An Axiom System for Feedback Centralities

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There are many axiomatizations of the feedback centrality measures proposed in the literature.

Palacios-Huerta & Volij (2004) : Seeley index, Eigenvector*
Altman & Tennenholtz (2005) : Seeley index
Kitti (2016) : Eigenvector
Dequiedt & Zenou (2017) : Katz, Eigenvector
Wąs & Skibski (2018) : Eigenvector, Katz
Wąs & Skibski (2020) : PageRank

* Considered, but without a complete characterization

However, each considers **at most 2** feedback centralities, hence it is difficult to compare properties between measures.

Thus, in this work we create a joint axiom system for **all** four main feedback centralities.

Feedback centralities are usually defined by their recursive equations (but can be also defined using walks on a graph).

Katz

$$K_G(v) = a \cdot \sum_{(u,v) \in E} c(u,v) \cdot K_G(u) + b(v)$$

Eigenvector

$$E_G(v) = \frac{1}{\lambda} \sum_{(u,v) \in E} c(u,v) \cdot E_G(u)$$

PageRank

$$P_G(v) = a \cdot \sum_{(u,v) \in E} \frac{c(u,v)}{deg(u)} \cdot P_G(u) + b(v)$$

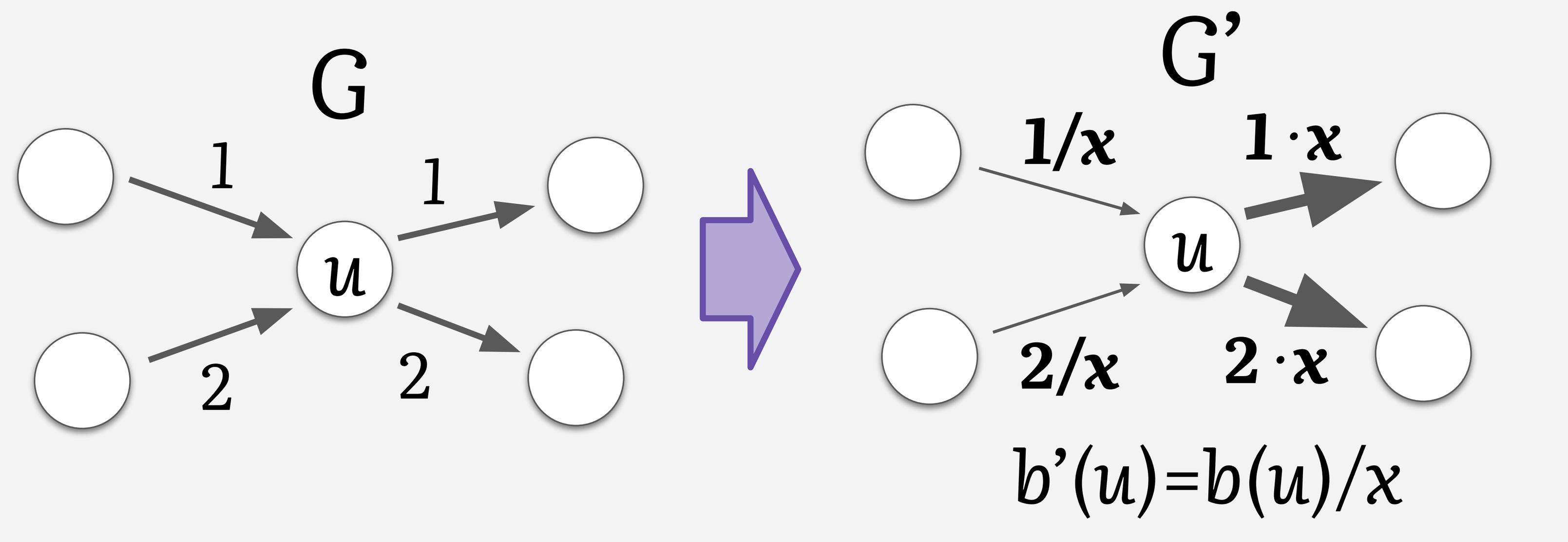
Seeley Index

$$S_G(v) = \sum_{(u,v) \in E} \frac{c(u,v)}{deg(u)} \cdot S_G(u)$$

K & EV

Axiom: Edge Compensation

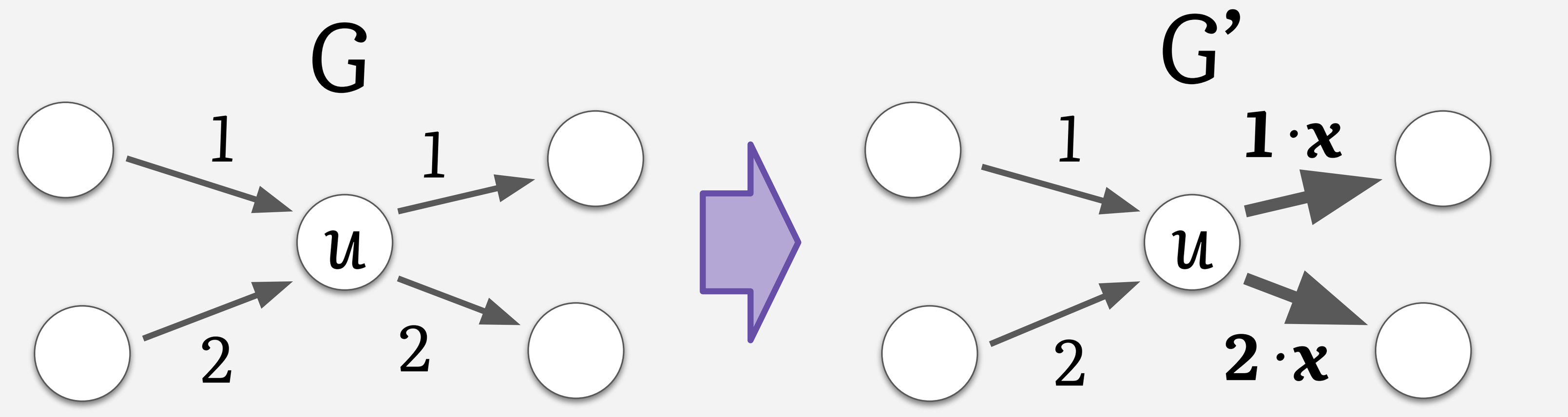
For every $x > 0$, graph G, and node u, if G' is graph G with the weights of outgoing edges of u multiplied by x and the weights of incoming edges of u and the weight of u divided by x, then $F_{G'}(v) = F_G(v)$ for every node $v \neq u$ and $F_{G'}(u) = F_G(u)/x$.



PR & SI

Axiom: Edge Multiplication

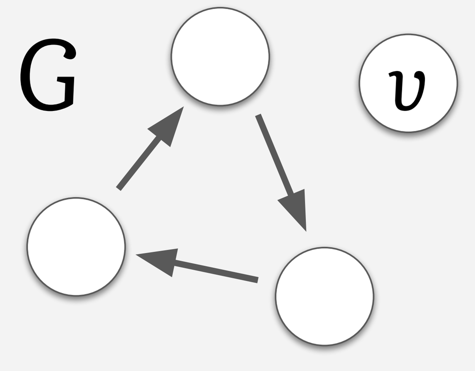
For every $x > 0$, graph G, and node u, if G' is graph G with weights of outgoing edges of u multiplied by x, then $F_{G'}(v) = F_G(v)$ for every node v.



Katz & PR

Axiom: Baseline

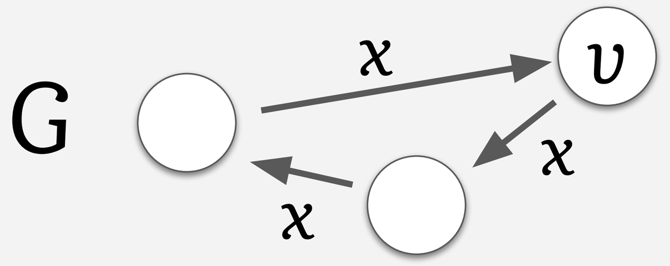
For every node v, if v is isolated, then $F_G(v) = b(v)$.



EV & SI

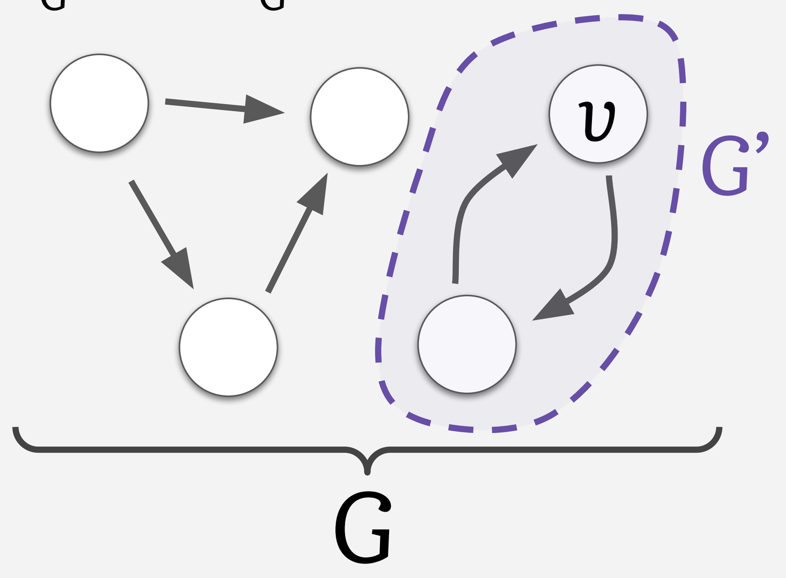
Axiom: Cycle

For every node v, if G is a cycle graph with equal edge weights, then $F_G(v) = \sum_{u \in V} b(u)/|V|$.



