# Supplementary materials for Hiding from Centrality Measures: A Stackelberg Game Perspective

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This document is structured as follows:

- **Section S1** (*page 2*) presents the proofs of our results concerning possible ranking changes,
- **Section S2** (*page 6*) presents the proofs of our computational complexity results,
- **Section S3** (*page 20*) presents the proof that MIQP and MILP formulations of finding the optimal strategies are equivalent,
- **Section S4** (*page 24*) presents the supplementary figures.

# S1 Proving the Possible Changes in Centrality Rankings

| | | Adding $e$ to the network | | Removing $e$ from the network | |
|---|---|---|---|---|---|
| | | Increase $v^\dagger$ rank | Decrease $v^\dagger$ rank | Increase $v^\dagger$ rank | Decrease $v^\dagger$ rank |
| $e \in \zeta_1^\dagger$ | Degree | $\Uparrow k$ ($G_2$) | $\Downarrow$ ✗ (T2) | $\Uparrow$ ✗ (T2) | $\Downarrow k$ ($G_2$) |
| | Closeness | $\Uparrow k$ ($G_8$) | $\Downarrow k$ ($G_5$) | $\Uparrow k$ ($G_5$) | $\Downarrow k$ ($G_8$) |
| | Betweenness | $\Uparrow k$ ($G_2$) | $\Downarrow k$ ($G_6$) | $\Uparrow k$ ($G_6$) | $\Downarrow k$ ($G_2$) |
| $e \in \zeta_2^\dagger$ | Degree | $\Uparrow$ ✗ (T2) | $\Downarrow 2$ (T4, $G_{10}$) | $\Uparrow 2$ (T4, $G_{10}$) | $\Downarrow$ ✗ (T2) |
| | Closeness | $\Uparrow$ ✗ (T3) | $\Downarrow k$ (T5,$G_1$) | $\Uparrow k$ (T5,$G_1$) | $\Downarrow$ ✗ (T3) |
| | Betweenness | $\Uparrow k$ ($G_7$) | $\Downarrow k$ ($G_4$) | $\Uparrow k$ ($G_4$) | $\Downarrow k$ ($G_7$) |
| $e \in \zeta_3^\dagger$ | Degree | $\Uparrow$ ✗ (T2) | $\Downarrow 2$ (T4, $G_{11}$) | $\Uparrow 2$ (T4, $G_{11}$) | $\Downarrow$ ✗ (T2) |
| | Closeness | $\Uparrow k$ ($G_9$) | $\Downarrow k$ ($G_3$) | $\Uparrow k$ ($G_3$) | $\Downarrow k$ ($G_9$) |
| | Betweenness | $\Uparrow k$ ($G_9$) | $\Downarrow k$ ($G_3$) | $\Uparrow k$ ($G_3$) | $\Downarrow k$ ($G_9$) |

Table S1: Summary of our results concerning possible ranking changes. For any given evader $v^\dagger \in V$, we study three classes of edges: $\zeta_1^\dagger$, $\zeta_2^\dagger$ and $\zeta_3^\dagger$, and three centrality measures: degree, closeness and betweenness. For every class and every measure, we investigate the potential impact of adding or removing an edge from that class on the centrality ranking of $v^\dagger$. The "$\Uparrow k$" (resp. "$\Downarrow k$") mark indicates that the increase (resp. decrease) in ranking can be arbitrarily large, while the "$\Uparrow 2$" (resp. "$\Downarrow 2$") mark indicates that the ranking can only increase (resp. decrease) by at most two positions. The "$\Uparrow$ ✗" (resp. "$\Downarrow$ ✗") mark indicates that the ranking increase (resp. decrease) is impossible. The "T$x$" indicates that the corresponding result is formally stated in Theorem $x$, whereas "$G_x$" indicates that an example of network where this change takes place is presented as $G_x$ in Figure S1.

Before presenting the formal proofs of the results in Table S1, let us first introduce the following proposition.

**Proposition 1.** *Let $G = (V, E)$ be a network, let $v^\dagger \in V$ be the evader, let $c$ be a centrality measure, and let $\zeta^\dagger$ be a class in $\{\zeta_1^\dagger, \zeta_2^\dagger, \zeta_3^\dagger\}$. Then, the ranking of $v^\dagger$ in $G$ according to $c$ can increase (resp. decrease) after adding to $G$ an edge from $\zeta^\dagger$ if and only if there exists a network $G' = (V, E')$ such that the ranking of $v^\dagger$ in $G'$ according to $c$ can decrease (resp. increase) after removing from $G'$ an edge from $\zeta^\dagger$.*

*Proof.* Given an edge $e \in \zeta^\dagger \setminus E$, assume that after the addition of $e$ to $G$ the ranking of $v^\dagger$ according to $c$ increases (resp. decreases). Let us denote by $G'$ the network that results from this addition, i.e., $G' = (V, E \cup \{e\})$. Then, removing $e$ from $G'$ will bring the ranking of $v^\dagger$ back to its original state, hence it will decrease (resp. increase). Proving the other direction of the equivalence statement is exactly analogous. □

As simple as it may sound, this proposition is very helpful, as it allows us to study only the impact of adding an edge (as opposed to adding and removing that edge). Before we prove our impossibility results, let us make two more observations.

**Observation 1.** *For a centrality measure $c \in \{c_{dg}, c_{cl}, c_{bt}\}$, an evader $v^\dagger$, and a class of edges $\zeta^\dagger \in \{\zeta_1^\dagger, \zeta_2^\dagger, \zeta_3^\dagger\}$, it is possible that adding or removing edge from class $\zeta^\dagger$ does not change the ranking of $v^\dagger$ according to $c$. An example of a network in which this take is presented as $G_0$ in Figure S1.*

**Observation 2.** *In order for the ranking of $v^\dagger$ (according to $c$) to increase (resp. decrease) as a result of adding to a network $G = (V, E)$ an edge $e \notin E$, at least one of the following conditions must hold, where $G' = (V, E \cup \{e\})$:*

1. *$c(G', v^\dagger) > c(G, v^\dagger)$ (resp. $c(G', v^\dagger) < c(G, v^\dagger)$);*
2. *there exists $v \in V$ such that $c(G, v) > c(G, v^\dagger)$ and $c(G', v) \leq c(G', v^\dagger)$ (resp. there exists $v \in V$ such that $c(G, v) < c(G, v^\dagger)$ and $c(G', v) \geq c(G', v^\dagger)$);*

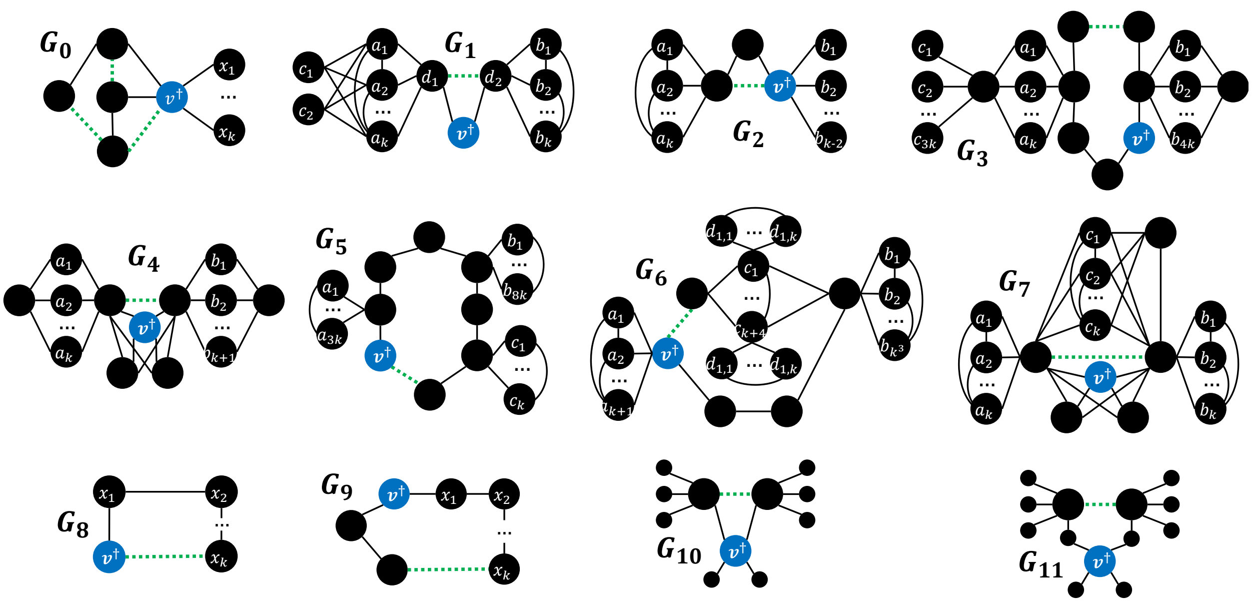


Figure S1: Networks used to show the possible changes in rankings. The dotted green edges have to be added to the network to observe the change in ranking. In case there are multiple dotted edges, they belong to different classes and are used to show different results from Table S1.

3. *there exists $v \in V$ such that $c(G, v) \geq c(G, v^\dagger)$ and $c(G', v) < c(G', v^\dagger)$ (resp. there exists $v \in V$ such that $c(G, v) \leq c(G, v^\dagger)$ and $c(G', v) > c(G', v^\dagger)$).*

Intuitively, the simplest way for the ranking of $v^\dagger$ to increase is for the centrality value of $v^\dagger$ to increase (condition 1). Otherwise, there has to exist a node $v$ that had greater centrality value than $v^\dagger$ before the addition $e$, and has smaller centrality value than $v^\dagger$ after the addition of $e$, taking into consideration the possibility of equal centrality values (conditions 2 and 3).

With these observations, we are ready to present the formal proofs of the results summarized in Table S1.

**Theorem 2.** *For any node $v^\dagger \in V$, any edge $e \in \zeta^\dagger_1$, and any network $G = (V, E)$ such that $e \notin E$, adding $e$ to $G$ cannot decrease the ranking of $v^\dagger$ according to degree centrality. Moreover, for any $e \in \zeta^\dagger_2 \cup \zeta^\dagger_3$ and any $G = (V, E)$ such that $e \notin E$, adding $e$ to $G$ cannot increase the ranking of $v^\dagger$ according to degree centrality.*

*Proof.* Adding an edge $e \in \zeta^\dagger_1$ increases the degree centrality value of $v^\dagger$ and the other end of $e$ (let us call it $v$) by exactly 1. Since the centrality value of $v^\dagger$ increases, condition 1 of Observation 2 does not hold. At the same time, since the centrality of $v$ increases by exactly the same amount as the centrality of $v^\dagger$, neither condition 2 nor condition 3 of Observation 2 holds. Therefore, the ranking of $v^\dagger$ cannot decrease.

Analogically, adding an edge $e \in \zeta^\dagger_2 \cup \zeta^\dagger_3$ increases the degree centrality value of two nodes in $\bar{V}$ by exactly 1. Since the centrality value of $v^\dagger$ does not change, condition 1 of Observation 2 does not hold. At the same time, since the centrality of the nodes other than $v^\dagger$ either increases or remains the same, neither condition 2 nor condition 3 of Observation 2 can hold. Therefore, the ranking of $v^\dagger$ cannot increase. □

**Theorem 3.** *For any node $v^\dagger \in V$, any edge $e \in \zeta^\dagger_2$, and any network $G = (V, E)$ such that $e \notin E$, adding $e$ to $G$ cannot increase the ranking of $v^\dagger$ according to closeness centrality.*

*Proof.* Adding an edge $e \in \zeta^\dagger_2$ does not change the closeness centrality value of $v^\dagger$, since it is added between two nodes within the same distance to $v^\dagger$, and that distance does not change after the addition of $e$. Therefore, condition 1 of Observation 2 does not hold. Moreover, since the addition of an edge can only create new shortest paths, it cannot decrease the value of the closeness centrality of any node in $\bar{V}$. Hence, neither condition 2 nor condition 3 of Observation 2 can hold. Consequently, the ranking of $v^\dagger$ cannot increase. □

**Theorem 4.** *For any $v^\dagger \in V$, any edge $e \in \zeta_2^\dagger \cup \zeta_3^\dagger$, and any network $G = (V, E)$ such that $e \notin E$, adding $e$ to $G$ can decrease the ranking of $v^\dagger$ according to degree centrality only by at most two positions.*

*Proof.* Notice that in order for the centrality ranking position of $v^\dagger$ to decrease by $k \in \mathbb{N}$, either the centrality value of $v^\dagger$ has to decrease, or the centrality value of at least $k$ other nodes has to increase. Adding an edge $e \in \zeta_2^\dagger \cup \zeta_3^\dagger$ to the network does not change the degree centrality value of $v^\dagger$, and it increases the degree centrality value of only two other nodes (namely the nodes connected with the new edge). □

The proof of Theorem 5 is presented as an example of how to construct a proof of existence for any of the results in Table S1 where a network $G_x$ is specified.

**Theorem 5.** *Let $v^\dagger$ be a node in $V$. There exists a network $G = (V, E)$ and an edge $e \in \zeta_2^\dagger \setminus E$ such that the addition of $e$ to $G$ decreases the ranking of $v^\dagger$ according to the closeness centrality. What is more, the change in ranking can be arbitrarily large.*

*Proof.* Consider the network $G_1$ depicted in Figure S1. In this network, we have:

- $c_{cl}(G_1, v^\dagger) = \frac{1}{4k+8}$,
- $c_{cl}(G_1, a_i) = \frac{1}{5k+7}$ for any $a_i \in V$,
- $c_{cl}(G_1, b_i) = \frac{1}{5k+15}$ for any $b_i \in V$,
- $c_{cl}(G_1, c_i) = \frac{1}{6k+11}$ for any $c_i \in V$,
- $c_{cl}(G_1, d_1) = \frac{1}{4k+7}$,
- $c_{cl}(G_1, d_2) = \frac{1}{4k+11}$.

Based on these values, $v^\dagger$ is ranked second in $G$ according to closeness centrality (only the node $d_1$ is ranked above $v^\dagger$).

