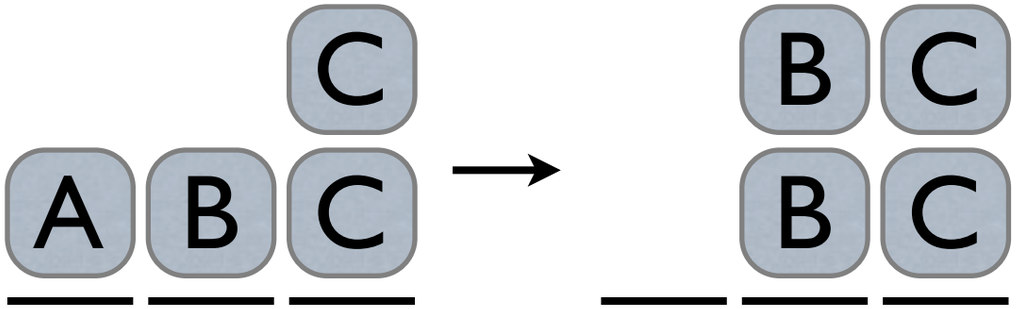


$C \longrightarrow (A, BD, C)$

$A \longrightarrow (\_, B, \_)$

$C \longrightarrow (\_, \_, \_)$

Multi-Stack Automaton

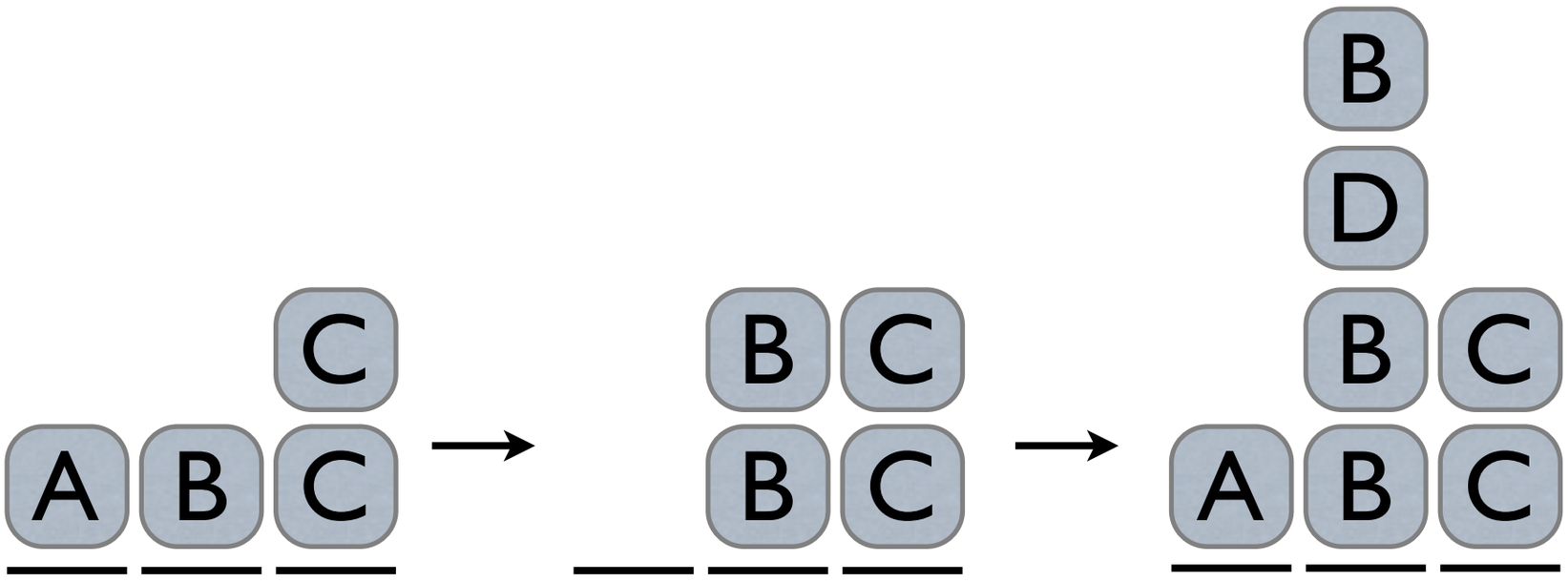


C ⟶ (A, BD, C)

A ⟶ (_, B, _)

C ⟶ (_, _, _)

Multi-Stack Automaton



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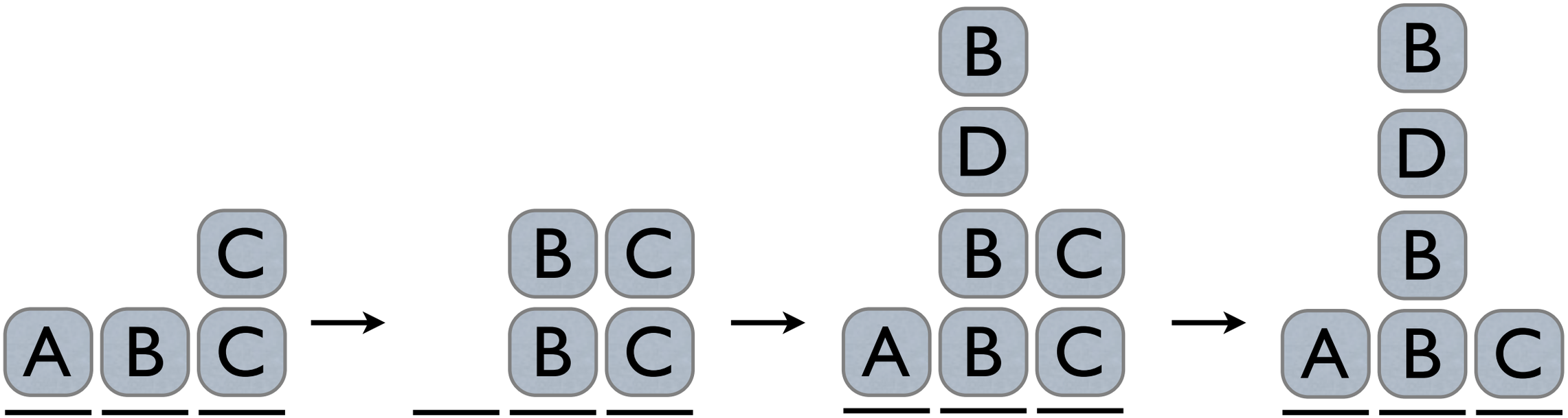
Multi-Stack Automaton



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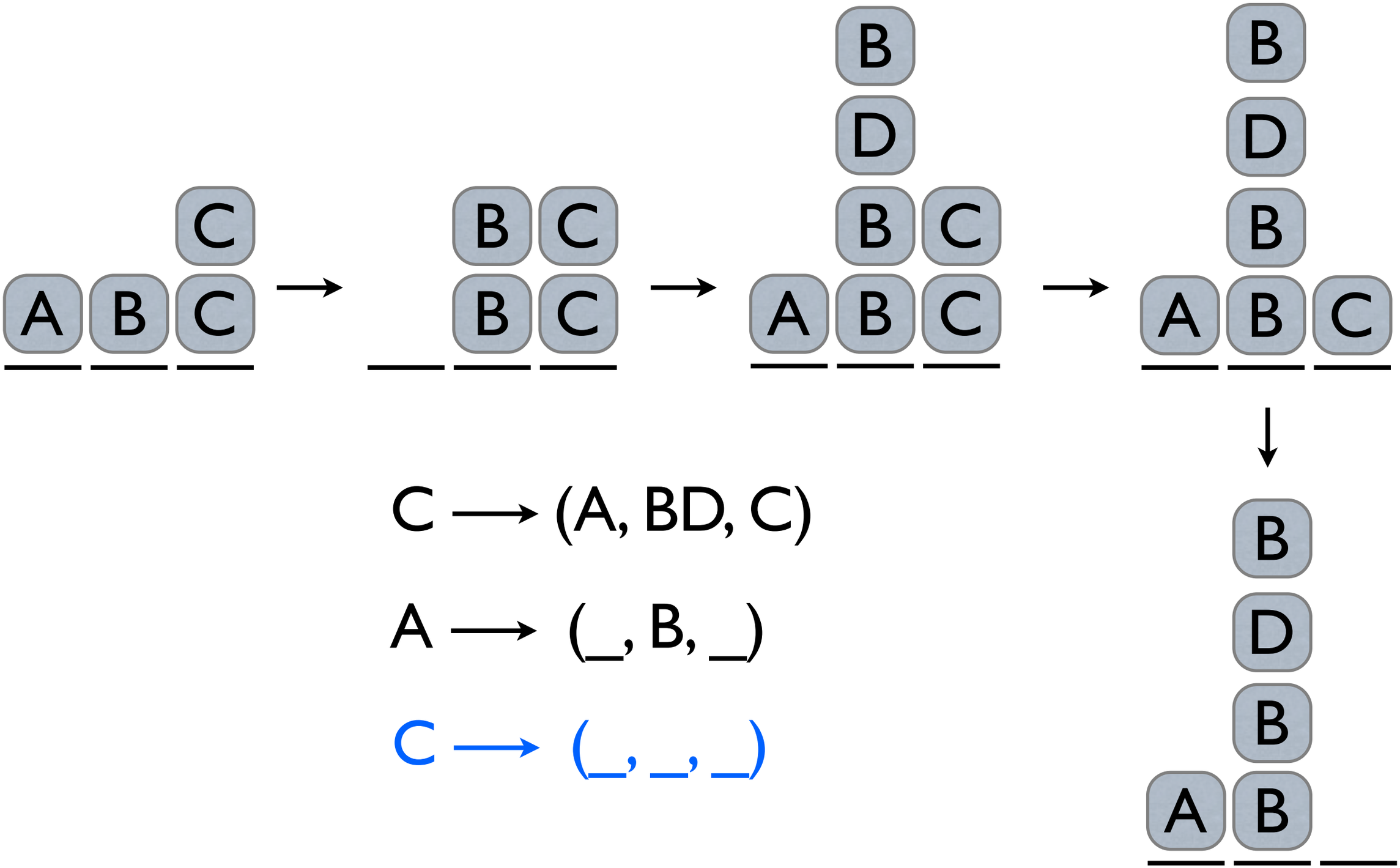

C ⟶ (A, BD, C)

A ⟶ (_, B, _)

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Motivation

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- errors in programs

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- errors in programs
- automatic detection of errors

