PageRank for Edges: Axiomatic Characterization

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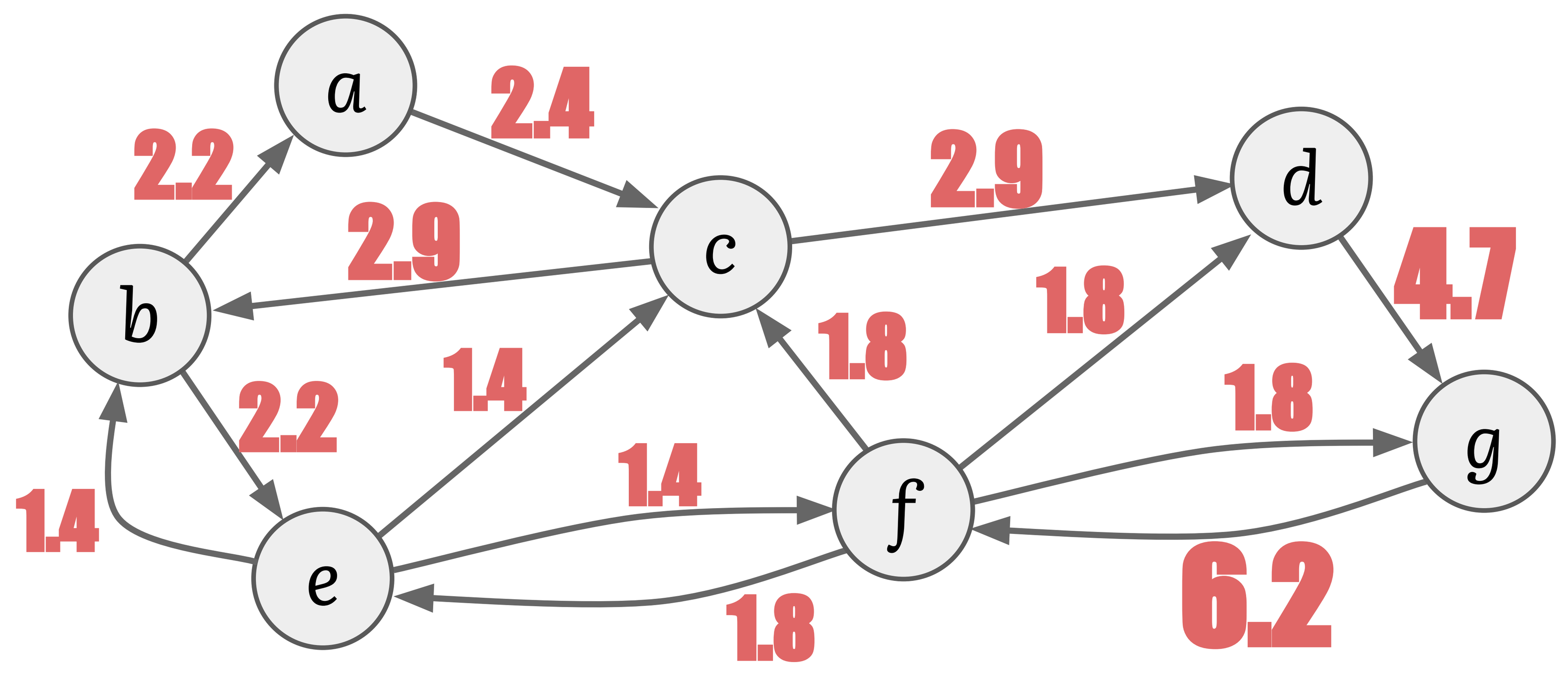
Edge centrality measures assign importance score to each edge in a graph and constitute a vital tool of network analysis.



While there exist axiomatizations of many node centrality measures, until now there were no axiomatizations of edge centrality measures.

We initiate the discussion on axiomatic analysis of edge centrality measures by creating an axiomatic characterization of Edge PageRank.

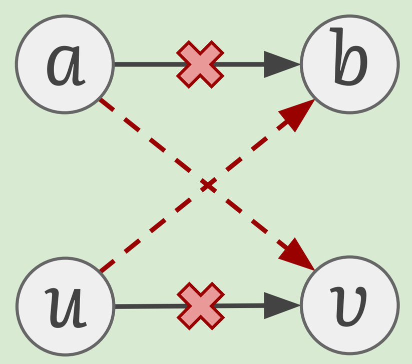
For $e=(u,v)$ and $a \in (0,1)$ Edge PageRank of e is given by:

$$PR_e(G) = \frac{1}{deg^+_x(G)}\left(\sum_{e' \in E^-_x} a \cdot PR_{e'}(G) + b(x)\right).$$

