



# Evolutionary dynamics of N-person snowdrift game



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## ABSTRACT

In this letter, we investigate the evolutionary dynamics of N-person snowdrift game in both well-mixed and structured populations. For well-mixed populations, we construct a double-threshold model considering both the necessary and the minimum cost players should pay for completing the task. We have explored the influences of these thresholds on both equilibrium points in infinite populations and the fixation probabilities in finite populations. Results present complicated behaviors that show characteristics of both stag-hunt game and snowdrift game. For structured populations, we use pair approximation and diffusion approximation to derive the critical benefit-to-cost ratio in favor of cooperation.

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## 1. Introduction

Cooperation is ubiquitous in natural and social systems. The emergence and persistence of cooperation remains an evolution riddle [1,2]. Evolutionary game theory has provided a powerful framework addressing this issue [3–5]. Most of the models on the evolution of cooperation consider pairwise interactions: a type C interacting with another type D receives payoff  $a$ ; two interacting D types get  $d$  each; Type C interacting with D gets  $b$ ; whereas D obtains  $c$ . This symmetric game can be described as the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{c} C \\ D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array}$$

The prisoner's dilemma game (PDG) [6–12] is defined by the payoff ranking  $c > a > d > b$ . The snowdrift game (SG) can be illustrated by the following scenario: Two drivers are trapped in a storm and on either side of a snowdrift. If they both cooperate to shovel the snow, they both get benefit  $b$  of getting home and share the cost of shoveling  $c$ . Thus, the payoff for each is  $b - c/2$ . If one defects (never gets out of the car) and the other cooperates, then the defector gets  $b$  and the cooperator obtains  $b - c$ . If both defect, then the payoff for each is 0. Both strategies are best replies to each other, which leads to a “coexistence game”. In the above mentioned general form for  $2 \times 2$  game, the payoff values satisfy  $c > a > b > d$ . While in the stag-hunt game, each strategy is the best reply to itself, corresponding to the coordination case where  $a > c > d > b$ . Besides the commonly used metaphor of prisoner's dilemma game (or other 2-person games) in pairwise interactions, researchers have adopted the public goods game (PGG) [13–19] as the representative for group interactions in such issues as “The tragedy of the commons” [20], where the group benefits increase with members' total contributions (formalized as the cost to cooperator (C), as opposed to the defector (D) who pays no cost), but are equally shared by each member irrespective of his or her contribution. However, there also exist other situations in the real world that every group member benefits if a public enterprise is accomplished but the costs are equally shared by the contributors only. These situations are well captured by the N-person snowdrift game (NSG) [21–24], which has so far received relatively little attention.

The NSG differs from classical PGG in that an extra cooperator does not add to any group benefit and thus often induces nonlinear payoff values naturally. (Note that nonlinear effect can also be observed in PGG when considering the “critical mass” of cooperators [25,26].) As a natural generalization of the traditional 2-person snowdrift game, Zheng et al. first studied the replicator dynamics of the N-person snowdrift game, where the costs of the collective effort are divided by the number of cooperators [27]. Souza et al. have

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proposed an N-person snowdrift model considering the minimum threshold  $M$  for the number of cooperators below which the task cannot be fulfilled and thus no benefit is generated [28]. However, in the real world critical mass plays an important role in collective actions [29,30], the cost of participating a public endeavor cannot decrease infinitely with increasing number of cooperators. It would be intriguing to consider the influence on evolutionary dynamics of the upper threshold, above which an extra cooperator does not further reduce the cost. Moreover, the works mentioned above have assumed well-mixed populations. That is to say, everyone is equally likely to interact with others. Network reciprocity [31,32], i.e., interactions among neighbors in structured population, as one of the most important mechanisms supporting cooperation, has received intensive studies in recent years [33–38]. Hauert et al. studied the snowdrift game on square lattices and found that spatial structure often inhibits the evolution of cooperation [39]. Ohtsuki et al. derived a concise condition for the evolution of cooperation on cycles [33] and on general graph [34] for general two-person games. For the multi-player game on structured population, only the situation on the cycle is considered by van Veelen [40]. In contrast to the extensive researches of PGG on a variety of social networks, studies of NSG on networks are relatively lacking [41]. Up till now, universal understanding of multi-player snowdrift game on structured populations is still lacking from a theoretical point of view. Therefore extending the multi-player snowdrift game to more general networks would be necessary.