Now, let $G'_1$ be the network that results from adding $(d_1, d_2)$ (which is an edge in $\zeta_2^\dagger$) to the network $G_1$. In this network, we have:

- $c_{cl}(G'_1, v^\dagger) = \frac{1}{4k+8}$,
- $c_{cl}(G'_1, a_i) = \frac{1}{4k+6}$ for any $a_i \in V$,
- $c_{cl}(G'_1, b_i) = \frac{1}{4k+12}$ for any $b_i \in V$,
- $c_{cl}(G'_1, c_i) = \frac{1}{5k+10}$ for any $c_i \in V$,
- $c_{cl}(G'_1, d_1) = \frac{1}{3k+6}$,
- $c_{cl}(G'_1, d_2) = \frac{1}{3k+8}$.

As such, $v^\dagger$ is ranked on position $k + 3$ in $G'$ according to the closeness centrality (all nodes $a_i$, as well as nodes $d_1$ and $d_2$ are ranked above $v^\dagger$). We have shown that it is possible to decrease the closeness-based ranking of $v^\dagger$ by an arbitrarily large number of positions by adding an edge $e \in \zeta_2^\dagger$. □

We end this section with the theorem showing that, under certain conditions concerning the knowledge of the evader, it is impossible for them to determine how rewiring even a single edge would influence the their centrality-based ranking.

**Theorem 6.** *Given a node $v^\dagger \in V$, a network $G = (V, E)$, and an edge $e \in V \times V$, by knowing only: (i) the set of neighbors of $v^\dagger$, (ii) whether $e \in E$, and (iii) the degrees of the ends of $e$, it is impossible to guarantee that the addition of $e$ (in case $e \notin E$) or the removal of $e$ (in case $e \in E$) would increase or decrease the centrality-based ranking of $v^\dagger$ when both the increase and decrease are possible.*

*Proof.* According to the results presented in Table S1, both the increase and decrease in ranking are possible for the following edge-centrality combinations:

1. adding $e \in \zeta_1^\dagger$ and the closeness centrality,
2. adding $e \in \zeta_1^\dagger$ and the betweenness centrality,
3. adding $e \in \zeta_2^\dagger$ and the betweenness centrality,
4. adding $e \in \zeta_3^\dagger$ and the closeness centrality,
5. adding $e \in \zeta_3^\dagger$ and the betweenness centrality.

Let $k_x$ be the value of $k$ set in the network $G_x$. One can easily verify that the following networks from Figure S1—which were used to demonstrate that changes in ranking for the aforementioned edge-centrality combinations are possible—cannot be distinguished given the knowledge stated in the theorem:

1. network $G_5$ and network $G_8$,
2. network $G_2$ and network $G_6$ where $k_2 = k_6 + 3$,
3. network $G_4$ and network $G_7$ where $k_4 = 2k_7$,
4. network $G_3$ and network $G_9$,
5. network $G_3$ and network $G_9$ (same as above).

To put it differently, for each of the aforementioned edge-centrality combinations, there exists a pair of networks that are not distinguishable given the knowledge assumed in the theorem, where adding edge $e$ of a particular class increases the centrality-based ranking of the evader in one network, while decreasing it in the other network. □

## S2 The Proofs of Computational Complexity Results

**Theorem 7.** *The problem of Local Hiding is NP-complete given the closeness centrality.*

*Proof.* The problem is trivially in NP, since after the addition of a given $A^*$, and the removal of a given $R^*$, it is possible to compute the closeness centrality of all nodes in polynomial time.

Next, we prove that the problem is NP-hard. To this end, we show a reduction from the NP-complete 3-Set Cover problem. An instance of this problem is defined by the universe $U = \{u_1, \dots, u_l\}$, the set of subsets of the universe $S = \{S_1, \dots, S_m\}$, where for every $S_i$ we have $|S_i| = 3$, and a number $k \in \mathbb{N}$. The goal is then to determine whether there exist $k$ elements of $S$ the union of which equals $U$.

Given an instance $(U, S, k)$ of the 3-Set Cover problem, let us construct a network, $G = (V, E)$, as follows (an example of this construction is presented in Figure S2):

- $V = \{v^\dagger, t\} \cup \bigcup_{S_i \in S} \{S_i\} \cup \bigcup_{u_i \in U} \{u_i, w_i\} \cup \bigcup_{i=1}^{l+m-k+1} \{x_i\}$,
- $E = \{(t, v^\dagger)\} \cup \bigcup_{x_i \in V} \{(x_i, t)\} \cup \bigcup_{w_i \in V} \{(w_i, v^\dagger), (w_i, u_i)\} \cup \bigcup_{S_i \in S} \{(S_i, v^\dagger)\}$ $\cup \bigcup_{u_j \in S_i} \{(S_i, u_j)\}$.

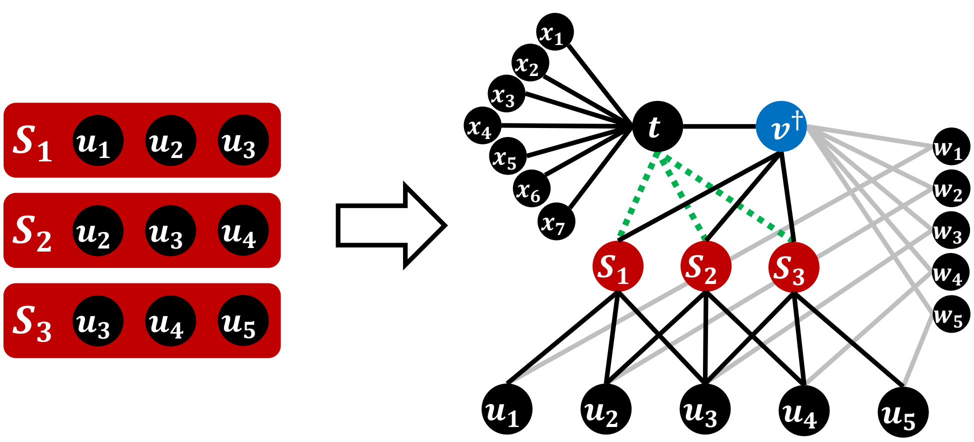


Figure S2: An example of the construction used in the proof of Theorem 7 for $k = 2$. Some edges are printed grey for better readability. Green dotted lines correspond to the edges allowed to be added.

Now, consider an instance of the Local Hiding problem, $(G, v^\dagger, b, c, \delta, \hat{A}, \hat{R})$, where $G$ is the network we just constructed, $v^\dagger$ is the evader, $b = k$ (where $k$ is the parameter of the 3-Set Cover problem), $c$ is the closeness centrality measure, $\delta = 1$, $\hat{A} = \{(t, S_i) : S_i \in S\}$, and $\hat{R} = \emptyset$.

From the definition of the problem, we see that the only edges that can be added to the graph are those between $t$ and the members of $S$. Notice that any such choice of $A^*$ corresponds to selecting a subset of $|A^*|$ elements of $S$ in the 3-Set Cover problem. In what follows, we will show that a solution to the above instance of Local Hiding corresponds to a solution to the 3-Set Cover problem.

First, we show that for every $v \in V \setminus \{t, v^\dagger\}$ and every $A^* \subseteq \hat{A}$ we either have $c(G', v) < c(G', t)$ or $c(G', v) < c(G', v^\dagger)$, where $G' = (V, E \cup A^*)$. To this end, let $D(G', v)$ denote the sum of distances from $v$ to all other nodes, i.e., $D(G', v) = \sum_{w \in V \setminus \{v\}} d(v, w)$. Given that $D(G', v) = \frac{n-1}{c(G', v)}$, we will show that the following holds:

$$\forall_{v \in V \setminus \{t, v^\dagger\}} \forall_{A^* \subseteq \hat{A}} \left( D(G', v) > D(G', t) \vee D(G', v) > D(G', v^\dagger) \right).$$

Let $d_t$ denote $\sum_{u_i \in U} d(t, u_i) + \sum_{S_i \in S} d(t, S_i)$. Notice also that $k \leq m$. Table S2 presents computation of distances between nodes in $G'$. Given these values we have that:

- $D(G', v^\dagger) = 5l + 3m - 2k + 3$;

| $v$ | $d(v, v^\dagger)$ | $d(v, t)$ | $\sum_{x_i \in X} d(v, x_i)$ | $\sum_{w_i \in W} d(v, w_i)$ | $\sum_{u_i \in U} d(v, u_i)$ | $\sum_{S_i \in S} d(v, S_i)$ |
|---|---|---|---|---|---|---|
| $v^\dagger$ | 0 | 1 | $2(m+l-k+1)$ | $l$ | $2l$ | $m$ |
| $t$ | 1 | 0 | $m+l-k+1$ | $2l$ | $\sum_{u_i \in U} d(t, u_i)$ | $\sum_{S_i \in S} d(t, S_i)$ |
| $x_i$ | 2 | 1 | $2(m+l-k)$ | $3l$ | $l + \sum_{u_i \in U} d(t, u_i)$ | $m + \sum_{S_i \in S} d(t, S_i)$ |
| $w_i$ | 1 | 2 | $3(m+l-k+1)$ | $2(l-1)$ | $1 + 3(l-1)$ | $2m$ |
| $u_i$ | 2 | $\geq 2$ | $3(m+l-k+1)$ | $1 + 3(l-1)$ | $3(l-1)$ | $\geq m$ |
| $S_i$ | 1 | $\geq 1$ | $2(m+l-k+1)$ | $2l$ | $3 + 3(l-3)$ | $2(m-1)$ |

Table S2: Distances between nodes in the graph constructed for the proof of Theorem 7.

- $D(G', t) = 3l + m - k + 2 + d_t$;
- $D(G', x_i) = 6l + 3m - 2k + 3 + d_t > D(G', t)$;
- $D(G', w_i) = 8l + 5m - 3k + 2 > D(G', v^\dagger)$;
- $D(G', u_i) \geq 9l + 4m - 3k + 2 > D(G', v^\dagger)$ as $\sum_{S_j \in S} d(u_i, S_j) \geq m$;
- $D(G', S_i) \geq 7l + 4m - 2k - 4 > D(G', v^\dagger)$ as $d(S_i, v^\dagger) \geq 1$.

Based on this, either $t$ or $v^\dagger$ has the greatest closeness centrality, therefore $A^* \subseteq \hat{A}$ is a solution to the problem of Local Hiding if and only if $D(G', t) < D(G', v^\dagger)$. This is the case when $d_t < 2l + 2m - k + 1$. Let $U_{A^*} = \{u_i \in U : \exists_{S_j \in S} u_i \in S_j \wedge (t, S_j) \in A^*\}$. We have that $d_t = |A^*| + 2(m - |A^*|) + 2|U_{A^*}| + 3(l - |U_{A^*}|)$ which gives us $d_t = 3l - |U_{A^*}| + 2m - |A^*|$. Since by definition $|U_{A^*}| \leq l$ and $|A^*| \leq k$, we have that $d_t < 2l + 2m - k + 1$ if and only if $|U_{A^*}| = l$ and $|A^*| = k$, i.e., $\forall_{u_i \in U} \exists_{S_j \in S} u_i \in S_j \wedge (t, S_j) \in A^*$.

Therefore, if there exists a solution to the given instance of the 3-Set Cover problem, i.e., $S^* \subseteq S$ such that $S^* \leq k$ and $\forall_{u_i \in U} \exists_{S_j \in S^*} u_i \in S_j$, then $\{(t, S_j) : S_j \in S^*\}$ is a solution the constructed instance of the Local Hiding problem. Moreover, if there exists a solution to the constructed instance of the Local Hiding problem, i.e., $A^* \subseteq \hat{A}$ such that $|U_{A^*}| = l$ and $|A^*| = k$, then we can construct a solution to the given instance of the 3-Set Cover problem as $\{S_i : (t, S_i) \in A^*\}$. We proved that a solution to the given instance of the 3-Set Cover problem exists if and only if there exists a solution to the constructed instance of the Local Hiding problem. □

**Theorem 8.** *The problem of Local Hiding is NP-complete given the betweenness centrality.*

*Proof.* The problem is trivially in NP, since after the addition of a given set of edges $A^*$, and the removal of a given set of edges $R^*$, it is possible to compute the betweenness centrality of all nodes in polynomial time.

Next, we prove that the problem is NP-hard. To this end, we show a reduction from the NP-complete 3-Set Cover problem. An instance of this problem is defined by the universe $U = \{u_1, \ldots, u_l\}$, the set of subsets of the universe $S = \{S_1, \ldots, S_m\}$, where for every $S_i$ we have $|S_i| = 3$, and a number $k \in \mathbb{N}$. The goal is then to determine whether there exist $k$ elements of $S$ the union of which equals $U$.

Given an instance of the 3-Set Cover problem, let us construct a network $G = (V, E)$ as follows (an example of this construction is presented in Figure S3):

- $V = \{v^\dagger, t, w_1, w_2\} \cup S \cup U \cup \bigcup_{i=1}^{\alpha} \{x_i\} \cup \bigcup_{i=1}^{\beta} \{y_i\}$, where $\alpha = m^2 l(m + l + 2)$ and $\beta = m^2 l(k + l + 2)$,
- $E = \{(t, v^\dagger), (w_1, w_2)\} \cup \bigcup_{x_i \in V} \{(x_i, t)\} \cup \bigcup_{y_i \in V} \{(y_i, v^\dagger)\} \cup \bigcup_{S_i \in V} \{(S_i, v^\dagger), (S_i, w_1)\} \cup \bigcup_{u_j \in S_i} \{(S_i, u_j)\} \cup \bigcup_{u_i \in V} \{(u_i, w_2)\} \cup \bigcup_{x_i, x_j \in V} \{(x_i, x_j)\} \cup \bigcup_{y_i, y_j \in V} \{(y_i, y_j)\}$.

Consider the instance $(G, v^\dagger, b, c, \delta, \hat{A}, \hat{R})$ of the problem of Local Hiding, where $G$ is the network we just constructed, $v^\dagger$ is the evader, $b = k$ (where $k$ is the parameter of the 3-Set Cover problem), $c$ is the betweenness centrality measure, $\delta = 1$, $\hat{A} = \{(t, S_i) : S_i \in S\}$, and $\hat{R} = \emptyset$.

