Cost Sharing in a Volunteer's Dilemma

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Many interesting situations of public good provision such as a bystander's decision to help a victim, a committee member's decision to veto, or a company's decision to develop innovative products can be described by the volunteer's dilemma (VOD). The authors analyze a variant of the VOD in which the costs of producing a public good are shared equally among the volunteers rather than paid in full by each of the volunteers. The game theoretic solution predicts that the probability of volunteering is larger under the condition of sharing than when each volunteer pays the full cost. It is predicted that, even when cost sharing, the individual probability to volunteer decreases with group size, and larger groups still underproduce the public good. Predictions are tested using data collected via a mailed questionnaire to students of Berne University. The quantitative predictions of the game-theoretic models do not describe the data well, even when the models are extended with risk preferences. However, the less informative qualitative prediction that cost sharing increases the individual probability to volunteer is supported by the data.

A volunteer's dilemma (VOD) (Diekmann 1985) is a simple n-player game in which n players can privately supply a public good at private costs c_i strictly smaller than the benefits b_i that are obtained by each of the players. In a VOD it is assumed that *each* of the volunteers pays the full costs c_i, even though only one volunteer would suffice to produce the public good. Consequently, in a VOD, the outcome is usually ex post inefficient for two reasons. First, if no player has volunteered, the public good is not produced, and the players miss the opportunity to gain utility from the public good. Second, if more than one player volunteers, the public good will be produced, but the total resources that are contributed to the production are too large. The resources invested by all but one player are simply wasted.

The assumption that each volunteer pays the full costs of producing the public good seems reasonable in some cases, such as summoning the police by phone to assist a victim of a crime. In other instances, however, it seems more reasonable to assume that costs are *shared* among the volunteers.

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- For example, if several bystanders of a crime intervene to protect the victim, the chances that a volunteer is hurt decrease with the number of volunteers. Hence, in such a situation volunteers could effectively share the cost of helping the victim.
- As another example, consider the maintenance of a community center that depends on voluntary contributions. The costs of producing the public good involve time mostly spent on *divisible* tasks such as cleaning and painting. Clearly, the more volunteers who show up the less each has to spend; that is, the lower the cost for each of them.
- Similarly, in a sanctioning dilemma, for instance, if the nonsmoking rule in a restaurant is violated by a smoker, other customers feel disturbed but may hesitate to complain because of the costs (e.g., due to embarrassment). However, less embarrassment will be felt if more nonsmokers complain.
- Another example is given in Diekmann (1994). Assume that a university committee of professors must decide whether to spend money on research projects or on student activities. The interests of the professors, who have the right to veto, is to spend the money on research. On the other hand, professors do not want to become unpopular in the eyes of their students. Thus, to veto spending money on student activities is costly. However, the individual popularity cost for casting a proresearch vote decreases with the number of other committee members who vote for research.

These examples illustrate that cost sharing is a frequent and central characteristic of VODs. The possibility of cost sharing raises two interesting questions. First, does cost sharing increase the micro-level probability that an individual will contribute in a VOD? Second, does cost sharing increase the macro-level probability that a public good will be provided, especially as a function of group size? In the second section, we introduce a game theoretic model of cost sharing in a VOD and analyze its effect on the micro and macro levels. Settings with and without cost sharing are compared in some detail. In addition, we analyze how behavior differs depending on how costs are shared; namely, via division of costs (each volunteer pays an equal share) or via a lottery (one randomly selected volunteer pays the full costs). In the third section, we describe an experimental setting to test some of the hypotheses derived from the theory. Finally, the findings are summarized and discussed in the fourth section.

SHARING COSTS IN THE VOD

DEFINITIONS AND MAIN RESULTS

The cost-sharing variant (VCS) of the VOD is a n-person game defined as follows.

VOD with Cost Sharing

Let $n > 1$ be the number of players. Each player chooses between volunteering to make a contribution and abstaining from making a contribution. Let $\mathbf{s} = (s_1, \ldots, s_n)$ be the (pure) strategy vector, and denote by $m = m(s)$ the number of volunteers in s, $0 \leq m \leq n$. The utility u_i of player i is given by

$$u_i(\mathbf{s}; b, c) = \begin{cases} b - \frac{c}{m} & \text{if } s_i = V_i, \\ b & \text{if } s_i = A_i \text{ and } m \geq 1, \\ 0 & \text{if } m = 0, \end{cases} \tag{1}$$

where $0 < c < b$.

The VCS as defined here assumes symmetry in two respects. First, players have identical interests in the public good. Second, the costs for producing the good are divided equally (or, in the case of lottery sharing, equally likely) among the volunteers. Both symmetry assumptions can be relaxed (Diekmann 1993; Weesie 1993) to reflect, for instance, that players who benefit more from the production of the public good also have to pay more.

Theorem 1 characterizes the noncooperative *solution* of the symmetric VCS; namely, the unique symmetric Nash equilibrium of the VCS. In the solution, the players use randomized strategies; that is, they volunteer with some probability α. According to game-theoretical rationality, the players choose α so that no player has an incentive to deviate, provided that all other players do not deviate. In addition, the theorem states some comparative statics results of the *micro-level* probability with which a player volunteers and of the *macro-* or *group-level* probability that the public good is produced in terms of the group size n, the costs c for producing the public good, and the benefits b that are attained by all players. Note that the comparative statics results with respect to group size assume that the costs c and individual benefits b are independent of group size (Vree 1997).