Inspired by all above, we propose a general model for multi-person snowdrift game considering both the minimum and maximum cost one individual pays for completing the task, and investigate its evolutionary dynamics in both infinite and finite well-mixed populations. For the structured population, we derive theoretically the critical benefit-to-cost values for selection to favor cooperation in NSG played on regular graphs. These values are also confirmed by computer simulations.

**2. Model**

In the following, we conduct our discussion in both well-mixed populations and structured populations.

*2.1. Well-mixed populations*

Here we consider a group consisting of  $N_g$  individuals playing the multi-person snowdrift game with two thresholds  $M_1$  and  $M_2$  ( $N_g \geq M_2 \geq M_1 \geq 1$ ). Assume that there are  $N_C$  cooperators in the group. For  $N_C < M_1$ , each cooperator pays a cost of  $c/M_1$  and defector nothing, but no benefit is produced. The lower threshold  $M_1$  indicates that the total cost  $c$  of the collective enterprise cannot be borne by a single person, and the maximum capability of a cooperator is  $c/M_1$ . For  $M_1 \leq N_C \leq M_2$ , the total cost  $c$  is equally shared by all the cooperators, and each individual in the group gains the benefit of  $b$ . While for  $N_C > M_2$ , each cooperator pays a cost of  $c/M_2$  and defector nothing. The upper threshold  $M_2$  indicates that the cost for each cooperator cannot decrease infinitely with increasing  $N_C$ , and  $c/M_2$  is the minimum cost of participation. In PGG behavior experiments, Capraro and Barcelo have proved that the benefit of full cooperation increases linearly for early contribution and then remains constant after a critical mass is reached [42]. Hence the upper threshold plays an important role in the evolution of cooperation, although in our model the payoff increases nonlinearly with number of cooperators.

For any composition of the group, the payoff for a cooperator or a defector is presented in the following payoff matrix:

	$N_g - 1$	...	$M_2$	$M_2 - 1$	...	$M_1$	$M_1 - 1$	...	1	0
C	$b - \frac{c}{M_2}$	...	$b - \frac{c}{M_2}$	$b - \frac{c}{M_2}$	...	$b - \frac{c}{M_1+1}$	$b - \frac{c}{M_1}$	...	$-\frac{c}{M_1}$	$-\frac{c}{M_1}$
D	b	...	b	b	...	b	0	...	0	0

where the left column indicates the strategy of this player, top column indicates the number of cooperators in the rest of  $N_g - 1$  players and each entry corresponds to the payoff this player acquires. We will use replicator dynamics equations to analyze its evolutionary dynamics in infinite populations, and study how the benefit-to-cost ratio  $b/c$ ,  $M_1$ , and  $M_2$  affect the equilibrium points of the dynamics. As for finite populations, we will use the stochastic methods to investigate the influences of  $b/c$ ,  $M_1$ ,  $M_2$ , and selection intensity  $\beta$  on the fixation probability of cooperation or defection in the population.

*2.2. Structured populations*

Structured populations take into account the fact that interactions among players are not random but are limited by spatial distribution of the players, and as such are best described by the networks. We consider a total population of size  $N$  consisting of cooperators and defectors on the network. The vertices of the network stand for game players. The edges denote the links between individuals in terms of game dynamic interactions. For the NSG, players participate in games organized by themselves and their neighbors, so that there will be  $k + 1$  rounds of games for each player on the homogeneous network of degree  $k$  [43]. After playing all games belonging to one individual, the individual turns the accumulated payoff  $P$  into his or her fitness by the principle  $f = 1 - w + wP$  ( $w \in [0, 1]$ ), where the parameter  $w$  is the selection intensity, i.e., the extent to which the payoff influences the fitness.  $w \rightarrow 0$  leads to the weak selection and  $w \rightarrow 1$  leads to the strong selection. As the evolutionary dynamics, here we adopt the ‘death-birth’ updating rule [34]. At each time step, a random individual is chosen to die, and subsequently the neighbors compete for the vacant site in proportion to their fitness. We focus on the fixation probability of cooperator (defector), i.e., the probability of a single cooperator (defector) turning the whole population from defection (cooperation) to cooperation (defection). By comparing it to  $1/N$ , the fixation probability of a neutral mutant, we can derive the critical benefit-to-cost ratio  $b/c$  for natural selection to favor cooperation or defection, which is dependent on the average degree  $k$  of the network on which the NSG is played.