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- errors in programs
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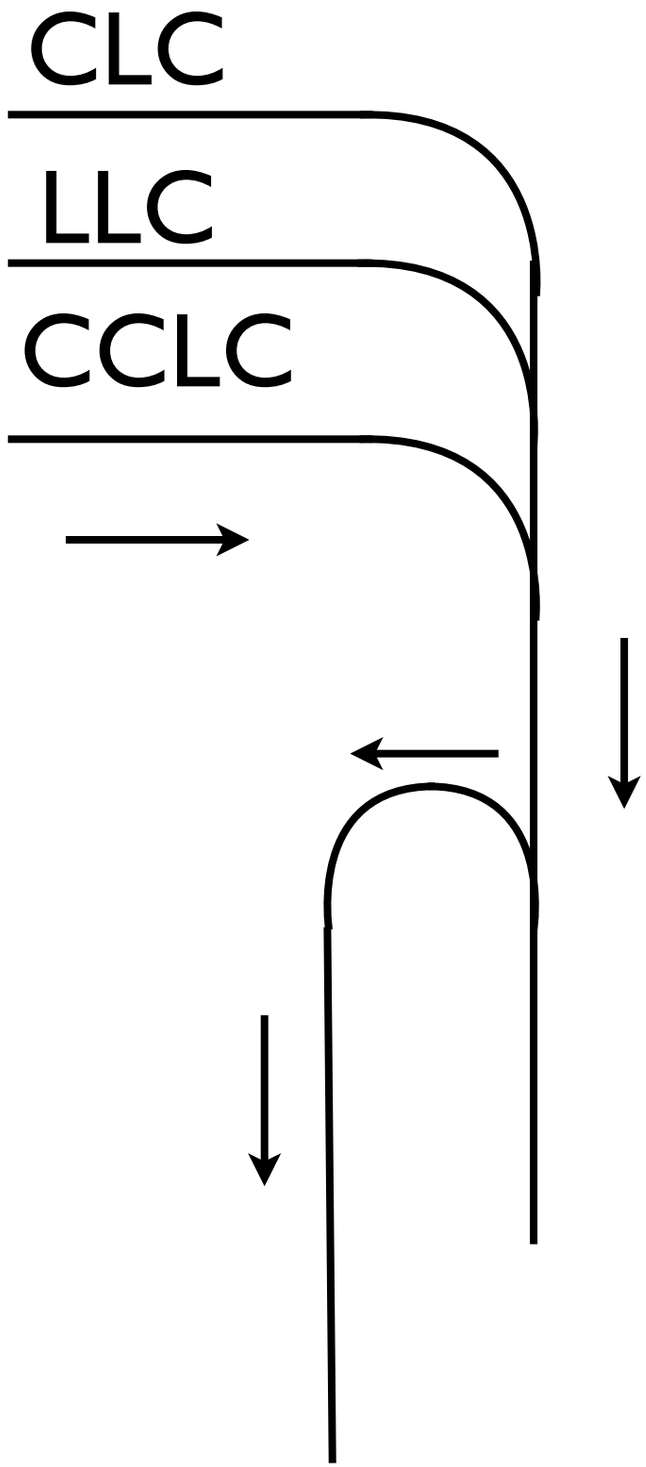
- model the program

Motivation

- errors in programs
- automatic detection of errors
- model the program
- Multi-Stack Automata as a model

Real life example



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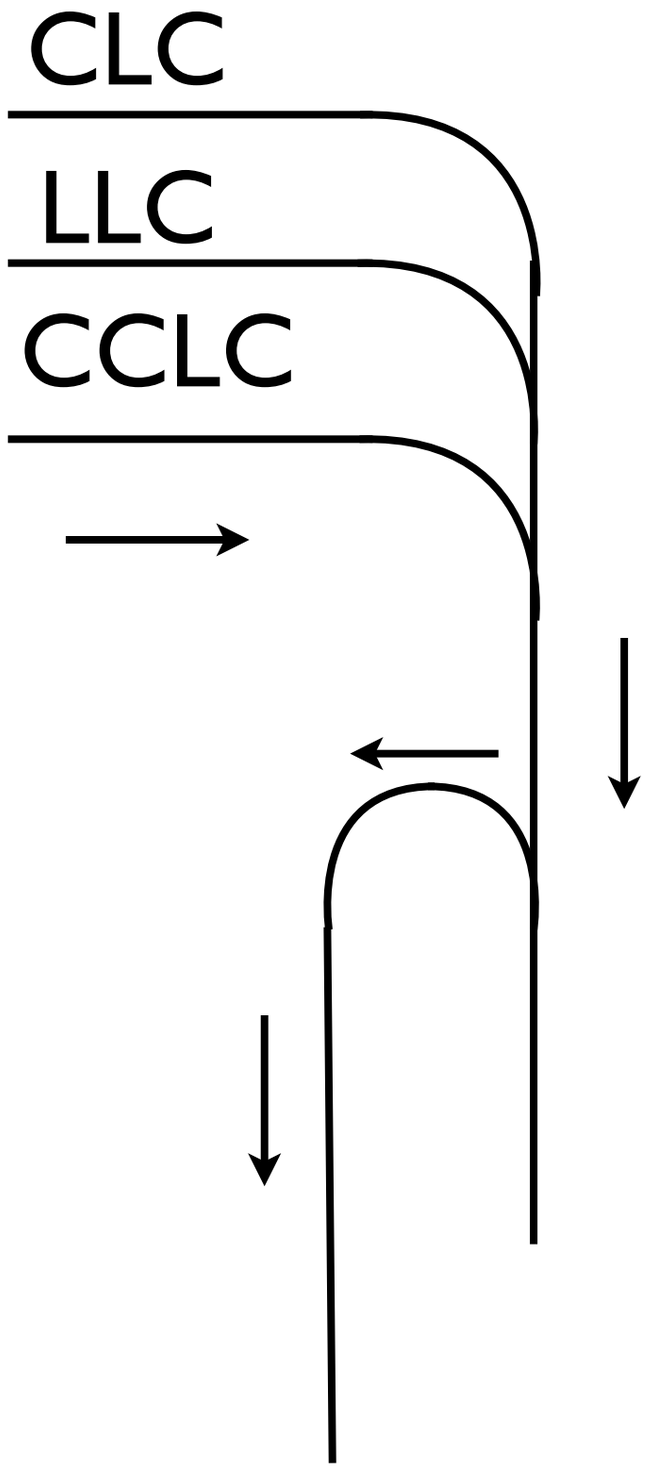
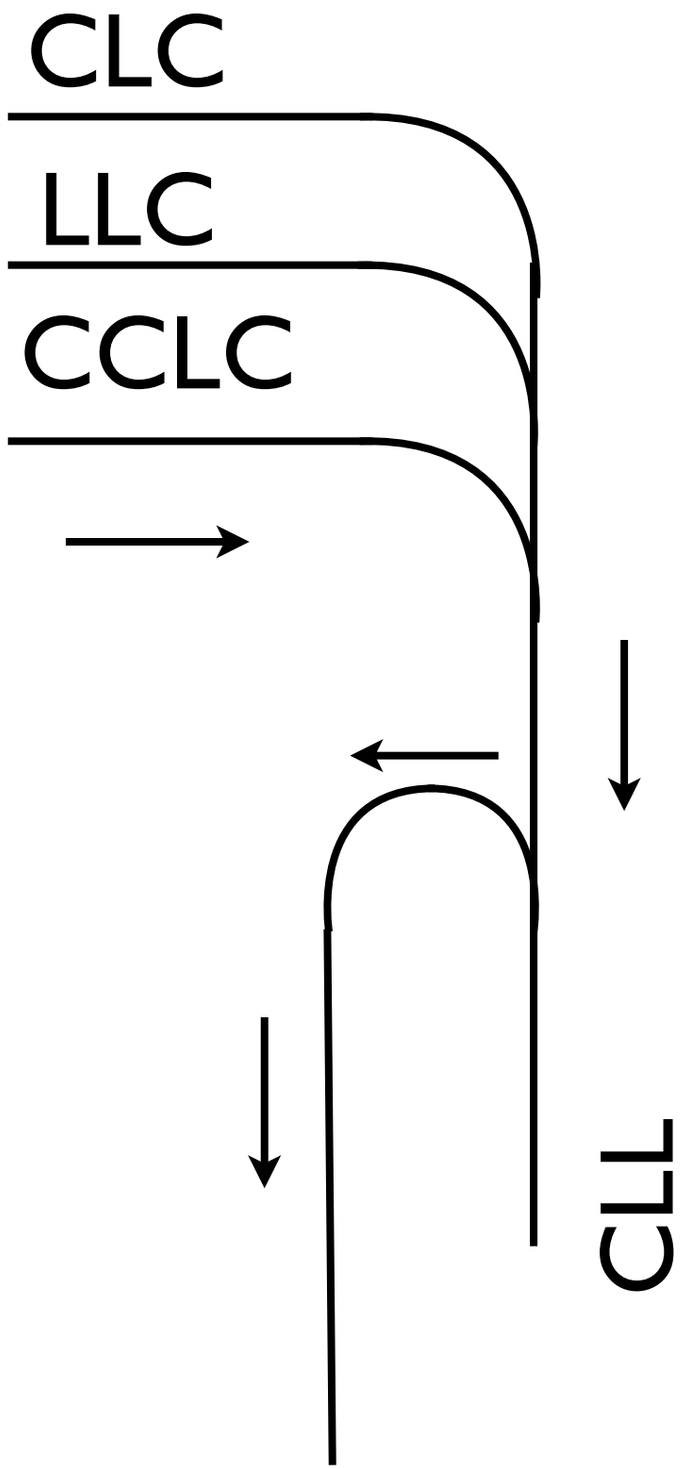


Can we produce trains with 2 cars and 1 locomotive?

Real life example



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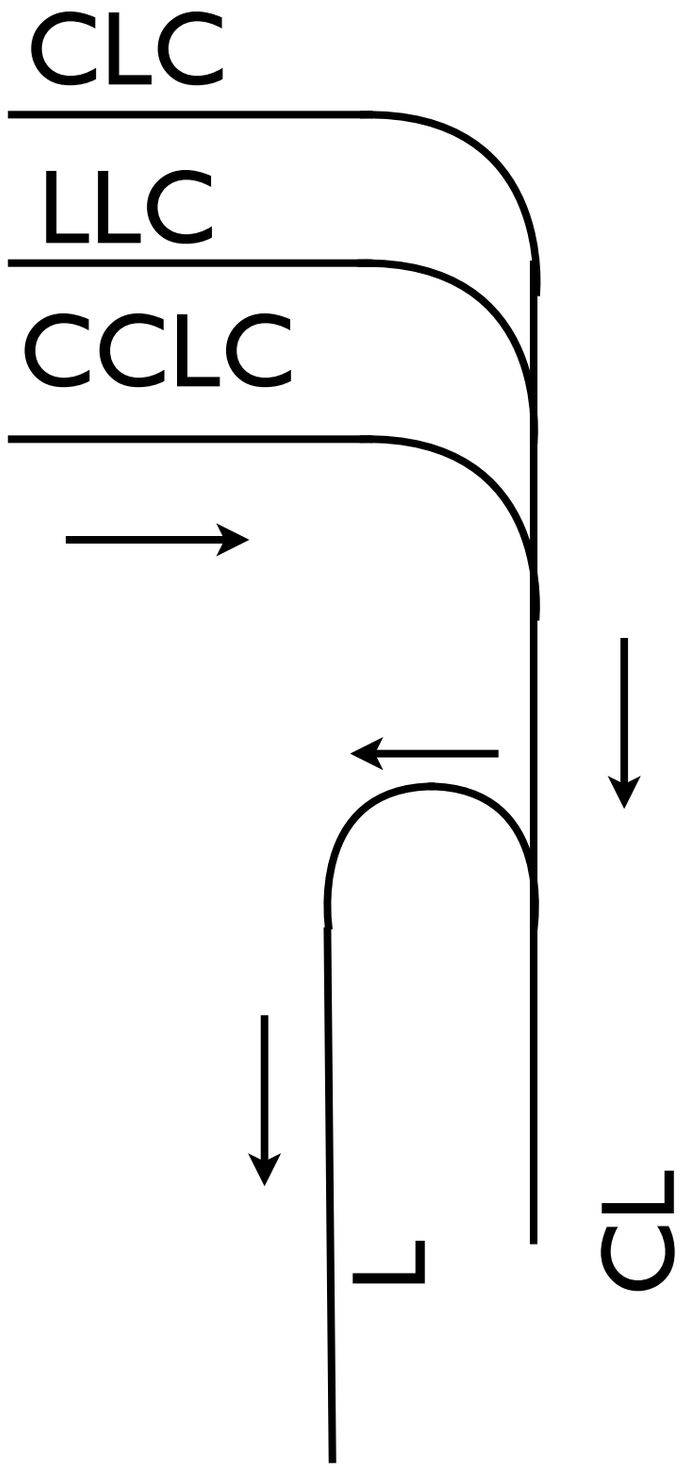