Theorem 1

The symmetric VOD with VCS has the following properties.

i. There exists a unique symmetric equilibrium in which a player volunteers with probability α^*, where α^* is the unique root in the open interval (0, 1) of the polynomial g,

$$g(\alpha) = (1-\alpha)^{n-1}\,(bn\alpha + c(1-\alpha)) - c = 0. \tag{2}$$

ii. The *micro-level* probability α^* increases in the benefits b, decreases in the costs c, and decreases in the group size n. Moreover, α^* satisfies the group size expansion

$$\alpha^* = \frac{\omega(b/c)}{n} + O(n^{-2}), \tag{3}$$

where the Riemann function $\omega(x)$ is defined as the unique strictly positive solution of

$$e^{\omega(x)} - x\omega(x) = 1. \tag{4}$$

Thus, the expected number of volunteers in a large group is approximately $\omega(b/c)$, and $\omega(b/c)$ decreases from 1 to 0 as c/b increases from 0 to 1.

iii. The *macro-level* probability $\theta^* = 1 - (1-\alpha^*)^n$ that the public good is produced increases in the benefits b, decreases in the costs c, and decreases with group size n. Moreover,

$$\theta^* = 1 - e^{-\omega(b/c)} + O(n^{-1}), \tag{5}$$

and so in large groups $\theta^* \approx 1 - e^{-\omega(b/c)}$.

See the technical appendix.

We conclude that the micro-level probability α^* with which a player volunteers decreases to zero with an increasing number n of players, decreases with the costs c of volunteering, and increases with benefits b. Because $\alpha^* = \frac{\omega}{n} + O(n^{-2})$, the number of volunteers in large groups is approximately *Poisson distributed* with mean number of volunteers $\omega = \omega(b/c)$. From Theorem 1, we conclude that the macro-level probability θ^* that the public good is produced in the VCS decreases with group size to a limiting value that depends on the cost-benefit ratio c/b. Thus, the limiting probability of θ^* under cost sharing is also strictly smaller than one. Consequently, even if costs are shared and hence tend to be potentially negligible for each player in larger groups, the behavior of the group will be inefficient. Even in large groups, there is a nonnegligible positive probability that the public good will not be produced. This implies that all players would ex ante rationally prefer to commit themselves to implement a (fair) lottery among all players over a (fair) lottery that is restricted to those players who volunteered to participate in the lottery.

A COMPARISON WITH THE VOD

In this subsection, we compare the VCS with the standard VOD in which each volunteer pays the full costs. Thus, substantively, we address whether and to what extent cost sharing affects the micro-level probability α^* that a player volunteers and the macro-level probability θ^* that the public good is produced. Moreover, we consider how the size of the effect of cost sharing is affected by the other model parameters; namely, the cost-benefit ratio $\eta = c/b$, and group size n. In this subsection, we will add subscripts VCS and VOD to quantities such as α to distinguish between the games. It is useful to recall (Diekmann 1985) that in the standard VOD, we have

$$\alpha^*_{\text{VOD}} = 1 - \eta^{1/(n-1)} \tag{6}$$

$$\theta^*_{\text{VOD}} = 1 - \eta^{n/(n-1)}, \tag{7}$$

where $\eta = \frac{c}{b}$, $0 < \eta < 1$. Qualitatively, the comparative statics of α^*_{VOD} and α^*_{VCS}, and similarly of θ^*_{VOD} and θ^*_{VCS}, are quite similar. Whether costs are shared or not, the probability α^* that a player volunteers and the probability θ^* that the public good is produced *increase* in the benefits b of the public good, and *decrease* in the costs c for producing it. Moreover, group size has similar effects as well. In larger groups, players are less likely to volunteer, and the public good is less likely to be produced. Thus, cost sharing does *not* affect the conclusion that the macro-level effect of increasing group size on the micro-level probability that an individual player volunteers is larger than the partially offsetting composition effect that a larger group has, by definition, a larger number of potential volunteers. Sharing the costs lowers the costs of producing the good in the case that other players volunteered as well, and hence reduces the *expected* costs for a volunteer. Because, in the VOD, lowering the costs increases the probability that a player volunteers, we expect that lowering the costs via sharing has a similar effect. It is relatively straightforward to verify formally that the probability to volunteer α^*, and, consequently, also θ^*, is indeed *strictly larger* in the VCS than in the VOD. [1]

To assess the effect of cost sharing on the probability to volunteer, we have to specify how two predicted probabilities can be compared. We consider three ways (statistics) for doing so: difference, ratio, and odds ratio.

1. The *difference* between probabilities. It can be shown that the micro-level difference $\alpha^*_{\text{VCS}} - \alpha^*_{\text{VOD}}$ as a function of η is nonnegative and inverted-U shaped, and is 0 for $\eta = 0$ and $\eta = 1$. For $n = 2$, $\alpha^*_{\text{VCS}} - \alpha^*_{\text{VOD}}$ has a maximum of .17 that is attained at $\eta = .586$. Both the maximal difference and the η for which this maximal difference is attained *decrease* with increasing group size. In a large group (i.e., asymptotically), both α^*_{VCS} and α^*_{VOD}, and hence also the difference $\alpha^*_{\text{VCS}} - \alpha^*_{\text{VOD}}$, are arbitrarily small. Similarly, the macro-level difference $\theta^*_{\text{VCS}} - \theta^*_{\text{VOD}}$ as a function of η is nonnegative and inverted-U shaped for each group size n. Now, $\theta^*_{\text{VCS}} - \theta^*_{\text{VOD}}$ has a maximum that increases from .20 to .21 with n increasing from 2 to infinity, whereas the maximizer *decreases* from $\eta = .74$ if $n = 2$ to $\eta = .54$ in large groups.
2. Because the probabilities α^* become very small in large groups in both the VOD and the VCS, the difference in these probabilities is actually not very meaningful. It is more interesting to compare the *ratio* of the probabilities. It can be shown that, for each group size n, both the micro-level ratio $\frac{\alpha^*_{\text{VCS}}}{\alpha^*_{\text{VOD}}}$ and the macro-level ratio $\frac{\theta^*_{\text{VCS}}}{\theta^*_{\text{VOD}}}$ *increase* from 1 to 2 if η increases from 0 to 1. Moreover, the ratios $\frac{\alpha^*_{\text{VCS}}}{\alpha^*_{\text{VOD}}}$ and $\frac{\theta^*_{\text{VCS}}}{\theta^*_{\text{VOD}}}$ increase in group size n. Finally, we note that $\frac{\alpha^*_{\text{VCS}}}{\alpha^*_{\text{VOD}}} > \frac{\theta^*_{\text{VCS}}}{\theta^*_{\text{VOD}}}$; that is, the effects in terms of the ratio of cost sharing is larger at the micro level than at the macro level.
3. Because the probabilities that we analyze may be close to 0 or 1 if η is close to 0 or 1, it is also useful to analyze the effects of cost sharing via the odds ratios $\tilde{\alpha}$ of the micro-level probabilities α^*_{VCS} and α^*_{VOD},

