Asymmetric Volunteer's Dilemma Game: Theory and Experiment[a]

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Abstract

This study explores asymmetric volunteers' dilemma (VOD) games where costs for volunteering is different among players. Diekmann (1993) conjectures that an equilibrium, in which a player with less costs contributes, is more likely to be played if it is risk dominant. We re-examined this hypothesis theoretically and experimentally to find that even though such equilibrium is risk dominant, it does not necessarily hold. Conducting an econometric comparison among several behavioral models, we find that the quantal response equilibrium (QRE) model fits the data best.

Keywords: Volunteers' dilemma, risk dominance, quantal response equilibrium (QRE), level-k model, inequality aversion, experiment

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1. Introduction

Natural disasters often devastate a country and ravage the lives of many. Still fresh in our memories are the huge earthquake and the resulting tsunami that hit the North-eastern part of Japan in 2011, as well as the ensuing tragic accident of the Fukushima Daiichi Nuclear Power Plant. The associated social and economic losses were huge, and the recovery from them has taken a long time. Such incidences make us acutely aware of the importance of volunteers, as we hear many stories about people who help others even at the risk of their own lives. However, the question of what kind of people volunteer and what their motivations are still remain unanswered.

Given that, by definition, no monetary rewards are given to volunteers, pecuniary incentives do not explain why they are so engaged. Sympathy or empathy is a strong candidate to serve as an explanation. If someone was hurt, we would feel, more or less, uneasy; we will be urged to take some actions and gain relief if the victim is, ultimately, saved. However, this explanation is also not conclusive. Since volunteering is a costly action, even empathetic people might think of free-riding on others, anticipating that someone else may do the job.

A famous parable in the New Testament best illustrates this complicated situation (*The Parable of the Good Samaritan*, Luke 10:25-37). Aware of an injured fellow citizen who fell on the street, a priest and a Levite passed him by. Then a Samaritan, who is, of course, a stranger, took action and saved him. The priest and the Levite, while feeling uncomfortable, ultimately, free-rode on the Samaritan's helping behavior. This story is even impressive given that the Israelites and the Samaritans were in an adversarial relationship in those days. This meant that helping an Israelite was costlier for the Samaritan not only in monetary terms but also politically and psychologically.

Even today, we see that many people help victims from very distant areas rather than nearby ones, and it is often the case that they themselves were once victims in other disasters. How can we understand such a phenomenon?

This study attempts to address this issue in terms of game theory as well as behavioral game theory[1]. The situations cited above are well captured by a game called Volunteer's Dilemma (VOD) game, which was firstly formulated by Diekmann (1985) to elucidate "social dilemmas" or "social traps," which are broader than those covered by the prisoner's dilemma.

While symmetric versions of VOD conduce to the elucidation of such interesting issues like the so-called "bystander effect,"[2] asymmetric versions of VOD are useful for exploring the issue of who most likely contribute (help) in the aforementioned situations. Diekmann (1993) introduced asymmetry in terms of costs of contribution among players and showed, theoretically, that in the completely mixed strategy equilibrium, a player with *more* costs volunteers *more* often (the behavior of the good Samaritan is, thus, explained!).

While this theoretical prediction seems to support some observations, we are still puzzled with this conclusion. For example, if we consider a bystander's rescue decisions in emergencies, is it not the case that the one with the least cost will help the

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[1] Some studies deal with essentially the same problem outside of game theory. Darly and Latané (1968) examine the helping behavior of people witnessing an accident or crime, as best exemplified by the murder case of Kitty Genovese. It is said that her life could have been saved if only one of the bystanders had paid a small amount of cost (such as making an emergency call to the police). A large amount of evidence has also been accumulated by political scientists and psychologists to investigate factors affecting the "bystander effect" and the effect of the group size on the tendency to cooperate or contribute in similar situations (Latané and Nida, 1981).

[2] Kawagoe et al. (2018) characterize all the symmetric equilibria of this class of games.

victim? This intuition, naturally, leads us to further investigations. With the same motivation of study, Diekmann (1993) and Healy and Pate (2009) conducted laboratory experiments and confirmed that the converse of the prediction by the mixed strategy equilibrium was really the case; a player with *less* cost volunteers *more* often. But, how can one provide a theoretical justification for such experimental findings? Diekmann (1993) suggested in a footnote an explanation based on risk dominance proposed by Harsanyi and Selten (1988), but his ‘analysis’ seems to be incomplete.

Thus, we first provide a rigorous theoretical analysis of the game based on risk dominance. We confirm Diekmann (1993)’s above-mentioned conjecture and give some characterizations of equilibria in a more general setting by utilizing an extension of the risk dominance concept to general n-person games (Güth 1990). Then, we analyze the game with other-regarding preference and bounded rationality. We use inequality aversion (Fehr and Schmidt, 1999), level-k model (Stahl and Wilson 1995; Crawford et al. 2013), and quantal response equilibrium (QRE: McKelvey and Palfrey 1995) as representative models.

Based on these analyses, S-equilibrium (where a player with less costs volunteers) is shown to be risk dominant for any treatment of Diekmann (1993)’s experiment. However, depending on parameter values of the games (QRE’s prediction depends on its noise parameter), the inequality aversion and the level-k model give different predictions than risk dominance. Thus, if Diekmann (1993)’s conjecture were right, as S-equilibrium was risk dominant in every treatment, we would observe S-equilibrium more frequently in his data. Our analysis showed that predictions by both mixed strategy and risk dominance were rejected. Other models captured important characteristics of the results, but no model explained all the data consistently. Finally, we, then, conducted an econometric horse race to determine which model better fits the data. The estimation results showed that QRE best explains Diekmann (1993)'s data. Thus, Diekmann (1993)'s conjecture that a player with *less* cost volunteers *more* often was falsified.

We, then, extended his experiment by adding a treatment to differentiate the prediction by inequality aversion from other models. We also added a more complicated three-person version of the game. Diekmann (1993)'s conjecture was also falsified in our experiment, and it was confirmed, again, that QRE best fits the data. Thus, Diekmann (1993)'s conjecture is not supported by his and our results.

Of course, this does not mean free-riding among volunteering players with *less* cost is dominant in the data. Rather, in some treatments, the majority of players with *less* cost volunteer *more* often. What we show, empirically, is that the condition that S-equilibrium is risk dominant is not a source of volunteering by players with *less* cost. Inequality aversion also fails. No single theory can explain the data. Roughly speaking, the fact that QRE was the best fit means that the subject behaviors might be a combination of mixed strategy equilibrium and random play, while its proportion depends on the parameters of the game. However, the understanding of volunteering decision by players with *less* cost needs a more careful scrutiny.

The organization of the study is as follows. In the next section, we formulate the VOD game with an asymmetric cost structure and present several candidate models that may explain the behavior of subjects in the experiments. This part includes extensive mathematical characterizations of the models with some new results. Section 3 re-examines the data reported by Diekmann (1993) with those models. In Section 4, we turn to the analyses of our experiments. We conclude in the final section.

2. Model

In the VOD game, if at least one of n players volunteers, certain public goods are provided. The benefit from the public goods for player i is denoted as V_i, and the cost of volunteering for player i is K_i. When public goods are not provided, the payoff for player i is L_i. Assume $V_i - L_i > K_i > 0$ for all i. Each player chooses between volunteering (C) and not volunteering (N). Obviously, there are multiple pure strategy Nash equilibria where one and only one player chooses C, and the rest of the players choose N. Note that a strategy profile where all players choose N is not a Nash equilibrium.

2.1 Mixed strategy equilibria

Next, we consider the mixed strategy equilibria of the game. The probability that player i chooses C is denoted by p_i, and the probability of choosing N is denoted by $q_i = 1 - p_i$. Then, the expected payoff of choosing C for player i is

$$E_i(C) = V_i - K_i,$$

while the expected payoff of choosing N is

$$E_i(N) = V_i\left(1 - \prod_{j \neq i} q_j\right) + L_i \prod_{j \neq i} q_j.$$

If player i chooses C with a probability of one, then $E_j(N) > E_j(C)$ for any other player $j \neq i$ by $K_j > 0$. Furthermore, if all the other players than i choose N with a probability of one, then $E_i(C) > E_i(N)$ by $V_i - L_i > K_i$. Therefore, in a mixed strategy equilibrium, $q_i = 0$ if and only if $q_j = 1$ for any $j \neq i$. This implies that if a player takes a completely mixed strategy (mixed strategy with full support) in a mixed strategy equilibrium, then there are no other players who choose *C* with a probability of one in this equilibrium.

Thus, the VOD game may have three kinds of mixed strategy equilibria. The first is that only one player chooses *C* and the rest of players choose *N* with a probability of one (that is, a pure strategy Nash equilibrium). The second is that some players (two or more players) take completely mixed strategies, and the rest of the players choose *N* with a probability of one. The third is the completely mixed strategy equilibrium (mixed strategy equilibrium where all players take completely mixed strategies).

Diekmann (1993) only considers the first and the last types of mixed strategy equilibrium and shows that the completely mixed strategy equilibrium always exists in a symmetric case ($V_i = V$, $K_i = K$, and $L_i = 0$ for any $i = 1,2,..n$) and in the case with $n = 2$. However, the completely mixed strategy equilibrium does not always exist in more general cases. Hence, we identify the completely mixed strategies of the players in a mixed strategy equilibrium (which is not necessary completely mixed).

Given any mixed strategy profile θ, let $C(\theta)$ be a set of players who do not choose N with a probability of one in θ. Then, we have the following proposition.

Proposition 1. *Suppose that player i takes a completely mixed strategy in a mixed strategy equilibrium θ ($i \in C(\theta)$). Then*

$$q_i = \left(\frac{V_i - L_i}{K_i}\right)\left[\prod_{j \in C(\theta)}\left(\frac{K_j}{V_j - L_j}\right)\right]^{\frac{1}{\#C(\theta)-1}}. \qquad (1)$$

where $\#C(\theta)$ is the number of $C(\theta)$.

Proof. In Appendix A.

It follows from this proposition that the completely mixed strategy equilibrium is unique if it exists, and the strategies of the players are given by equation (1) if $C(\theta)$ is the set of all players.

Hereafter, we consider a special case in which there are only two different values for benefit V_i and cost K_i, and characterize the set of the mixed strategy equilibria. Following Diekmann (1993), we call the players with low cost "*strong* players" (hereafter S-players) and the players with high cost "*weak* players" (hereafter W-players). That is, the cost for an S-player K_S is strictly smaller than that for a W-player K_W ($K_S < K_W$). Here, let V_S and V_W be the benefit for an S-player and a W-player, respectively. L_S and L_W are defined similarly. We also, then, assume that these parameters satisfy the inequality

$$\frac{V_S - L_S}{K_S} > \frac{V_W - L_W}{K_W}, \tag{2}$$

which implies that the marginal per capita cost for volunteering for an S-player is less than the one for a W-player. Diekmann (1993) considers the case where $K_S < K_W$, $0 < V_W \leq V_S$, and $L_S = L_W = 0$, which satisfies condition (2). Moreover, as for the number of S-players, m, only the case of $m = 1$ is considered. We generalize those analyses to an arbitrary $m \leq n$.

Note that Proposition 1 implies that if two players are of the same type, and they take completely mixed strategies in equilibrium, then their probabilities of volunteering are the same. Furthermore, by condition (2), we have the following.

Corollary 1. *Suppose that an S-player* i *and a W-player* j *take completely mixed strategies in a mixed strategy equilibrium, then the probability that S-player* i *will volunteer is less than that of W-player* j*; that is,* $q_i > q_j$.

As mentioned above, Diekmann (1993) shows that the completely mixed strategy equilibrium always exists in a symmetric case ($m = n$ or $m = 0$) and in the case with $n = 2$. The following proposition gives the necessary and sufficient condition for the existence of the completely mixed strategy equilibrium when $n \geq 2$ and $1 \leq m \leq n$.