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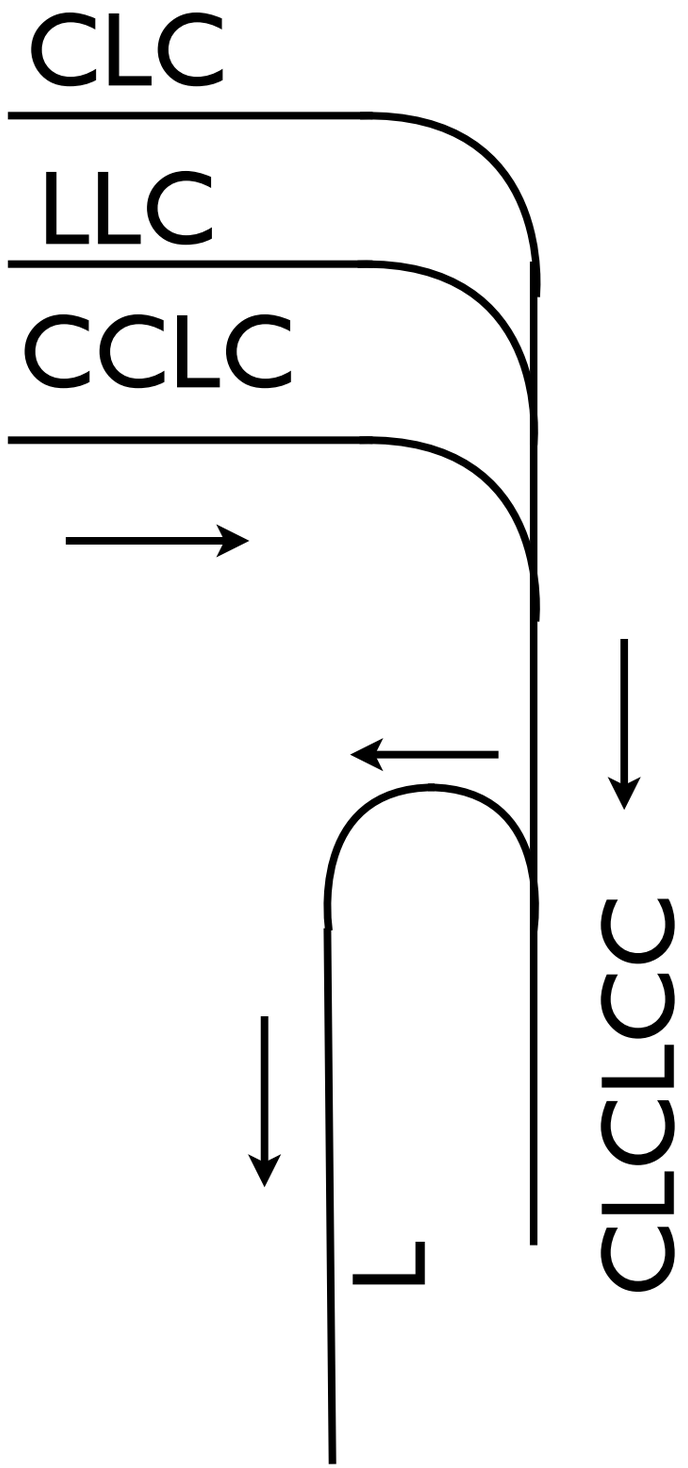

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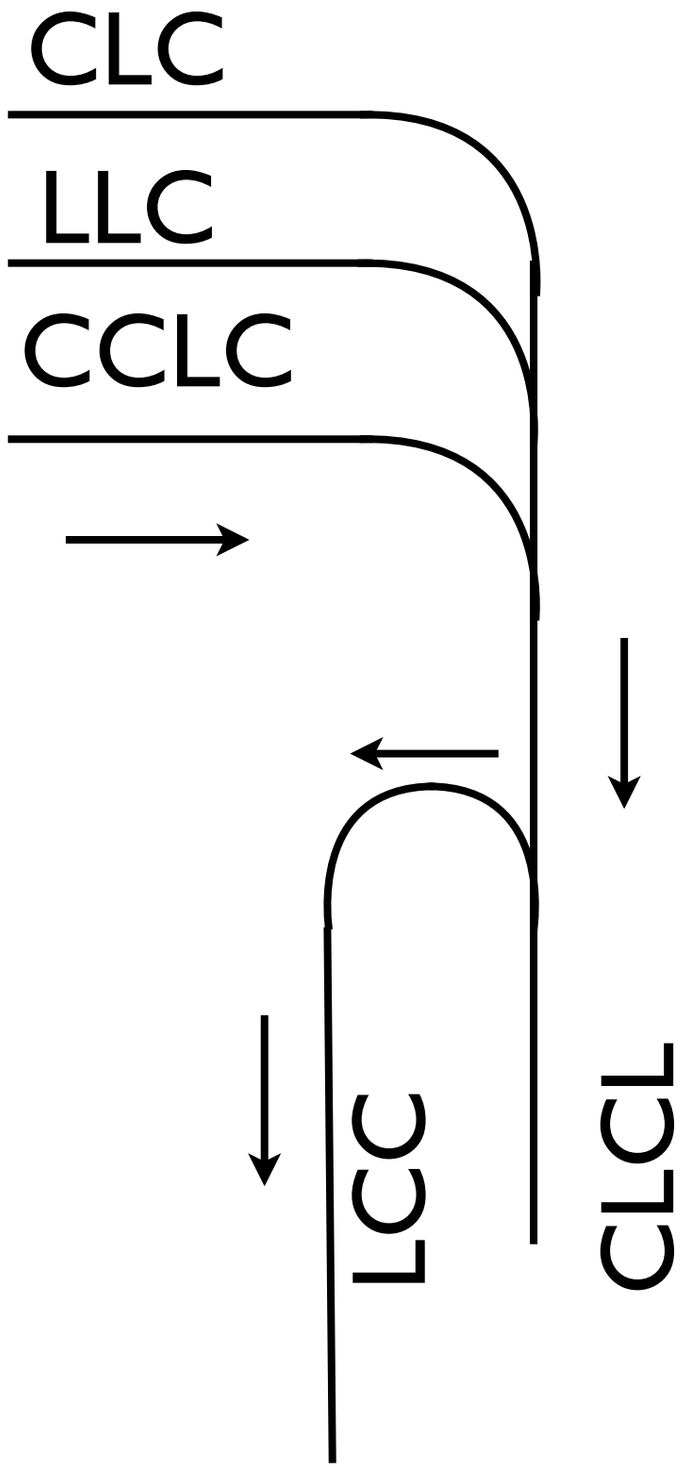

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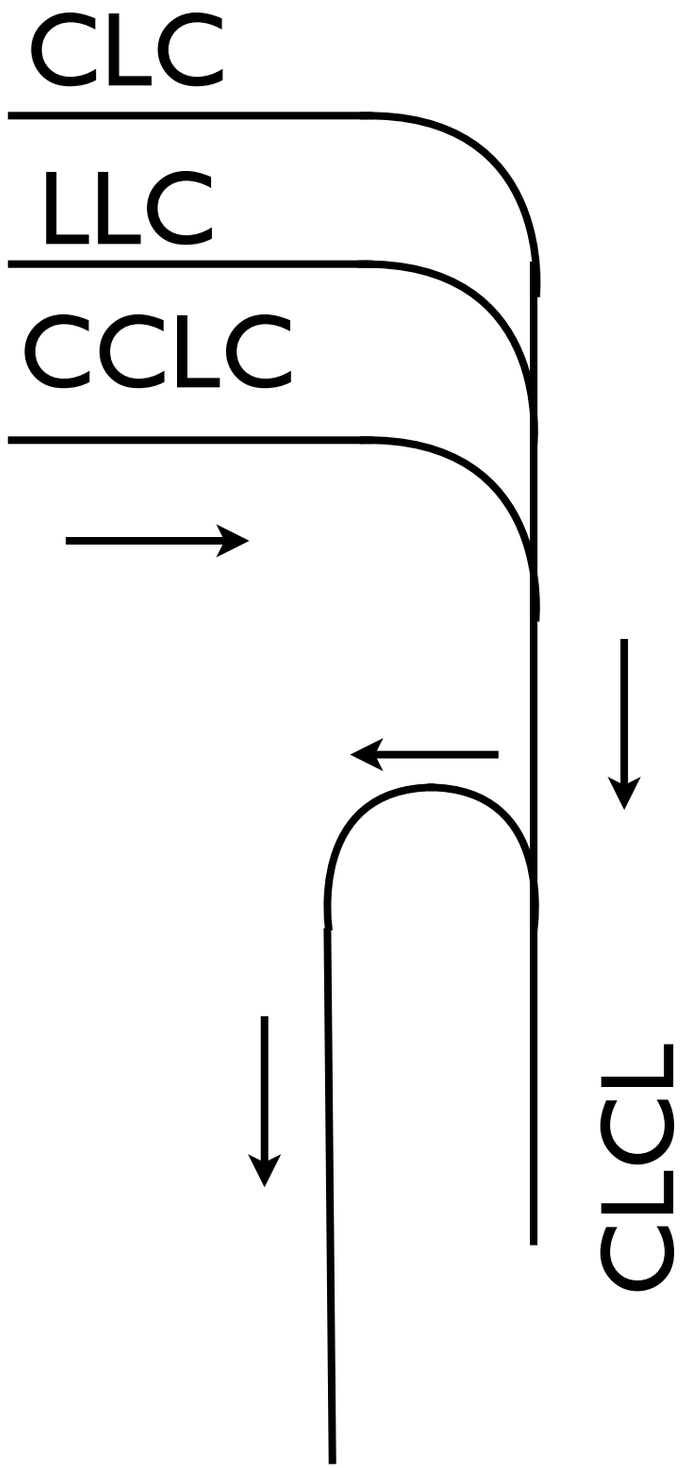
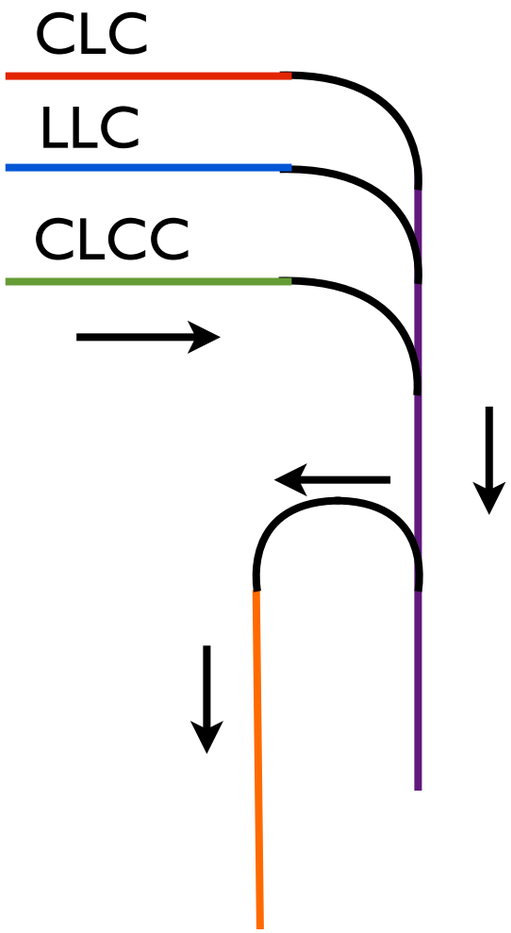