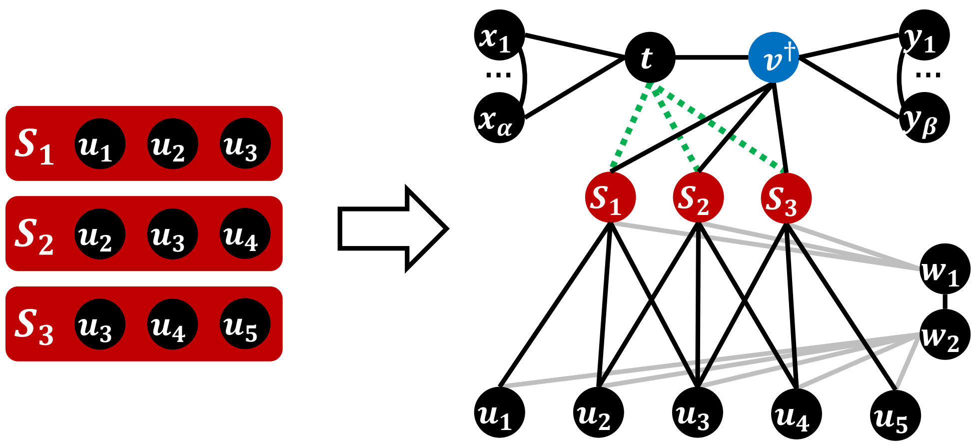


Figure S3: An example of the construction used in the proof of Theorem 8 for $k = 2$. Some edges are printed grey for better readability. Green dotted lines correspond to the edges allowed to be added.

From the definition of the problem, we can see that the only edges that can be added to the graph are those between $t$ and the members of $S$. Notice that any such choice of $A^*$ corresponds to selecting a subset of $|A^*|$ elements of $S$ in the 3-Set Cover problem. In what follows, we will show that a solution to the above instance of Local Hiding corresponds to a solution to the 3-Set Cover problem.

First, we show that for every node $v \in V \setminus \{t, v^\dagger\}$ and every $A^* \subseteq \hat{A}$ we have $c(G', v) < c(G', t)$, where $G' = (V, E \cup A^*)$. To this end, let $B(v)$ denote the sum of percentages of shortest paths controlled by $v$ between pairs of other nodes in $G'$, i.e.:

$$B(v) = \sum_{w,w' \in V \setminus \{v\}} \frac{|\{p \in \Pi(w, w') : v \in p\}|}{|\Pi(w, w')|}.$$

Note that $B(v) = \frac{(n-1)(n-2)}{2} c(G', v)$. Next, we will show that the following holds:

$$\forall_{v \in V \setminus \{t, v^\dagger\}} \forall_{A^* \subseteq \hat{A}} B(v) < B(t).$$

Since $t$ controls all shortest paths between the nodes in $X$ and those in $\{v^\dagger, w_1, w_2\} \cup Y \cup S \cup U$, we have:

$$B(t) \geq \alpha(\beta + m + l + 3) \geq m^4 l^3 (m + l + 2) + m^2 l (m + l + 2)^2$$

Moreover, since $\alpha = m^2 l(m + l + 2)$, $\beta = m^2 l(k + l + 2)$, and $k < m$, then $\alpha + \beta < 2m^2 l(m + l + 2)$.

For nodes other than $t$ we have:

- $B(x_i) = B(y_i) = 0 < B(t)$, since the nodes in $X \cup Y$ do not control any shortest paths.
- $B(w_1) \leq (\alpha+\beta+m+2)+\frac{m(m-1)}{2}+ml \leq 2m^2 l(m+l+2)+m^2+m+ml < (2m^2 l+m)(m+l+2) < B(t)$, because $w_1$ controls some shortest paths between $w_2$ and nodes in $\{t, v^\dagger\} \cup X \cup Y \cup S$ (there are $\alpha + \beta + m + 2$ such pairs), some shortest paths between pairs of nodes in $S$ (there are at most $\frac{m(m-1)}{2}$ such pairs), and some shortest paths between nodes in $U$ and nodes in $S$ (there are at most $ml$ such pairs).
- $B(w_2) \leq \frac{l(l-1)}{2} + l + ml < \frac{l^2+l}{2} + ml < B(t)$, because $w_2$ controls some shortest paths between pairs of nodes in $U$ (there are at most $\frac{l(l-1)}{2}$ such pairs), some shortest paths between nodes in $U$ and $w_1$ (there are at most $l$ such pairs), and some shortest paths between nodes in $U$ and nodes in $S$ (there are at most $ml$ such pairs).

- $B(u_i) \leq (\alpha + \beta + m + 2) + \frac{m(m-1)}{2} < B(t)$, because $u_i$ controls some shortest paths between $w_2$ and nodes in $\{t, v^\dagger\} \cup X \cup Y \cup S$ (there are $\alpha + \beta + m + 2$ such pairs), and some shortest paths between pairs of nodes in $S$ (there are at most $\frac{m(m-1)}{2}$ such pairs).
- $B(S_i) \leq 3(\alpha+\beta+l+m+2)+l+2(\alpha+\beta+2) \leq 5(\alpha+\beta+l+m+2) \leq (10m^2l+5)(m+l+2) < B(t)$, because $S_i$ controls some shortest paths between the nodes in $U$ that are connected to $S_i$ and the nodes in $\{t, v^\dagger\} \cup X \cup Y \cup S \cup U$ (there are at most $3(\alpha + \beta + l + m + 2)$ such pairs), some shortest paths between $w_1$ and the nodes in $U$ (there are at most $l$ such pairs), and some of the shortest paths between nodes in $\{w_1, w_2\}$ and nodes in $\{t, v^\dagger\} \cup X \cup Y$ (there are at most $2(\alpha + \beta + 2)$ such pairs).

Therefore, either $t$ or $v^\dagger$ has the greatest betweenness centrality. Hence, $A^* \subseteq \hat{A}$ is a solution to the problem of Local Hiding if and only if $B(t) > B(v^\dagger)$. We now compute the values of $B(t)$ and $B(v^\dagger)$. We have that:

$$B(t) = \alpha(\beta + m + l + 3) + \sum_{\substack{S_i, S_j \in S: \\ (t,S_i)\in E \wedge (t,S_j)\in E}} \frac{1}{|N(S_i, S_j)|} + \sum_{S_i \in N(t)} \sum_{u_j \in U \setminus N(S_i)} \frac{|N(t, u_j)|}{|N(t, u_j)| + |N(v^\dagger, u_j)| + 1}$$

as $t$ controls all shortest paths between every pair $(x_i, v)$ where $x_i \in X$ and $v \in V \setminus (X \cup \{t\})$ (there are $\alpha(\beta + m + l + 3)$ such pairs), one shortest path between each pair of nodes in $N(t) \cap S$, and the shortest paths between every pair $(v, w)$ where $v \in N(t) \cap S$ and $w \in U : N(t) \cap N(w) \neq \emptyset$ (other paths run through $v^\dagger$ and nodes in $S$, or through $w_1$ and $w_2$). On the other hand, we have that:

$$B(v^\dagger) = \beta(\alpha + m + l + 3) + \sum_{S_i, S_j \in S} \frac{1}{|N(S_i, S_j)|} + \sum_{S_i \notin N(t)} (\alpha + 1) + \sum_{u_i \in U : N(t, u_i) = \emptyset} (\alpha + 1) + \sum_{S_i \in S} \sum_{u_j \in U \setminus N(S_i)} \frac{|N(v^\dagger, u_j)|}{|N(t, u_j)| + |N(v^\dagger, u_j)| + 1}$$

as $v^\dagger$ controls all shortest paths between nodes in $Y$ and all other nodes (there are $\beta(\alpha + m + l + 3)$ such pairs), one shortest path between each pair of nodes in $S$, paths between nodes in $S$ and nodes in $U$, and all shortest paths between $\{t\} \cup X$ and nodes $\{S_i \in S : S_i \notin N(t)\} \cup \{u_i \in U : N(t, u_i) = \emptyset\}$. Thus, we have:

$$B(v^\dagger) - B(t) = (\beta - \alpha)(m + l + 3) + \sum_{\substack{S_i, S_j \in S: \\ (t,S_i)\notin E \vee (t,S_j)\notin E}} \frac{1}{|N(S_i, S_j)|} + \sum_{S_i \notin N(t)} (\alpha + 1) + \sum_{u_i \in U : N(t, u_i) = \emptyset} (\alpha + 1) + \Delta SU$$

where $0 < \Delta SU \leq ml$.

Next, we prove that:

1. If $|A^*| \geq k$ and for every $u_i \in U$ there exists $S_j \in N(t)$ such that $u_i \in S_j$, then $B(v^\dagger) < B(t)$;
2. If $|A^*| < k$ or there exists $u_i \in U$ such that for every $S_j \in N(t)$ we have $u_i \notin S_j$, then $B(v^\dagger) \geq B(t)$.

Regarding point 1, we have:

$$B(v^\dagger) - B(t) = (\beta - \alpha)(m + l + 3) + (m - |A^*|)(\alpha + 1) + \sum_{\substack{S_i, S_j \in S: \\ (t,S_i)\notin E \vee (t,S_j)\notin E}} \frac{1}{|N(S_i, S_j)|} + \Delta SU.$$

Let $\gamma = |\{S_i, S_j \in S : (t, S_i) \notin E \vee (t, S_j) \notin E\}|$. Now, since $|A^*| \geq k$ we have that:

$$\gamma = \frac{m(m-1) - |A^*|(|A^*| - 1)}{2} \leq \frac{m(m-1) - k(k-1)}{2} = \frac{(m-k)(m+k-1)}{2}.$$

Given the above and $|N(S_i, S_j)| \geq 2$ we have that:

$$B(v^\dagger) - B(t) \leq (\beta - \alpha)(m + l + 3) + (m - k)\left(\alpha + 1 + \frac{\Delta SU}{m - k} + \frac{m + k - 1}{4}\right).$$

By substituting the values of $\alpha$ and $\beta$, and observing that $\Delta SU < ml$ and $k < m$, we get:

$$B(v^\dagger) - B(t) < m^2 l(k - m)(m + l + 3) + (m - k)(m^2 l(m + l + 2) + 1 + ml + 2m - 1),$$

which gives us:

$$B(v^\dagger) - B(t) < (k - m)m^2 l + (m - k)(ml + 2m) = (k - m)m(ml - l - 2) < 0.$$

Hence, if $|A^*| \geq k$ and for every $u_i \in U$ there exists $S_j \in N(t)$ such that $u_i \in S_j$, then $B(v^\dagger) < B(t)$.

Regarding point 2, since either there exists $u_i \in U$ such that for every $S_j \in N(t)$ we have $u_i \notin S_j$ or we have that $|A^*| < k$, then:

$$B(v^\dagger) - B(t) \geq (\beta - \alpha)(m + l + 3) + (m - k)(\alpha + 1) + (\alpha + 1) + \sum_{\substack{S_i, S_j \in S: \\ (t,S_i) \notin E \vee (t,S_j) \notin E}} \frac{1}{|N(S_i, S_j)|} + \Delta SU.$$

Since $\sum_{\substack{S_i, S_j \in S: \\ (t,S_i) \notin E \vee (t,S_j) \notin E}} \frac{1}{|N(S_i,S_j)|} > 0$ and $\Delta SU > 0$, then we have:

$$B(v^\dagger) - B(t) > (\beta - \alpha)(m + l + 3) + (m - k + 1)(\alpha + 1).$$

By substituting the values of $\alpha$ and $\beta$ we get:

$$B(v^\dagger) - B(t) > m^2 l(k - m)(m + l + 3) + (m - k + 1)(m^2 l(m + l + 2) + 1)$$

which gives us:

$$B(v^\dagger) - B(t) > m^2 l(k - m) + m^2 l(m + l + 2) = m^2 l(k + l + 2) > 0$$

Hence, if $|A^*| < k$ or there exists $u_i \in U$ such that for every $S_j \in N(t)$ we have $u_i \notin S_j$, then $B(v^\dagger) > B(t)$.

From the points 1 and 2 we have that $B(v^\dagger) < B(t)$ if and only if $|A^*| \geq k$ and for every $u_i \in U$ there exists $S_j \in N(t)$ such that $u_i \in S_j$.

Therefore, if there exists a solution to the given instance of the 3-Set Cover problem, i.e., $S^* \subseteq S$ for every $u_i \in U$ there exists $S_j \in S^*$ such that $u_i \in S_j$, then $\{(t, S_j) : S_j \in S^*\}$ is a solution the constructed instance of the Local Hiding problem (if $|\S^*| < k$ we can connect $t$ to additional $k - |A^*|$ nodes $S_j$, as $B(v^\dagger)$ decreases with $|A^*|$, while $B(t)$ increases with it). Moreover, if there exists a solution to the constructed instance of the Local Hiding problem, i.e., $A^* \subseteq \hat{A}$ such that $B(v^\dagger) < B(t)$ after the addition of $A^*$, then we can construct a solution to the given instance of the 3-Set Cover problem as $\{S_i : (t, S_i) \in A^*\}$ (notice that $|A^*| = k$, as we have $|A^*| \geq k$ and $|A^*| \leq b = k$). We proved that a solution to the given instance of the 3-Set Cover problem exists if and only if there exists a solution to the constructed instance of the Local Hiding problem. □

**Theorem 9.** *Algorithm 1 is a 2-approximation for the Minimum Local Hiding problem given the degree centrality.*

*Proof.* We first give the idea behind Algorithm 1, before moving to the detailed description. The algorithm will go over all possible sizes $r$ of the set of removed edges. For a given $r$ it will identify a solution where the size of the set of removed edges is exactly $r$, and the size of the set of added edges is at most twice as big as in an optimal solution, which makes this algorithm a 2-approximation.

In more detail, let $(A^*, R^*)$ be an optimal solution to a given $(G, v^\dagger, c_{dg}, \delta, \hat{A}, \hat{R})$ problem instance, and let $W$ be the set of nodes that contribute towards the safety threshold, i.e., $W = \{v \in V : \kappa_{G^*}(v) > \kappa_{G^*}(v^\dagger)\}$,

**Algorithm 1** A 2-approximation algorithm for the Minimum Local Hiding problem given the degree centrality. The operator $\sigma(k, S)$ selects any $\min(k, |S|)$ elements of set $S$. We define $\hat{A}(v)$ as the edges in $\hat{A}$ incident with $v$.