Proposition 2. *For any* $n \geq 2$ *and* $1 \leq m \leq n$*, the volunteer's dilemma game has the completely mixed strategy equilibrium if and only if*

$$\left(\frac{V_W - L_W}{K_W}\right)^{n-m} > \left(\frac{V_S - L_S}{K_S}\right)^{n-m-1}. \quad (3)$$

Proof. In Appendix A.

For any $n \geq 2$ and $m \leq n$, let $\Theta(n,m)$ be the set of mixed strategy equilibria of the VOD game, where the number of players is n and the number of S-players is m. By the above arguments, we can easily identify all mixed strategy profiles in $\Theta(2,1)$ (and also $\Theta(2,2)$ and $\Theta(2,0)$), which has only two kinds of mixed strategy equilibria. One is that one player chooses *C* and the other player chooses *N* with a probability of one, and the other is the completely mixed strategy equilibrium in which the strategies of the players are given by (1) with $\#C(\theta) = 2$. By the following proposition, we can also identify $\Theta(n,m)$ where $n > 2$, inductively.

Proposition 3. *Suppose that* $\theta \in \Theta(n, m)$ *is the completely mixed strategy equilibrium, and* $q_S(n, m)$ *and* $q_W(n, m)$ *are the probabilities of choosing* N *by an S-player and a W-player in* θ*, respectively. For any two non-negative integers,* l *and* h*, consider the VOD game where the number of players is* $n + l + h$*, and the number of S-players is* $m + l$*. In this game, a mixed strategy profile* θ' *such that* l *S-players and* h *W-players are choosing* N *with a probability of one,* m *S-players take* $q_S(n, m)$*, and* $n - m$ *W-players take* $q_W(n, m)$ *is a mixed strategy equilibrium; that is,* $\theta' \in \Theta(n + l + h, m + l)$.

Proof. In Appendix A.

For example, $\Theta(3,1)$ consists of two or three kinds of mixed strategy equilibria. First is that a player chooses *C* and the other two players choose *N* with a probability of one. Second is that a W- (an S-) player chooses *N* with a probability of one, and the other two players take completely mixed strategies in the completely mixed strategy equilibrium in $\Theta(2,1)$ ($\Theta(2,0)$). Third is the completely mixed strategy equilibrium in which two W-players take $q_W(3,1)$ and the S-player takes $q_S(3,1)$ if (3) is satisfied for $n = 3$ and $m = 1$.

We now focus on the completely mixed strategy equilibrium. As in the above proposition, let $q_S(n, m)$ and $q_W(n, m)$ be the probabilities of choosing N of an S-player and a W-player in the completely mixed strategy equilibrium in $\Theta(n, m)$, respectively. The following proposition states how these probabilities change with the group size and the number of S-players in it.

Proposition 4. *Suppose that* $\Theta(n,m)$, $\Theta(n+1,m)$, *and* $\Theta(n+1,m+1)$ *have the completely mixed strategy equilibria. Then*

(i) $q_S(n+1,m) > q_S(n,m)$,

(ii) $q_W(n+1,m) > q_W(n,m)$,

(iii) $q_S(n+1,m) > q_S(n+1,m+1)$, *and*

(iv) $q_W(n+1,m) > q_W(n+1,m+1)$.

Proof. In Appendix A.

Goeree et al. (2017) explored the relationship between group size and volunteering in symmetric VOD games, where the completely mixed strategy equilibrium predicts the probability of volunteering to be a decreasing function of group size and that the probability of a no-volunteer outcome increases with the number of players. Proposition 4 considers their results in more generalized cases. Thus, it shows that the probability of a volunteer outcome increases with the number of S-players, and the probability of a no-volunteer outcome increases with the number of W-players.

2.2 Equilibrium selection based on risk dominance

As we have shown, there are multiple equilibria in VOD games in general. As for pure strategy Nash equilibria, there are two kinds of them: S-equilibrium where only one of the S-players chooses *C* and W-equilibrium where only one of the W-players chooses *C*. Then, asking a question of which equilibrium will be selected is important. For the case of $m = 1$, Diekmann (1993) suggests that S-equilibrium is only a risk dominant equilibrium. Suppose, for example, $m = 1$ and $n = 2$. Then, the game becomes as follows (Table 1).

Table 1. Asymmetric volunteer's dilemma when $m = 1$ and $n = 2$

Strong \ Weak	***C***	***N***
C	$V_S - K_S,\ V_W - K_W$	$V_S - K_S,\ V_W$
N	$V_S,\ V_W - K_W$	$L_S,\ L_W$

In this game, there are two pure strategy Nash equilibria, (*C*, *N*) and (*N*, *C*), since $V_i - L_i > K_i$ for all i. If (*C*, *N*) risk dominates (*N*, *C*), the product of deviation loss for (*C*, *N*) is greater than that for (*N*, *C*) (Harsanyi and Selten, 1988). This implies

$$(V_S - L_S - K_S)K_W > K_S(V_W - L_W - K_W).$$

Note that this condition is equivalent to condition (2). Thus, we have the following proposition.

Proposition 5. *In the case of* $n = 2$ *and* $m = 1$, *S-equilibrium is risk dominant.*

We now consider a risk dominant equilibrium in the case of many players. Here we consider Güth (1990)'s notion of *unilateral deviation stability (UDS)*. UDS satisfies Harsanyi and Selten's axioms for characterizing risk dominance.[3] Suppose that an n-

[3] The notion of p-dominance, introduced by Morris, Rob, and Shin (1995), generalizes the notion of risk dominance in Harsanyi and Selten (1988) differently than Güth (1990); that is, 1/2-dominance coincides with the latter in symmetric 2x2 games. Peski (2010), also, proposes a related

person game has several strict Nash equilibria. Take two of them, $s = (s_1, \cdots, s_n)$ and $t = (t_1, \cdots, t_n)$. Let $\mathrm{M}(s,t)$ be a set of players whose equilibrium strategy is different in s and t. For players i and j in $\mathrm{M}(s,t)$, construct a comparison game $G_{ij}(s,t)$, where the set of strategies is $v = (s_k, t_k)$ for $k = i, j$, and the payoff function is also restricted by this set of strategies (players other than i and j use the same strategies in both s and t by the assumption of $\mathrm{M}(s,t)$.). Thus, $G_{ij}(s,t)$ can be represented by the following game in Table 2.

Table 2. Comparison game $G_{ij}(s,t)$

	$\boldsymbol{s_j}$	$\boldsymbol{t_j}$
$\boldsymbol{s_i}$	$\pi_i(s), \pi_j(s)$	$\pi_i(s_i, t_j), \pi_j(s_i, t_j)$
$\boldsymbol{t_i}$	$\pi_i(t_i, s_j), \pi_j(t_i, s_j)$	$\pi_i(t), \pi_j(t)$

The relative strength of equilibrium s against t, $R_{ij}(s,t)$, is defined as follows.

$$R_{ij}(s,t) = \frac{\{\pi_j(s) - \pi_j(s_i, t_j)\}\{\pi_i(s) - \pi_i(t_i, s_j)\}}{\{\pi_j(t) - \pi_j(t_i, s_j)\}\{\pi_i(t) - \pi_i(s_i, t_j)\}}$$

Thus, the product of losses resulting from the unilateral deviation of players i and j from equilibrium s and t is compared. Finally, the aggregated value of $R_{ij}(s,t)$ for any pair of players in $\mathrm{M(s,t)}$ is given by

$$R_*(s,t) = \prod_{\substack{i,j \in M(s,t) \\ i > j}} R_{ij}(s,t)$$

Then, if $R_*(s,t) > 1$, equilibrium s risk dominates t. With this notion, we have a generalization of Proposition 5.

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concept for n-person symmetric games based on 1/2-dominance. However, as the game we study is not symmetric, we did not use these concepts.

Proposition 5'. *In the case of* $n \geq 2$ *and* $1 \leq m < n$, *S-equilibrium risk dominates W-equilibrium in the sense of unilateral deviation stability.*

Proof. In Appendix A.

2.3 Alternative theories

In what follows, we also examine the explanatory power of alternative theories that take into account the possibility that agents have a non-selfish motivation or act irrationally. In fact, we will see that the explanatory power of both mixed strategy equilibria and risk dominance is not satisfactory concerning the explanation of the experimental data, Diekmann (1993)'s and ours. Among many alternative models so far proposed for defining non-selfish motivation, we choose inequality aversion (Fehr and Schmidt, 1999) as the most appropriate in the present context. As for irrational behavior, we investigate the level-k model (Stahl and Wilson 1995; Crawford et al. 2013) and the quantal response equilibrium (QRE: McKelvey and Palfrey 1995), which are both simple, but known to have been very successful in the experimental economics literature.

A. Inequity aversion

The concept of inequity aversion (resistance to inequitable outcomes) in explaining experimental regularities was developed in Fehr and Schmidt (1999).[4] They postulated

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[4] Bolton and Ockenfels (2000), also, proposed a similar kind of concept. One of the distinguished features in Fehr and Schmidt (1999)'s model as compared to Bolton and Ockenfels (2000)'s is that

that people make decisions to minimize inequity in outcomes. Specifically, consider a setting where player i $(i = 1,2,\cdots,n)$ receives a pecuniary payoff x_i. Then the utility of an inequity averse player i for the allocation $(x_{1,}x_2,\dots,x_n)$ is given by

$$U_i(x_{1,}x_2,\dots x_n) = x_i - \frac{\alpha}{n-1}\sum_{j\neq i} max\{x_j - x_i, 0\} - \frac{\beta}{n-1}\sum_{j\neq i} max\{x_i - x_j, 0\}, \qquad (4)$$

where α is player i's disutility of having less than the others and β is player i's disutility of having more than the others. Fehr and Schmidt (1999) assume that $0 \leq \beta < 1$, and $\beta \leq \alpha$.

If there is a player having the above utility function, S-equilibrium or W-equilibrium would no longer be a Nash equilibrium, and other outcomes would be a Nash equilibrium depending on values of the parameters α and β. We will list equilibrium conditions for each outcome later in examining Diekmann (1993)'s data.

However, the following proposition implies that the outcome that every player chooses *C*, All-*C*, cannot be an equilibrium irrespective of the values of the parameters.

Proposition 6. *Suppose that player i is an inequity averse player. Then player i never chooses C when all other players choose C.*

Proof. In Appendix A.

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it allows a distinction between advantageous and disadvantageous inequalities. As this feature plays a role in explaining our data, we adopted Fehr and Schmidt (1999)'s model.

B. Level-k model

Level-k model is a non-equilibrium model that reflects strategic thinking by boundedly rational players. It assumes that each player adopts a strategy that corresponds to some level of strategic thinking. Level-k models have so far been applied to many games, and have succeeded in explaining a number of anomalous behaviors found in the laboratory (for a survey, see Crawford et al., 2013).

Assume that $L0$ player, who is the least rational among the players, chooses C and N with probability 1/2, respectively. In the level-k model, Lk player chooses the best response to the actions taken by $L(k-1)$ players. The outcome the level-k model predicts also depends on the payoff structure of the game; we will show its predictions later in analyzing Diekmann (1993)'s data.

C. Quantal response equilibrium (QRE)

QRE is an equilibrium concept based on boundedly rational strategic behavior, assuming that players play a noisy best response (McKelvey and Palfrey, 1995). For player i, the stochastic best response in terms of his choice probability of C is given by

$$p_i = \frac{\exp(\lambda \cdot E_i(C))}{\exp(\lambda \cdot E_i(C)) + \exp(\lambda \cdot E_i(N))}$$
$$= \frac{1}{1 + \exp[\lambda \cdot \{E_i(N) - E_i(C)\}]},$$

where parameter $\lambda \in [0, \infty)$ represents the degree of rationality such that $\lambda = 0$ implies complete randomizing over pure strategies. QRE is a fixed point of this mapping. If $\lambda = 0$, then $p_i = 1/2$ for any i, which is usually called the centroid of the simplex of the strategy space.