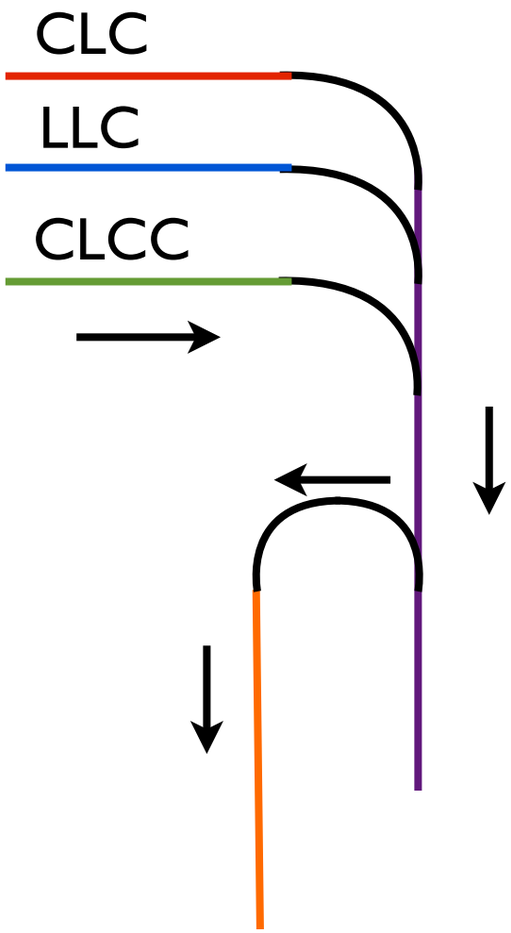
Can we produce trains with 2 cars and 1 locomotive?

Modelling



Modelling

$$T \longrightarrow (T, \_, \_, CLC, \_)$$



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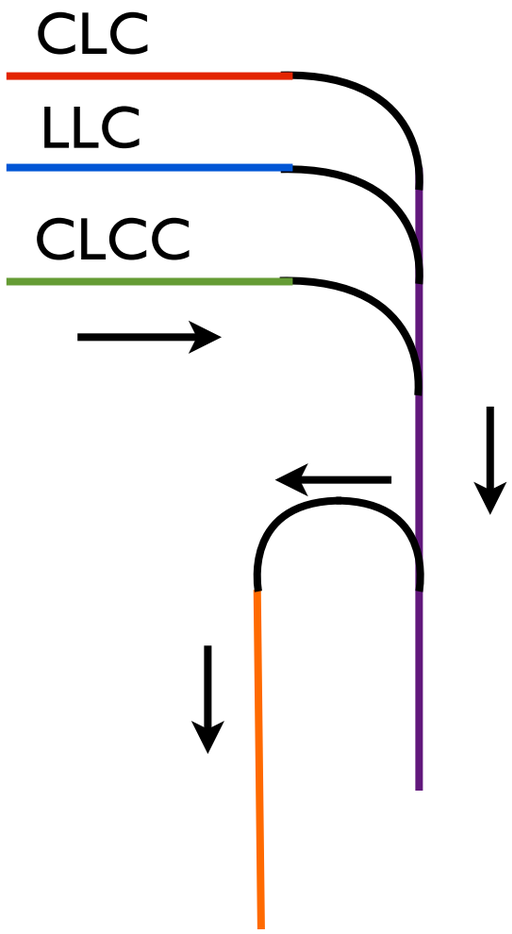
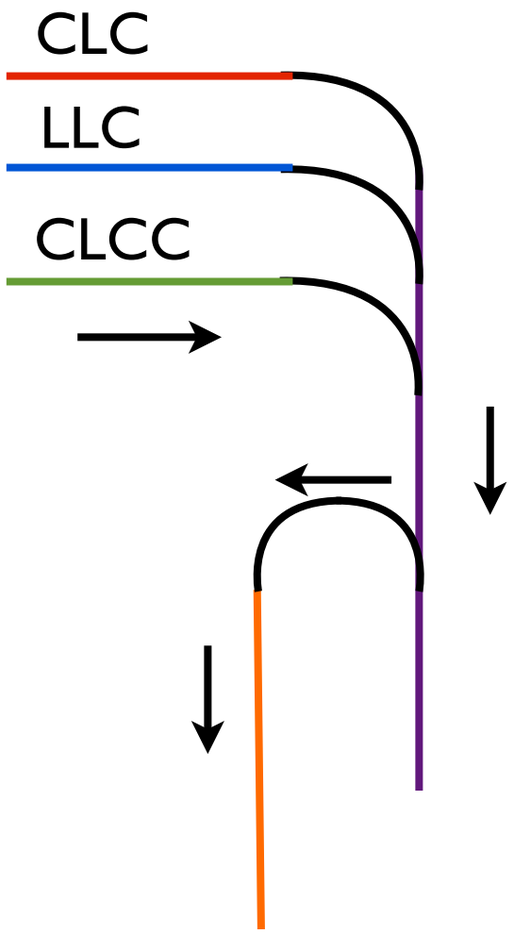


T ⟶ (T, _, _, CLC, _)

T ⟶ (_, T, _, LLC, _)

Modelling

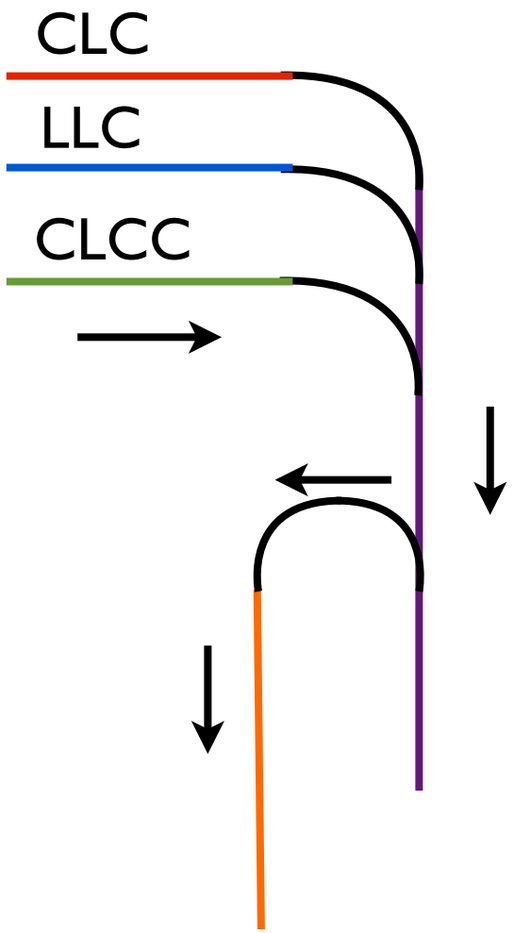


T ⟶ (T, _, _, CLC, _)

T ⟶ (_, T, _, LLC, _)

T ⟶ (_, _, T, CLCC, _)

Modelling



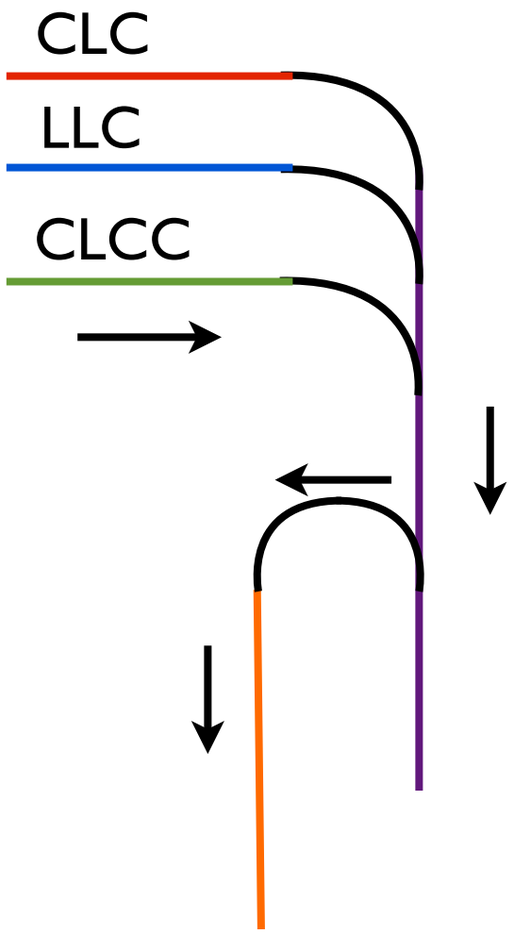
$T \longrightarrow (T, \_, \_, CLC, \_)$

$T \longrightarrow (\_, T, \_, LLC, \_)$

$T \longrightarrow (\_, \_, T, CLCC, \_)$

$C \longrightarrow (\_, \_, \_, \_, C)$

Modelling



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$T \longrightarrow (\_, T, \_, LLC, \_)$