$$\tilde{\alpha} = \frac{\alpha^*_{\text{VCS}}}{1 - \alpha^*_{\text{VCS}}} \frac{1 - \alpha^*_{\text{VOD}}}{\alpha^*_{\text{VOD}}},$$

and similarly for $\tilde{\theta}$. The following results can be demonstrated $\tilde{\alpha} = 2$ for all η if $n = 2$. For $n > 2$, $\tilde{\alpha}$ decreases in n and increases to 2 if η increases from 0 to 1. Similarly, $\tilde{\theta}$ *decreases* to 2 if η increases from 0 to 1. Generally, $\tilde{\theta}$ will *decrease* in group size n, but for η around .20 (specifics depending on group size), $\tilde{\theta}$ may actually increase in group size.

These qualitative comparative statics provided a useful way to come up with testable hypotheses, namely, as *interaction effects* between group size and the cost-benefit ratio on the size of the effect of cost sharing. Note, however, that the actual

[1]Theorem 1 characterizes α^*_{VCS} as the unit solution of a polynomial g that is inverted-U shaped on (0,1), while $g(0) = 0$ (see the technical appendix). Thus, to prove that $\alpha^*_{\text{VOD}} < \alpha^*_{\text{VCS}}$, one only has to show that $g(\alpha^*_{\text{VOD}}) > 0$, which is straightforward.

hypotheses depend on the statistical model used in testing the hypotheses. Thus, in a linear probability model, we have to consider (1), whereas a logistic regression (logit) model requires the odds-ratio hypothesis (3).

Finally, we point to an interesting distinction between the VOD and the VCS. In the VOD, the expected utility of a player is $b - c$ which also equals his maximin utility. Thus, equilibrium behavior is not profitable. Some authors have argued that, under these conditions, equilibrium behavior may not constitute rational behavior (e.g., Harsanyi 1977, 133-38, 273). According to Harsanyi's (1977) solution concept, players should surely volunteer in the VOD. In contrast, the VCS has a unique symmetric equilibrium that *is* profitable; that is, the equilibrium payoff is strictly larger than the maximin payoff. Thus, we may expect that players have stronger incentives to follow equilibrium behavior in the VCS than in the VOD—even if in both cases the equilibria are in mixed strategies. Thus, in VOD the players have no incentive to deviate but no strict incentive not to deviate.

SHARING VIA A LOTTERY VERSUS REAL COST SHARING: THE EFFECTS OF RISK PREFERENCES

The examples that set the stage for the definition of the VCS illustrate that costs for producing a public good may be shared in two distinct ways. First, the costs may be divided among the volunteers (division sharing): the load is divided equally among the volunteers. Alternatively, a lottery may be used to select one of the volunteers who carries the full burden of the production of the public good (i.e., the full costs c), whereas the other volunteers, just like the players who did not volunteer, pay nothing (see Elster 1993 for an insightful comparison of institutions for allocating costs; see Elster 1989, ch. 2, for an extensive discussion of the use of lotteries to allocate burdens). How does the sharing rule—that is, the way in which the costs are shared (division vs. lottery) among the volunteers—affect the behavior of the players? Taking the perspective of some focal player, let k be the number of volunteers among the other players. Clearly, the utility of the focal player does not depend on the institutional rule if he does not volunteer. If he does volunteer, the rule matters. If the costs are shared via a lottery, the focal player receives b with probability $\frac{k}{k+1}$ and $b - c$ with probability $\frac{1}{k+1}$. On the other hand, if the costs are divided among the volunteers, the focal player receives the outcome $b - \frac{c}{k+1}$. Note that

$$\frac{k}{k+1}b + \frac{1}{k+1}(b - c) = b - \frac{c}{k+1},$$

and so the *expected* outcome of the lottery equals the sure outcome under sharing by division. Under risk neutrality—that is, the utility $U(x)$ of outcome x is just x (or an increasing affine transformation of x)—the player's payoffs in the VCS clearly do not depend on the sharing rule. Hence, the behavior of players does not depend on the sharing rule under risk neutrality.

However, this does not hold true if the players have nontrivial risk attitudes. Consider a risk averse player. More specifically, let $U(x)$ be a concave increasing utility function of x; that is, we assume decreasing marginal utility. A player with a concave utility function is called risk averse because, by Jensen's inequality, he prefers a sure outcome over some lottery with the same expected outcome (Hirshleifer and Riley 1992). In the context of cost sharing, risk preferences have interesting implications. If costs are divided among volunteers, the utility of the focal player is $U(b - \frac{c}{k+1})$. If costs are shared via a lottery, the focal player's utility is uncertain, but his expected utility equals

$$\frac{k}{k+1}U(b) + \frac{1}{k+1}U(b-c) < U\left(\frac{k}{k+1}b + \frac{1}{k+1}(b-c)\right) = U\left(b - \frac{c}{k+1}\right),$$

where the inequality follows by the assumed concavity of U. Thus, under risk aversion (concavity), conditional on the behavior of the other players, the expected costs for volunteering are larger under lottery sharing than under division sharing. Thus, again under risk aversion, we may expect that the probability that a rational player volunteers under lottery sharing is *smaller* than under division sharing. In Theorem 2 of the appendix, we show that this somewhat informal analysis—we took the behavior of other players to be given—can indeed be supported by a strict game-theoretical analysis.