McKelvey and Palfrey (1995) showed that for any normal-form game, (i) the correspondence $\text{QRE}(\lambda)$ is upper hemi-continuous, (ii) the number of QREs is odd for generic values of λ, and, (iii) generically, the graph $(\lambda, \text{QRE}(\lambda))$ contains a unique branch which starts at the centroid and converges to a unique mixed strategy equilibrium as λ approaches infinity. The limiting point of this principal branch is called limiting (logit) QRE. Thus, limiting QRE can serve as an equilibrium selection criterion in generic cases.

Turocy (2005) showed that with the homotopy method (under an intuitive monotonicity assumption), the limiting QRE is the risk dominant equilibrium in generic 2x2 games with two strict Nash equilibrium. However, the cases with more than two players may be problematic. It is easy to see that these games have multiple S-equilibria, depending on which S-player finally contribute. Thus, the principal branch of the QRE graph that starts from the centroid may bifurcate somewhere on the way, meaning that these cases are not generic. This actually happens in our case. However, in the statistical estimation that follows, the log-likelihood function is maximized well before the bifurcation occurs.

3. Re-examination of Diekmann's experiment

3.1 The experiment

For empirically testing our theoretical predictions in previous section, we begin with the reexamination of Diekmann (1993)'s experiment. Diekmann (1993) concludes that the predictions of the risk-dominance theory accord better with his data than that of the completely mixed strategy equilibrium. Thus, unlike the completely mixed strategy equilibrium prediction which states that a player with *less* cost contributes *less* often, an S-player in the experiment contributes a significantly higher proportion.

However, once we closely consider his experimental result, it seems the explanatory power of the risk-dominance theory is not so high. Therefore, it is worthwhile to reexamine the data with econometric comparisons, including the alternative theories in Section 2.3, which was not available when his research was conducted.

His experiment consists of ten different sessions, including symmetric and asymmetric versions of VOD. The total of 328 subjects were recruited and allotted randomly into the sessions. However, as a result of the preliminary test, twenty-seven subjects were excluded from the proper experiment. In what follows, we will only focus on the part of his experiment that concerns asymmetric VOD.

Table 3. Diekmann's experimental games (Sessions)

Games	**D-1**		**D-2**		**D-3**		**D-4**	
Sessions	B	D	C	E	F	G	I	J
# of players	2	2	2	2	2	2	5	5
# of S-players	1	1	1	1	1	1	1	1
V(common)	100	100	100	100	100	100	100	100
K_S	40	40	10	10	20	20	40	40
K_W	50	50	50	50	80	80	50	50
Subjects' role	W	S	W	S	W	S	W	S
# of subjects	29	30	32	39	27	32	27	27

Table 3 shows the sessions in his experiment that corresponds to asymmetric versions of VOD. It is important to notice that the subjects did not played the game at all; that is, they were not matched with one another to play the games. Instead, subjects were simply asked to make a decision as a W-player or an S-player before the experiment ended. Each subject could participate in only one session. How the game outcome was determined, and, thus, how the rewards were determined, is not reported in the study.

We classify each session by games and give each game a name, from D-1 to D-4. Note that Sessions D-1 to D-3 are two-player games, and only Session D-4 is a five-player game. In every session, the number of S-players is one. Note also that for parameter values V, K_S and K_W $(L_S = L_W = 0)$, the condition in Proposition 5 and 5' is satisfied; that is, S-equilibrium is risk dominant.

3.2 Hypotheses

Testing Diekmann's conjecture is the primary purpose here. As S-equilibrium is risk dominant in all the session, it is expected that S-equilibrium should be observed most frequently. If it is not the case, our next task is to judge whether the completely mixed strategy equilibrium prediction is observed in his experiment.

As shown in Corollary 1, if players adopt completely mixed strategies, the probability that an S-player volunteers is less than that of a W-player. However, it is counterintuitive because the marginal per capita cost for volunteering for an S-player, $K_S/(V_S - L_S)$, is less than the one for a W-player, $K_W/(V_W - L_W)$. Thus, our first hypothesis concerns whether S-equilibrium is attained in the laboratory.

Hypothesis 1. S-equilibrium is more frequently observed than W-equilibrium.

Of course, subjects in the experiment may not always play rationally. Sometimes they deviate from the equilibrium either consciously or unconsciously. The former results from different motivations subjects have; for example, other-regarding preferences. The latter is due to the subjects' misunderstanding or confusion, lack of concentration, among others. Thus, we cannot expect that a particular equilibrium is played with a probability of one. If it is the case, a completely mixed strategy might predict the subjects' behavior better than the pure strategy predicted by risk dominance. Then, is the subjects' choice frequency of volunteering close to the mixed strategy equilibrium prediction in Section 2? This is our second hypothesis to be tested.

Hypothesis 2. A subject's choice frequency of volunteering coincides with the completely mixed strategy in a mixed strategy equilibrium.

As we have shown in Section 2, there are two types of mixed strategy equilibria where some players take completely mixed strategies. The first type is the completely mixed strategy equilibrium. The second type is mixed strategy equilibria where some players choose N with a probability of 1 and the other players choose the completely mixed strategies. As the latter exists only in D-4 (five-person game) and our objective is to differentiate counter intuitive completely mixed strategy prediction from that of risk dominance, we use the former in testing Hypothesis 2. Table 4 summarizes completely mixed strategy equilibria in Diekmann (1993)'s experiment.

As both hypotheses are mutually inconsistent, it is an easy task to judge which theory, mixed strategy equilibrium and risk dominance, predicts better subject behavior in the experiment.

Table 4. Completely mixed strategy equilibrium prediction for Diekmann's Games

	D-1	D-2	D-3	D-4
P_S	0.500	0.500	0.200	0.006
P_W	0.600	0.900	0.800	0.205

Table 5. The prediction of inequality aversion

	D-1	D-2	D-3	D-4
S-eq.	$\alpha \leq 3/2$	$\alpha \leq 9$, $\beta \leq 5$	$\alpha \leq 4$, $\beta \leq 4$	$\alpha \leq 6$
W-eq.	$\alpha \leq 1$, $\beta \leq 4/5$	$\alpha \leq 1$, $\beta \leq 1/5$	$\alpha \leq 1$, $\beta \leq 1/4$	$\alpha \leq 8$, $\beta \leq 16/5$
All-*N*	$\alpha \geq 1$	$\alpha \geq 9$	$\alpha \geq 4$	$\alpha \geq 6$

If the date supports neither hypothesis, we have to resort to different theories other than rational ones such as mixed strategy equilibrium and risk dominance. Thus, to obtain theoretical predictions by inequity aversion for each experimental session, we assume that all players have the same utility function given by (4). Then, for each session, either S-equilibrium, W-equilibrium, or the outcome that every player chooses *N*, All-*N*, can be a Nash equilibrium depending on the values of the parameters. The ranges of the values of α and β that each outcome will be a Nash equilibrium for each session are summarized in Table 5.

As for the level-k model, the best responses of Lk S-player and Lk W-player to the actions of $L(k-1)$ players up to $k=4$ are summarized in Table 6.

Table 6. Best response of Lk to L(k-1)[5]

		D-1	**D-2**	**D-3**	**D-4**
***L*1**	**S**	*C*	*C*	*C*	*N*
	W	*C,N*	*C,N*	*N*	*N*
***L*2**	**S**	*C*	*C*	*C*	*C*
	W	*N*	*N*	*N*	*C*
***L*3**	**S**	*C*	*C*	*C*	*N*
	W	*N*	*N*	*N*	*N*
***L*4**	**S**	*C*	*C*	*C*	*C*
	W	*N*	*N*	*N*	*C*

We, then, econometrically measure the goodness-of-fit of these alternative theories, including QRE.

3.3 Results

[5] "C, N" means that both strategies are indifferent. The identification of a higher-level strategy assumed that both are played with equal probability.

Table 7 shows Diekman's experimental data, where *n* is the number of groups, n_S and n_W are the number of *C* chosen by an S-player and a W-player, respectively, and f_S and f_W are the choice frequency of *C* by an S-player and a W-player respectively. If Diekmann's conjecture is right, as S-equilibrium is risk dominant in every session, we have to observe $f_S > f_W$. In two-person game sessions (D-1, D-2 and D-3) $f_S > f_W$ is observed (chi squared test, $p < 0.05$ except Session D-3). However, in Session D-4, a five-player game, we have a counter fact: $f_S = 0.30 < f_W = 0.56$ (chi squared test, $p < 0.05$). Thus, we conclude that Diekmann's conjecture is only applicable to two-person games. Therefore, Hypothesis 1 is only partially confirmed.

Table 7. Choice Frequencies of Each Strategy

	D-1	**D-2**	**D-3**	**D-4**
n_S	20	37	27	8
n_W	13	6	2	15
f_S	0.67	0.95	0.84	0.30
f_W	0.45	0.19	0.07	0.56

As for the completely mixed strategy equilibrium, when we compare Tables 4 and 7, there are huge discrepancies between them. Significant differences are visible in D-2, D-3, and D-4 Sessions. The null hypothesis stating that the choice frequency of C and the probabilities obtained from the completely mixed strategy equilibrium are equal is rejected for S-players in Sessions D-2, D-3, and D-4 (binomial test, $p < 0.01$ for each). On the contrary, W-players chose *C* less frequently than the completely mixed strategy equilibrium. The null hypothesis stating that the choice frequency of C and the probabilities obtained from the completely mixed strategy equilibrium are equal is rejected for W-players in Sessions D-2, D-3, and D-4 (binomial test, $p < 0.01$ for each). Thus, the mixed strategy equilibrium prediction also fails.[6]

Furthermore, to find the model that best fits Diekmann (1993)'s data among alternative theories, we will resort to an econometric horse race; that is, conducting maximum likelihood estimation for obtaining the parameter values such as α, β, and λ in these models and comparing explanatory powers among them.

Moreover, to make a comparison, we need to make the models "statistical" in the sense that they "contain adjustable parameters" (Sober 2008, p.79). Recall that the level-k analysis in Section 2.3B usually gives a "point prediction" with a probability of one assigned to a specific strategy. This can be problematic when we conduct an econometric comparison because the value of the log-likelihood of a probability of zero is minus infinity. Thus, we decided to introduce the same kind of noises across all the models, which may be simply interpreted as the errors in the implementation of the actions agents choose. Specifically, we assume that the choice probability of C for each player i is given by the following logit form,

$$p_i = \frac{\exp\left(\frac{EU_i(C)}{\mu}\right)}{\exp\left(\frac{EU_i(C)}{\mu}\right) + \exp\left(\frac{EU_i(N)}{\mu}\right)}, \qquad i = 1,2,\dots,n, \tag{5}$$

where μ is the noise parameter, and $EU_i(C)$ ($EU_i(N)$) is the expected utility of player i for choosing C (N). In the model of inequality aversion, player i's expected utility is calculated by (4); that is, the utility of an inequality averse player. In the level-k model,

[6] Note that in Session D-1, the null hypothesis is not rejected (binomial test, $p < 0.05$ for S-players and W-players).

if player i is Lk player, his expected utility is the expected payoff when all other players are $L(k-1)$ players. In our estimation of QRE, we also use $\mu = 1/\lambda$ for ease of comparison.

In each model, the probabilities of each player's choices are obtained in a fixed point of the above n equations for any given values of parameters. Let ω be the vector of parameters for each model (for example, $\omega = (\mu, \alpha, \beta)$ in inequality aversion), $p_S^{Dj}(\omega)$ be the choice probability of the S-player, and n_S^{Dj} be the number of S-players who chose C in game D-j. $p_W^{Dj}(\omega)$, n_W^{Dj}, and notations for the other sessions are defined similarly. Then, for each model, the maximum likelihood estimates are obtained by maximizing the following log likelihood function with respect to ω.

$$LogL^D(\omega) = \sum_{j=1}^{4} n_S^{Dj} \ln\left(p_S^{Dj}(\omega)\right) + \sum_{j=1}^{4} \left(g_S^{Dj} - n_S^{Dj}\right) \ln\left(1 - p_S^{Dj}(\omega)\right)$$

$$+ \sum_{j=1}^{4} n_W^{Dj} \ln\left(p_W^{Dj}(\omega)\right) + \sum_{j=1}^{4} \left(g_w^{Dj} - n_W^{Dj}\right) \ln\left(1 - p_W^{Dj}(\omega)\right),$$

where g_S^{Dj} (g_W^{Dj}) is the number of the S-players (W-players) in D-j. Since the subjects in Diekmann (1993) were not matched with one another to play the games, g_S^{Dj} is not necessarily the same as g_W^{Dj}. For example, $g_S^{D1} = 30$, whereas $g_W^{D1} = 29$.