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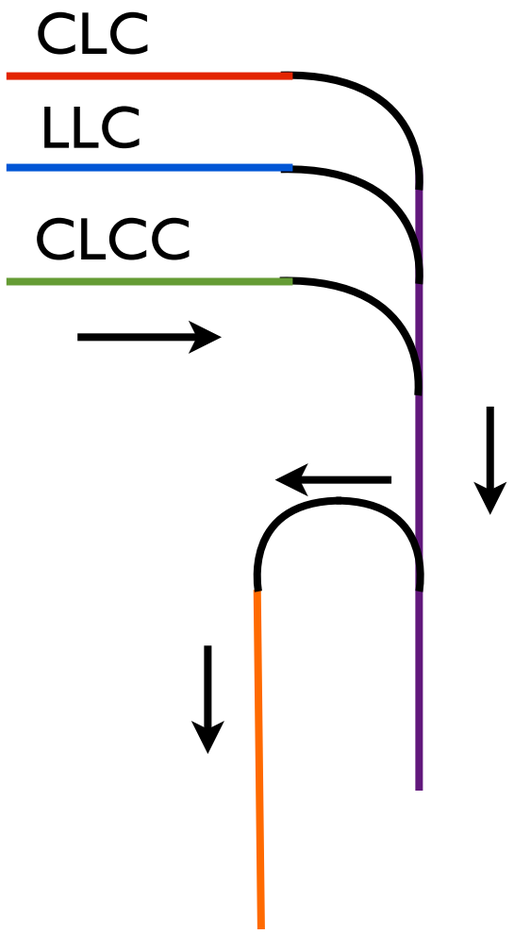
$C \longrightarrow (\_, \_, \_, \_, C)$

$L \longrightarrow (\_, \_, \_, \_, L)$

Modelling



(T, T, T, _, _)

T ⟶ (T, _, _, CLC, _)

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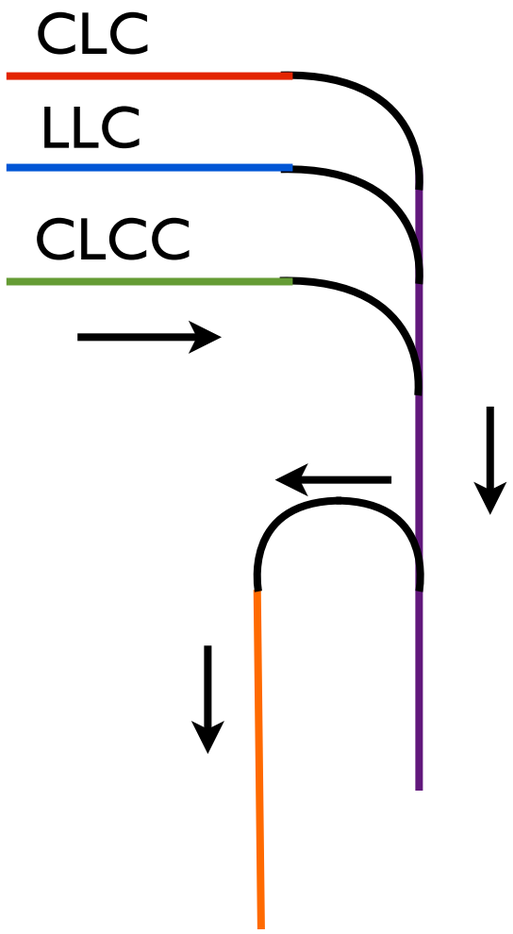
T ⟶ (_, _, T, CLCC, _)

C ⟶ (_, _, _, _, C)

L ⟶ (_, _, _, _, L)

Modelling



T ⟶ (T, _, _, CLC, _)

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T ⟶ (_, _, T, CLCC, _)

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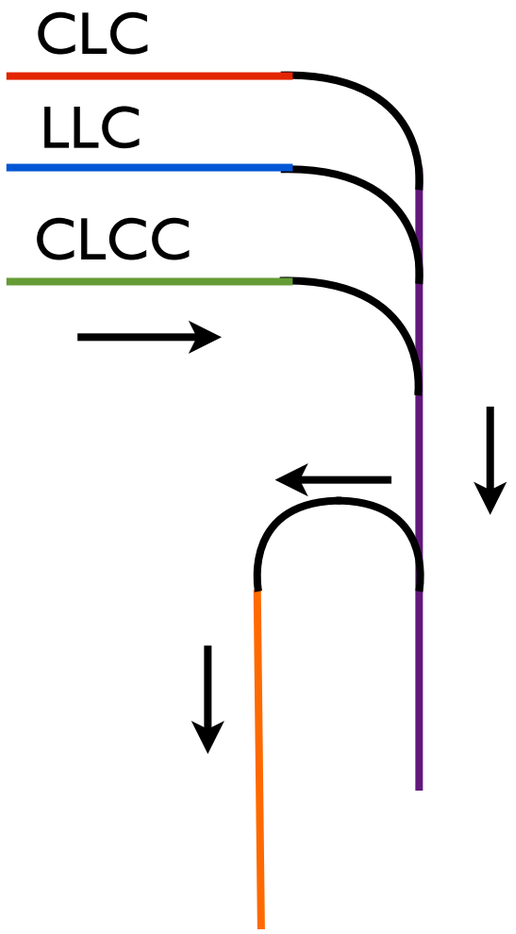
L ⟶ (_, _, _, _, L)

(T, T, T, _, _) ⟶ ⟶ ? ... ⟶

Modelling



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$C \longrightarrow (\_, \_, \_, \_, C)$

$L \longrightarrow (\_, \_, \_, \_, L)$

$(T, T, T, \_, \_) \longrightarrow \longrightarrow \overset{?}{\dots} \longrightarrow (T, T, T, \_, (CCL)^{+})$

Problems

Problems

Expressivity

Problems

Expressivity

Reachability problem

Problems

Expressivity

Reachability problem

Equivalence problem

Main result (expressivity)

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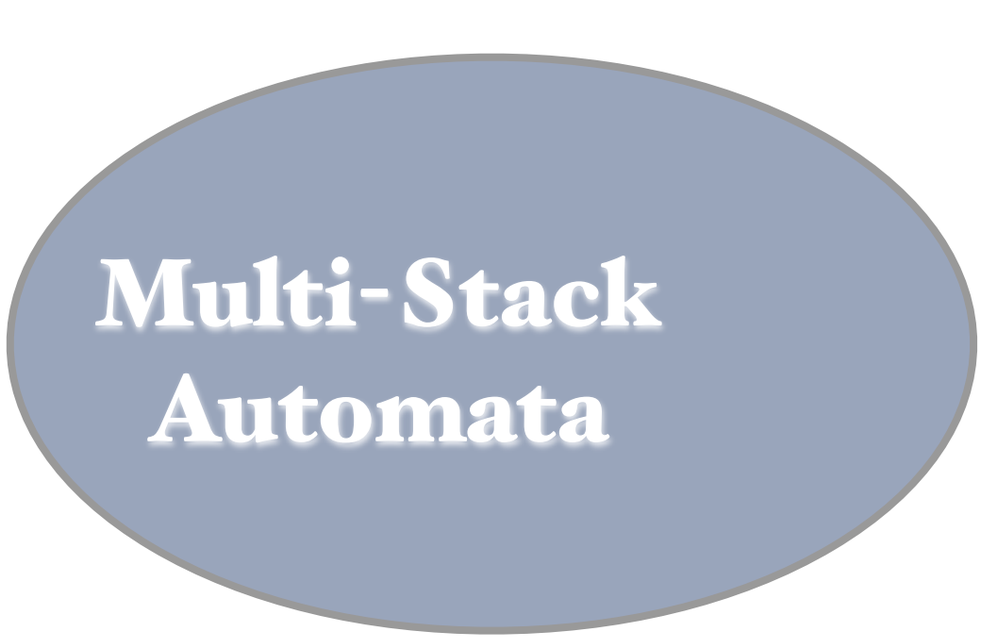
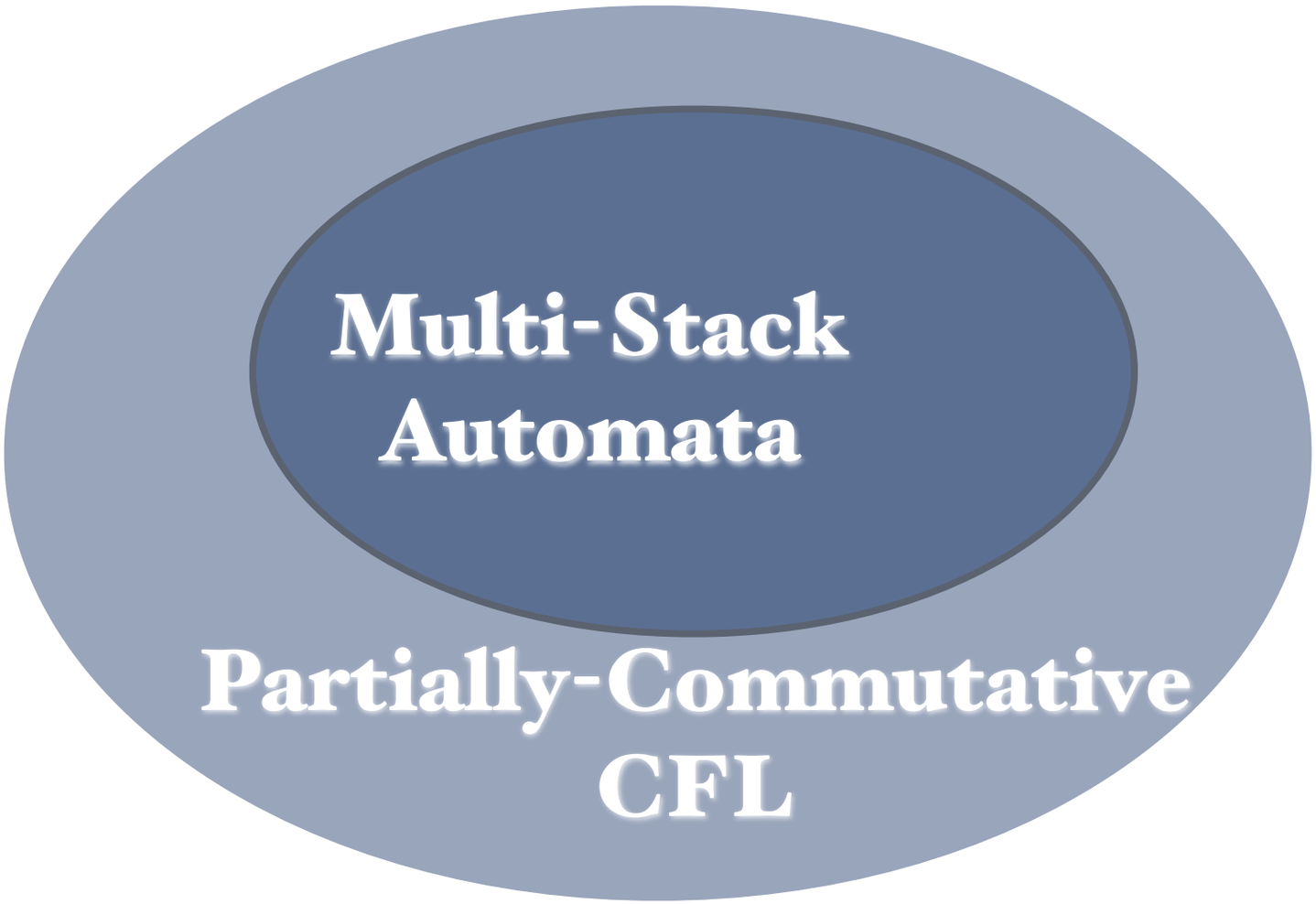