**Input:** Network $G = (V, E)$, evader $v^\dagger \in V$, safety threshold $\delta \in \mathbb{N}$, the set of edges allowed to be added $\hat{A} \subseteq N(v^\dagger) \times N(v^\dagger)$, the set of edges allowed to be removed $\hat{R} \subseteq \{v^\dagger\} \times N(v^\dagger)$.
**Output:** A solution $(A^\circ, R^\circ)$ to instance $(G, v^\dagger, c_{dg}, \delta, \hat{A}, \hat{R})$ of the Minimum Local Hiding problem such that $|A^\circ| + |R^\circ| \leq 2(|A^*| + |R^*|)$, where $(|A^*|, |R^*|)$ is an optimal solution, or $\perp$ if there is no solution.

$(A^\circ, R^\circ) \leftarrow \perp$
**for** $r \leftarrow 0, \ldots, |\hat{R}|$ **do**
  $\kappa^* \leftarrow \kappa(v^\dagger) - r$
  $R' \leftarrow \sigma(r, \{(v, v^\dagger) \in \hat{R} : \kappa(v) > \kappa^* + 1 \vee \kappa(v) + |\hat{A}(v)| \leq \kappa^*\})$
  **if** $|R'| < r$ **then**
    $X \leftarrow \hat{R} \setminus R'$ sorted increasingly based on $\kappa(v)$
    $Y \leftarrow \{(v, v^\dagger) \in \hat{R} : \kappa(v) \leq \kappa^* + 1 \wedge \kappa(v) + |\hat{A}(v)| = \kappa^* + 1\}$
    $y \leftarrow |Y| - (\delta - |\{v \in \bar{V} : \kappa(v) + |\hat{A}(v)| > \kappa^* \wedge (v, v^\dagger) \notin Y\}|)$
    **while** $|R'| < r \wedge |X| > 0$ **do**
      $(v^*, v^\dagger) \leftarrow \mathrm{pop}(X)$
      **if** $(v^*, v^\dagger) \notin Y \vee R' \cap Y < y$ **then**
        $R' \leftarrow R' \cup \{v^*\}$
  **if** $|R'| = r$ **then**
    $G' = (V, E \setminus R')$
    $a = \delta - |\{v \in V : \kappa_{G'}(v) > \kappa^*\}|$
    $Z \leftarrow \{v \in N(v) : \kappa_{G'}(v) \leq \kappa^* \wedge \kappa_{G'}(v) + |\hat{A}(v)| > \kappa^*\}$ sorted decreasingly based on $\kappa(v)$
    $A' \leftarrow \emptyset$
    **while** $a > 0 \wedge |Z| > 0$ **do**
      $v^* \leftarrow \mathrm{pop}(Z)$
      $A' \leftarrow A' \cup \sigma(\kappa^* + 1 - \kappa_{G'}(v^*), \hat{A}(v^*))$
      $G' = (V, E \cup A' \setminus R')$
      $a \leftarrow a - 1$
    **if** $a \leq 0 \wedge ((A^\circ, R^\circ) = \perp \vee |A'| + |R'| < |A^\circ| + |R^\circ|)$ **then**
      $(A^\circ, R^\circ) \leftarrow (A', R')$
**return** $(A^\circ, R^\circ)$

where $G^* = (V, E \cup A^* \setminus R^*)$. The set $W$ can be divided into nodes $W_0$ that would contribute to the safety threshold even without adding any edges from $\hat{A}$, i.e., $W_0 = \{v \in W : \kappa_{(V,E \setminus R^*)}(v) > \kappa_{G^*}(v^\dagger)\}$, and nodes $W_1$ that required adding edges from $\hat{A}$ in order to contribute to the threshold, i.e., $W_1 = \{v \in W : \kappa_{(V,E \setminus R^*)}(v) \leq \kappa_{G^*}(v^\dagger)\}$. Notice that the lower bound of the size of $A^*$ is:

$$m^* = \frac{1}{2} \sum_{v \in W_1} \kappa_{G^*}(v^\dagger) + 1 - \kappa_{(V,E \setminus R^*)}(v)$$

as the minimum degree of a node $v$ necessary for it to contribute to the safety thresholds is $\kappa_{G^*}(v^\dagger) + 1$, while every added edge increases degree of at most two nodes from $W_1$ (hence the $\frac{1}{2}$ factor).

Algorithm 1 selects for $r = |R^*|$ a set of edges to be removed $R'$ such that $|R'| = r$ and the choice of $R'$ does not increase the minimal possible size of the set of edges to be added (notice that if the evader gets disconnected with a node $v \in W_1$, the degree of $v$ has to increased by adding an additional edge, we will comment on this further when describing the algorithm). The algorithm then increases the degrees of $|W_1|$ nodes in the order of decreasing degrees. The reason for this order is that nodes with greater degrees require fewer edges to be added to them in order for their degrees to reach $\kappa_{G^*}(v^\dagger) + 1$. Let $W_1'$ be the set of nodes the degrees of which are increased by Algorithm 1 to satisfy the safety threshold. Since they are selected as the nodes with maximal degrees, we have:

$$\sum_{v \in W_1'} \kappa_{(V,E \setminus R')}(v) \geq \sum_{v \in W_1} \kappa_{(V,E \setminus R^*)}(v).$$

Therefore, we have that:

$$\begin{aligned}
|R'| + |A'| &= r + \sum_{v \in W_1'} \kappa_{G^*}(v^\dagger) + 1 - \kappa_{(V,E \setminus R')}(v) \\
&\leq r + \sum_{v \in W_1} \kappa_{G^*}(v^\dagger) + 1 - \kappa_{(V,E \setminus R^*)}(v) \\
&= r + 2m^* \leq 2(r + m^*) \\
&\leq 2(|R^*| + |A^*|).
\end{aligned}$$

Hence, we have that Algorithm 1 is a 2-approximation. We will now describe in detail how Algorithm 1 identified sets of edges $R'$ and $A'$ characterized above.

The algorithm begins in line 1 with initializing the $(A^\circ, R^\circ)$ variable pair with $\perp$ (this is the value that will be returned if the particular instance of the problem does not have any solutions). The loop in line 2 iterates over all possible sizes of the set of removed edges. For a given size $r$ we compute the degree of the evader after removing $r$ edges as $\kappa^*$ in line 3. Notice that after removing $r$ edges from $\hat{R}$ and possibly adding some edges from $\hat{A}$, we need to have at least $\delta$ nodes with degrees with degrees greater than $\kappa^*$.

In lines 4-12 the algorithm selects the set of $r$ edges to be removed from the network $R'$. First, in line 4 it selects as many edges as possible connecting the evader either to nodes that will contribute to safety threshold even if we decrease their degree by one ($\kappa(v) > \kappa^* + 1$) or nodes that will not contribute to the safety threshold even if we add all edges from $\hat{A}$ incident with them ($\kappa(v) + |\hat{A}(v)| \leq \kappa^*$). If the algorithm still needs to remove more edges (condition tested in the 5, the removal takes place in lines 6-12), they have to be selected from nodes that require increasing their degrees by adding the edges from $\hat{A}$. In line 6 we sort all the remaining edges that can be removed based on the degree of the non-evader end into the list $X$. Further, in line 7 we collect the set $Y$ of edges that can be removed between the evader and nodes, but if they are removed, the non-evader end can no longer contribute to the safety threshold. Notice that if we remove too many edges from the set $Y$, we encounter a risk of not reaching a solution for a given $r$, even if it exists. To avoid this risk, we compute the maximal number of edges from $Y$ that we can safely remove $y$ in line 8. Having made sure that we will not miss a possible solution, we select edges to remove in the loop in lines 9-12, starting with those connecting the evader with nodes with lowest degrees (the pop operator removes the head of the list). These nodes will be the last to be selected as contributing to the threshold by adding incident edges from $\hat{A}$ in the latter part of the algorithm, which allows us to minimize the total cost of the solution.

In line 13 we test whether we managed to identify $r$ edges from $\hat{R}$ to remove. If not, there exist no solutions that remove $r$ edges and satisfy the safety threshold at the same time. Otherwise, we select the edges to add from $\hat{A}$ in lines 14-22. To this end, we generate the structure of the network $G'$ after the removal of edges from $R'$ in line 14, and compute the number of nodes the degrees of which we have to increase $a$ in line 15. To minimize the total cost of the solution, we select the candidates based on decreasing degree, using the list $Z$ generated in line 16. The edges that are to be added to the network as part of the solution are gathered in set $A'$, initialized in line 17. In the loop in lines 18-22 we add edges necessary for the nodes with greatest degrees to contribute to the safety threshold. Finally, if we managed to increased the degree of the necessary number of nodes, i.e., if $a \leq 0$, we update the best identified solution in lines 23-24. Said solution is returned at the end of the algorithm in line 25.

Finally, let us comment on the time complexity of the algorithm. The main loop of the algorithm, i.e., the loop in line 2 is executed $\mathcal{O}(|V|)$ times (notice that the size of $\hat{R}$ is at most $|V| - 1$). Within the loop, the most expensive operations are sorting the list $X$ in line 4 ($\mathcal{O}(|V| \log |V|$ operations per main loop iteration), executing the loop in lines 9-12 ($\mathcal{O}(|V| \log |V|$ operations per main loop iteration), and executing the loop in lines 18-22 ($\mathcal{O}(|V||E|$ operations per main loop iteration). Therefore, the time complexity of Algorithm 1 is $\mathcal{O}(|V|^2(\log |V| + |E|))$. □

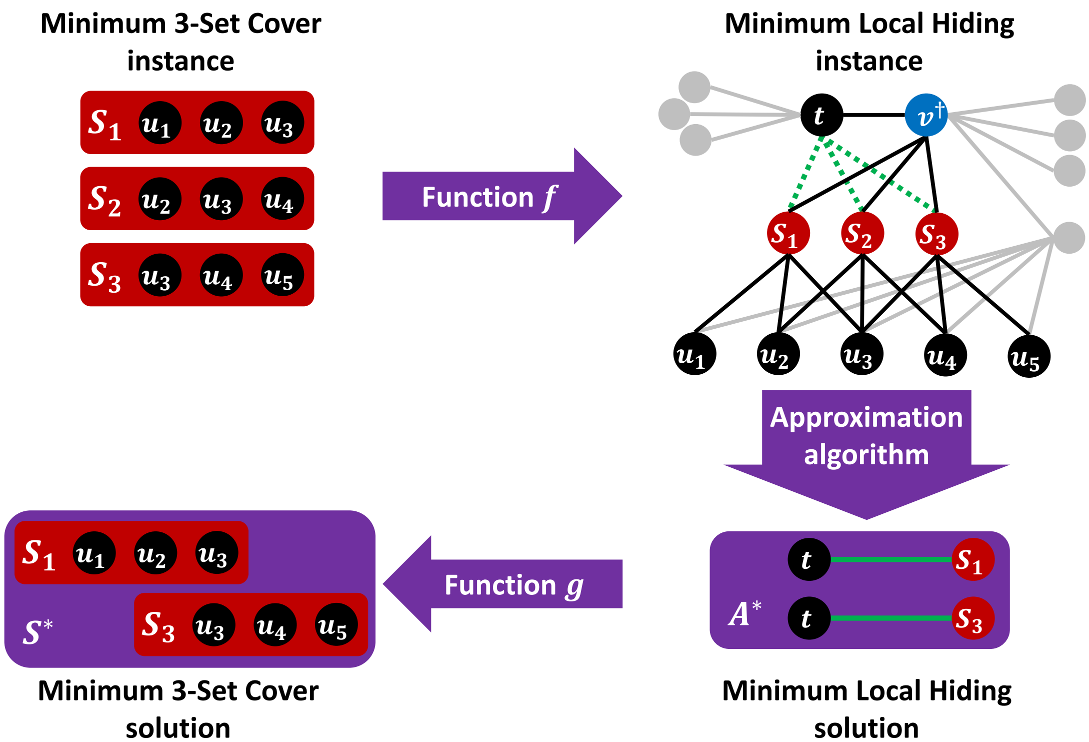


Figure S4: An overview of the approximation process in the proofs of Theorems 10 and 11.

**Theorem 10.** *The Minimum Local Hiding problem given the closeness centrality cannot be approximated within a ratio of* $(1-\epsilon)\ln|\hat{A} \cup \hat{R}|$ *for any* $\epsilon > 0$*, unless* $P = NP$.

*Proof.* In our proof we will use the result by Dinur and Steurer [1] that the *Minimum 3-Set Cover* problem cannot be approximated better than logarithmically, unless $P = NP$. More precisely, we will show that if there exists an efficient approximation algorithm for the Minimum Local Hiding problem then there also exists an efficient approximation algorithm for the Minimum 3-Set Cover problem. An overview of the process is presented in Figure S4.

An instance of the Minimum 3-Set Cover problem is defined by a universe of elements $U = \{u_1, \ldots, u_{|U|}\}$ and a set of subsets of the universe $S = \{S_1, \ldots, S_{|S|}\}$ such that $\forall_{S_i} S_i \subset U \wedge |S_i| = 3$. The goal of the problem is to identify a subset $S^* \subseteq S$ that covers entire universe, i.e., $\bigcup_{S_i \in S^*} S_i = U$, and the size of $S^*$ is minimal.

We now define two functions: function $f$ that translates and instance of the Minimum 3-Set Cover problem to an instance of the Minimum Local Hiding problem, and function $g$ that translates a Minimum Local Hiding solution (i.e., sets of edges added $A^*$ and removed $R^*$ from the network) to a Minimum 3-Set Cover solution (i.e., a subset of $S$).