Table 8 shows the estimated parameters.[7] In Level-k, we estimate two types of models. In Level-1, it is assumed that all players are L1 players. Another model is Level-2 where all players are assumed to be L2-players. The log likelihood for completely mixed strategy equilibrium (in Table 4) is also shown in this table. For model comparisons, the following Akaike information criterion (AIC) is used,

$$\text{AIC} = -2LogL + 2k,$$

[7] Our estimation procedures follow the method described in Chapter 16 in Moffatt (2016).

where k is the number of parameters in the model. The smaller the value of AIC, the better the model fit the data.

Table 8. Estimated parameters based on Diekmann (1993)

	Completely mixed strategy	Inequality aversion	Level-1	Level-2	QRE
α	-----	0.145	-----	-----	-----
β	-----	0.778	-----	-----	-----
μ	-----	30.829**	28.590**	78.750**	32.755**
LogL	-280.382	-124.989	-143.165	-157.774	-125.957
AIC	560.764	255.978	288.330	317.548	253.915

** denotes significance at the 1% level.

First, note that parameters α and β in the inequality aversion model are not significant. Thus, altruistic motivation does not play a role in explaining the data. Then, among the rest of the models, the value of AIC for QRE is minimum. Thus, we conclude that QRE is the best fit model in the data reported in Diekmann (1993). This result and the fact that S-equilibrium is frequently observed for two-person games are mutually consistent because limiting QRE converges to risk dominant S-equilibrium in two-person games according to Turocy (2005).

4. Our Experiment

In the previous section, we showed that Diekmann's conjecture (which states that when S-equilibrium is risk dominant, it is frequently observed) is only confirmed in two-person games. However, our theoretical analysis in Section 2 proved that S-equilibrium was also risk dominant in the five-person game in his experiment. Thus, the explanatory power of the risk dominance theory is not universal. Rather, QRE is the best fit model in his data. Interestingly, inequality aversion also fails. As Diekmann's experiment is not an experiment in the exact sense, and for generalizing our findings, we need to run a proper experiment with more variations. Thus, we decided to conduct our own experiment.

4.1 Design and procedures

The experiments were conducted in November 2016 at Chuo University in Japan. The subjects were undergraduates of the university and were recruited via a university e-mail list. Most of them were not from the Economics Department; only a few knew of game theory, and none had previously participated in any experiment.

The games we used in the experiment were two-person and three-person Volunteer's dilemma games. In the two-person game, one player was an S-player, and the other was a W-player. In the three-person game, we had two variations: In one, there were one S-player and two W-players; In the other, there were two S-players and one W-player. We call the two-person game session as Session A, the first variation of the three-person game session as Session B, and the second variation of the three-person game session as Session C.

In every session, $L_S = L_W = 0$, and the cost for volunteering is common for players of the same type; that is, $K_S = 200$ and $K_W = 400$. There were three sessions for each of two-person and three-person games, which have different pair of benefits (for S- and W-players) from the public goods. Note that the condition (2) assumed in the model in Section 2 is always satisfied with these parameter values. Thus, S-equilibrium is always risk dominant. Table 9 summarizes the details of each Session.

The experiments were conducted manually. Each subject was randomly assigned to a seat in the room. There was sufficient physical distance between seats to prevent eye contact, and no oral communication was allowed among the subjects. The experimental instructions were distributed, and the experimenter read them aloud in front of the subjects.

Table 9. Summary of experimental sessions

Session	**A-1**	**A-2**	**A-3**	**B-1**	**B-2**	**B-3**	**C-1**	**C-2**	**C-3**
# of players	2	2	2	3	3	3	3	3	3
# of S-player	1	1	1	1	1	1	2	2	2
$\boldsymbol{V_S}$	600	600	350	1000	1000	600	1000	1000	600
$\boldsymbol{V_W}$	1000	600	600	1800	1000	1000	1800	1000	1000

Some subjects played the two-person game first and, then, without knowing the results, played three-person game. The others played the three-person game first and, then, the two-person game. Each game was played only once in each session.

A total of 30 subjects participated in each two-person game session, forming 15 groups. A total of 36 subjects participated in each three-person game with one S-player session, and 12 groups were created. For the three-person game with two S-players session, the number of participants differs in each session due to no-show subjects: 13 groups in two sessions and 10 groups in one session.

At the end of the experiment, the total payoffs each participant earned in both games was paid in cash. The conversion rate was one-to-one: each point earned was exchanged for JPY 1 at the end of the experiments. No participation fee was paid to each subject. For the one-hour experiment, the average reward was around JPY1,276 (approximately 10 US dollars at the time). Details of the experimental procedure and instructions are given in Appendix C.

Table 10 summarizes the following two types of symmetric mixed strategy equilibria in the VOD games in our experiment. Eq-1 is the completely mixed strategy equilibrium, and Eq-2 is the mixed strategy equilibrium where two players of the same type take the same completely mixed strategy and the rest one takes *N* with a probability of one.

Table 10. Symmetric mixed strategy equilibrium prediction in the experiment.

Eq.	**Session**	**A-1**	**A-2**	**A-3**	**B-1**	**B-2**	**B-3**	**C-1**	**C-2**	**C-3**
1	p_S	0.600	0.333	0.333	0.503	0.106	0.307	0.529	0.368	0.368
	p_W	0.667	0.667	0.429	0.553	0.553	0.423	0.576	0.684	0.473
2	p_S	---	---	---	0.000	0.000	0.000	0.800	0.800	0.667
	p_W	---	---	---	0.778	0.600	0.600	0.000	0.000	0.000

4.2 Results

Table 11 summarizes our experimental data, where *n* is the number of groups, n_S and n_W are the number of *C* chosen by an S-player and a W-player, respectively, and f_S

and f_W are the choice frequency of C by an S-player and a W-player, respectively.[8] Note that in Sessions B-1, B-2, and B-3, there are one S-player and two W-players in a group, and that in Sessions C-1, C-2, and C-3, there are two S-players and one W-player in a group.

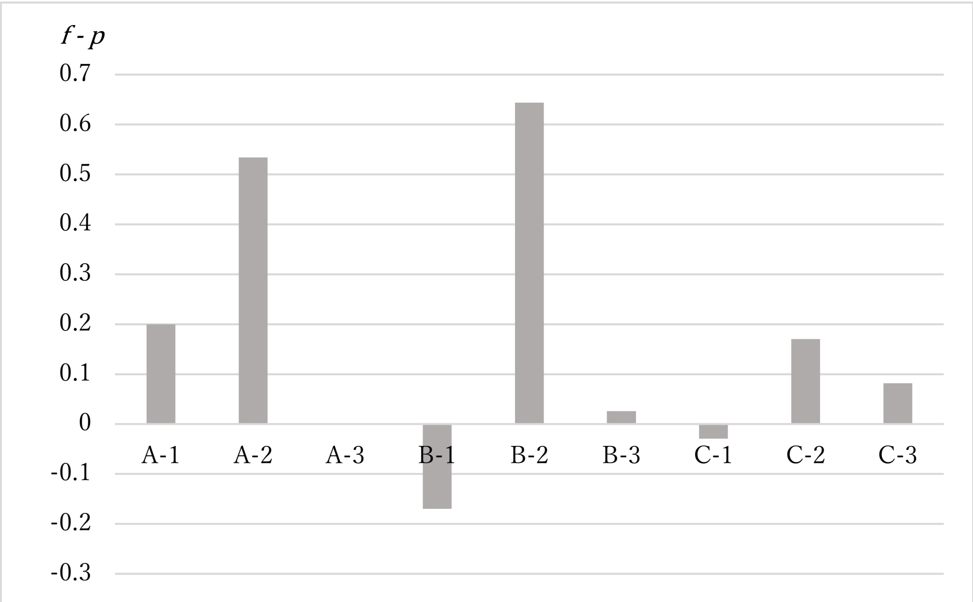
Table 11. Choice frequencies of each strategy.

Session	A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
n	15	15	15	12	12	12	13	13	10
n_S	12	13	5	4	9	4	13	14	9
n_W	6	2	7	13	9	11	8	3	5
f_S	0.800	0.867	0.333	0.333	0.750	0.333	0.500	0.538	0.450
f_W	0.400	0.133	0.467	0.542	0.375	0.458	0.615	0.231	0.500

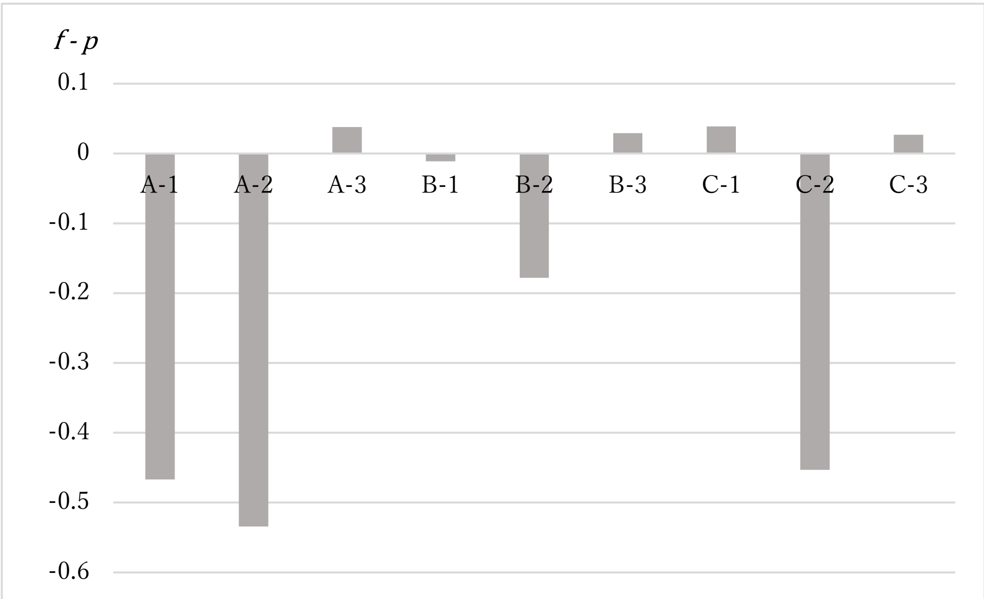
Figure 1 shows the differences between the probabilities obtained from the symmetric mixed strategy equilibrium (Eq-1 and Eq2 in Table 10) and the choice frequencies.

In most Sessions, S-players chose C more frequently than the mixed strategy equilibrium. Significant differences are visible in Sessions A-2 and B-2 for Eq1 and Sessions B1-B3 for Eq2, and, in fact, the null hypothesis which states that the choice frequency of C and the probabilities obtained from the symmetric mixed strategy equilibrium are equal is rejected for S-players in Sessions A-2 and B-2 for Eq1 and Sessions B1-B3 for Eq2 (binomial test, $p < 0.01$ for each).