A similar argument can be made under the assumption that the players are risk seeking; that is, endowed with a convex utility function. Then a player prefers a lottery over the sure expected value of the lottery. In that case, we may expect that the probability that a rational player volunteers under lottery sharing is *larger* than under division sharing.

Raub and Snijders (1997) analyze the consequences of risk preferences in repeated prisoner's dilemmas. They obtain the surprising conclusion that the conditions for continued mutual cooperation, supported by conditional cooperation, are more restrictive with risk-seeking than with risk averse preferences. In Weesie (1998), the analogous result is obtained for the standard VOD, a nonrepeated game. It can be shown that the analogous result also holds for the VOD with cost sharing under division sharing as well as with lottery sharing.[2] We conclude that the probability of volunteering can be ranked over the four conditions obtained by crossing risk preference and sharing rule as follows:

	Division Sharing	Lottery Sharing
Risk seeking	4 = smallest	3
Risk averse	1 = largest	2

Kahneman and Tversky (1979) report that people are risk aversive in decision problems that involve gains, and risk seeking if the situations involve losses. Under the assumption that the outcome of nonproduction of the public good is the relevant reference point here, one would predict that division sharing creates more favorable incentives to produce the public good (gains) than lottery sharing, whereas lottery sharing would be more effective to public goods problems that constitute the prohi-

2. The proof can be obtained by demonstrating that, for each of the sharing rules, the defining polynomial for α shifts up (down) with risk-seeking (risk averse) preferences.

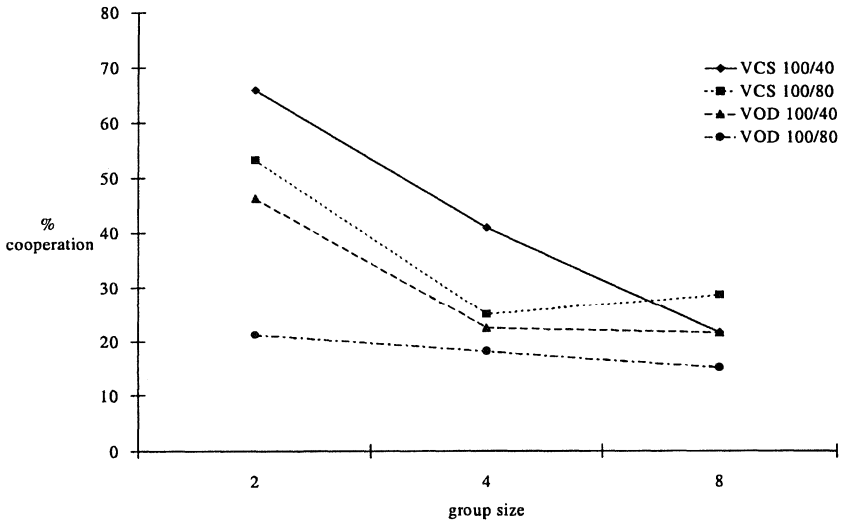

Figure 1: Experimental Results in the Volunteer's Dilemma with Cost Sharing (VCS) and without Cost Sharing (VOD)
NOTE: Benefits (b) were set to 100 points in all experimental conditions, whereas costs (c) varied between 40 (low cost) and 80 (high cost) points (N = 465).

bition of the production of a public bad. Moreover, we predict that the production of public goods (gains) is easier than the prohibition of public bads (losses).

Finally, we emphasize that these theoretical predictions about interaction effects between risk attitudes and the sharing arrangement on the production of public goods ignore some of the other features of real situations that are important to understand why sharing via division or lotteries is used in different situations. For instance, sharing the work among a couple of volunteers involves the costly coordination of the contributions of the volunteers. Such coordination costs depend on, for example, the task structure, such as the degree of interdependency of tasks. Also, incentive and monitoring problems among the volunteers may lead to additional costs that lead to suboptimal production of the public good.

EXPERIMENTAL TEST

DESIGN OF THE EXPERIMENT

To test the hypothesis that the individual probability to cooperate in the VOD with VCS is an increasing function of group size and the cost-benefit ratio, we conducted an experiment. Group size was varied between 2, 4, and 8 players. The benefits of the public good were set at 100 (b = 100), whereas the costs of cooperation were varied between a high-cost and a low-cost situation with $c = 80$ and $c = 40$, respectively (see Figure 1). To compare behavior in the VCS with behavior in the VOD without cost sharing, both variations were also conducted for the symmetric VOD. Thus, we have a full-factorial design with three factors; namely, group size (2, 4, and 8) by cost structure (low vs. high) by dilemma type (VOD vs. VCS). The number of experimental conditions is 12. In this experiment, we did not compare the effects of the sharing rule (division vs. lottery), nor did we compare risk preferences (e.g., producing a public goods vs. preventing a public bad).

A separate questionnaire describing the experimental condition and the task for the subjects was produced for each of the 12 conditions. Participants were told that a given number of coplayers (1, 3, or 7) received the same questionnaire and their payoff depended not only on their own choice but also on the choice of their coplayers. The situation was both verbally explained in the questionnaire and presented in matrix form. Cost sharing was described as cost division; that is, the total cost would be divided equally among the volunteers. Subjects were asked to make a choice between the two alternatives "cooperate" (volunteer) and "defect" (abstain), and to guess how many of their coplayers would choose either alternative. Thereafter, subjects were asked to calculate the number of points they would receive if their guess concerning the choices of coplayers was correct. Similarly, subjects had to calculate the points each of the coplayers would receive. This computation of payoffs had a twofold purpose. First, it was intended to remind subjects of the number of coplayers. Second, incorrect calculation of own or others' payoffs might be an indication that the task had not been completely understood. Finally, participants were asked to describe in their own words the reason for their decision. This open question was included to enforce thoughtful consideration about subjects' decisions.