Main result (expressivity)



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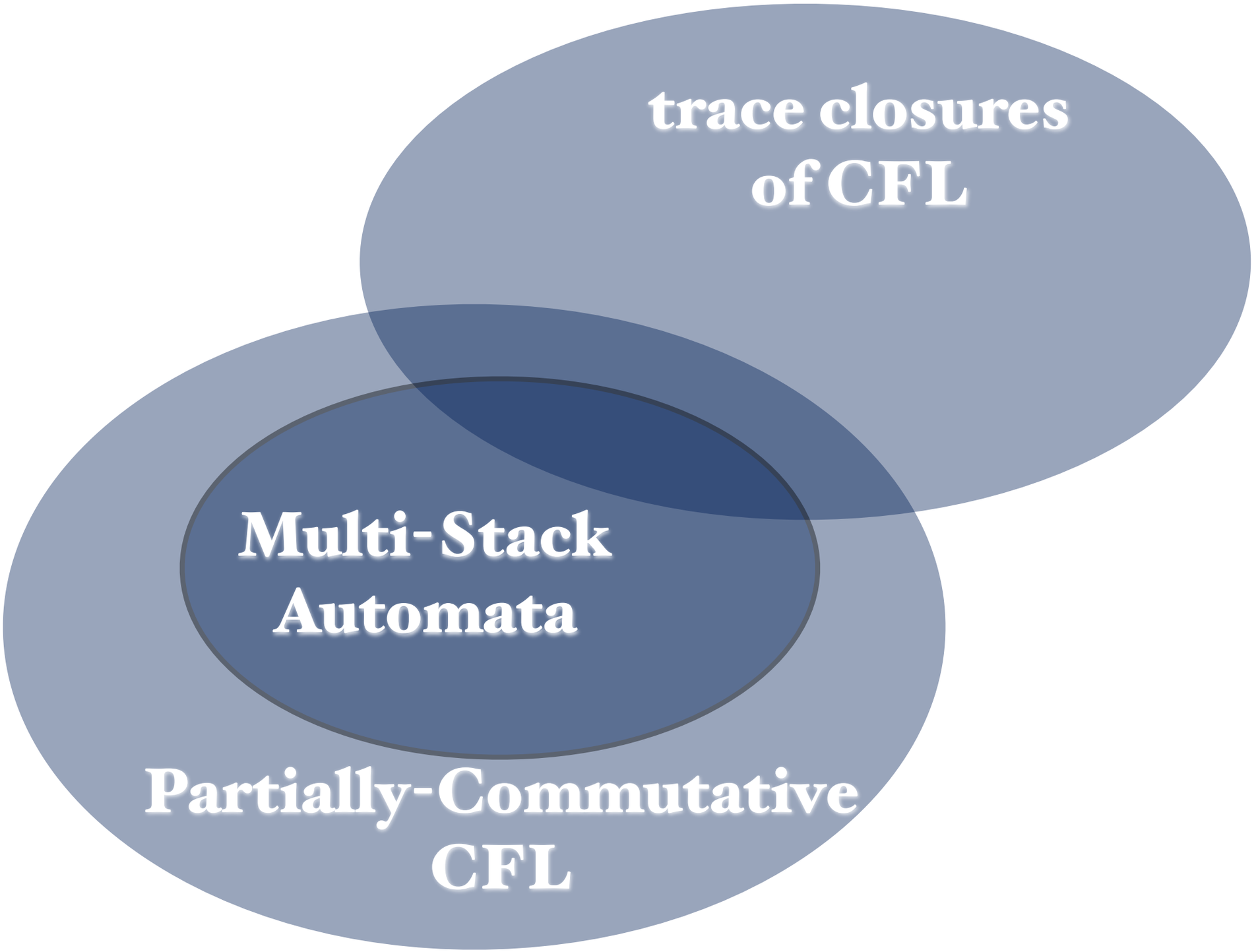

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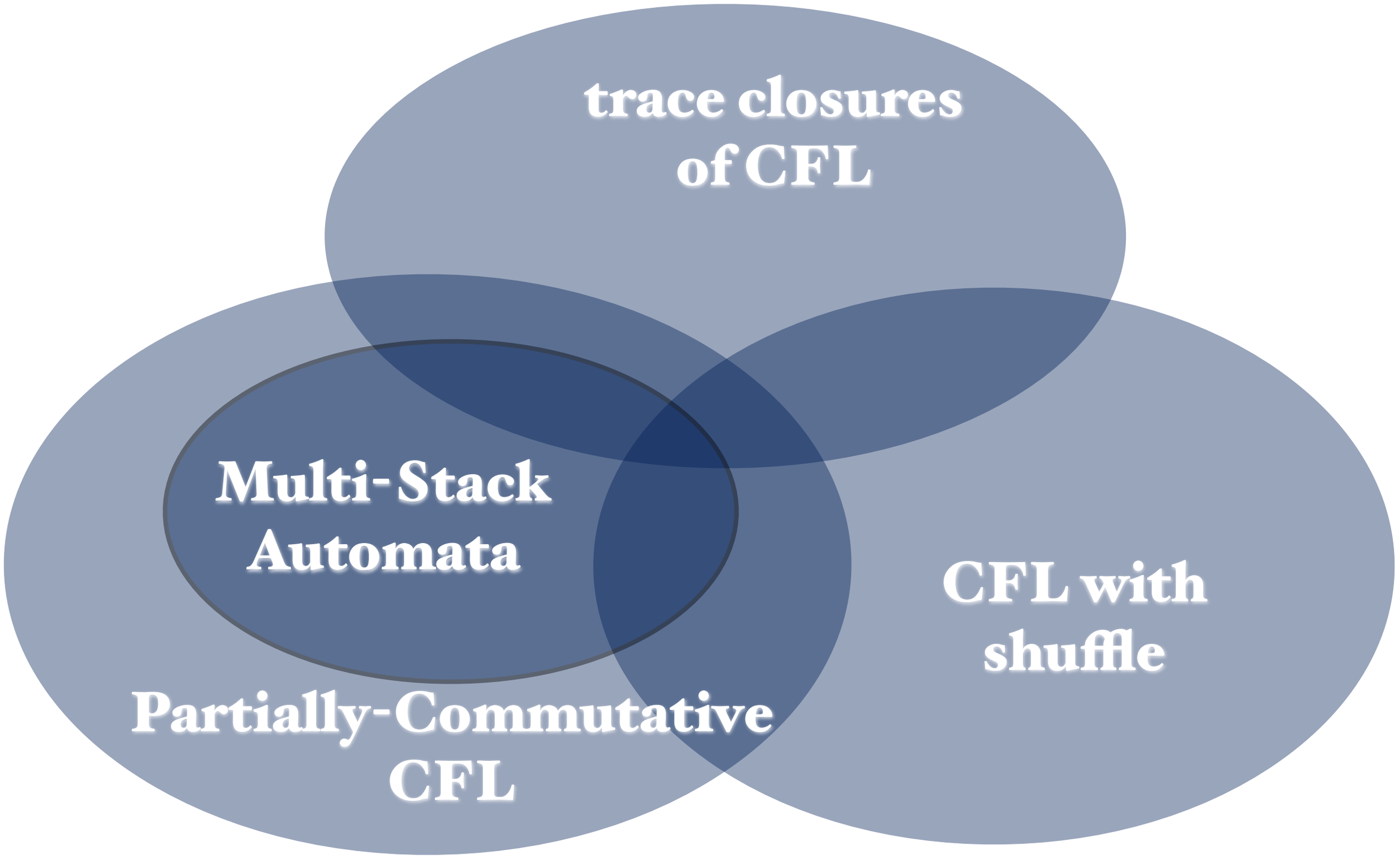

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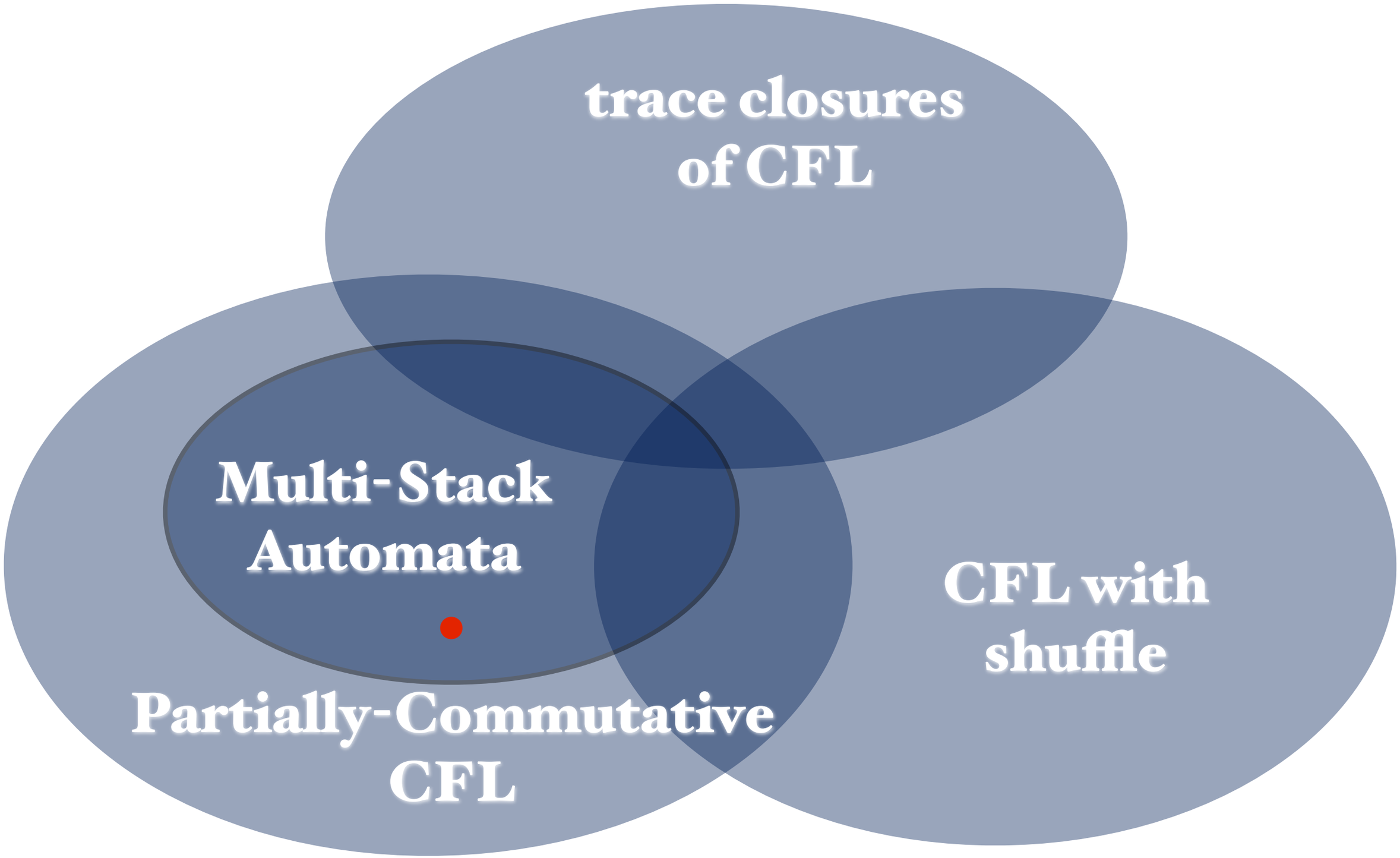