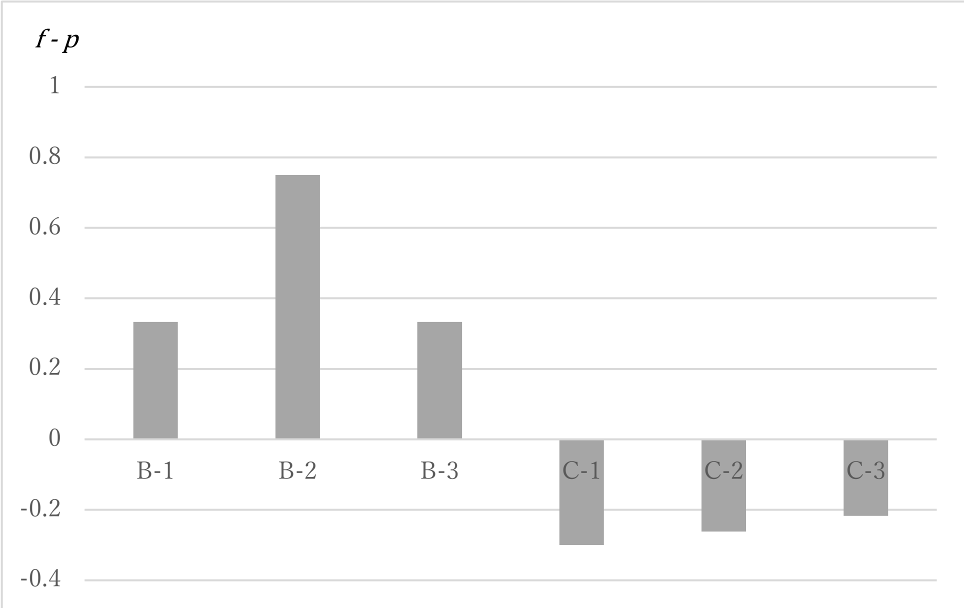
8 Raw data is given in Appendix B.



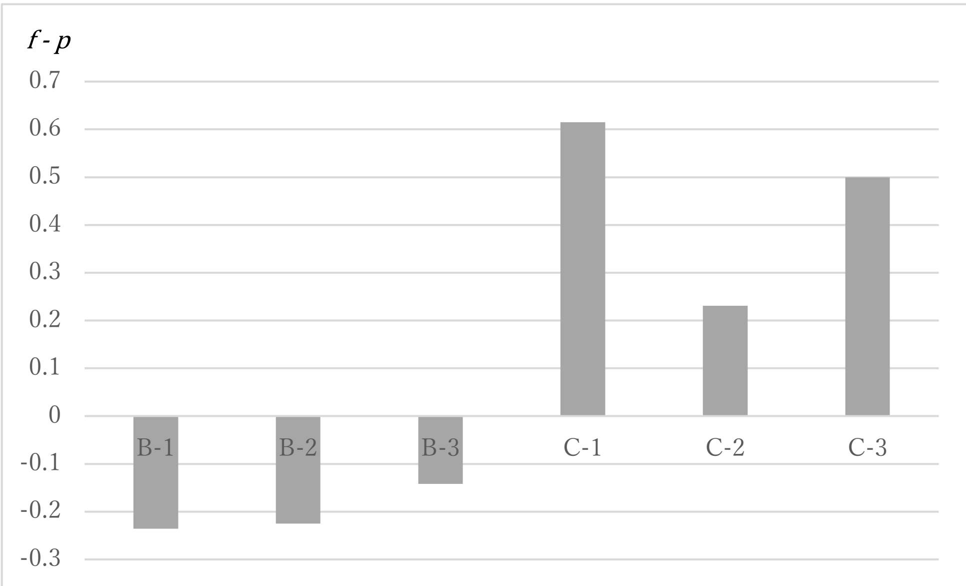
(a) S-player in Eq1



(b) W-player in Eq1



(c) S-player in Eq2



(d) W-player in Eq2

Figure 1. Difference between mixed strategy and the choice frequencies.

Moreover, S-players chose C most often in the second treatment in every Session (86.7%, 75.0%, and 53.8% in Session A-2, B-2, and C-2 respectively for Eq1 and

75.0% in B2 for Eq2). This fact contradicts the symmetric mixed strategy equilibrium prediction, other than in C-2 for Eq2, since the lowest probabilities (33.3%, 10.6%, and 36.8% in Session A-2, B-2, and C-2 for Eq1 and 0.0% for Eq2 respectively) should be assigned in the equilibrium to the second treatment among the three treatments.

On the contrary, W-players chose *C* less frequently than the symmetric mixed strategy equilibrium in many Sessions. The null hypothesis stating that the choice frequency of C and the probabilities obtained from the symmetric mixed strategy equilibrium are equal is rejected for W-players in Sessions A-1, A-2, and C-2 for Eq1 and Session B1 and B2 for Eq2 (binomial test, $p < 0.01$ for Eq1 and $p < 0.05$ for Eq2). Especially, as for Eq1, W-players chose *C* least often in the second treatment in every Session (13.3%, 37.5%, and 23.1% in Session A-2, B-2, and C-2 respectively for Eq1). However, in Eq1, the highest probabilities (66.7%, 55.3%, and 68.4% in Session A-2, B-2, and C-2 respectively for Eq1) should be assigned to the second treatment among the three treatments. From these, we conclude that the symmetric mixed strategy equilibria fails to explain our data. Next, we will examine whether asymmetric mixed strategy equilibrium (other than S- or W-equilibrium) can explain our data. In Sessions B-1, B-2, and B-3, there are asymmetric mixed strategy equilibria in which one of the W-players chooses *N* with a probability of 1 and the other W-players and S-players choose *C* with a probability of less than 1. Likewise, in Sessions C-1, C-2, and C-3, there are asymmetric mixed strategy equilibria in which one of the S-players chooses *N* with a probability of 1 and the other S-players and W-players choose *C* with a probability of less than 1.

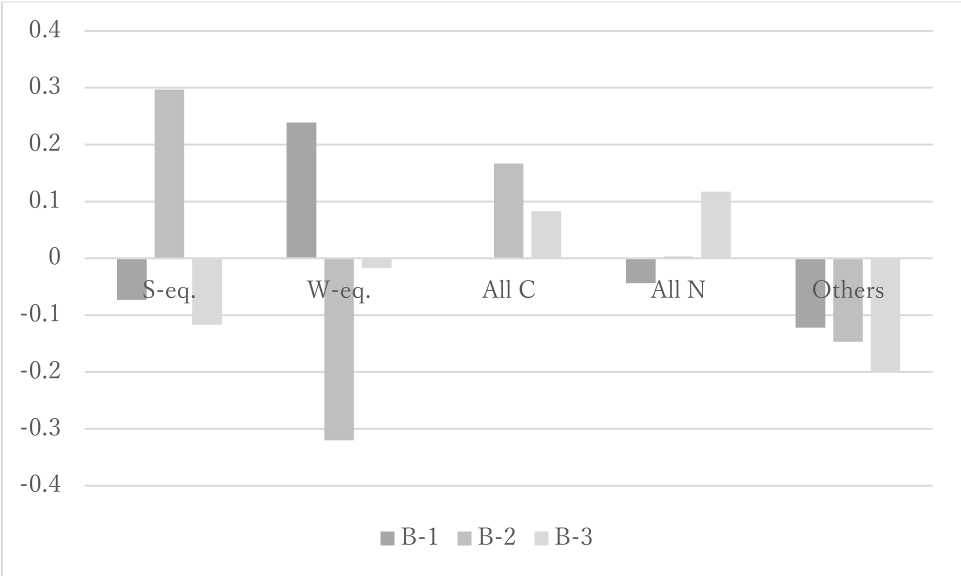
Asymmetric mixed strategy equilibrium predictions in the experiment are summarized in Table 12. The percentages of every possible outcome implied by the asymmetric mixed strategy equilibrium are shown in Table 13. Differences of outcome frequencies between predictions of asymmetric mixed strategy equilibrium in the experiment are also presented in Figure 2.

Table 12. Asymmetric mixed strategy equilibria.

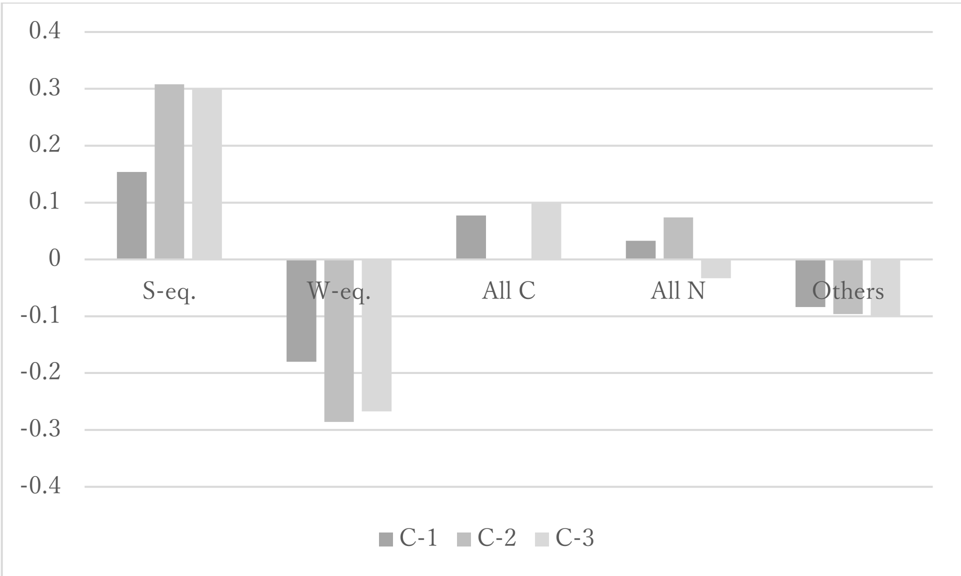
	B-1	B-2	B-3		C-1	C-2	C-3
p_S	0.778	0.600	0.600	p_S	0.000	0.000	0.000
p_W	0.000	0.000	0.000	p_S	0.778	0.600	0.600
p_W	0.800	0.800	0.667	p_W	0.800	0.800	0.667

Table 13. The frequencies of each outcome implied by asymmetric mixed strategy equilibria.

	B-1	B-2	B-3	C-1	C-2	C-3
S-eq.	0.156	0.120	0.200	0.000	0.000	0.000
W-eq.	0.178	0.320	0.267	0.334	0.440	0.467
All-*C*	0.000	0.000	0.000	0.000	0.000	0.000
All-*N*	0.440	0.800	0.133	0.440	0.800	0.133
Others	0.622	0.480	0.400	0.622	0.480	0.400



(a) Session B



(b) Session C

Figure 2. Differences of outcome frequencies between predictions of asymmetric mixed strategy equilibria in the experiment.

In Session B-1, the frequency of S-equilibrium in the experiment is slightly lower than theoretical predictions, while the frequency of W-equilibrium is significantly higher. The null hypothesis stating that the frequency of W-equilibrium is equal to the probability implied by the asymmetric mixed strategy equilibrium is rejected for Session B-1 (binomial test, $p < 0.01$), but the null hypothesis that the frequency of S-equilibrium is equal to the probability implied by the asymmetric mixed strategy equilibrium is not rejected. The opposite is the case in Session B-2 where the frequency of S-equilibrium in the experiment is significantly higher than the theoretical predictions (binomial test, $p < 0.01$), while the frequency of W-equilibrium is significantly lower (binomial test, $p < 0.01$). In Session B-3, non-equilibrium outcomes, All-*C*, All-*N*, and others, due to coordination failures, are prominent.

In Session C-1, C-2, and C-3, frequencies of S-equilibrium in the experiment are significantly higher than the theoretical predictions ($p < 0.01$ for each), while the frequencies of W-equilibrium are significantly lower. The null hypothesis stating that the frequency of W-equilibrium is equal to the probability implied by asymmetric mixed strategy equilibrium is not rejected for every session. In summary, we must conclude that the asymmetric mixed strategy equilibrium prediction also fails in our experimental data.

From these, we conclude that the mixed strategy equilibrium prediction fails in our data. Thus, we reject Hypothesis 2.

Then, how about equilibrium selection from risk dominance? We count the number of occurrences for each equilibrium and summarize them in Table 14. Here we count it as an S-equilibrium (W-equilibrium) when exactly one S-player (W-player) choose *C* and the other players choose *N*. Otherwise, the subjects made coordination failures, which are either over-cooperation where more than one player chooses *C* including every player choosing *C* (“All-*C*”) or played mutually non-cooperatively by choosing *N* (“All-N”). This is never an equilibrium in our setting.