The theoretical approximations are confirmed by Monte Carlo simulations, and the flow diagram of which is given in Fig. 1. We construct a network of size  $N$  and each node is occupied by one individual. Initially the whole population is assigned to be defectors except one as a cooperator. Firstly, individuals play NSG with their direct neighbors and accumulate the payoffs. Secondly, individuals change their strategies according to the updating rule. These two processes are repeated until the whole population is turned into cooperators (fixation) or defectors (extinction). If cooperators neither fix nor go extinct after a given time steps, the process will be terminated and the result

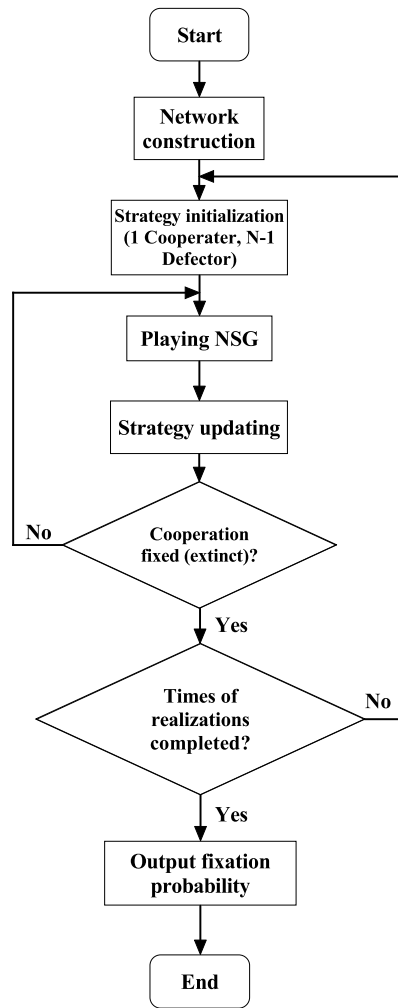


Fig. 1. The flow diagram for the Monte Carlo simulations of calculating the fixation probability of one cooperator on the network.

is discarded. To calculate the fixation probability of a cooperator, we perform the above realizations for  $run_{count}$  times, and sum up the times that cooperators are fixed as  $fix_{count}$ . The fixation probability is given by  $\rho_c = fix_{count}/run_{count}$ . We have ensured that each realization runs for steps long enough so that the discarded times will not affect the accuracy of results. The benefit-to-cost value  $b/c$  is adjusted increasingly until the fixation probability of one single cooperator exceeds the neutral case  $1/N$ . We take this as the critical benefit-to-cost value for benefiting the fixation of cooperation. Similarly, the critical benefit-to-cost value benefiting defection can also be obtained. Our simulations are not only carried out on regular graphs, but also on heterogeneous network like Erdős–Renyi random network and BA scale-free network.

### 3. Analysis and results

#### 3.1. Well-mixed population

##### 3.1.1. Infinite population

The time evolution of the fraction of cooperators  $x$  is given by the replicator equation [4,44]