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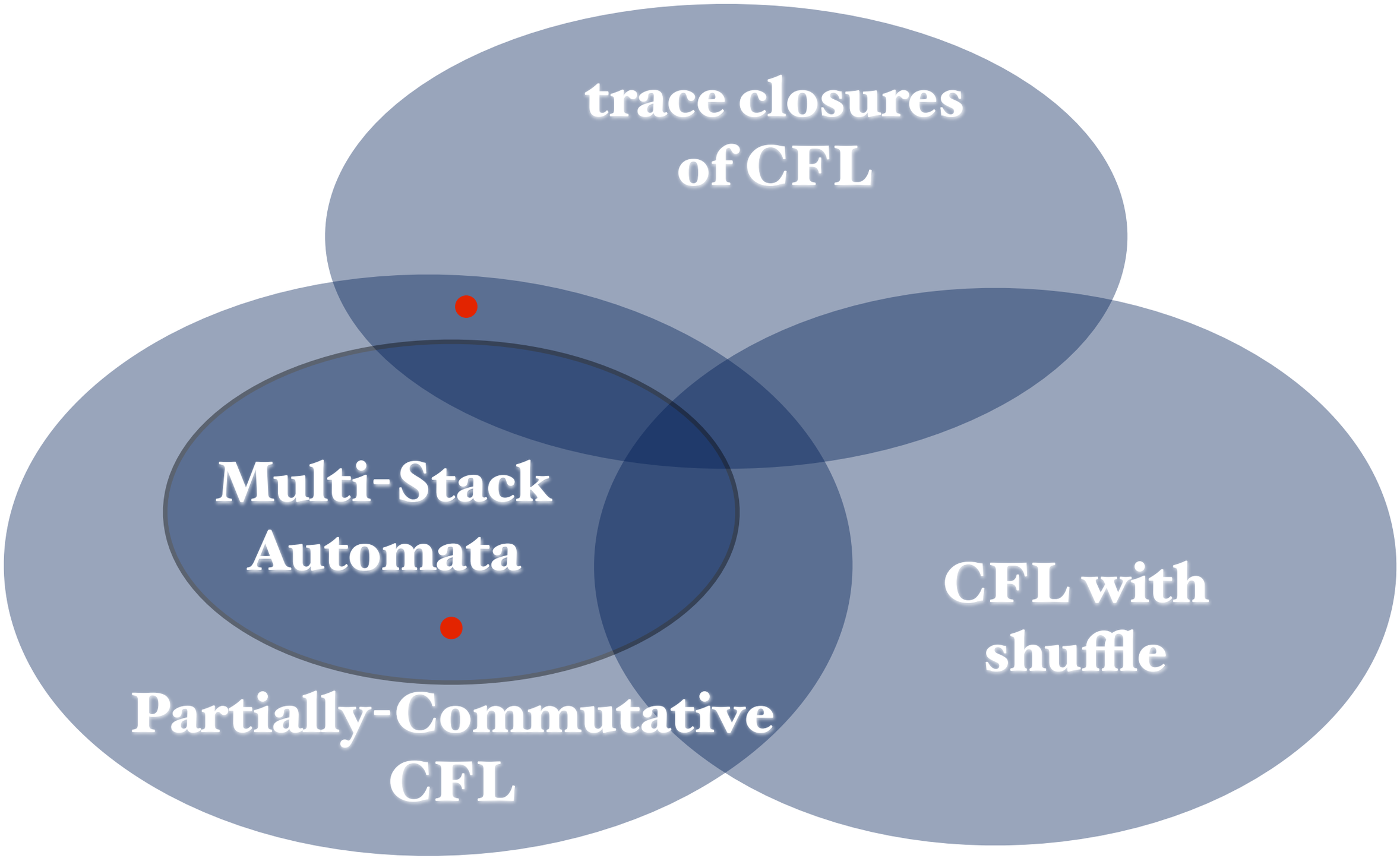

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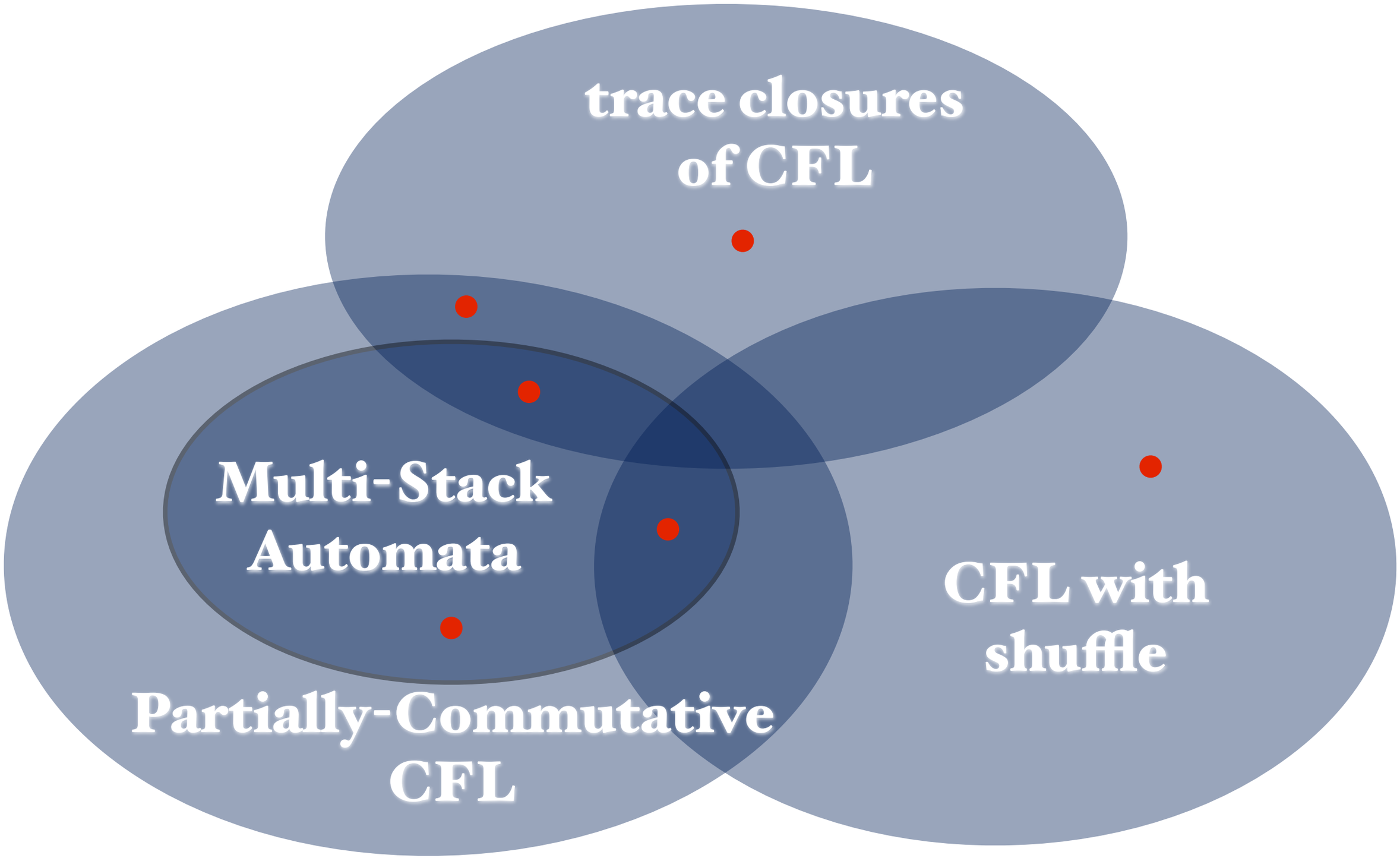

Reachability problem

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Given: A Multi-Stack Automaton and two its configurations: s (source) and t (target)

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Question: Decide whether there is a path from s to t

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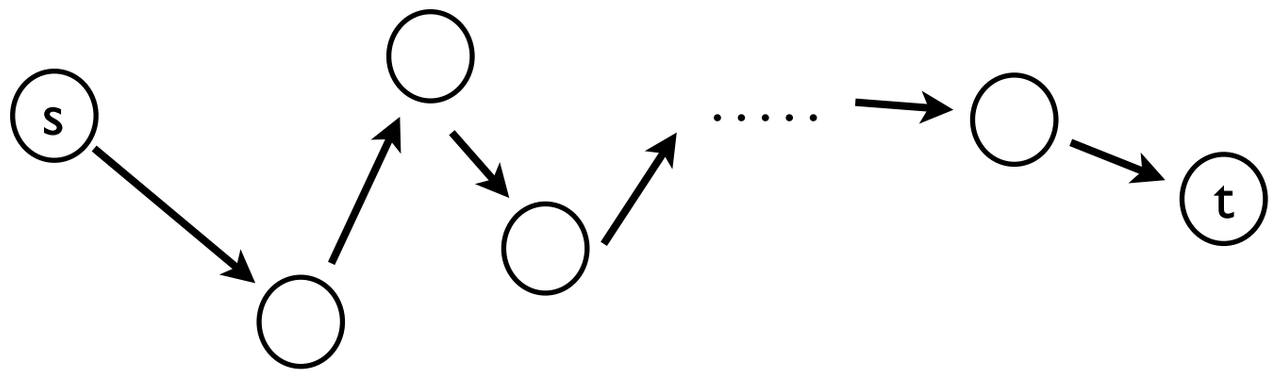

