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A Few Remarks on “Fixed-Width Output Analysis for Markov Chain Monte Carlo” by Jones et al.

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Our aim is to relax assumptions and simplify proofs in results given by Jones, Haran, Caffo, and Neath in the recent paper “Fixed-Width Output Analysis for Markov Chain Monte Carlo.”

KEY WORDS: Batch means; Geometric ergodicity; Markov chain; Regeneration; Regenerative simulation

In the following discussion, we refer to the setting and notation introduced in Jones, Haran, Caffo, and Neath (2006) where the following lemma is stated and used repeatedly.

Lemma 1 (Lemma 1 of Jones et al. 2006). Let $X$ be a Harris ergodic Markov chain on $X$ with invariant distribution $\pi$ and suppose that $g : X \rightarrow \mathbb{R}$ is a Borel function. Assume $X$ is geometrically ergodic and the minorization condition holds; that is, there exists a function $s : X \rightarrow [0, 1]$ for which $E_{\pi} s > 0$ and a probability measure $Q$ such that

$$P(x, A) \geq s(x)Q(A) \quad \text{for all } x \in X \text{ and } A \in B(X). \quad (1)$$

Then, for every integer $p \geq 1$,

1. If $E_{\pi} |g|^{p \delta + 1} < \infty$ for some $0 < \delta < 1$, then $E_{\pi} N_1^p < \infty$ and $E_{\pi} S_1^p < \infty$.

2. If $E_{\pi} |g|^{p+\delta} < \infty$ for some $0 < \delta < 1$, then $E_{\pi} N_1^p < \infty$ and $E_{\pi} S_1^{p+\delta} < \infty$.

Here $N_i = \tau_i - \tau_{i-1}$, $S_1 = \sum_{i=0}^{\tau_1} g(X_i)$, and $0 = \tau_0 < \tau_1 < \cdots$ are the regeneration times of the chain (see sec. 2.1 of Jones et al. 2006 for detailed definitions).

The lemma generalizes the main theoretical result of Hobert, Jones, Pressnell, and Rosenthal (2002) and is also of independent interest. However, the following stronger result holds true.

Lemma 2. Under the assumptions of Lemma 1, if $E_{\pi} |g|^{p+\delta} < \infty$ for some $p > 0$ and $\delta > 0$, then $E_{\pi} N_1^p < \infty$ and $E_{\pi} S_1^p < \infty$.

Proof. It is enough to show that $E_{\pi} S_1^p < \infty$, because the remaining part of the original proof is valid under the relaxed assumption. To this end, first note that

$$C := (E_{\pi} |g(X)|^{p+\delta})^{1/(p+\delta)} < \infty. \quad (2)$$

For $p \geq 1$, we first use the triangle inequality in $L^p$, then the Hölder inequality, then (2), and, finally, corollary A.1 of Jones et al. (2006).

$$(E_{\pi} S_1^p)^{1/p} \leq \left[ E_{\pi} \left( \sum_{i=0}^{\tau_1-1} |g(X_i)| \right)^p \right]^{1/p} \leq \left[ E_{\pi} \left( \sum_{i=0}^{\tau_1} N_i \right) \right]^{1/p} = \left[ E_{\pi} \left( \sum_{i=0}^{\tau_1} (i+1) |g(X_i)| \right) \right]^{1/p}.$$
for some $\delta > 0$ and some $\varepsilon > 0$, then there exist a constant $0 < \sigma_\varepsilon < \infty$ and a sufficiently large probability space such that

$$\left| \sum_{i=1}^{n} g(X_i) - nEg - \sigma_\varepsilon B(n) \right| = O(\gamma(n)),$$

with probability 1 as $n \to \infty$, where $\gamma(n) = n^{\alpha} \log n$, $\alpha = 1/(2 + \delta)$, and $B = \{B(t), t \geq 0\}$ denotes a standard Brownian motion.

**Proposition 4** (Proposition 3 of Jones et al. 2006). Let $X$ be a Harris ergodic Markov chain with invariant distribution $\pi$. Further, suppose $X$ is geometrically ergodic, (1) holds, and $E_x |g|^{2+\delta+\varepsilon} < \infty$ for some $\delta > 0$ and some $\varepsilon > 0$. If

1. $a_n \to \infty$ as $n \to \infty$,
2. $b_n \to \infty$ and $b_n/n \to 0$ as $n \to \infty$,
3. $b_n^{-1} n^{2\alpha} [\log n]^3 \to 0$ as $n \to \infty$, where $\alpha = 1/(2 + \delta)$,
4. there exists a constant $c \geq 1$, such that $\sum_{n=1}^{\infty} (b_n/n)^c < \infty$,

then $\sigma_{BM}^2 \to \sigma_g^2$ w.p.1 as $n \to \infty$.

**Concluding Remark.** Compare the foregoing result with proposition 1 of Jones et al. (2006) to see that both methods described by Jones et al., that is, regenerative simulation (RS) and batch means (CBM), provide strongly consistent estimators of $\sigma_g^2$ under the same assumption for the target function $g$.

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**REFERENCES**