Let $(U, S)$ be an instance of the Minimum 3-Set Cover problem. Assume that $|U| \geq 5$ (all smaller instances can easily be solved in polynomial time). Function $f$ is then defined as

$$f(U, S) = (G, v^\dagger, c, \delta, \hat{A}, \hat{R}),$$

where:

- $G = (V, E)$ is a network (an example of its construction is presented in Figure S5) where:
  - $V = \{v^\dagger, t\} \cup \bigcup_{S_i \in S}\{S_i\} \cup \bigcup_{u_i \in U} \bigcup_{S_j \in S}\{u_{i,j}\} \cup \bigcup_{u_i \in U}\{w_i\} \cup \bigcup_{i=1}^{|U|+|S|}\{x_i\}$,

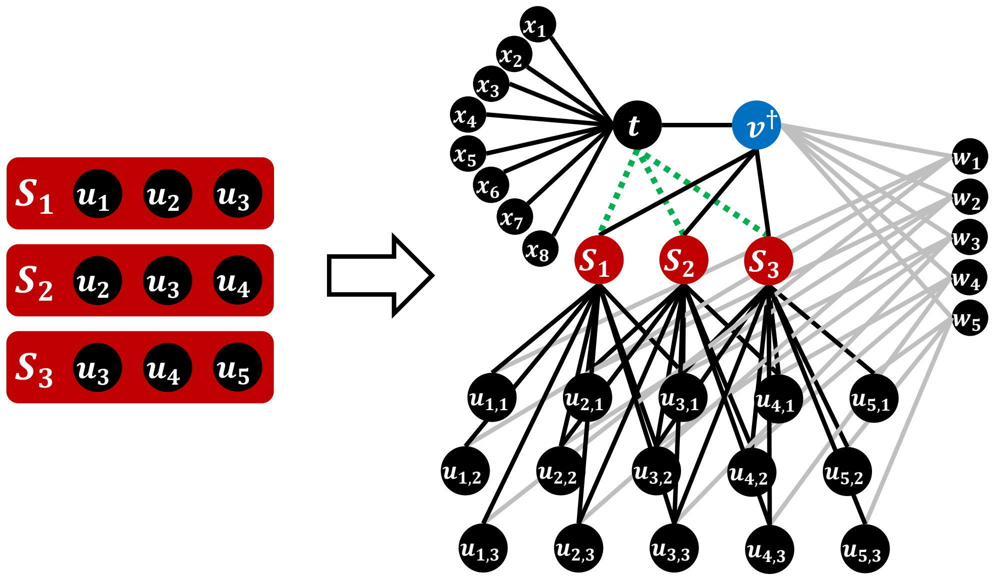


Figure S5: An example of the construction used in the proof of Theorem 10. Some edges are printed grey for better readability. Green dotted lines correspond to the edges allowed to be added.

– $E = \{(t, v^\dagger)\} \cup \bigcup_{x_i \in V} \{(x_i, t)\} \cup \bigcup_{w_i \in V} \{(w_i, v^\dagger)\} \cup \bigcup_{S_i \in S} \{(S_i, v^\dagger)\}$
$\cup \bigcup_{u_{i,j} \in V} \{(w_i, u_{i,j})\} \cup \bigcup_{u_j \in S_i} \bigcup_{u_{j,k} \in V} \{(S_i, u_{j,k})\}$,

- $v^\dagger \in V$ is the evader,
- $c = c_{cl}$ is the closeness centrality measure;
- $\delta = 1$ is the safety margin,
- $\hat{A} = \{t\} \times S$,
- $\hat{R} = \emptyset$.

Let $(A^*, R^*)$ be a solution to the $f(U, S)$ instance of the Minimum Local Hiding problem. Function $g$ is then defined as:

$$g(A^*, R^*) = \{S_i \in S : (t, S_i) \in A^*\}.$$

Notice that since $\hat{R} = \emptyset$ then necessarily $R^* = \emptyset$. Hence, in the remainder of the proof we will call $A^*$ the solution of $f(U, S)$.

From the definition of the problem, we see that the only edges that can be added to the network are those between $t$ and the members of $S$. Notice that any such choice of $A^*$ corresponds to selecting a subset of $|A^*|$ elements of $S$ in the Minimum 3-Set Cover problem. In what follows, we will show what are the necessary condition for $A^*$ to be a solution to the $f(U, S)$ problem instance.

First, we show that for every $v \in V \setminus \{t, v^\dagger\}$ and every $A^* \subseteq \hat{A}$ we have $c(G', v) < c(G', v^\dagger)$, where $G' = (V, E \cup A^*)$. To this end, let $D(G', v)$ denote the sum of distances from $v$ to all other nodes, i.e., $D(G', v) = \sum_{w \in V \setminus \{v\}} d(v, w)$. Given that $D(G', v) = \frac{n-1}{c(G', v)}$, we will show that the following holds:

$$\forall_{v \in V \setminus \{t, v^\dagger\}} \forall_{A^* \subseteq \hat{A}} D(G', v) > D(G', v^\dagger).$$

Let $d_t$ denote $\sum_{u_i} d(t, u_i) + \sum_{S_i} d(t, S_i)$. Table S3 presents computation of distances between nodes in $G'$. Given these values we have that:

- $D(G', v^\dagger) = 2|U||S| + 3|U| + 3|S| + 1$;

| $v$ | $d(v, v^\dagger)$ | $d(v,t)$ | $\sum_{x_i} d(v,x_i)$ | $\sum_{w_i} d(v,w_i)$ | $\sum_{u_{i,j}} d(v,u_{i,j})$ | $\sum_{S_i} d(v,S_i)$ |
|---|---|---|---|---|---|---|
| $v^\dagger$ | 0 | 1 | $2(\|U\|+\|S\|)$ | $\|U\|$ | $2\|U\|\|S\|$ | $\|S\|$ |
| $t$ | 1 | 0 | $\|U\|+\|S\|$ | $2\|U\|$ | $\sum_{u_{i,j}} d(t,u_{i,j})$ | $\sum_{S_i} d(t,S_i)$ |
| $x_i$ | 2 | 1 | $2(\|U\|+\|S\|-1)$ | $3\|U\|$ | $\geq 3\|U\|\|S\|$ | $\geq 2\|S\|$ |
| $w_i$ | 1 | 2 | $3(\|U\|+\|S\|)$ | $2(\|U\|-1)$ | $\|S\|+3\|S\|(\|U\|-1)$ | $2\|S\|$ |
| $u_{i,j}$ | 2 | $\geq 2$ | $\geq 3(\|U\|+\|S\|)$ | $1+3(\|U\|-1)$ | $\geq 2(\|U\|\|S\|-1)$ | $\geq \|S\|$ |
| $S_i$ | 1 | $\geq 1$ | $\geq 2(\|U\|+\|S\|)$ | $2\|U\|$ | $3\|S\|+3\|S\|(\|U\|-3)$ | $2(\|S\|-1)$ |

Table S3: Distances between nodes in the graph constructed for the proof of Theorem 10.

- $D(G',t) = 3|U| + |S| + 1 + d_t$;
- $D(G',x_i) \geq 3|U||S| + 5|U| + 4|S| + 1 > D(G',v^\dagger)$;
- $D(G',w_i) = 3|U||S| + 5|U| + 3|S| + 1 > D(G',v^\dagger)$;
- $D(G',u_i) \geq 2|U||S| + 6|U| + 4|S| > D(G',v^\dagger)$;
- $D(G',S_i) \geq 3|U||S| + 4|U| - 2|S| > D(G',v^\dagger)$ as $|U| \geq 5$.

Based on this, either $t$ or $v^\dagger$ has the greatest closeness centrality, therefore $A^* \subseteq \hat{A}$ is a solution to $f(U,S)$ if and only if $D(G',t) < D(G',v^\dagger)$. This is the case when $d_t < 2|U||S|+2|S|$. Let $U_{A^*} = \{u_i \in U : \exists_{S_j \in S} u_i \in S_j \wedge (t,S_j) \in A^*\}$. We have that:

$$d_t = |A^*| + 2(|S| - |A^*|) + 2|U_{A^*}||S| + 3(|U| - |U_{A^*}|)|S| = 2|U||S| + 2|S| + (|U| - |U_{A^*}|)|S| - |A^*|.$$

Since by definition $|U_{A^*}| \leq |U|$ and $|A^*| \leq |S|$, we have that $d_t < 2|U||S| + 2|S|$ if and only if $|U_{A^*}| = |U|$, i.e., $\forall_{u_i \in U} \exists_{S_j \in S} u_i \in S_j \wedge (t,S_j) \in A^*$. Hence, $A^*$ is a solution to $f(U,S)$ if and only if the sets corresponding to the nodes that $t$ is connected to via edges in $A^*$ cover entire universe. In particular, it implies that the optimal solutions to the both instances are of the same size.

Now, assume that there exists an approximation algorithm for the Minimum Local Hiding problem with ratio $(1-\epsilon)\ln|\hat{A} \cup \hat{R}|$ for some $\epsilon > 0$. This algorithm can be used to solve the constructed instance $f(U,S)$, acquiring solution $A^*$. Since $A^*$ is a solution to $f(U,S)$, the sets corresponding to the nodes with which $t$ is connected via edges in $A^*$ cover the universe $U$. Therefore, $g(A^*)$ (which extracts exactly these sets) is a solution to the given instance of the Minimum 3-Set Cover problem. What is more, we have that $|A^*| = |g(A^*)|$ and the optimal solutions to the both instances are of the same size. Since we also have that $|S| = |\hat{A} \cup \hat{R}|$, we obtained an approximation algorithm that solves the Minimum 3-Set Cover problem to within $(1-\epsilon)\ln|S|$ for $\epsilon > 0$. However, Dinur and Steurer [1] showed that the Minimum 3-Set Cover problem cannot be approximated to within $(1-\epsilon)\ln|S|$ for any $\epsilon > 0$, unless $P = NP$. Therefore, such an approximation algorithm for the Minimum Local Hiding problem cannot exist, unless $P = NP$. This concludes the proof. □

**Theorem 11.** *The Minimum Local Hiding problem given the betweenness centrality cannot be approximated within a ratio of* $(1-\epsilon)\ln|\hat{A} \cup \hat{R}|$ *for any* $\epsilon > 0$*, unless* $P = NP$.

*Proof.* In our proof we will use the result by Dinur and Steurer [1] that the *Minimum 3-Set Cover* problem cannot be approximated better than logarithmically, unless $P = NP$. More precisely, we will show that if there exists an efficient approximation algorithm for the Minimum Local Hiding problem then there also exists an efficient approximation algorithm for the Minimum 3-Set Cover problem. An overview of the process is presented in Figure S4.

An instance of the Minimum 3-Set Cover problem is defined by a universe of elements $U = \{u_1, \dots, u_{|U|}\}$ and a set of subsets of the universe $S = \{S_1, \dots, S_{|S|}\}$ such that $\forall_{S_i} S_i \subset U \wedge |S_i| = 3$. The goal of the problem is to identify a subset $S^* \subseteq S$ that covers entire universe, i.e., $\bigcup_{S_i \in S^*} S_i = U$, and the size of $S^*$ is minimal.

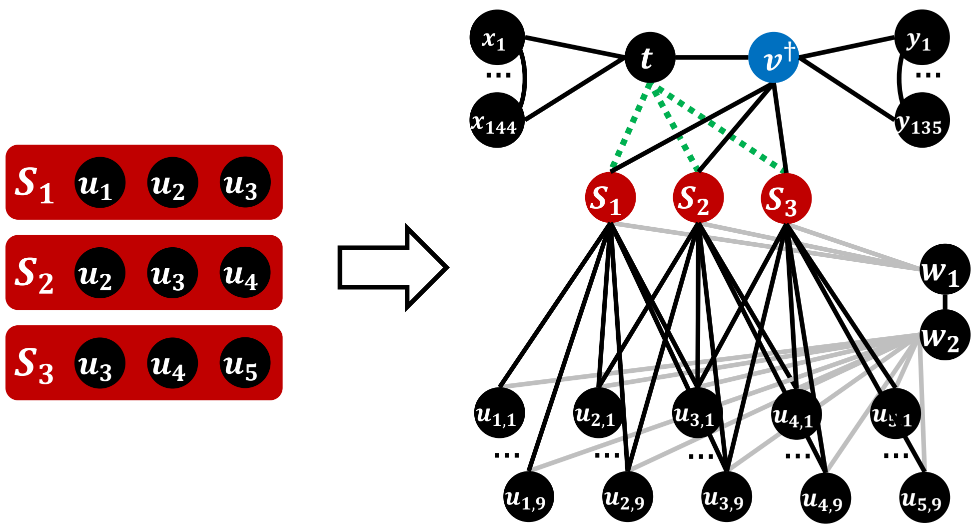


Figure S6: An example of the construction used in the proof of Theorem 11. Some edges are printed grey for better readability. Green dotted lines correspond to the edges allowed to be added.

We now define two functions: function $f$ that translates an instance of the Minimum 3-Set Cover problem to an instance of the Minimum Local Hiding problem, and function $g$ that translates a Minimum Local Hiding solution (i.e., sets of edges added $A^*$ and removed $R^*$ from the network) to a Minimum 3-Set Cover solution (i.e., a subset of $S$).

Let $(U, S)$ be an instance of the Minimum 3-Set Cover problem. Assume that $|S| \geq 3$ and $|U| \geq 3$ (all smaller instances can easily be solved in polynomial time). Function $f$ is then defined as

$$f(U, S) = (G, v^\dagger, c, \delta, \hat{A}, \hat{R}),$$

where:

- $G = (V, E)$ is a network (an example of its construction is presented in Figure S6) where:
  - $V = \{v^\dagger, t, w_1, w_2\} \cup \bigcup_{S_i \in S}\{S_i\} \cup \bigcup_{u_i \in U} \bigcup_{j=1}^{|S|^2}\{u_{i,j}\} \cup \bigcup_{i=1}^{|U||S|^3+|S|^2}\{x_i\} \cup \bigcup_{i=1}^{|U||S|^3}\{y_i\}$,
  - $E = \{(t, v^\dagger)\} \cup \{(w_1, w_2)\} \cup \bigcup_{x_i \in V}\{(x_i, t)\} \cup \bigcup_{x_i, x_j \in V}\{(x_i, x_j)\} \cup \bigcup_{y_i \in V}\{(y_i, v^\dagger)\} \cup \bigcup_{y_i, y_j \in V}\{(y_i, y_j)\} \cup \bigcup_{S_i \in V}\{(S_i, v^\dagger), (S_i, w_1)\} \cup \bigcup_{u_{i,j} \in V}\{(u_{i,j}, w_2)\} \cup \bigcup_{u_j \in S_i} \bigcup_{u_{j,k} \in V}\{(S_i, u_{j,k})\}$,
- $v^\dagger \in V$ is the evader,
- $c = c_{bt}$ is the betweenness centrality measure;
- $\delta = 1$ is the safety margin,
- $\hat{A} = \{t\} \times S$,
- $\hat{R} = \emptyset$.