An equal number of copies of the 12 questionnaires were randomly assigned to a random sample of 850 first- and third-year students of the University of Berne in Switzerland. The questionnaire was returned by 489 subjects, of whom 465 answered all control questions correctly. About half of the participants were female. About one third of the students were enrolled in economic or law programs, one third in history or language programs, and one third in medicine and natural science. Participants were told the questionnaires would be randomly matched within each condition, and that 100 points would be transformed into 10,- Sfr (about 9 U.S. dollars). However, to ease our work, and because a 10,- Sfr bill is easiest to send by mail, all participants received 10,- Sfr in return for their participation.

A RIGOROUS TEST

The data of the experiment are shown in Table 1. At first, we seek a very stringent test of the models in which the *detailed numerical predictions* from the game- theoretic models VCS and VOD are confronted with the data. Clearly, such a test requires assumptions about how the theoretical concepts of the models, namely the costs and benefits, relate to the empirical conditions. To begin, we make the standard assumption that

$$\text{utility} = \text{own money} : U(x) = x.$$

Thus, it is assumed that subjects are interested only in the money (points) that they receive themselves, and are indifferent to the monetary outcomes of the other subjects. We refer to Weesie (1998) for a related theoretical analysis and empirical tests of a VOD in which the assumption of selfishness is relaxed. Moreover, it is assumed that utility is linear in money, thereby ignoring for now the possible effects of risk attitudes on behavior.

Given this specification of utility, the computation of the predictions of the models VCS and VOD is straightforward. Recall that in the case of risk neutrality, division sharing and lottery sharing should be the same. A test for these models can be based on a deviance statistic (McCullagh and Nelder 1989)

$$\text{Deviance} = \sum_{i=1}^{12} f_i \log\left(\frac{p_i}{1-p_i}\frac{1-\pi_i}{\pi_i}\right) + n_i \log\left(\frac{1-p_i}{1-\pi_i}\right), \tag{8}$$

where n_i is the number of subjects in condition i, f_i is the observed number of volunteers, $p_i = f_i/n_i$ is the observed proportion of volunteers, and π_i is the predicted probability. The deviance is, of course, only a likelihood ratio statistic of a model that predicts probabilities π against the saturated model. If the π model is true, the deviance statistic is approximately (asymptotically) χ^2 distributed with 12 degrees of freedom. We stress that, in this case, the predictions are very exact and strictly theoretical; they do not involve the estimation of any parameter. For our VOD/VCS models, the deviance statistic equals 54.3, with $p = \Pr(\chi^2_{12} > 54.3) < .001$. We conclude that our game-theoretic models VOD and VCS, combined with the auxiliary utility = money assumption, do not describe the data well.

What is wrong? Logically, both the formal game-theoretical predictions of behavior and the auxiliary assumptions about utility (or, more generally, the assumptions about what games human subjects are actually playing in these experiments) could be the culprits. Let us start by addressing the first possibility. We argued above that, due to maximin properties of the equilibria of VOD in contrast to VCS, we might expect that the game-theoretic predictions would fit the data for the VCS better than for the VOD. Thus, we might expect large residuals for the VOD. Of course, conflicting arguments that distinguish VODs and VCSs can easily be developed. For instance, the VOD is probably a less complex game to understand than the VCS: for a volunteer, the payoffs are unconditionally $b - c$ in case of the VOD, whereas in the VCS, the payoffs depend on the behavior of the other players. In addition, to the extent that numerical complexity resembles cognitive complexity, the solution of the VOD can be obtained explicitly, whereas the solution of the VCS can be obtained only as the solution of a polynomial of degree n. Thus, one might expect that a human subject may fail to arrive at the correct game-theoretic solution of the VCS more often than in the case of the VOD. However, the data support neither of these additional

TABLE 1
Empirical results for the Volunteer's Dilemma with and without Cost Sharing (VOD vs. VCS) for three group sizes n ($n = 2$, 4, and 8), and for low versus high costs ($c = 40$ vs. $C = 80$).

	VOD (no cost sharing)		VCS (cost sharing)	
	low cost	high cost	low cost	high cost
	.46 (41)	.21 (33)	.66 (44)	.53 (45)
n = 2	.60	.20	.75	.33
	-1.75	.17	-1.27	2.69
	.24 (42)	.18 (34)	.41 (39)	.25 (40)
n = 4	.26	.07	.40	.13
	-.38	1.60	.16	1.74
	.24 (38)	.15 (33)	.22 (37)	.29 (42)
n = 8	.12	.03	.20	.06
	1.66	1.92	.22	3.25

NOTE: Listed are the observed proportions and, between parentheses, the number of observations (first row), predictions of the probabilities of volunteering based on the game-theoretic models and risk neutrality (second row), and the standardized residuals (third row).

and rather ad hoc hypotheses. Table 1 includes standardized residuals, based on the game-theoretic predictions, that are approximately normal distributed if the models are indeed true. The large residuals simply do not appear to be concentrated in the residuals for either the VOD or the VCS.

The second possible reason for the lack of fit could be an incorrect assumption about what games the subjects are playing. In particular, we want to address the possibility that the risk neutrality implied by the utility = money assumption is not satisfied in our data. A simple and suitable specification for utility that allows us to test the utility = money assumption is a power function,

$$U(x) = x^{\beta} \qquad x > 0, \beta > 0. \tag{9}$$

Note that the power function allows risk averse ($0 < \beta < 1$), risk neutral ($\beta = 1$), and risk-seeking ($\beta > 1$) preferences. Here, the risk preference parameter β is unknown and has to be estimated from the data. In our empirical test, we make the very restrictive assumption that all subjects have the same risk preference parameter β.