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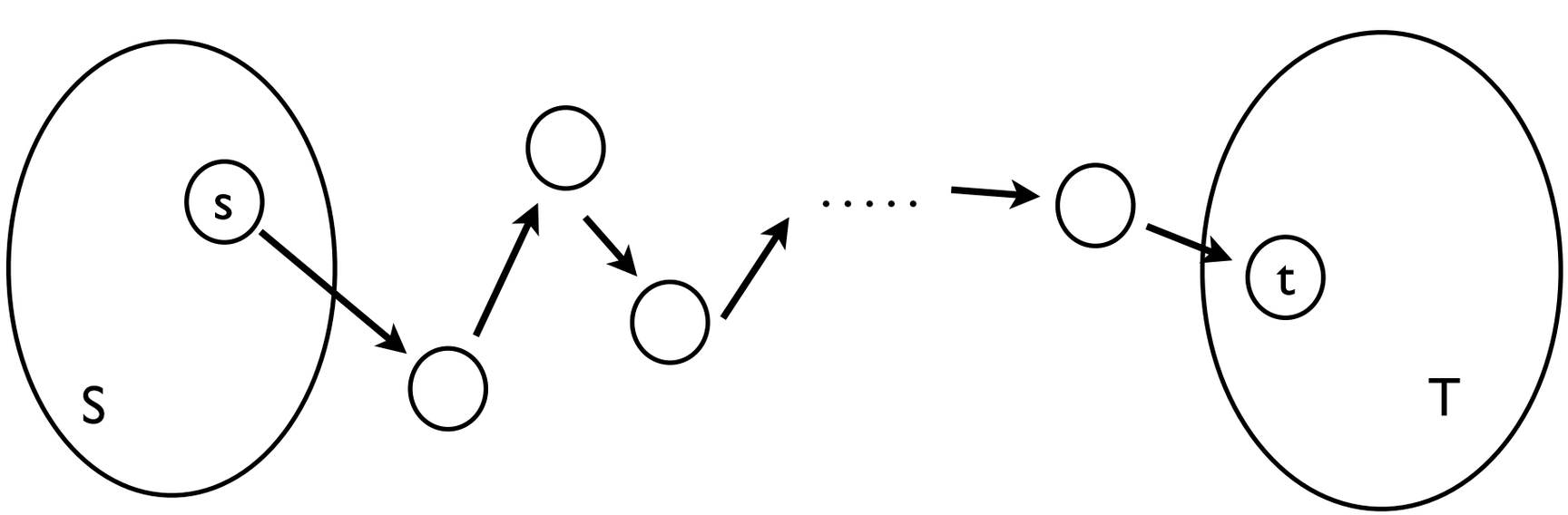

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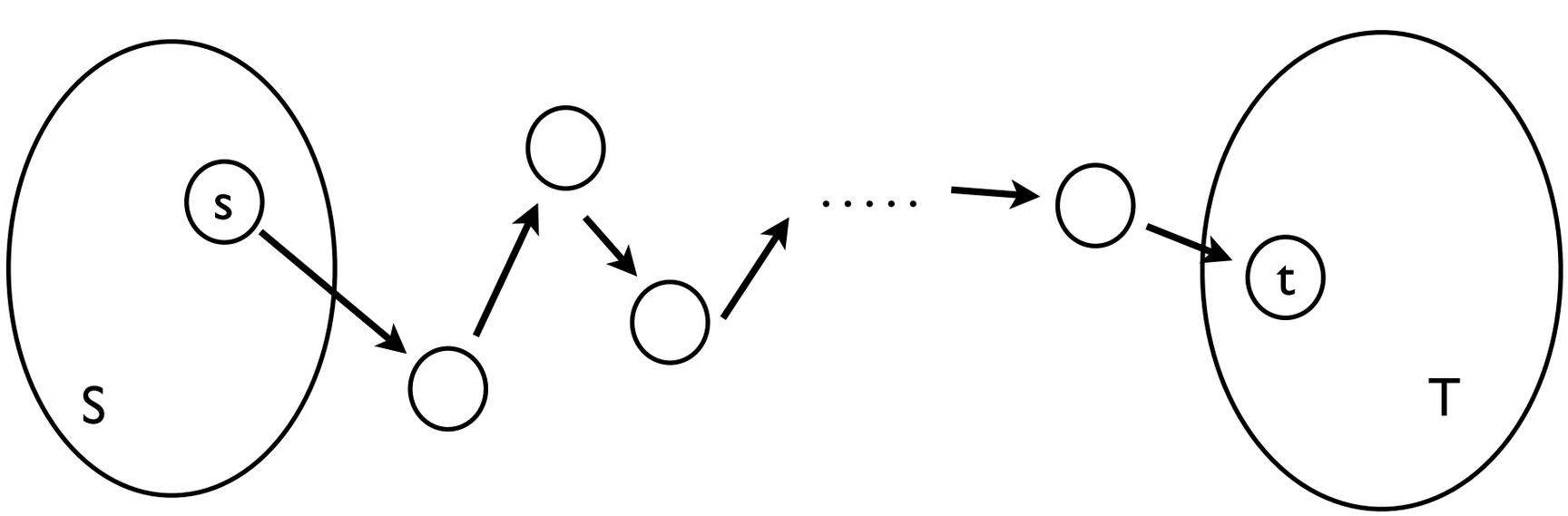

Regular: ((AB)*, CD) ∪ (A*B, C*D*)

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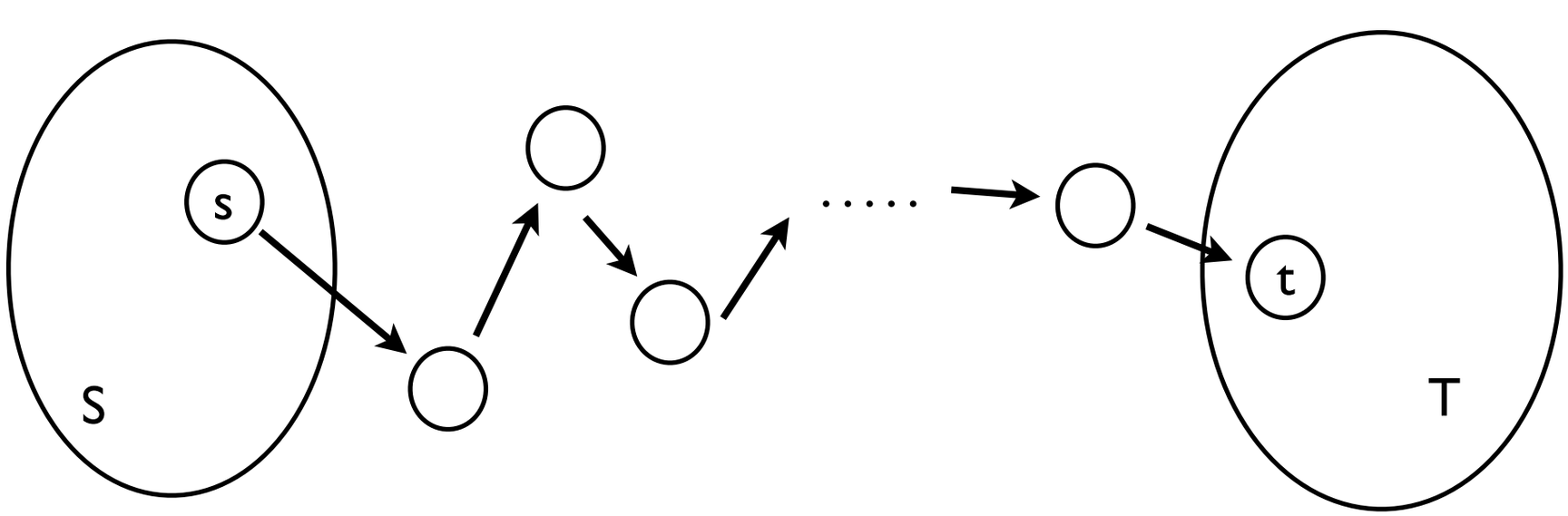

Regular: ((AB)*, CD) ∪ (A*B, C*D*)

Not regular: (A^n, B^n)

Main result (reachability)

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Theorem:
Reachability problem for *normed* MSA with regular source and target sets is NP-complete

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Reachability problem for *normed* MSA with regular source and target sets is NP-complete

Normedness: for every letter A

$$\underline{A} \longrightarrow \longrightarrow \cdots \longrightarrow \_$$

Equivalence

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Rules are labelled:

Equivalence

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Equivalence

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The idea: type of the action (c - moving of a car)

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Bisimulation equivalence:

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s

t

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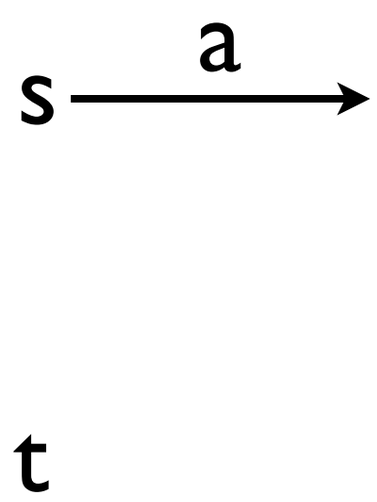

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Bisimulation equivalence:

$s \xrightarrow{a} s'$

t

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$t \xrightarrow{a}$

Equivalence

Rules are labelled: $A \xrightarrow{a} (B, C)$

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Bisimulation equivalence:

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$$t \xrightarrow{a} t'$$

Equivalence

Rules are labelled: $A \xrightarrow{a} (B, C)$

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Bisimulation equivalence:

$$s \xrightarrow{a} s'$$

$$t \xrightarrow{a} t' \xrightarrow{b}$$

Equivalence

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Bisimulation equivalence:

$$s \xrightarrow{a} s'$$

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Equivalence

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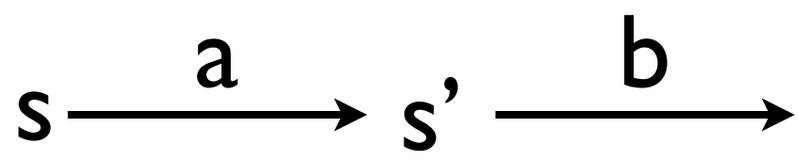

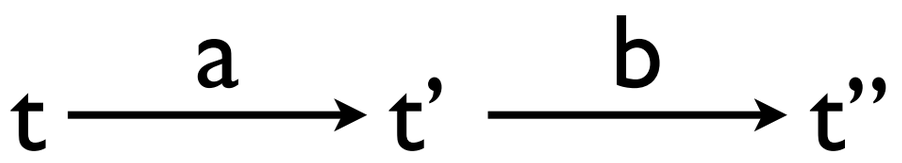

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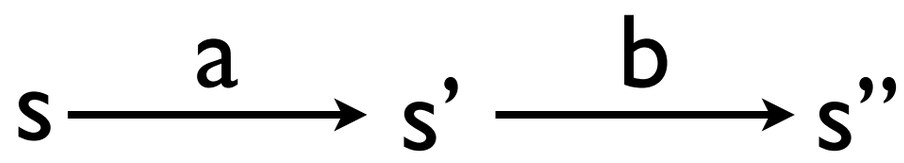

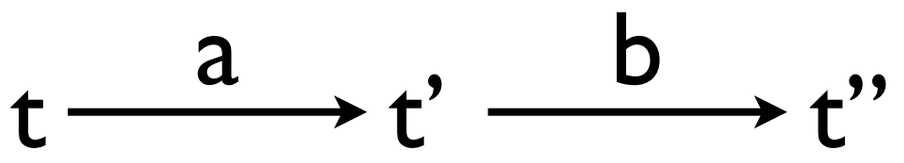

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Question: Decide whether s and t are equivalent

Main results (equivalence)

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Bisimulation equivalence is decidable

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- in polynomial time for normed *disjoint* Multi-Stack Automata

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Bisimulation equivalence is decidable

- in polynomial time for normed *disjoint* Multi-Stack Automata
- in time $O(n^4)$ for normed Single-Stack Automata

Main results (equivalence)

Bisimulation equivalence is decidable

- in polynomial time for normed *disjoint* Multi-Stack Automata
- in time $O(n^4)$ for normed Single-Stack Automata
- in time $O(n^3)$ for Simple Grammars

Thank you!