Table 14. The percentage of each outcome in the experiment.

Session	A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
S-eq.	60.0	73.3	13.3	8.3	41.7	8.3	15.4	30.8	30.0
W-eq.	13.3	0.0	26.7	41.7	0.0	25.0	15.4	15.4	20.0
All-*C*	26.7	13.3	20.0	0.0	16.7	8.3	7.7	0.0	10.0
All-*N*	0.0	13.3	40.0	0.0	8.3	25.0	7.7	15.4	10.0
Others	---	---	---	50.0	33.3	20.0	53.8	38.4	30.0

While S-equilibrium always risk dominates W-equilibrium, theoretically, in every Session, sometimes the percentages of W-equilibrium are higher than that of S-equilibrium in our data (Sessions A-3, B-1, and B-3). More so, the percentages of non-cooperative outcomes (“All-*N*”) are higher than that of S-equilibrium in some Sessions (Sessions A-3 and B-3). In fact, the null hypothesis stating that the frequency of S-equilibrium is equal among three treatments (such as among A-1, A-2, and A-3) is rejected in both Sessions A and B (Chi-squared test, $p < 0.01$ for Session A and $p < 0.05$ for Session B).

If the sum of the percentages of W-equilibrium and the non-cooperative outcome is seen as counter-evidence against S-equilibrium prediction, risk dominance prediction fails in more than a half of Sessions (except for Sessions A-1, A-2, B-2 and C-3). Thus, we have to conclude from these data that the explanatory power of the risk dominance concept is very limited. Therefore, we reject Hypothesis 1.

Thus, Diekmann's conjecture that S-equilibrium is played more frequently when it is risk dominant also fails in our experiment. The mixed strategy equilibrium prediction fails as well. Therefore, we have to examine alternative theories with altruistic motivation or bounded rationality.

4.3 Econometric estimation

As in Section 3, we examine three alternative theories, inequality aversion, level-k model, and QRE as representative models in behavioral game theory. Model structures and estimation strategy are identical in Section 3.

A. Inequity aversion

Ranges of parameter values consistent with each equilibrium are shown in Table 15. Moreover, remember that the most prominent regularity in our data is that the frequencies of S-equilibrium vary among sessions, unlike the prediction of risk dominance.

In fact, from Table 14, S-equilibrium was observed most frequently in the second treatment in every Session (73.3%, 41.7% and 30.8% in Session A-2, B-2 and C-2 respectively), while W-equilibrium was observed most frequently in the third treatment in Sessions A and C (26.7% and 20.0% in Session A-3 and C-3 respectively) and the first treatment in Session B (41.7%).

In fact, the most frequent outcome was an S-equilibrium in sessions A-1, A-2, B-2, C-1, C-2, and C-3. However, All-N was the most frequent in sessions A-3 and B-3, and W-equilibrium was the most frequent in session B-3.

But these regularities are not consistent with inequality aversion for any parameter values shown in Table 15.

Table 15. The prediction of inequity aversion.[9]

	A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
S-eq.	$\alpha \leq \frac{2}{3}$	$\alpha \leq 2$	$\alpha \leq \frac{1}{3}$	$\alpha \leq \frac{4}{5}$	$\alpha \leq 4$	$\alpha \leq \frac{2}{3}$	$\alpha \leq \frac{4}{3}$	$\alpha \leq 4$	$\alpha \leq 1$
W-eq.	$any\ \alpha$	$\alpha \leq \frac{1}{2}$	$\alpha \leq \frac{4}{3}$	$\alpha + \beta \leq 7$	$\alpha \leq \frac{3}{2}$	$\alpha \leq 3$	$any\ \alpha$	$\alpha \leq \frac{3}{2}$	$any\ \alpha$
All-*N*	--	$2 \leq \alpha$	$\frac{4}{3} \leq \alpha$	$7 \leq \alpha + \beta$	$4 \leq \alpha$	$3 \leq \alpha$	--	$4 \leq \alpha$	--

B. Level-k model

Assume that $L0$ player, who is the least rational player, chooses C and N with a probability of 1/2 respectively. In the level-k model, Lk player chooses the best response to the actions taken by $L(k-1)$ players. If we assume $L_S = L_W = 0$ as in the experimental sessions, the best responses of Lk S-player and Lk W-player to the actions of $L(k-1)$ players up to $k = 4$ are summarized in Table 16.

As $L1$ and $L3$, as well as L2 and L4, choose exactly the same responses in every Session, it is enough to consider up to $L2$ in the analysis. Then, suppose that both an S-player and a W-player have the same level of strategic thinking. This assumption is natural and justifiable, as the subjects were in the same population, and each player's role was randomly determined in the experiment.

9 The symbol "—" implies that the outcome cannot be a Nash equilibrium irrespective of the values of α and β.

Table 16. Best response of *Lk* to *L(k-1)*

		A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
***L*1**	**S-player**	*C*	*C*	*N*	*C*	*C*	*N*	*C*	*C*	*N*
	W-player	*C*	*N*	*N*	*C*	*N*	*N*	*C*	*N*	*N*
***L*2**	**S-player**	*N*	*C*	*C*	*N*	*C*	*C*	*N*	*N*	*C*
	W-player	*N*	*N*	*C*	*N*	*N*	*C*	*N*	*N*	*C*
***L*3**	**S-player**	*C*	*C*	*N*	*C*	*C*	*N*	*C*	*C*	*N*
	W-player	*C*	*N*	*N*	*C*	*N*	*N*	*C*	*C*	*N*
***L*4**	**S-player**	*N*	*C*	*C*	*N*	*C*	*C*	*N*	*N*	*C*
	W-player	*N*	*N*	*C*	*N*	*N*	*C*	*N*	*N*	*C*

Remember that S-equilibrium was observed most frequently in the second treatment in every session in our experiment. The fact can be obtained if all the players are *L*1 (In this case, an S-player chooses *C* and a W-player chooses *N*.). Thus, the level-k model can explain a part of the regularities found in our experiment.

Note, however, that whatever level the players find themselves in, the level-k model predicts that a W-equilibrium where an S-player chooses *N* and a W-player chooses *C* never occur in every session. This contradicts the fact that the percentages of W-equilibrium are higher than that of S-equilibrium in some sessions in our experimental data.

C. Estimation results

Similar to the previous section, we conduct a maximum likelihood estimation for obtaining the parameter values for inequality aversion, the level-k model, and QRE. For each model, the maximum likelihood estimates are obtained by maximizing the following log-likelihood function with respect to ω.

$$LogL(\omega) = LogL^A(\omega) + LogL^B(\omega) + LogL^C(\omega),$$

where each term is defined similarly to $LogL^D(\omega)$ in Section 3.[10] Table 17 shows the estimated parameters. The log likelihood for the completely mixed strategy equilibrium (Eq-1 in Table 10) is also shown in this table.[11]

From Table 17, one can see that the value of AIC for QRE is minimum among them. Thus, we conclude that QRE is the best fit model in our experimental data. However, since $\lambda = 1/\mu$, a relatively higher value of $\hat{\mu} = 25.781$ means that players' choices are close to the ones made via a low level of rationality, as $\lambda = 0.039$. Next, the value of AIC for inequality aversion is lowest, but none of the parameters concerning inequality aversion, α and β, is significant, and only μ is significant. Finally, Level-1's performance is better than Level-2. All these facts indicate that our data is generated by players with a relatively low level of rationality.

But this does not necessarily imply that subject choice were random. QRE captures some of the regularities in our experimental data. In fact, Table B1 in Appendix B

[10] In this definition, it is implicitly assumed that players of the same type have the same choice probabilities in a fixed point. Even if we redefine the log likelihood function to consider the possibility in which players of the same type have different choice probabilities, no result of the estimation changes.

[11] As shown in Tables 10 and 12, there are the other mixed strategy equilibria in Session B and C. However, the AIC when assuming Eq-1 in Table 10 in all sessions is smaller than those for other combinations of equilibria.

shows choice probabilities implied by the estimated parameter values, which are closer to the data than the completely mixed strategy equilibrium prediction.

Table 17. Estimated parameters

	Completely mixed strategy	Inequality aversion	Level-1	Level-2	QRE
α	-----	0.0241	-----	-----	-----
β	-----	0.0547	-----	-----	-----
μ	-----	25.738**	22.752**	186.228+	25.781**
LogL	-237.324	-202.188	-205.784	-210.384	-202.351
AIC	474.648	410.377	413.569	422.767	406.702

** and + denote significance at the 1% and 10% levels, respectively.

6. Conclusion

We have analyzed a generalized version of an asymmetric VOD game where the cost for volunteering is different among players. In this game, there is S-equilibrium where a player with *less* cost contributes *more* often. S-equilibrium is intuitively appealing, and, in fact, it is an efficient outcome. Under certain plausible conditions, S-equilibrium is risk dominant for a general n-person game, which was, firstly, proved in this study. As many researchers accept the risk dominance concept as an equilibrium selection criterion, the plausibility of S-equilibrium is reinforced by that fact. Thus, our first task was to closely examine Diekmann's conjecture which states that when S-equilibrium is risk dominant, it is observed more frequently. However, this conjecture was not fully confirmed in both Diekmann’s and our experiments, even though S-equilibrium was risk dominant in every session.

As for the prediction by the mixed strategy equilibrium, which sometimes contradicts with our intuition, if we consider people’s volunteering decision in the time of a natural disaster, we can say that it is not an unacceptable prediction. Sometimes a player with *more* cost contributes *more* often, as the Parable of the Good Samaritan illustrates. However, again, it failed in explaining Diekmann’s and our data.

Thus, we further analyzed the game with alternative theories including inequality aversion, level-k model, and QRE as representative models in behavioral game theory. Interestingly, our econometric comparison results show that altruistic motivation reflected by the inequality aversion model play no role in explaining the data. Instead, QRE best fits the data. In fact, choice probabilities implied by QRE is closer than the mixed strategy equilibrium prediction.

For two-person games, it is shown that limiting QRE converges to risk dominant equilibrium. Thus, for two-person games, the frequency of S-equilibrium is relatively higher when S-equilibrium is risk dominant. However, for a general n-person game, whether limiting QRE converges to risk dominant equilibrium is unknown. Especially, if there are more than two S-players in the game, as in our experiment, the principal branch of QRE correspondence bifurcates because there exits at least two symmetric S-equilibria. Analyzing, theoretically, such complicated nonlinear dynamics is quite difficult at this moment. The numerical simulation also shows quite exotic behaviors and no convergence. Fortunately, as the best fit parameter of QRE was found before the principal branch of QRE correspondence bifurcates, our conclusion is intact, a close examination of limiting QRE in this game requires more elaboration in the future research.

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Appendix A. Proofs

Proof of Proposition 1

Proof. If some players take completely mixed strategies, then the rest of players choose N with probability one in an equilibrium θ. Thus $0 < q_i < 1$ for any $i \in C(\theta)$. Therefore, $E_i(C) = E_i(N)$ must hold, and we have

$$\prod_{j \neq i} q_j = \frac{K_i}{V_i - L_i},$$

for any $i \in C(\theta)$. Therefore, we have

$$\prod_{i \in C(\theta)} \left(\prod_{j \neq i} q_j \right) = \prod_{i \in C(\theta)} \left(\frac{K_i}{V_i - L_i} \right).$$

Note that for any player $h \in C(\theta)$, $C(\theta) \setminus \{h\} \neq \emptyset$ since at least two players belong to $C(\theta)$. Solving the left-hand side of the above equation for q_h, we have

$$\begin{aligned}
\prod_{i \in C(\theta)} \left(\prod_{j \neq i} q_j \right) &= \left(\prod_{j \neq h} q_j \right) \prod_{i \in C(\theta) \setminus \{h\}} \left(q_h \prod_{j \neq i,h} q_j \right) \\
&= {q_h}^{\#C(\theta)-1} \left(\prod_{j \neq h} q_j \right) \prod_{i \in C(\theta) \setminus \{h\}} \left(\prod_{j \neq i,h} q_j \right) \\
&= {q_h}^{\#C(\theta)-1} \left(\prod_{j \neq h} q_j \right) \left(\prod_{j \neq h} q_j \right)^{\#C(\theta)-2} \\
&= {q_h}^{\#C(\theta)-1} \left(\frac{K_h}{V_h - L_h} \right)^{\#C(\theta)-1}.
\end{aligned}$$

The third equality follows from the fact that $q_k = 1$ for any $k \notin C(\theta)$. Thus, we have equation (1) for any player $h \in C(\theta)$.