Let $(A^*, R^*)$ be a solution to the $f(U, S)$ instance of the Minimum Local Hiding problem. Function $g$ is then defined as:

$$g(A^*, R^*) = \{S_i \in S : (t, S_i) \in A^*\}.$$

Notice that since $\hat{R} = \emptyset$ then necessarily $R^* = \emptyset$. Hence, in the remainder of the proof we will call $A^*$ the solution of $f(U, S)$.

From the definition of the problem, we see that the only edges that can be added to the network are those between $t$ and the nodes $S_i$. Notice that any such choice of $A^*$ corresponds to selecting a subset of $|A^*|$ elements of $S$ in the Minimum 3-Set Cover problem. In what follows, we will show what are necessary condition for $A^*$ to be solution to the $f(U,S)$ problem instance.

First, we show that for every node $v \in V \setminus \{t, v^\dagger\}$ and every $A^* \subseteq \hat{A}$ we have $c(G', v) < c(G', t)$, where $G' = (V, E \cup A^*)$. To this end, let $B(v)$ denote the sum of percentages of shortest paths controlled by $v$ between pairs of other nodes in $G'$, i.e.:

$$B(v) = \sum_{w,w' \in V \setminus \{v\}} \frac{|\{p \in \Pi(w,w') : v \in p\}|}{|\Pi(w,w')|}.$$

Notice that $B(v) = \frac{(n-1)(n-2)}{2} c(G', v)$. Next, we will show that the following holds:

$$\forall_{v \in V \setminus \{t, v^\dagger\}} \forall_{A^* \subseteq \hat{A}} B(v) < B(t).$$

Since $t$ controls all shortest paths between the nodes $x_i$ and nodes $v^\dagger, w_1, w_2, y_j, S_j, u_{j,k}$, we have:

$$\begin{aligned} B(t) \geq &(|U||S|^3 + |S|^2)(|U||S|^3 + |U||S|^2 + |S| + 3) \\ &= |U|^2|S|^6 + |U|^2|S|^5 + |U||S|^5 + 2|U||S|^4 + 3|U||S|^3 + |S|^3 + 3|S|^2 \end{aligned}$$

For nodes other than $t$ and $v^\dagger$ we have:

- $B(x_i) = B(y_i) = 0 < B(t)$, since the nodes $x_i$ and $y_i$ do not control any shortest paths;
- $B(w_1) \leq 3|U||S|^3 + \frac{3|S|^2+|S|}{2} + 2 < B(t)$, because $w_1$ controls some shortest paths between $w_2$ and nodes $t, v^\dagger, x_i, y_i, S_i$ (there are at most $2|U||S|^3 + |S|^2 + |S| + 2$ such pairs), some shortest paths between pairs of nodes $S_i, S_j$ (there are at most $\frac{|S|(|S|-1)}{2}$ such pairs), and some shortest paths between nodes $u_{i,j}$ and nodes $S_k$ (there are at most $|U||S|^3$ such pairs);
- $B(w_2) \leq \frac{|U|2|S|^4+|U||S|^2}{2} + |U||S|^3 < B(t)$, because $w_2$ controls some shortest paths between pairs of nodes $u_{i,j}, u_{k,l}$ (there are at most $\frac{|U||S|^2(|U||S|^2-1)}{2}$ such pairs), some shortest paths between nodes $u_{i,j}$ and $w_1$ (there are at most $|U||S|^2$ such pairs), and some shortest paths between nodes $u_{i,j}$ and nodes $S_k$ (there are at most $|U||S|^3$ such pairs);
- $B(u_i) \leq 2|U||S|^3 + \frac{3|S|^2+|S|}{2} + 2 < B(t)$, because $u_i$ controls some shortest paths between $w_2$ and nodes $t, v^\dagger, x_j, y_j, S_j$ (there are $2|U||S|^3 + |S|^2 + |S| + 2$ such pairs), and some shortest paths between pairs of nodes $S_j, S_k$ (there are at most $\frac{|S|(|S|-1)}{2}$ such pairs);
- $B(S_i) \leq 6|U||S|^5 + 3|U||S|^4 + 4|U||S|^3 + 3|S|^4 + 3|S|^3 + 11|S|^2 + 4 < B(t)$, because $S_i$ controls some shortest paths between the nodes $u_{j,k}$ that are connected to $S_i$ and the nodes $t, v^\dagger, w_1, x_l, y_l, S_l, u_{l,o}$ (there are at most $3|S|^2(2|U||S|^3 + |S|^2 + |U||S|^2 + |S| + 3)$ such pairs), and some of the shortest paths between nodes $w_1, w_2$ and nodes $t, v^\dagger, x_j, y_j$ (there are at most $2(2|U||S|^3 + |S|^2 + 2)$ such pairs), and because we assumed $|S| \geq 3$ and $|U| \geq 3$.

Therefore, either $t$ or $v^\dagger$ has the greatest betweenness centrality. Hence, $A^* \subseteq \hat{A}$ is a solution to the problem of Minimum Local Hiding if and only if $B(t) > B(v^\dagger)$. We now compute the values of $B(t)$ and $B(v^\dagger)$. We have that:

$$\begin{aligned} B(t) = &(|U||S|^3 + |S|^2)(|U||S|^3 + |U||S|^2 + |S| + 3) + \sum_{\{S_i,S_j\} \subseteq N(t)} \frac{1}{|N(S_i,S_j)|} \\ &+ \sum_{S_i \in N(t)} \sum_{u_{j,k} \in U \setminus N(S_i)} \frac{|N(t,u_{j,k})|}{|N(t,u_{j,k})| + |N(v^\dagger,u_{j,k})| + 1} \end{aligned}$$

as $t$ controls all shortest paths between all nodes $x_i$ and all other nodes in the network (there are $(|U||S|^3 + |S|^2)(|U||S|^3 + |U||S|^2 + |S| + 3)$ such pairs), one shortest path between each pair of nodes $S_i, S_j$ connected to $t$, and some shortest paths between nodes $S_i$ connected to $t$ and nodes $u_{j,k}$ such that $N(t) \cap N(u_{j,k}) \neq \emptyset$ (other paths run through $v^\dagger$ and nodes $S_l$, or through $w_1$ and $w_2$). On the other hand, we have that:

$$\begin{aligned} B(v^\dagger) =& |U||S|^3(|U||S|^3 + |S|^2 + |U||S|^2 + |S| + 3) + \sum_{S_i,S_j} \frac{1}{|N(S_i,S_j)|} \\ &+ \sum_{S_i} \sum_{u_{j,k} \in U \setminus N(S_i)} \frac{|N(v^\dagger, u_{j,k})|}{|N(t,u_{j,k})| + |N(v^\dagger,u_{j,k})| + 1} \\ &+ \sum_{S_i \notin N(t)} (|U||S|^3 + |S|^2 + 1) + \sum_{u_{i,j}: N(t,u_{i,j}) = \emptyset} (|U||S|^3 + |S|^2 + 1) \end{aligned}$$

as $v^\dagger$ controls all shortest paths between nodes $y_i$ and all other nodes in the network (there are $|U||S|^3(|U||S|^3 + |S|^2 + |U||S|^2 + |S| + 3)$ such pairs), one shortest path between each pair of nodes in $S_i, S_j$, some shortest paths between nodes $S_i$ and nodes in $u_{j,k}$ not connected with $S_i$, all shortest paths between nodes $t, x_i$ and nodes $S_j$ not connected to $t$, and all shortest paths between nodes $t, x_i$ and nodes $u_{j,k}$ such that none of their neighbor is connected to $t$.

Thus, we have:

$$\begin{aligned} B(v^\dagger) - B(t) = & \sum_{S_i \notin N(t)} (|U||S|^3 + |S|^2 + 1) + \sum_{u_{i,j}: N(t,u_{i,j}) = \emptyset} (|U||S|^3 + |S|^2 + 1) \\ & - |S|^2(|U||S|^2 + |S| + 3) + \Delta SU \end{aligned}$$

where $0 < \Delta SU \leq |U||S|^3 + |S|^2$.

Next, we prove that:

1. If for every $u_i \in U$ there exists $S_j \in N(t)$ such that $u_i \in S_j$, then $B(v^\dagger) < B(t)$;
2. If there exists $u_i \in U$ such that for every $S_j \in N(t)$ we have $u_i \notin S_j$, then $B(v^\dagger) \geq B(t)$.

Regarding point 1, we have:

$$B(v^\dagger) - B(t) = \sum_{S_i \notin N(t)} (|U||S|^3 + |S|^2 + 1) - |S|^2(|U||S|^2 + |S| + 3) + \Delta SU.$$

Since $\Delta SU \leq |U||S|^3 + |S|^2$ and we can assume that $|A^*| > 0$, i.e., $t$ is connected with at least one node $S_i$, we have that:

$$B(v^\dagger) - B(t) \leq (|S| - 1)(|U||S|^3 + |S|^2 + 1) - |S|^2(|U||S|^2 + |S| + 3) + |U||S|^3 + |S|^2,$$

which after simplifying terms gives us:

$$B(v^\dagger) - B(t) \leq |S| - 1 - 3|S|^2 < 0.$$

Hence, if for every $u_i \in U$ there exists $S_j \in N(t)$ such that $u_i \in S_j$, then $B(v^\dagger) < B(t)$.

Regarding point 2, since there exists at least one $u_i \in U$ such that for every $S_j \in N(t)$ we have $u_i \notin S_j$, and $\Delta SU > 0$:

$$B(v^\dagger) - B(t) > |S|^2(|U||S|^3 + |S|^2 + 1) - |S|^2(|U||S|^2 + |S| + 3),$$

which after simplifying terms, and from assumption that $|S| \geq 3$, gives us:

$$B(v^\dagger) - B(t) > |U||S|^5 + |S|^4 - |U||S|^4 - |S|^3 - 2|S|^2 > 0.$$

Hence, if there exists $u_i \in U$ such that for every $S_j \in N(t)$ we have $u_i \notin S_j$, then $B(v^\dagger) > B(t)$.

From the points 1 and 2 we have that $A^* \subseteq \hat{A}$ is a solution to $f(U, S)$ if and only if for every $u_i \in U$ there exists $S_j \in N(t)$ such that $u_i \in S_j$, i.e., the sets corresponding to the nodes that $t$ is connected to via edges in $A^*$ cover entire universe. In particular, it implies that optimal solutions to both instances are of the same size.

Now, assume that there exists an approximation algorithm for the Minimum Local Hiding problem with ratio $(1 - \epsilon) \ln |\hat{A} \cup \hat{R}|$ for some $\epsilon > 0$. This algorithm can be used to solve the constructed instance $f(U, S)$, acquiring solution $A^*$. Since $A^*$ is a solution to $f(U, S)$, the sets corresponding to the nodes with which $t$ is connected via edges in $A^*$ cover the universe $U$. Therefore, $g(A^*)$ (which extracts exactly these sets) is a solution to the given instance of the Minimum 3-Set Cover problem. What is more, we have that $|A^*| = |g(A^*)|$ and the optimal solutions to the both instances are of the same size. Since we also have that $|S| = |\hat{A} \cup \hat{R}|$, we obtained an approximation algorithm that solves the Minimum 3-Set Cover problem to within $(1 - \epsilon) \ln |S|$ for $\epsilon > 0$. However, Dinur and Steurer [1] showed that the Minimum 3-Set Cover problem cannot be approximated to within $(1 - \epsilon) \ln |S|$ for any $\epsilon > 0$, unless $P = NP$. Therefore, such an approximation algorithm for the Minimum Local Hiding problem cannot exist, unless $P = NP$. This concludes the proof. $\square$

## S3 The Proof of Equivalency between MIQP and MILP Formulations

For the convenience of the reader, we repeat the definitions of the MIQP and MILP formulations from the main article.

**Definition 3** (MIQP formulation)**.** *The mixed-integer quadratic program finding the optimal strategies of the seeker and the evader is:*

$$\begin{aligned}
&\max_{p,q,a} \sum_{\phi\in\Phi}\sum_{c\in C_S}\sum_{\xi\in\Xi_E} t(\phi)p(c)q(\phi,\xi)U_s(\phi,c,\xi) \\
\text{subject to } & \sum_{c\in C_S} p(c) = 1 && \text{(i)}\\
\forall_{\phi\in\Phi} & \sum_{\xi\in\Xi_E} q(\phi,\xi) = 1 && \text{(ii)}\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & a(\phi) \geq \sum_{c\in C_S} p(c)U_e(\phi,c,\xi) && \text{(iii)}\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & a(\phi) \leq (1-q(\phi,\xi))\eta + \sum_{c\in C_S} p(c)U_e(\phi,c,\xi) && \text{(iv)}\\
\forall_{c\in C_S}\ & p(c)\in[0,1]\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & q(\phi,\xi)\in\{0,1\}\\
\forall_{\phi\in\Phi}\ & a(\phi)\in\mathbb{R}
\end{aligned}$$

**Definition 4** (MILP formulation)**.** *The mixed-integer linear program finding the optimal strategies of the seeker and the evader is:*

$$\begin{aligned}
&\max_{z,q,a} \sum_{\phi\in\Phi}\sum_{c\in C_S}\sum_{\xi\in\Xi_E} t(\phi)z(\phi,c,\xi)U_s(\phi,c,\xi) \\
\text{subject to } \forall_{\phi\in\Phi}\ & \sum_{c\in C_S}\sum_{\xi\in\Xi_E} z(\phi,c,\xi) = 1 && (1)\\
\forall_{\phi\in\Phi}\ & \sum_{\xi\in\Xi_E} q(\phi,\xi) = 1 && (2)\\
\forall_{\phi\in\Phi}\forall_{c\in C_S}\ & \sum_{\xi\in\Xi_E} z(\phi,c,\xi) = \sum_{\xi\in\Xi_E} z(0,c,\xi) && (3)\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & \sum_{c\in C_S} z(\phi,c,\xi) = q(\phi,\xi) && (4)\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & a(\phi) \geq \sum_{c\in C_S} U_e(\phi,c,\xi)\sum_{\xi'\in\Xi_E} z(\phi,c,\xi') && (5)\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & a(\phi) \leq (1-q(\phi,\xi))\eta + \sum_{c\in C_S} U_e(\phi,c,\xi)\sum_{\xi'\in\Xi_E} z(\phi,c,\xi') && (6)\\
\forall_{\phi\in\Phi}\forall_{c\in C_S}\forall_{\xi\in\Xi_E}\ & z(\phi,c,\xi)\in[0,1]\\
\forall_{\phi\in\Phi}\forall_{\xi\in\Xi_E}\ & q(\phi,\xi)\in\{0,1\}\\
\forall_{\phi\in\Phi}\ & a(\phi)\in\mathbb{R}
\end{aligned}$$

**Theorem 12.** *Program formulations in Definitions 3 and 4 are equivalent.*

*Proof.* Consider $p(c), q(\phi,\xi), a(\phi)$ that is a feasible solution to the formulation in Definition 3. We will show that $z(\phi,c,\xi) = p(c)q(\phi,\xi), q(\phi,\xi), a(\phi)$ is a feasible solution to the formulation in Definition 4 with the same objective function value.