As argued above, the predictions about behavior in VODs, in the version both with and without cost sharing, depend on the risk preference parameter β. For the VOD, the predicted probability can be written explicitly in terms of the monetary outcomes b and c and the risk preference parameter β by careful substitution as

$$\begin{aligned}\alpha^*_{\mathrm{VOD}}(\beta) &= 1-\left(\frac{U(b)-U(b-c)}{U(b)-U(0)}\right)^{1/(n-1)} \\ &= 1-\left(\frac{b^\beta-(b-c)^\beta}{b^\beta-0^\beta}\right)^{1/(n-1)} \\ &= 1-\left(1-\left(1-\frac{c}{b}\right)^\beta\right)^{1/(n-1)}. \end{aligned} \tag{10}$$

The consequences of introducing risk preferences in a VCS are more complicated, mainly because the subjects were instructed that cost sharing would occur via *division sharing* rather than the theoretically simpler method of *lottery sharing*. Frankly, we had given this issue insufficient attention when developing the written instruction, and so we had to theoretically analyze division sharing under nontrivial risk attitudes. As explained in the second section and the technical appendix (Theorem 2), the prediction $\alpha^*_{\mathrm{VCS-D}}(\beta)$ is *implicitly* defined as the root of a polynomial of degree $n-1$, the coefficients of which depend on the group size n, the monetary payoffs b and c, and the utility function $U(x)$, and therefore also on the risk preference parameter β. In the technical appendix, we explain briefly how we estimated the risk attitude parameter. We obtained a maximum likelihood estimator of $\hat{\beta} = .74$ with 95% support interval (.63, .87). The deviance of this model was 38.14, with $p < .001$. To test the null hypothesis of risk neutrality, a likelihood ratio test can be used against the saturated model. Under H_0, the likelihood ratio test statistic LR is approximately χ^2 with 1 degree of freedom. In this case, $LR = 16.2$ with $p < .0001$, and so we find evidence against risk neutrality and in favor of risk aversion. We also estimated a variety of other specifications of utility with similar conclusions. On the other hand, the deviance of 38.1 of the model with estimated risk parameter is still quite high ($p = \Pr(\chi^2_{11} > 38.1) < .001$), and so we have to reject the detailed numerical predictions from the game-theoretic models. Next, we fitted the game-theoretic models with risk preferences separately for the data on the VOD ($\hat{\beta} = .85$, with 95% support interval [.66, 1.08] and deviance = 17.86, $p = .003$) and on the VCS ($\hat{\beta} = .69$, with 95% support interval [.56, .83] and deviance =18.55, $p = .002$). We conclude that the precise game-theoretic predictions, extended via risk references, do not fit the data very well. We can test the hypothesis that β is the same under VOD and VCS—β is a characteristic of utility of money, not of the structure of interaction. Although the β estimates are somewhat different, the support intervals have much overlap, and according to a likelihood ratio test ($LR = 38.14 - 17.86 - 18.55 = 1.73$, $p = .19$), the difference is indeed not significant.

TESTING THE QUALITATIVE PREDICTIONS

As a weaker test of the models VCS and VOD, we now consider the signs (directions) of the comparative statics analyses (Theorem 1, A Comparison with the VOD section) as our hypotheses to be tested rather than the quantified predictions tested above. Summarizing these findings, we hypothesize the following:

TABLE 2
Logit-Linear Analyses of Volunteering Decisions (model selection)

model	*deviance*	*df*	
factor(n) + (high costs)	19.1	8	
(high costs) + (VOD)	33.3	9	
factor(n) + (VOD)	12.6	8	
factor(n) + (high costs) + (VOD)	6.6	7	
n + (high costs) + (VOD)	12.8	8	
(n=2) + (high costs) + (VOD)	7.4	8	selected model

NOTE: The expression (x) denotes a dummy variable that is one if expression x is true and zero otherwise. Factor(n) denotes seperate effects for each level of n.

TABLE 3
Parameter Estimates for the Selected Logit-Linear Model of Volunteer Decisions

Variable	$\hat{\beta}$	$\hat{se}(\hat{\beta})$
(VOD)	-0.731	.210
(cost = high)	-0.509	.210
(n = 2)	1.059	.210
Constant	-0.548	.186

- *Hypothesis 1:* The probability of volunteering α decreases with increasing group size n.
- *Hypothesis 2:* The probability of volunteering α decreases with increasing costs c.
- *Hypothesis 3:* The probability of volunteering α is larger in the VCS than in the VOD.

Note that no hypothesis is formulated about b because it was fixed in our design. In Table 2, summary statistics are shown for logistic regression analyses with a few simple predictor variables. As noted before, the deviance statistic is the likelihood ratio test statistic of a model against the saturated model. It is approximately (asymptotically) χ^2 distributed if the hypothesized model is true. Moreover, the difference between the deviances of nested models is χ^2 distributed (with the difference in the dimension of the models as the degrees of freedom) if the smaller model is indeed true. Both from the results in this table and from unreported analyses of residuals, we conclude that a very simple model describes the data well. The details of this model are shown in Table 3. The predictions that the probability to volunteer is larger if costs are shared and if the costs of producing the collective good are lower are indeed supported. The predictions of the effects of group size with respect to the individual probability to volunteer are only partially supported. Whereas a group of size 2 provides more favorable conditions for the production of collective goods than the larger groups, the expected difference between groups of size 4 and 8 were not found. Because this small and simple model describes the data so well already—recall, we

TABLE 4
Expected and Observed Macro Probabilities of Public Good Production

	VOD (no cost sharing)		VCS (cost sharing)	
	low cost	*high cost*	*low cost*	*high cost*
n = 2	(.84)	(.36)	(.94)	(.55)
	.71	.38	.88	.78
n = 4	(.70)	(.25)	(.87)	(.43)
	.67	.55	.88	.68
n = 8	(.64)	(.22)	(.83)	(.39)
	.89	.73	.86	.94

NOTE: The first row denotes the expected probabilities, second row denotes the observed probabilities.

have an absolute goodness-of-fit statistic here, there is no need to discuss in detail the fate of the hypotheses about interaction effects that were based on the formal analyses in the second section—they are not supported by the data. With respect to the macro probability of public goods production, game-theoretic predictions are refuted, as can be seen from Table 4. The game-theoretic analysis concludes that macro probabilities should decline with increasing group size. However, this hypothesis is clearly refuted by the data. The observed macro probabilities of public goods production are almost always larger in the 8-person groups than in the smaller groups.[3]

CONCLUSIONS

A game-theoretic analysis of the VCS, a VOD in which the costs of providing the good are equally shared among the volunteers, yielded three major conclusions.