Q.E.D.

Proof of Proposition 2

Proof. By Proposition1, the probability of choosing N of a W-player, q_W, in the completely mixed strategy equilibrium is given by

$$q_W = \left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{V_W - L_W}{K_W}\right)^{m-1}\right]^{\frac{1}{n-1}}$$

This is always a positive number. Moreover, $1 > q_W$ holds by (2) since

$$1 > q_W \Leftrightarrow \left(\frac{V_S - L_S}{K_S}\right)^m > \left(\frac{V_W - L_W}{K_W}\right)^{m-1}.$$

The probability of choosing N of a S-player, q_S, in the completely mixed strategy equilibrium is given by

$$q_S = \left[\left(\frac{V_S - L_S}{K_S}\right)^{n-m-1} \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{1}{n-1}}.$$

This is always a positive number. Moreover,

$$1 > q_S \Leftrightarrow \left(\frac{V_W - L_W}{K_W}\right)^{n-m} > \left(\frac{V_S - L_S}{K_S}\right)^{n-m-1}.$$

Thus, q_W always satisfy $1 > q_W > 0$ and (3) gives the necessary and sufficient condition for $1 > q_S > 0$.

Q.E.D.

Proof of Proposition 3

Proof. For any S-player i with $q_i = q_S(n, m)$ in θ',

$$\prod_{j \neq i} q_j = \left[\left(\frac{V_S - L_S}{K_S}\right)^{n-m-1} \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{m-1}{n-1}} \left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{V_W - L_W}{K_W}\right)^{m-1}\right]^{\frac{n-m}{n-1}}$$

$$= \left(\frac{K_S}{V_S - L_S}\right).$$

Therefore, $E_i(C) = E_i(N)$. Similarly, for any W-player i with $q_i = q_W(n,m)$ in θ', $E_i(C) = E_i(N)$. For any player i with $q_i = 1$ in θ',

$$\prod_{j \neq i} q_j = \left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{1}{n-1}}.$$

Therefore, for any S-player i with $q_i = 1$ in θ',

$$E_i(N) > E_i(C) \Leftrightarrow \prod_{j \neq i} q_j < \left(\frac{K_S}{V_S - L_S}\right)$$

$$\Leftrightarrow \left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{1}{n-1}} < \left(\frac{K_S}{V_S - L_S}\right)$$

$$\Leftrightarrow \left(\frac{V_W - L_W}{K_W}\right)^{n-m} > \left(\frac{V_S - L_S}{K_S}\right)^{n-m-1}.$$

The last inequality is always true by (3) since $\Theta(n,m)$ has the completely mixed strategy equilibrium. For any W-player i with $q_i = 1$ in θ',

$$E_i(N) > E_i(C) \Leftrightarrow \prod_{j \neq i} q_j < \left(\frac{K_W}{V_W - L_W}\right)$$

$$\Leftrightarrow \left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{1}{n-1}} < \left(\frac{K_W}{V_W - L_W}\right)$$

$$\Leftrightarrow \left(\frac{V_S - L_S}{K_S}\right)^{m} > \left(\frac{V_W - L_W}{K_W}\right)^{m-1}.$$

The last inequality is always true by (2).

Q.E.D.

Proof of Proposition 4

Proof. Suppose that $\theta \in \Theta(n,m)$ is the completely mixed strategy equilibrium. Then, we have

$$\ln q_S = \frac{n-m-1}{n-1} \ln\left(\frac{V_S - L_S}{K_S}\right) - \frac{n-m}{n-1} \ln\left(\frac{V_W - L_W}{K_W}\right)$$

Differentiate both sides by n, we have

$$\frac{\mathrm{d}q_S}{\mathrm{d}n} = \frac{q_S}{(n-1)^2}\left[m\ln\left(\frac{V_S-L_S}{K_S}\right)-(m-1)\ln\left(\frac{V_W-L_W}{K_W}\right)\right] > 0.$$

The last inequality follows from the fact that

$$\frac{V_S-L_S}{K_S} > \frac{V_W-L_W}{K_W} > 1.$$

On the other hand, differentiate both sides by m, we have

$$\frac{\mathrm{d}q_S}{\mathrm{d}m} = -\frac{q_S}{(n-1)}\left[\ln\left(\frac{V_S-L_S}{K_S}\right)-\ln\left(\frac{V_W-L_W}{K_W}\right)\right] < 0$$

For W-player, we have

$$\ln q_W = -\frac{m}{n-1}\ln\left(\frac{V_S-L_S}{K_S}\right)+\frac{m-1}{n-1}\ln\left(\frac{V_W-L_W}{K_W}\right).$$

Differentiate both sides by n, we have

$$\frac{\mathrm{d}q_W}{\mathrm{d}n} = \frac{q_W}{(n-1)^2}\left[m\ln\left(\frac{V_S-L_S}{K_S}\right)-(m-1)\ln\left(\frac{V_W-L_W}{K_W}\right)\right] > 0$$

On the other hand, differentiate both sides by m, we have

$$\frac{\mathrm{d}q_W}{\mathrm{d}m} = -\frac{q_W}{(n-1)}\left[\ln\left(\frac{V_S-L_S}{K_S}\right)-\ln\left(\frac{V_W-L_W}{K_W}\right)\right] < 0.$$

Q.E.D.

Proof of Proposition 5'

Proof. Note that there are m S-equilibrium and $\mathrm{n}-m$ W-equilibrium. For any pair (s,t) where s is a S-equilibrium and t is a W-equilibrium, $M(s,t)=\{i,j\}$ where i is a S-player and j is a W-player, respectively, and players other than i and j choose *N* in both s and t. Therefore, $G_{ij}(s,t)$ is similar to Table 1, and we have

$$R_*(s,t) = R_{ij}(s,t) = \frac{K_W(V_S-L_S-K_S)}{K_S(V_W-L_W-K_W)}.$$

Thus, $R_*(s,t) > 1$ if and only if

$$\left(\frac{V_S - L_S}{K_S}\right) > \left(\frac{V_W - L_W}{K_W}\right).$$

Q.E.D.

Proof of Proposition 6

Proof. For player i, let $H(i)$ be the set of players such that $V_j - K_j \geq V_i$, $L(i)$ be the set of players such that $V_i - K_i > V_j - K_j$, and $M(i)$ be the set of players other than i such that $V_i > V_j - K_j \geq V_i - K_i$ (the players of same type as i belong to $M(i)$). Note that $H(i) \cap L(i) = \emptyset \; and \; H(i) \cap M(i) = \emptyset$ by their definition. Then, if all players choose *C*, the utility of player i is given by

$$(V_i - K_i) - \frac{\alpha}{n-1} \sum_{j \in H(i) \cup M(i)} \left(V_j - K_j - V_i + K_i\right) - \frac{\beta}{n-1} \sum_{j \in L(i)} \left(V_i - K_i - V_j + K_j\right).$$

On the other hand, if player i chooses *N* and the other players choose *C*, then player i's utility is

$$V_i - \frac{\alpha}{n-1} \sum_{j \in H(i)} \left(V_j - K_j - V_i\right) - \frac{\beta}{n-1} \sum_{j \in M(i) \cup L(i)} \left(V_i - V_j + K_j\right).$$

Therefore, if player i deviates from All-*C*, player i's utility will increase by

$$K_i + \frac{\alpha}{n-1} \sum_{j \in H(i)} \left\{\left(V_j - K_j - V_i + K_i\right) - \left(V_j - K_j - V_i\right)\right\}$$
$$+ \frac{\beta}{n-1} \sum_{j \in L(i)} \left\{\left(V_i - K_i - V_j + K_j\right) - \left(V_i - V_j + K_j\right)\right\}$$
$$+ \frac{\alpha}{n-1} \sum_{j \in M(i)} \left(V_j - K_j - V_i + K_i\right) - \frac{\beta}{n-1} \sum_{j \in M(i)} \left(V_i - V_j + K_j\right).$$

If $\beta = 0$, then the above equation is positive since $K_i > 0$ and the other terms are nonnegative. Suppose that $\beta > 0$ and rearrange this equation, we have,

$$K_i\left(1-\frac{\#M(i)}{n-1}+\frac{\alpha\#H(i)}{n-1}-\frac{\beta\#L(i)}{n-1}\right)+\frac{(1+\alpha)\#M(i)}{n-1}K_i$$

$$-\frac{\alpha+\beta}{n-1}\sum_{j\in M(i)}\left(V_i-V_j+K_j\right)$$

$$\geq K_i\left(1-\frac{\#M(i)}{n-1}+\frac{\beta}{n-1}\big(\#H(i)-\#L(i)\big)\right)+\frac{(1+\alpha)\#M(i)}{n-1}K_i$$

$$-\frac{\alpha+1}{n-1}\sum_{j\in M(i)}\left(V_i-V_j+K_j\right)$$

$$=K_i\left(1-\frac{\#M(i)}{n-1}+\frac{\beta}{n-1}\big(\#H(i)-\#L(i)\big)\right)$$

$$+\frac{1+\alpha}{n-1}\sum_{j\in M(i)}\left(K_i-V_i+V_j-K_j\right)$$

$$\geq K_i\left(1-\frac{\#M(i)}{n-1}+\frac{\beta}{n-1}\big(\#H(i)-\#L(i)\big)\right),$$

where $\#H(i), \#L(i), and\ \#M(i)$ be the number of their set, respectively. The first inequality follows from $\beta < 1$ and $\beta \leq \alpha$. The last inequality follows from $K_i - V_i + V_j - K_j \geq 0$ for any $j \in M(i)$. Since there are only two types of players, S and W, if $\#H(i) > 0$ then $\#L(i) = 0$, and vice versa. Suppose that $\#H(i) > 0$, then the last equation is positive since $\mathrm{n} - 1 \geq \#M(i)$ and $\beta > 0$. Suppose that $\#L(i) > 0$, then we have

$$K_i\left(1-\frac{\#M(i)}{n-1}+\frac{\beta}{n-1}\big(\#H(i)-\#L(i)\big)\right)=K_i\left(1-\frac{\#M(i)}{n-1}-\frac{\beta\#L(i)}{n-1}\right)$$

$$>K_i\left(1-\frac{\#M(i)}{n-1}-\frac{\#L(i)}{n-1}\right)=K_i\left(1-\frac{\#M(i)+\#L(i)}{n-1}\right)\geq 0,$$

since $\beta < 1$ and $n - 1 \geq \#M(i) + \#L(i)$.

Q.E.D.

Appendix B.