- As for the objective function value we have:

$$\sum_{\phi\in\Phi}\sum_{c\in C_S}\sum_{\xi\in\Xi_E} t(\phi)z(\phi,c,\xi)U_s(\phi,c,\xi) = \sum_{\phi\in\Phi}\sum_{c\in C_S}\sum_{\xi\in\Xi_E} t(\phi)p(c)q(\phi,\xi)U_s(\phi,c,\xi)$$

straight from the definition of $z(\phi,c,\xi)$.

- As for the constraint (1) we have:

$$\sum_{c\in C_S}\sum_{\xi\in\Xi_E} z(\phi,c,\xi) = \sum_{c\in C_S}\sum_{\xi\in\Xi_E} p(c)q(\phi,\xi) = \sum_{c\in C_S} p(c)\sum_{\xi\in\Xi_E} q(\phi,\xi) = \sum_{c\in C_S} p(c) = 1$$

where the first equality comes from the definition of $z(\phi,c,\xi)$, the second from simply putting $p(c)$ outside the sum sign, the third from the constraint (ii), and the fourth from the constraint (i).

- As for the constraint (2) we have:

$$\sum_{\xi\in\Xi_E} q(\phi,\xi) = 1$$

straight from the constraint (ii).

- As for the constraint (3) we have:

$$\sum_{\xi\in\Xi_E} z(\phi,c,\xi) = p(c)\sum_{\xi\in\Xi_E} q(\phi,\xi) = p(c) = p(c)\sum_{\xi\in\Xi_E} q(0,\xi) = \sum_{\xi\in\Xi_E} z(0,c,\xi)$$

where the first and the fourth equalities come from the definition of $z(\phi,c,\xi)$, while the second and the third from the constraint (ii).

- As for the constraint (4) we have:

$$\sum_{c\in C_S} z(\phi,c,\xi) = \sum_{c\in C_S} p(c)q(\phi,\xi) = q(\phi,\xi)\sum_{c\in C_S} p(c) = q(\phi,\xi)$$

where the first equality comes from the definition of $z(\phi,c,\xi)$, the second from simply putting $q(\phi,\xi)$ outside the sum sign, and the third from the constraint (i).

- As for the constraint (5) we have:

$$\begin{aligned}\sum_{c\in C_S} U_e(\phi,c,\xi)\sum_{\xi'\in\Xi_E} z(\phi,c,\xi') &= \\ \sum_{c\in C_S} U_e(\phi,c,\xi)p(c)\sum_{\xi'\in\Xi_E} q(\phi,\xi') &= \\ \sum_{c\in C_S} U_e(\phi,c,\xi)p(c) &\leq a(\phi)\end{aligned}$$

where the first equality comes from the definition of $z(\phi,c,\xi)$, the second from the constraint (ii), and the final inequality comes from the constraint (iii).

- As for the constraint (6) we have:

$$\begin{aligned}(1-q(\phi,\xi))\eta + \sum_{c\in C_S} U_e(\phi,c,\xi)\sum_{\xi'\in\Xi_E} z(\phi,c,\xi') &= \\ (1-q(\phi,\xi))\eta + \sum_{c\in C_S} U_e(\phi,c,\xi)p(c)\sum_{\xi'\in\Xi_E} q(\phi,\xi') &= \\ (1-q(\phi,\xi))\eta + \sum_{c\in C_S} U_e(\phi,c,\xi)p(c) &\geq a(\phi)\end{aligned}$$

where the first equality comes from the definition of $z(\phi, c, \xi)$, the second from the constraint (ii), and the final inequality comes from the constraint (iv).

Now, consider $p(c), z(\phi, c, \xi), a(\phi)$ that is a feasible solution to the formulation in Definition 4. We will show that $p(c) = \sum_{\xi \in \Xi_E} z(0, c, \xi), q(\phi, \xi), a(\phi)$ is a feasible solution to the formulation in Definition 3 with the same objective function value.

- As for the objective function value we have:

$$\sum_{\phi \in \Phi} \sum_{c \in C_S} \sum_{\xi \in \Xi_E} t(\phi) p(c) q(\phi, \xi) U_s(\phi, c, \xi) =$$
$$\sum_{\phi \in \Phi} \sum_{c \in C_S} \sum_{\xi \in \Xi_E} t(\phi) q(\phi, \xi) U_s(\phi, c, \xi) \sum_{\xi' \in \Xi_E} z(0, c, \xi') =$$
$$\sum_{\phi \in \Phi} \sum_{c \in C_S} \sum_{\xi \in \Xi_E} t(\phi) q(\phi, \xi) U_s(\phi, c, \xi) \sum_{\xi' \in \Xi_E} z(\phi, c, \xi') =$$
$$\sum_{\phi \in \Phi} \sum_{c \in C_S} t(\phi) U_s(\phi, c, \xi^*_\phi) \sum_{\xi' \in \Xi_E} z(\phi, c, \xi') = \sum_{\phi \in \Phi} \sum_{c \in C_S} \sum_{\xi' \in \Xi_E} t(\phi) z(\phi, c, \xi') U_s(\phi, c, \xi')$$

  where the first equality comes from the definition of $p(c)$, while the second comes from the constraint (3). As for the third equality, notice that from the fact that $q(\phi, \xi) \in \{0, 1\}$ and from the constraint (2) we get that for any given $\phi \in \Phi$ we have only one strategy $\xi^*_\phi \in \Xi_E$ for which $q(\phi, \xi^*_\phi) = 1$, while for any $\xi \neq \xi^*_\phi$ we have $q(\phi, \xi) = 0$. As for the fourth equality, notice that from the fact that $z(\phi, c, \xi) \geq 0$ and from the constraint (4) we get that for a given $\phi \in \Phi, c \in C_S$ and for any $\xi \neq \xi^*_\phi$ we have $z(\phi, c, \xi) = 0$. As a result, we get:

$$U_s(\phi, c, \xi^*_\phi) \sum_{\xi' \in \Xi_E} z(\phi, c, \xi') = U_s(\phi, c, \xi^*_\phi) z(\phi, c, \xi^*_\phi) = \sum_{\xi' \in \Xi_E} z(\phi, c, \xi') U_s(\phi, c, \xi').$$

- As for the constraint (i) we have:

$$\sum_{c \in C_S} p(c) = \sum_{c \in C_S} \sum_{\xi \in \Xi_E} z(0, c, \xi) = 1$$

  where the first equality comes from the definition of $p(c)$, and the second comes from the constraint (1).

- As for the constraint (ii) we have:

$$\sum_{\xi \in \Xi_E} q(\phi, \xi) = 1$$

  straight from the constraint (2).

- As for the constraint (iii) we have:

$$\sum_{c \in C_S} p(c) U_e(\phi, c, \xi) =$$
$$\sum_{c \in C_S} U_e(\phi, c, \xi) \sum_{\xi' \in \Xi_E} z(0, c, \xi') =$$
$$\sum_{c \in C_S} U_e(\phi, c, \xi) \sum_{\xi' \in \Xi_E} z(\phi, c, \xi') \leq a(\phi)$$

  where the first equality comes from the definition of $p(c)$, the second comes from the constraint (3), and the final inequality comes from the constraint (5).

- As for the constraint (iv) we have:

$$
\begin{aligned}
(1 - q(\phi, \xi))\eta + \sum_{c \in C_S} p(c) U_e(\phi, c, \xi) = \\
(1 - q(\phi, \xi))\eta + \sum_{c \in C_S} U_e(\phi, c, \xi) \sum_{\xi' \in \Xi_E} z(0, c, \xi') = \\
(1 - q(\phi, \xi))\eta + \sum_{c \in C_S} U_e(\phi, c, \xi) \sum_{\xi' \in \Xi_E} z(\phi, c, \xi') \geq a(\phi)
\end{aligned}
$$

  where the first equality comes from the definition of $p(c)$, the second comes from the constraint (3), and the final inequality comes from the constraint (6).

Hence, we have shown that the two formulations are equivalent. $\square$

# S4 Supplementary Figures

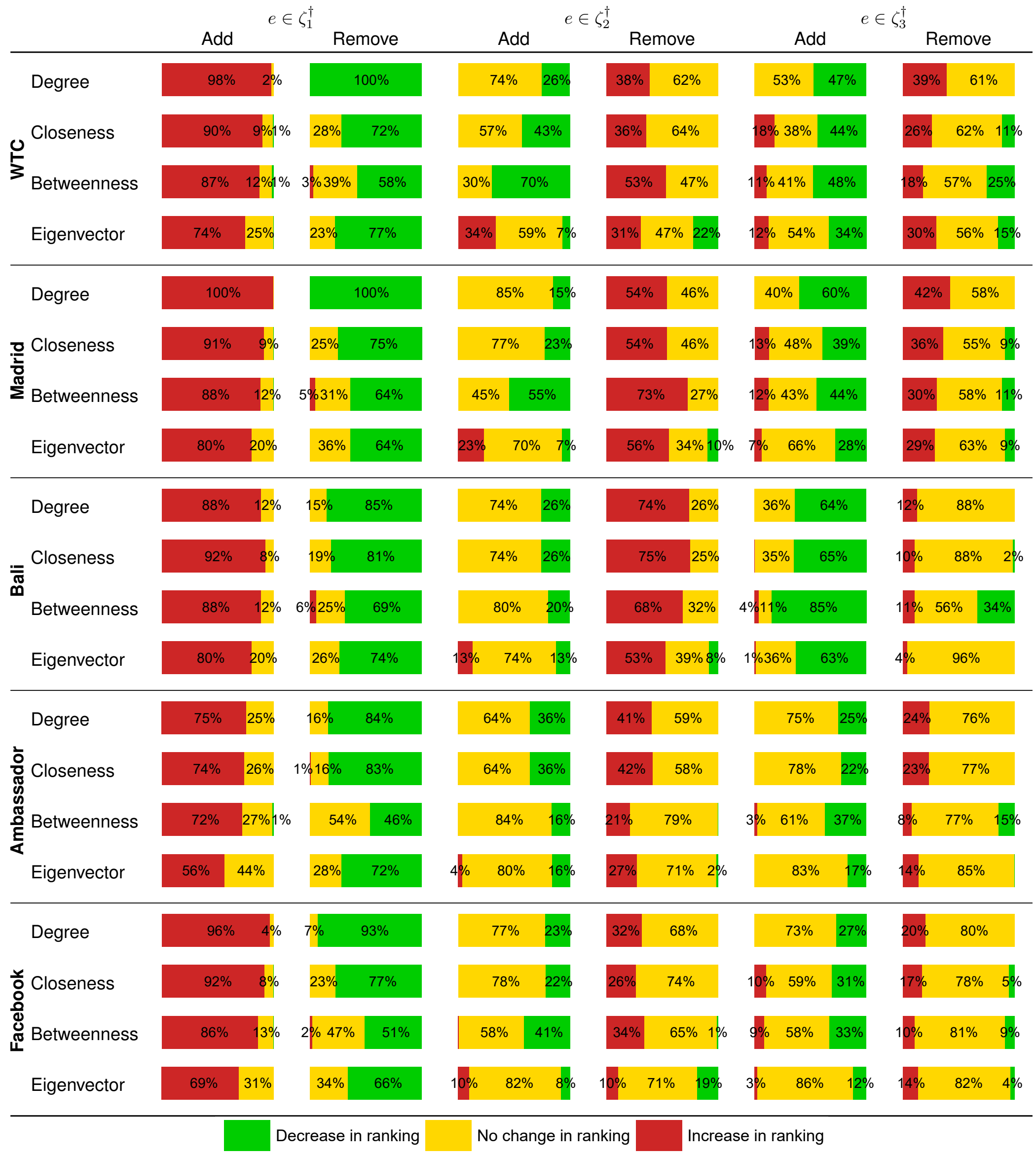


Figure S7: Percentage of the edge modifications that result in a given change in the evader's centrality ranking in real-life networks. Labels present values rounded to the nearest percent, values below 0.5% have been omitted for readability.