1. The individual probability to cooperate as well as the macro-level probability that the public good will be provided are ceteris paribus higher for the VOD with cost sharing than the for the standard VOD without VCS.
2. The individual probability that a player volunteers in the cost sharing situation is a decreasing function of group size and of the cost-benefit ratio, as in the standard VOD.
3. The macro-level probability that a public good will be produced in the VOD with cost sharing is a decreasing function of group size and the cost-benefit ratio as in the standard VOD.

Thus, according to game theory, VCS enhances cooperation and increases the chances that a public good is provided as compared to the standard VOD. But even when costs are shared, and tend to become negligible in larger groups, the behavior of the group will be inefficient because there remains a positive probability that the public good will not be produced. Hence, VCS enhances cooperation but does not eliminate the inefficiency problem.

[3]Note that the observed macro probabilities are calculated from the observed micro probabilities depicted in Table 2.

To test the empirical validity of these hypotheses, a three-factorial design with $2x2x3 = 12$ experimental conditions was conducted. The detailed numerical predictions of the game-theoretic model do not fit the data very well, even if we take the effects of risk preferences into account. However, the qualitative hypotheses that the probability of volunteering decreases with increasing group size, decreases with increasing costs, and is larger in the VCS than in the VOD, were supported by the data. Furthermore, subjects in the VOD and VCS volunteered more often in larger groups than predicted by game-theoretic models. As a consequence, the likelihood that the public good would have been provided increases with group size. Therefore, in real life a victim in need should hope for a large group of bystanders, not for small groups as game theory suggests.

TECHNICAL APPENDIX

This appendix contains a number of proofs and additional technical remarks on cost sharing in VODs (second section) and on estimating models with risk preferences (third section).

Proof of Theorem 1

Due to the symmetry of the VCS, it is natural to assume that the solution is symmetric as well; that is, all players volunteer with the same probability α. We want to derive equilibrium conditions on α. Thus, we have to compute the expected utility that a player obtains by playing V or A, where the expectation is taken with respect to the random decisions of the other players. We have

$$U(A;\alpha) = b\left(1-(1-\alpha)^{n-1}\right). \tag{11}$$

The expected utility associated with playing V for $\alpha = 0$ is $U(V;0) = b - c$ and for $\alpha > 0$, by straightforward computation,

$$\begin{aligned} U(C;\alpha) &= b - \sum_{k=0}^{n-1} \frac{c}{k+1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k} \\ &= b - \frac{c}{n\alpha}\sum_{k=0}^{n-1}\binom{n}{k+1}\alpha^{k+1}(1-\alpha)^{n-(k+1)} \\ &= b - \frac{c}{n\alpha}\left(1-(1-\alpha)^n\right). \end{aligned} \tag{12}$$

It is easily seen than $U(V;0) > U(A;0)$ and $U(V;1) < U(A;1)$. Thus, there does not exist a symmetric equilibrium in pure strategies. A symmetric game has at least one symmetric Nash equilibrium that may involve mixed strategies. Thus, there should exist a solution of $U(V;\alpha) = U(A;\alpha)$ with $\alpha \in (0,1)$. By substitution of (11) and (12) and some simplification, we conclude that such an equilibrium should be a root α^* of the polynomial equation $g(\alpha) = 0$ with g as defined in defg. To show that there exists one and only one solution in $(0, 1)$, note that

$$g'(\alpha) = n(1-\alpha)^{n-2}((b-c) - \alpha(nb-c)), \tag{13}$$

and so g' changes sign exactly once in the interval (0, 1); namely, at $\frac{b-c}{nb-c}$. Because $g(0) = 0$ and $g'(0) > 0$, there is exactly one solution α^* of $g(\alpha) = 0$, and $\alpha^* > \frac{b-c}{nb-c}$. Note that $g'(\alpha^*) < 0$. This finishes the proof of (i).

To prove (ii), we note that $cg(\alpha^*) = (1-\alpha^*)^n - 1 < 0$, and so

$$c\alpha^* = -cg \Big/ \alpha g < 0.$$

Similarly, $b\alpha^* > 0$ because $bg(\alpha^*) = n\alpha^*(1-\alpha^*)^{n-1} > 0$. To show that α^* decreases with group size n, we need to demonstrate that

$$n\alpha^* = (1-\alpha^*)^{n-1}\left(\log(1-\alpha^*)(bn\alpha^* + c(1-\alpha^*)\right) < 0, \tag{14}$$

that is

$$-\log(1-\alpha^*) > \frac{b\alpha^*}{bn\alpha^* + c(1-\alpha^*)}.$$

By direct computation, it easily follows that the inequality $\alpha^* > \frac{b-c}{bn-c}$ is equivalent to

$$\alpha^* > \frac{b\alpha^*}{bn\alpha^* + c(1-\alpha^*)}.$$

Because $-\log(1-\alpha) > \alpha$ for $\alpha \in (0, 1)$, we conclude that $n\alpha^* < 0$, and so α^* decreases in n.