Table B1. Choice probabilities implied by the estimated parameters

Session		Actual Choice frequencies	Completely mixed strategy	QRE
A-1	p_S	0.800	0.600	0.591
A-1	p_w	0.400	0.667	0.509
A-2	p_S	0.867	0.333	0.703
A-2	p_w	0.133	0.667	0.298
A-3	p_S	0.333	0.333	0.510
A-3	p_w	0.467	0.429	0.399
B-1	p_S	0.333	0.503	0.542
B-1	p_w	0.542	0.553	0.507
B-1	p_w	0.542	0.553	0.507
B-2	p_S	0.750	0.106	0.768
B-2	p_w	0.375	0.553	0.287
B-2	p_w	0.375	0.553	0.287
B-3	p_S	0.333	0.307	0.518
B-3	p_w	0.458	0.423	0.396
B-3	p_w	0.458	0.423	0.396
C-1	p_S	0.500	0.529	0.537
C-1	p_S	0.500	0.529	0.537
C-1	p_w	0.615	0.576	0.487

C-2	p_S	0.538	0.368	0.588
C-2	p_S	0.538	0.368	0.588
C-2	p_w	0.231	0.684	0.290
C-3	p_S	0.450	0.368	0.493
C-3	p_S	0.450	0.368	0.493
C-3	p_w	0.500	0.473	0.365

Table B2. Raw data

Group	**Player**	**Choice**	**Group**	**Player**	**Choice**
1	**Strong**	**C**	**9**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
2	**Strong**	**C**	**10**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
3	**Strong**	**C**	**11**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
4	**Strong**	**N**	**12**	**Strong**	**C**
	Weak	**C**		**Weak**	**C**
5	**Strong**	**C**	**13**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
6	**Strong**	**N**	**14**	**Strong**	**C**
	Weak	**C**		**Weak**	**C**
7	**Strong**	**N**	**15**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**

8	**Strong**	**C**
	Weak	**N**

(a) Session A-1

Group	Player	Choice	Group	Player	Choice
1	**Strong**	**C**	**9**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
2	**Strong**	**C**	**10**	**Strong**	**C**
	Weak	**N**		**Weak**	**C**
3	**Strong**	**N**	**11**	**Strong**	**N**
	Weak	**N**		**Weak**	**N**
4	**Strong**	**C**	**12**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
5	**Strong**	**C**	**13**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
6	**Strong**	**C**	**14**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
7	**Strong**	**C**	**15**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
8	**Strong**	**C**			
	Weak	**N**			

(b) Session A-2

Group	Player	Choice	Group	Player	Choice
1	**Strong**	**C**	**9**	**Strong**	**C**

	Weak	**C**		**Weak**	**C**
2	**Strong**	**N**	**10**	**Strong**	**N**
	Weak	**N**		**Weak**	**N**
3	**Strong**	**N**	**11**	**Strong**	**N**
	Weak	**N**		**Weak**	**C**
4	**Strong**	**C**	**12**	**Strong**	**C**
	Weak	**N**		**Weak**	**C**
5	**Strong**	**C**	**13**	**Strong**	**N**
	Weak	**N**		**Weak**	**N**
6	**Strong**	**N**	**14**	**Strong**	**N**
	Weak	**N**		**Weak**	**C**
7	**Strong**	**N**	**15**	**Strong**	**N**
	Weak	**C**		**Weak**	**C**
8	**Strong**	**N**			
	Weak	**N**			

(c) Session A-3

Group	Player	Choice	Group	Player	Choice
1	**Strong**	**N**	**7**	**Strong**	**N**
	Weak	**N**		**Weak**	**C**
	Weak	**C**		**Weak**	**C**
2	**Strong**	**N**	**8**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
	Weak	**C**		**Weak**	**N**

3	**Strong**	**N**	**9**	**Strong**	**N**
	Weak	**N**		**Weak**	**N**
	Weak	**N**		**Weak**	**N**
4	**Strong**	**C**	**10**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
	Weak	**N**		**Weak**	**C**
5	**Strong**	**N**	**11**	**Strong**	**N**
	Weak	**C**		**Weak**	**C**
	Weak	**C**		**Weak**	**N**
6	**Strong**	**C**	**12**	**Strong**	**N**
	Weak	**C**		**Weak**	**N**
	Weak	**N**		**Weak**	**C**

(d) Session B-1

Group	Player	Choice	Group	Player	Choice
1	**Strong**	**N**	**7**	**Strong**	**C**
	Weak	**N**		**Weak**	**C**
	Weak	**N**		**Weak**	**C**
2	**Strong**	**N**	**8**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
	Weak	**C**		**Weak**	**C**
3	**Strong**	**C**	**9**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
	Weak	**N**		**Weak**	**C**

4	**Strong**	**N**	**10**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
	Weak	**C**		**Weak**	**N**
5	**Strong**	**C**	**11**	**Strong**	**C**
	Weak	**C**		**Weak**	**N**
	Weak	**C**		**Weak**	**N**
6	**Strong**	**C**	**12**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
	Weak	**N**		**Weak**	**N**

(e) Session B-2

Group	**Player**	**Choice**	**Group**	**Player**	**Choice**
1	**Strong**	**C**	**7**	**Strong**	**N**
	Weak	**C**		**Weak**	**N**
	Weak	**C**		**Weak**	**C**
2	**Strong**	**N**	**8**	**Strong**	**N**
	Weak	**N**		**Weak**	**C**
	Weak	**C**		**Weak**	**C**
3	**Strong**	**C**	**9**	**Strong**	**N**
	Weak	**C**		**Weak**	**N**
	Weak	**N**		**Weak**	**N**
4	**Strong**	**N**	**10**	**Strong**	**N**
	Weak	**C**		**Weak**	**N**
	Weak	**N**		**Weak**	**N**

5	**Strong**	**C**	**11**	**Strong**	**C**
	Weak	**N**		**Weak**	**N**
	Weak	**N**		**Weak**	**C**
6	**Strong**	**N**	**12**	**Strong**	**N**
	Weak	**C**		**Weak**	**N**
	Weak	**C**		**Weak**	**N**

(f) Session B-3

Group	Player	Choice	Group	Player	Choice
1	**Strong**	**N**	**8**	**Strong**	**N**
	Strong	**N**		**Strong**	**C**
	Weak	**C**		**Weak**	**C**
2	**Strong**	**N**	**9**	**Strong**	**C**
	Strong	**N**		**Strong**	**C**
	Weak	**N**		**Weak**	**C**
3	**Strong**	**C**	**10**	**Strong**	**C**
	Strong	**N**		**Strong**	**N**
	Weak	**C**		**Weak**	**N**
4	**Strong**	**C**	**11**	**Strong**	**C**
	Strong	**C**		**Strong**	**N**
	Weak	**N**		**Weak**	**N**
5	**Strong**	**N**	**12**	**Strong**	**C**
	Strong	**C**		**Strong**	**C**
	Weak	**C**		**Weak**	**C**

6	**Strong**	**C**	**13**	**Strong**	**N**
	Strong	**C**		**Strong**	**N**
	Weak	**N**		**Weak**	**C**
7	**Strong**	**N**			
	Strong	**C**			
	Weak	**C**			

(g) Session C-1

Group	Player	Choice	Group	Player	Choice
1	**Strong**	**N**	**8**	**Strong**	**C**
	Strong	**N**		**Strong**	**C**
	Weak	**C**		**Weak**	**N**
2	**Strong**	**C**	**9**	**Strong**	**C**
	Strong	**C**		**Strong**	**C**
	Weak	**N**		**Weak**	**C**
3	**Strong**	**C**	**10**	**Strong**	**N**
	Strong	**N**		**Strong**	**N**
	Weak	**N**		**Weak**	**C**
4	**Strong**	**C**	**11**	**Strong**	**N**
	Strong	**C**		**Strong**	**C**
	Weak	**N**		**Weak**	**N**
5	**Strong**	**N**	**12**	**Strong**	**N**
	Strong	**C**		**Strong**	**C**
	Weak	**N**		**Weak**	**N**

6	**Strong**	**C**	**13**	**Strong**	**N**
	Strong	**C**		**Strong**	**N**
	Weak	**N**		**Weak**	**N**
7	**Strong**	**N**			
	Strong	**N**			
	Weak	**N**			

(h)Session C-2

Group	**Player**	**Choice**	**Group**	**Player**	**Choice**
1	**Strong**	**N**	**6**	**Strong**	**N**
	Strong	**N**		**Strong**	**N**
	Weak	**N**		**Weak**	**C**
2	**Strong**	**N**	**7**	**Strong**	**C**
	Strong	**C**		**Strong**	**C**
	Weak	**C**		**Weak**	**C**
3	**Strong**	**C**	**8**	**Strong**	**C**
	Strong	**N**		**Strong**	**N**
	Weak	**N**		**Weak**	**N**
4	**Strong**	**N**	**9**	**Strong**	**N**
	Strong	**C**		**Strong**	**N**
	Weak	**C**		**Weak**	**C**
5	**Strong**	**C**	**10**	**Strong**	**C**
	Strong	**C**		**Strong**	**N**
	Weak	**N**		**Weak**	**N**

(i) Session C-3

Appendix C. Instructions (originally in Japanese)

Session A-1: Instructions

In this experiment, you are paired with another player and make a decision.

Pairing is randomly determined and you are not informed of whom you are paired with during and after the experiment.

Your role in the experiment is either X or Y, which is randomly determined. As a result, there are a player whose role is X and a player whose role is Y in each pair.

Please check in the recording sheet about which role do you play in the experiment.

In the experiment, each player in a pair chooses option A or B simultaneously and independently. If a player X chooses A, he/she has to pay 200 JPY. If a player Y chooses A, he/she has to pay 400 JPY. If each player chooses B, no one needs to pay anything.

Depending on the number of players who chose A, each player's payoff is determined by the following table.

Payoff table

	# of player who chose A other than you	**0**	**1**
Player X	**Option A**	400	400
	Option B	0	600
Player Y	**Option A**	600	600

	Option B	0	1000

Thus, if one member in your pair chooses A, a player X gains 600 JPY and a player Y 1000 JPY. Please note that if your role is X and choose A, as you have to pay 200 JPY, your payoff becomes 400 JPY and that if your role is Y and choose A, as you have to pay 400 JPY, your payoff becomes 600 JPY.

If no one chooses A including yourself, each player gains nothing.

Then, once you decide your choice, draw a circle on A or B in the recording sheet. After everyone's choice is made, experimenters collect your recording sheet.

If this is your first experiment today, please wait for a while for next experiment. Your outcome in the experiment is not informed before all the experiments ends. If this is your second experiment today, we sum up your outcomes in both experiments and pay the total to you in cash.

Session A-1: Recording sheet

Your identity：Pair (　　) Role (　　) Player (　　)

Payoff table

	# of player who chose A other than you	**0**	**1**
Player X	**Option A**	400	400
	Option B	0	600
Player Y	**Option A**	600	600
	Option B	0	1000

Your choice	A or B

Session B-1: Instructions

In this experiment, you are paired with other two players and make a decision. Pairing is randomly determined and you are not informed of whom you are paired with during and after the experiment.

Your role in the experiment is either X or Y, which is randomly determined. As a result, there are a player whose role is X and two players whose role are Y in each group. Please check in the recording sheet about which role do you play in the experiment.

In the experiment, each player in a group chooses option A or B simultaneously and independently. If a player X chooses A, he/she has to pay 200 JPY. If a player Y chooses A, he/she has to pay 400 JPY. If each player chooses B, no one needs to pay anything.

Depending on the number of players who chose A, each player's payoff is determined by the following table.

Payoff table

	# of player who chose A other than you	**0**	**1**	**2**

Player X	**Option A**	800	800	800
	Option B	0	1000	1000
Player Y	**Option A**	1400	1400	1400
	Option B	0	1800	1800

Thus, if one member in your group chooses A, a player X gains 1000 JPY and a player Y 1800 JPY. Please note that if your role is X and choose A, as you have to pay 200 JPY, your payoff becomes 800 JPY and that if your role is Y and choose A, as you have to pay 400 JPY, your payoff becomes 1400 JPY.

If no one chooses A including yourself, each player gains nothing.

Then, once you decide your choice, draw a circle on A or B in the recording sheet. After everyone's choice is made, experimenters collect your recording sheet.

If this is your first experiment today, please wait for a while for next experiment. Your outcome in the experiment is not informed before all the experiments ends. If this is your second experiment today, we sum up your outcomes in both experiments and pay the total to you in cash.

Session B-1: Recording sheet

Your identity：Pair (　　)　Role (　　)　Player (　　)

Payoff table

	# of player who chose A other than you	**0**	**1**	**2**

Player X	**Option A**	800	800	800
	Option B	0	1000	1000
Player Y	**Option A**	1400	1400	1400
	Option B	0	1800	1800

Your choice	A or B