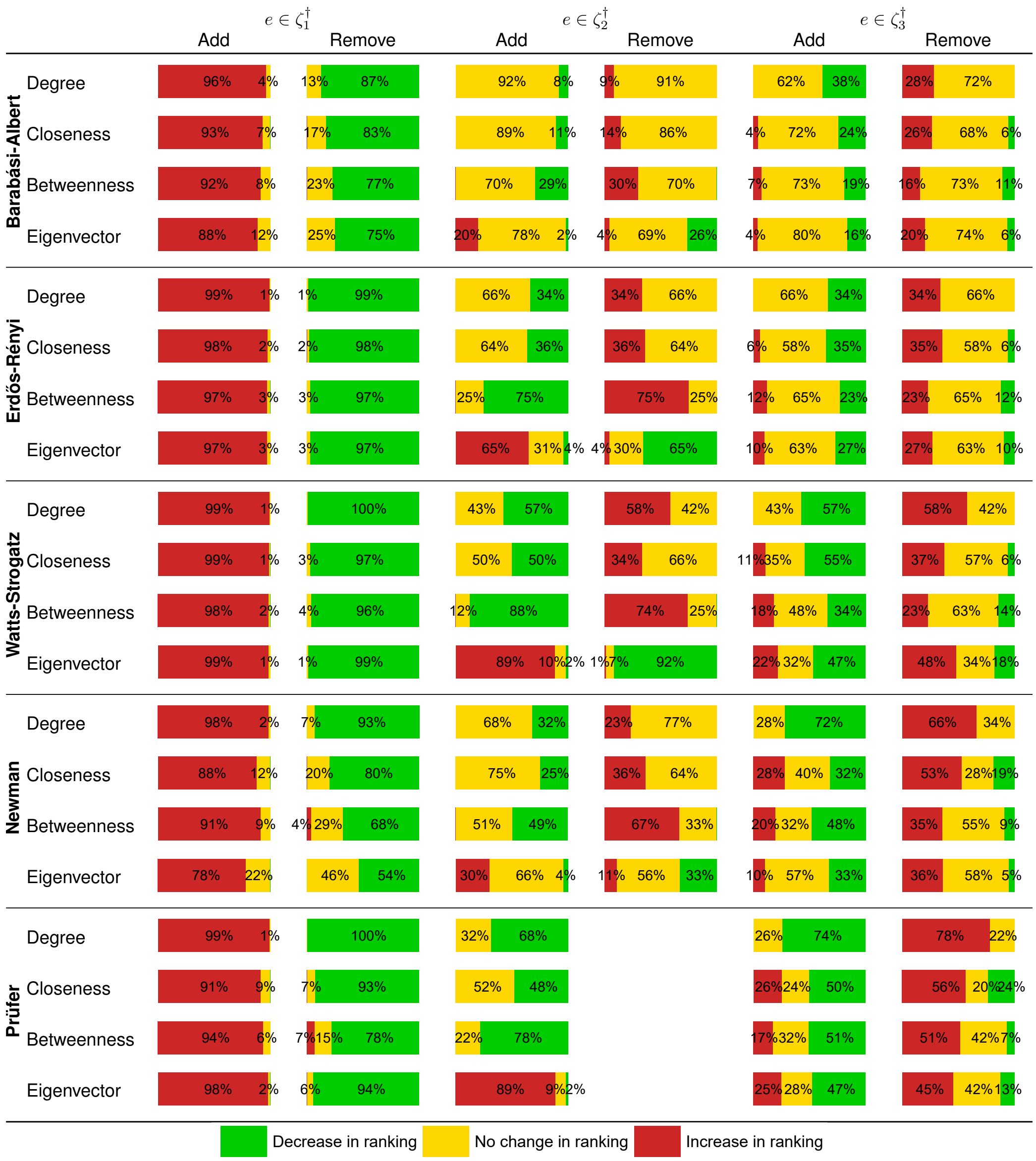

Figure S8: Percentage of the edge modifications that result in a given change in the evader's centrality ranking in random networks. Results are presented as an average over 1000 networks with 100 nodes and the average degree of 10 generated using each model. Labels present values rounded to the nearest percent, values below 0.5% have been omitted for readability. There are no edges between neighbors in Prüfer trees.

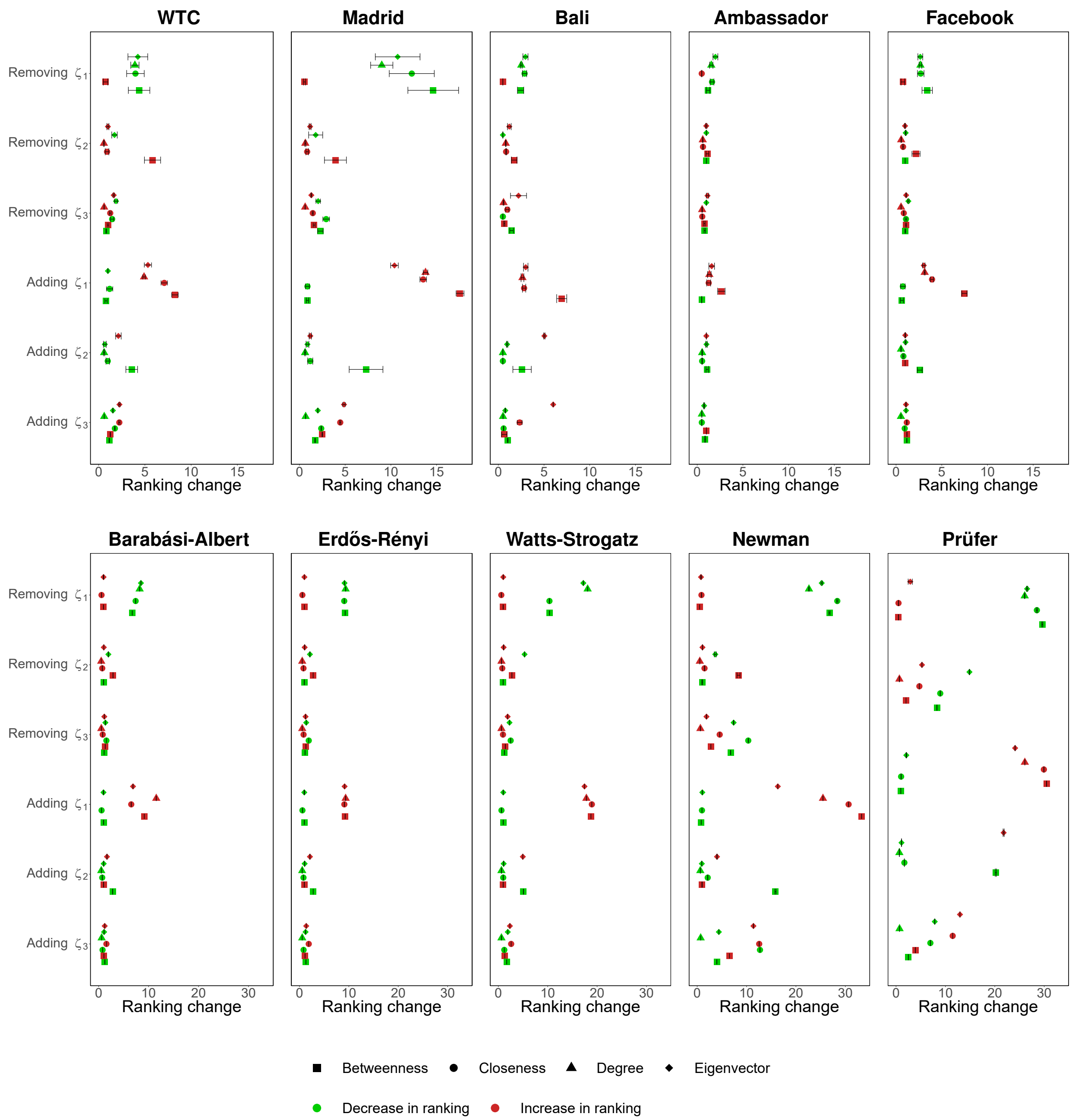


Figure S9: Magnitude of the change in the evader's centrality ranking. The first row presents results for real-life networks. Results in the second row are presented as an average over 1000 networks with 100 nodes and the average degree of 10 generated using each model. Error bars correspond to 95% confidence intervals. Scales in each row are fixed for easier comparison.

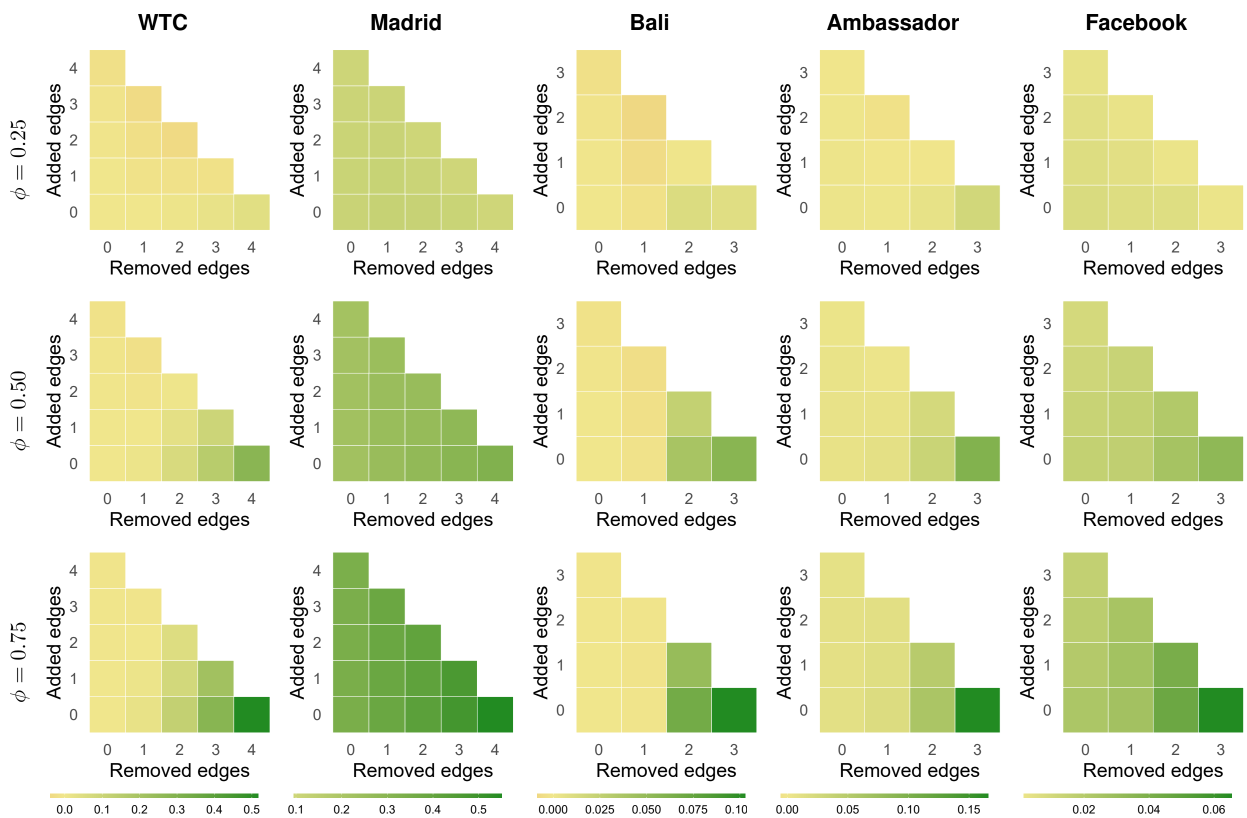


Figure S10: Utility of the evader in real-life networks, computed as an average over all possible strategies. Each column corresponds to a different network, while each row corresponds to a different type of the evader. In each heatmap the x-axis corresponds to the number of edges removed between the evader and their neighbors, while the y-axis corresponds to the number of edges added between the neighbors of the evader. The color of each cell corresponds to the utility of the evader, with color closer to green indicating greater utility.

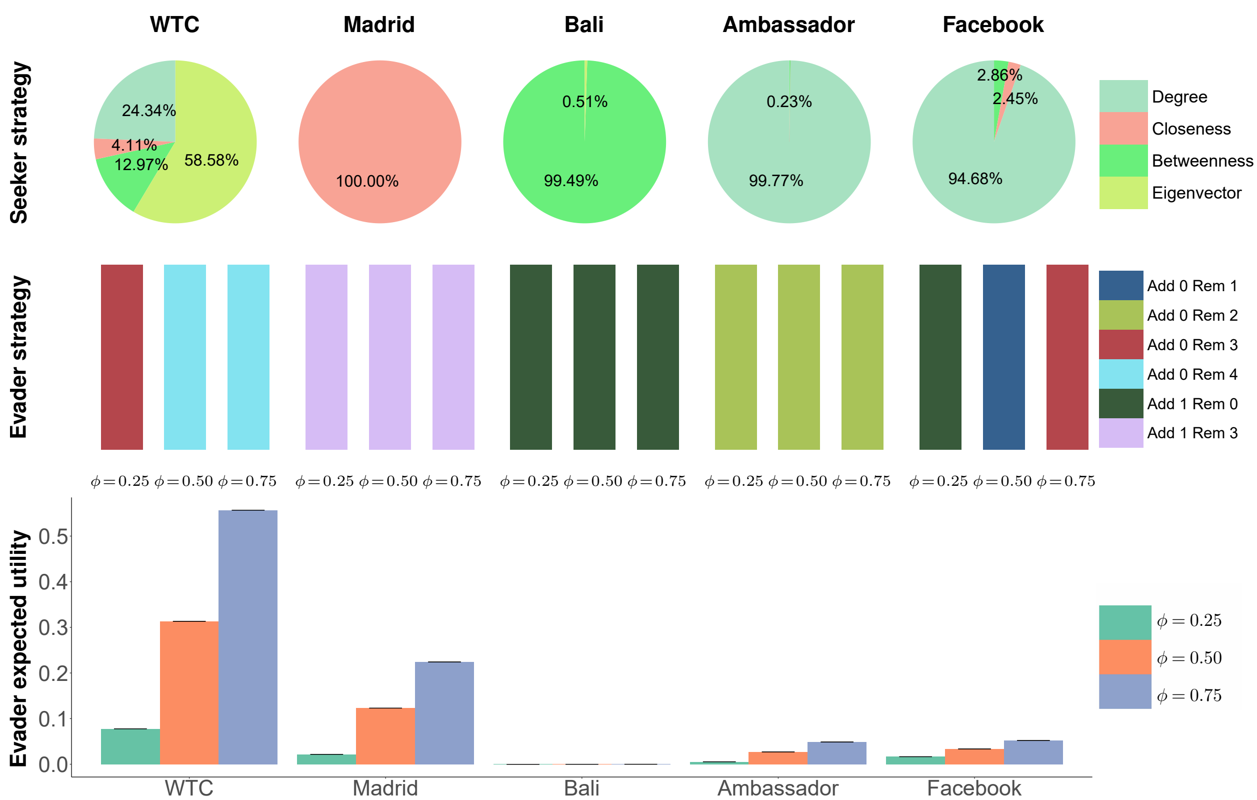


Figure S11: The seeker-evader game equilibria in real-life networks. Each column corresponds to a different network. Each pie chart in the first row presents the mixed strategy selected by the seeeker in an equilibrium of the seeker-evader game. Each group of bars in the second row presents the strategy the evader, with each bar corresponding to a different type of the evader. Each group of bars in the third row presents the expected utility of the evader, with each bar corresponding to a different type of the evader.

## References

[1] I. Dinur and D. Steurer. Analytical approach to parallel repetition. In *Proceedings of the forty-sixth annual ACM symposium on Theory of computing*, pages 624–633, New York, USA, 2014. ACM.