To derive the series expansion of α^* in group size n, we should show first that $n\alpha^*$ actually converges to a positive value. The formal argument can be provided using the upperbound $n\alpha^* \leq \omega(\frac{nb}{(n-1)c})$. Now write $\alpha^* = \frac{\gamma}{n} + O(\frac{1}{n^2})$ for some $\gamma > 0$. It follows that $(1-\alpha^*)^{n-1} \to e^{-\gamma}$ and $n\alpha \to \gamma$, so γ should be a root of Riemann's ω function defined in defg. Uniqueness and the stated properties of the solution $\gamma > 0$ are easily established. This finishes the proof of (ii).

Finally, we prove (iii). Because θ^* increases with α^*, by (ii), θ^* decreases in c and increases in b. The assertion that θ^* decreases in n does not follow directly from (ii). Note that

$$\theta^* = 1 - (1-\alpha^*)^n = \frac{nb\alpha^*}{nb\alpha^* + c(1-\alpha^*)},$$

and so by straightforward computation,

$$n\theta^* < 0 \quad \Longleftrightarrow \quad n\alpha^* > -\frac{\alpha^*(1-\alpha^*)}{n}.$$

Substituting ga and gn, we get

$$n\alpha^* = -ng/\alpha g = -\frac{1-\alpha*}{n}\frac{\log(1-\alpha^*)((nb-c)\alpha^*+c)+b\alpha^*}{\alpha^*(c-nb)+(b-c)}.$$

The inequality $n\alpha^* > -\frac{\alpha^*(1-\alpha^*)}{n}$ readily simplifies to $-\log(1-\alpha^*) > \alpha^*$, which, as we have seen before, holds true. The limiting value of θ^* follows by substitution. This finishes the proof of (iii).

In the next theorem, we formally state and prove the results discussed in the second section on the effects of risk attitudes on behavior in volunteer dilemmas with VCS via division sharing. Following Raub and Snijders (1997), we introduce risk attitudes into the game-theoretic model by interpreting the stated outcomes not as cardinal payoffs; that is, directly in terms of the goals of actors. Rather, the stated outcomes refer to objective outcomes, whereas the payoffs of a player are nonlinear utility functions of these outcomes. Note that for VCS via a lottery, the outcomes b, $b-c$, and 0 are already payoffs. Consequently, the game-theoretic predictions about the volunteering decision of players depend both on the objective outcomes and on the subjective utility functions U of the players.

Theorem 2

The VOD with VCS via division sharing with strictly increasing and continuous utility functions $U(x)$, the same for all players, has a unique symmetric equilibrium in which players volunteer with probability $\alpha^*_{\text{VCS-Division}}$ that can be characterized as the unit root in α of

$$\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}U\left(b-\frac{c}{k+1}\right) = U(b)-(U(b)-U(0))(1-\alpha)^{n-1}. \tag{15}$$

The solution $\alpha^*_{\text{VCS-Division}}$ decreases in group size n, and decreases in the costs c, but increases in benefits b.

If the utility function $U(x)$ is concave—that is, $U(\lambda x+(1-\lambda)y) \leq \lambda U(x)+(1-\lambda)U(y)$ for all x, y, and for $\lambda \in [0, 1]$—then

$$\alpha^*_{\text{VCS-Lottery}} < \alpha^*_{\text{VCS-Division}}. \tag{16}$$

The equilibrium condition (15) is easily derived by the following observations. First, it is easily verified that there exists no symmetric equilibrium in pure strategies. Thus, we know that there should exist a symmetric equilibrium in mixed strategies. Second, the left-hand (right-hand) side is just a player's expected utility if he volunteers (does not volunteer), whereas all other $n-1$ players volunteer independently with probability α. Thus, (15) is nothing but the familiar characterization of mixed equilibria; namely, that players should be indifferent between all alternatives they play with positive probability. Uniqueness and the stated comparative statics properties of the unique equilibrium are demonstrated analogously to Theorem 1.

To demonstrate (16), note that the defining equation (15) for the case of division sharing is a polynomial of degree $n-1$, say $h(\alpha)$, just like the defining polynomial $g(\alpha)$ (see defga) for the lottery-sharing case. Note that utilities have to be substituted for outcomes in the case of lottery sharing; for example, $U(b)$ for b. It is easy to verify that $g(0) = h(0)$ and $g(1) = h(1)$. Moreover, by concavity of U,

$$\sum_{k=0}^{n-1} \left(\frac{1}{k+1} U(b-c) + \frac{k}{k+1} U(b) \right] \binom{n-1}{k} \alpha^k (1-\alpha)^{n-1-k}$$
$$< \sum_{k=0}^{n-1} U \left(\frac{1}{k+1}(b-c) + \frac{k}{k+1} b \right) \binom{n-1}{k} \alpha^k (1-\alpha)^{n-1-k}$$

so $g(\alpha) < h(\alpha)$ for $0 < \alpha < 1$. Because g and h are decreasing, the roots should satisfy $\alpha^*_{\text{VCS-Lottery}} < \alpha^*_{\text{VCS-Division}}$.

ON THE ESTIMATION OF MODELS WITH RISK PREFERENCE PARAMETERS

In the third section, we argued that estimating statistical models with a risk attitude parameter was complicated due to the implicit nature of the predictions for the VCS. Note that in estimating the model, all subjects are assumed to have the same risk preferences β (and, moreover, that this is common knowledge of the players). Conceptually, according to standard statistical theory, one can compute the maximum likelihood estimator (or, equivalently, the minimum deviance estimator) β and an approximate (asymptotic) confidence or support interval as follows:

- For all possible values of β:
  - Compute π_{VOD} for the eight VOD conditions using (10).
  - Compute π_{VCS} for the eight VCS conditions by solving for the root of equation (15) using a numerical secant method (Press et al. 1989).
  - Compute the deviance using deviance.
- Select $\hat{\beta}$, the β with minimal deviance.
- Estimate the 95% support interval for $\hat{\beta}$ as

$$SI(\hat{\beta} = \{\beta | Deviance(\beta) - Deviance(\hat{\beta}) < 3.84\}.$$

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