Axiom: Locality

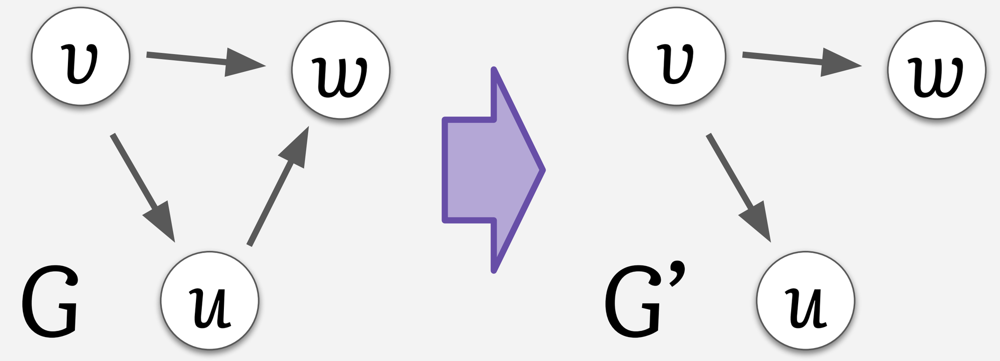
For every graph G and node v, if G' is a connected component of v, then $F_{G'}(v) = F_G(v)$.



All: K, EV, PR, & SI

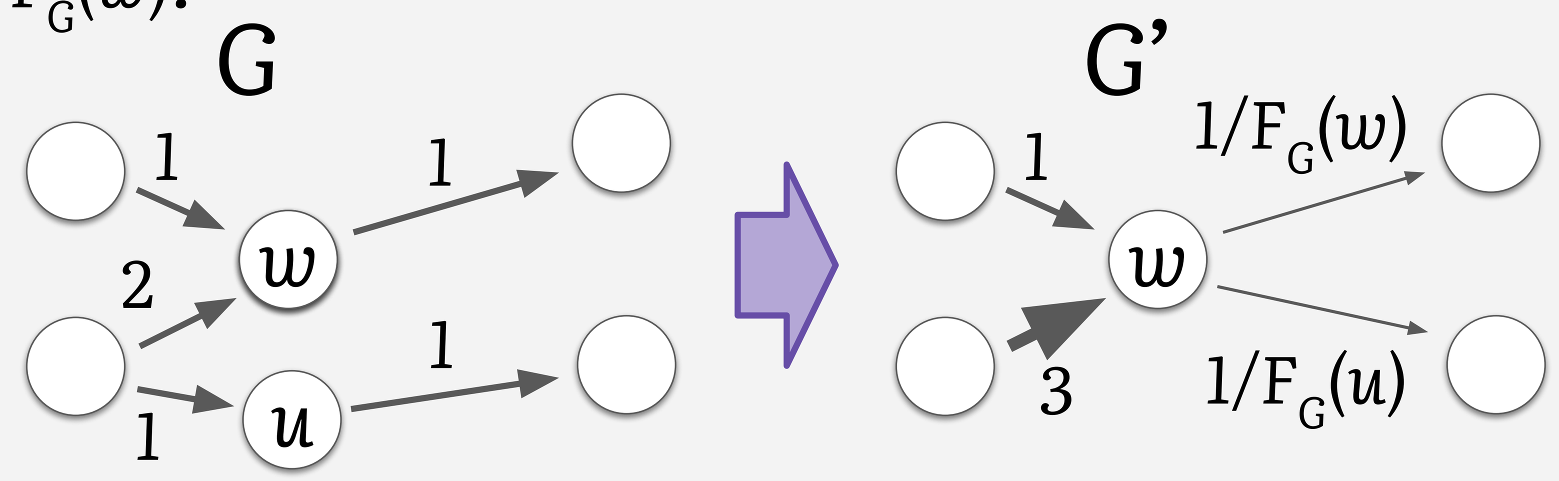
Axiom: Edge Deletion

For every graph G and edge (u,w), if G' is graph G with (u,w) removed, then $F_{G'}(v) = F_G(v)$ for every v that is not a successor of u in G.



Axiom: Node Combination

For every graph G and nodes u, w, such that $deg(u) = deg(w) = deg(s)$ for every successor s of u or w, if G' is graph G with proportionally combined u into w, then $F_{G'}(v) = F_G(v)$ for all $v \notin \{u,w\}$ and $F_{G'}(w) = F_G(u) + F_G(w)$.



Each of the four main feedback centralities is **uniquely characterized** by the subset of five of our axioms.