Six axioms that uniquely characterize Edge PageRank

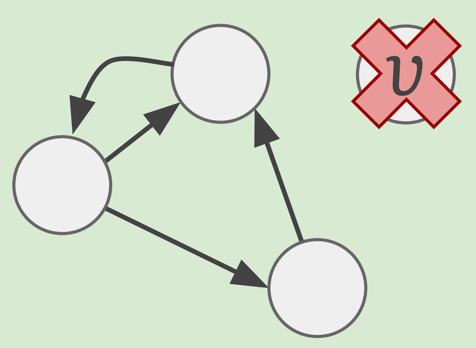
(based on axioms for node PageRank by Wąs & Skibski, 2020)

Edge Swap



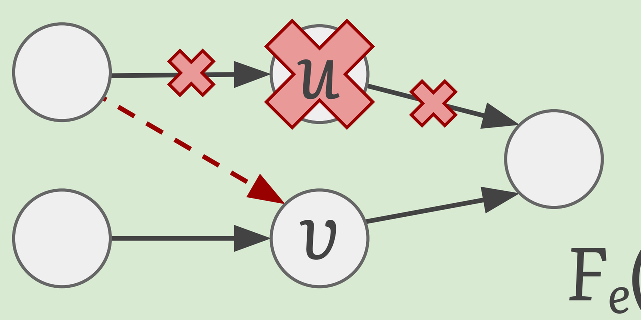
If $F_{(a,b)}(G)=F_{(u,v)}(G)$, then $F_e(G-(a,b)-(u,v)+(a,v)+(u,b)) = F_e(G)$, for every $e \in E$.

Node Deletion



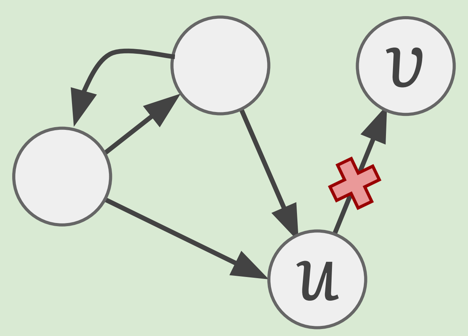
If node v is isolated, then $F_e(G-v) = F_e(G)$, for every $e \in E$.

Node Redirect



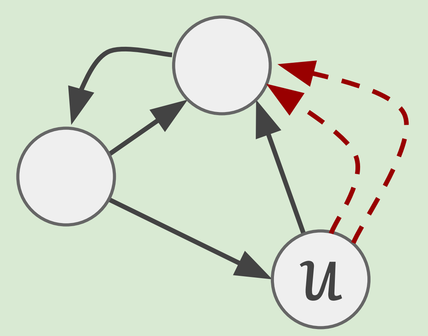
If there's a bijection $\psi : E^+_v \to E^+_u$ s.t. $end(\psi(e)) = end(e)$ for every $e \in E^+_v$, then $F_e(R_{a\to x}(G)) = F_e(G)$, for every $e \in E/(E^+_v \cup E^+_u)$, $F_e(R_{a\to x}(G)) = F_e(G) + F_{\psi(e)}(G)$, for every $e \in E^+_v$.

Edge Deletion



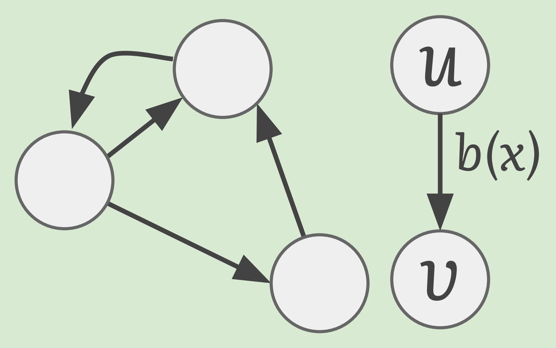
For every $(u,v) \in E$, $F_e(G-(u,v)) = F_e(G)$, for every $e \in E$ s.t. the start of e is not a successor of u.

Edge Multiplication



If $\{e\}=E^+_u$, then $F_e(G+(k-1)\cdot copy(e)) = F_e(G)/k$ and $F_{e'}(G+(k-1)\cdot copy(e)) = F_{e'}(G)$, for every $e' \in E/\{e\}$.

Baseline



If $e=(u,v)$ is an isolated edge, then $F_e(G) = b(x)$.

Axioms can show where a measure can be used. Example: Edge Multiplication makes sense in a WWW network, but not necessarily in an air traffic network.