$$\dot{x} = x(1-x)(\pi_C - \pi_D), \quad (3.1)$$

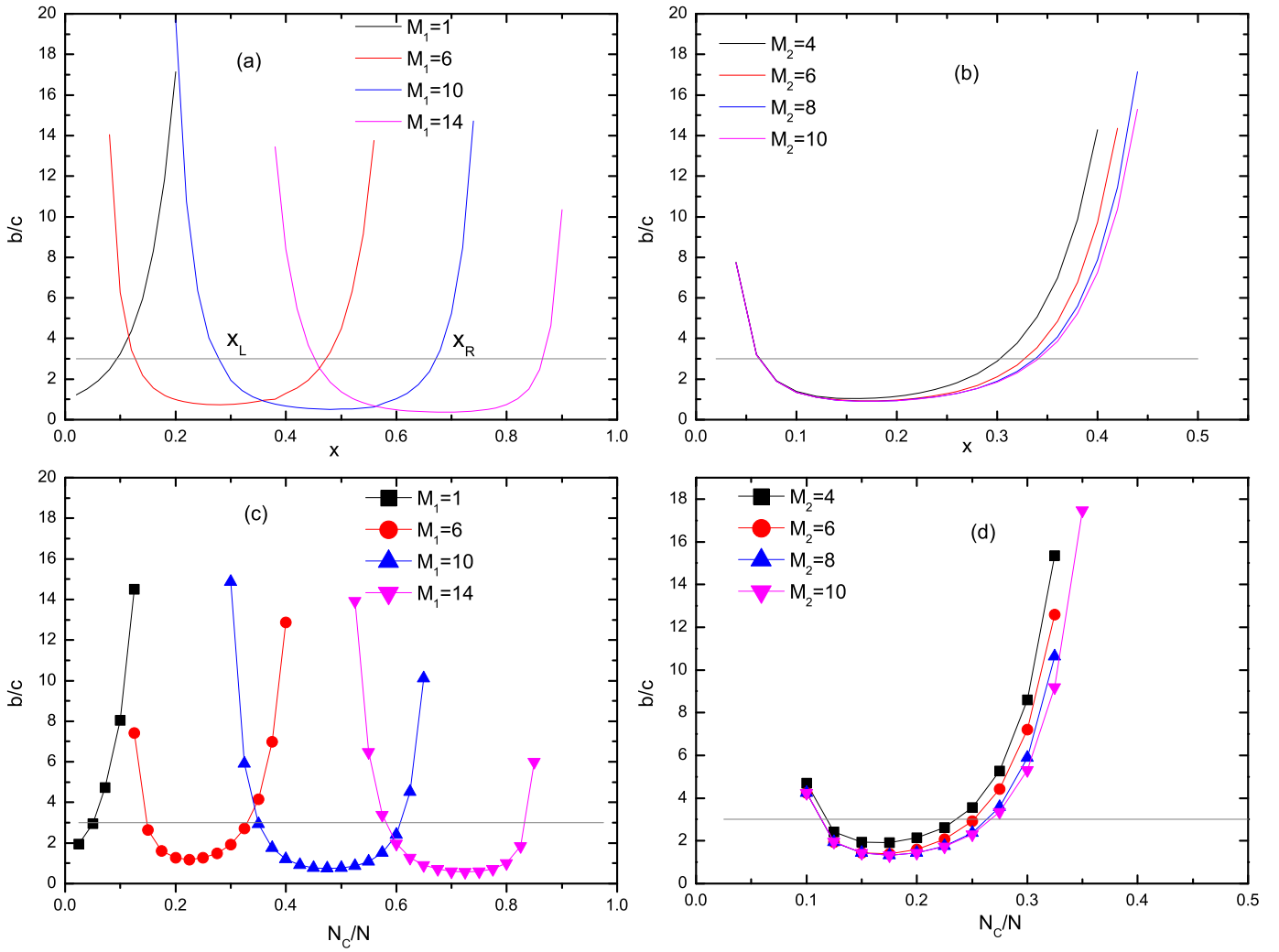
where

$$\pi_C = \sum_{N_C=0}^{N_g-1} C_{N_g-1}^{N_C} x^{N_C} (1-x)^{N_g-1-N_C} \Pi_C(N_C+1) \quad (3.2)$$

and

$$\pi_D = \sum_{N_C=0}^{N_g-1} C_{N_g-1}^{N_C} x^{N_C} (1-x)^{N_g-1-N_C} \Pi_C(N_C). \quad (3.3)$$

In each group of size  $N_g$  with  $N_C$  cooperators, the payoff of Cs and Ds are as follows:



**Fig. 2.** Equilibrium points of multi-player snowdrift game with double thresholds in well-mixed populations.  $x$  and  $N_c/N$  denote the fraction of cooperators in infinite populations ((a) and (b)) and finite populations ((c) and (d)), respectively. Panels (a) and (c) show the influence of lower threshold with fixed upper threshold  $M_2 = 14$ . Panels (b) and (d) demonstrate the effect of the upper threshold with fixed lower threshold  $M_1 = 4$ . Group size is  $N_g = 20$  for the above panels, and population size is  $N = 40$  for Panels (c) and (d).

$$\Pi_D(N_c) = b\Theta(N_c - M_1) \quad (3.4)$$

$$\Pi_C(N_c) = \Pi_D(N_c) - \frac{c}{N_c} [\Theta(N_c - M_1) - \Theta(N_c - M_2)] - \frac{c}{M_1} [1 - \Theta(N_c - M_1)] - \frac{c}{M_2} \Theta(N_c - M_2), \quad (3.5)$$

where the Heaviside function  $\Theta(N_c - L)$  is defined as the characteristic function of the nonnegative numbers  $\chi_{[0, \infty)}$ . Therefore

$$\begin{aligned} \pi_C - \pi_D = \frac{c}{xN_g} \left\{ \frac{b}{c} N_g C_{N_g-1}^{M_1-1} x^{M_1} (1-x)^{N_g-M_1} - \left[ 1 + \sum_{N_c=0}^{M_1-1} C_{N_g}^{N_c} x^{N_c} \right. \right. \\ \left. \left. \times (1-x)^{N_g-N_c} \left( \frac{N_c}{M_1} - 1 \right) + \sum_{N_c=M_2}^{N_g} C_{N_g}^{N_c} x^{N_c} (1-x)^{N_g-N_c} \left( \frac{N_c}{M_2} - 1 \right) \right] \right\}. \end{aligned} \quad (3.6)$$

Let  $\pi_C - \pi_D = 0$ , we obtain the relation between benefit-to-cost ratio  $b/c$  and the equilibrium point  $x^*$ :

$$\frac{b}{c} = \frac{1 + \sum_{N_c=0}^{M_1-1} C_{N_g}^{N_c} x^{*N_c} (1-x^*)^{N_g-N_c} \left( \frac{N_c}{M_1} - 1 \right) + \sum_{N_c=M_2}^{N_g} C_{N_g}^{N_c} x^{*N_c} (1-x^*)^{N_g-N_c} \left( \frac{N_c}{M_2} - 1 \right)}{N_g C_{N_g-1}^{M_1-1} x^{*M_1} (1-x^*)^{N_g-M_1}}. \quad (3.7)$$

We first consider the case with fixed upper threshold  $M_2$ . When  $M_1 = 1$  (no lower threshold), the system has only one equilibrium point for any given value of  $b/c$  and it is a stable co-existence point, as illustrated in Fig. 2(a). When  $1 < M_1 \leq M_2$ , the number of equilibrium points depends on the value of  $b/c$ . There exists a minimum value  $\eta$  along the line of critical  $b/c$ . If  $b/c > \eta$ , it will lead to two interior equilibrium points  $x_L$  (the left one) and  $x_R$  (the right one).  $x_L$  is an unstable point and  $x_R$  is a stable one. For initial frequency of cooperators  $x < x_L$ ,  $x$  will stabilize at 0. For  $x > x_L$ , it will stabilize at  $x_R$ . If  $b/c = \eta$ ,  $x_L = x_R$  and it is an unstable point. Cooperation level will decline to 0 ( $x < x_L$ ) or increase to 1 ( $x > x_L$ ). If  $b/c < \eta$ , there will be no interior equilibrium point. The above conclusion is similar to that in [28], and the appearance of  $M_1$  will add one coordination point  $x_L$  to the replicator dynamics, as we can see from Fig. 2(a). With increasing  $M_1$ , both  $x_L$  and  $x_R$  move rightwards. This indicates that larger lower threshold  $M_1$  increases the level of cooperation (i.e.,

$x_R$ ) once it is stabilized. Meanwhile, it becomes harsher for cooperation to be established since larger initial fraction of cooperators (i.e.,  $x_L$ ) is required.

When the lower threshold  $M_1$  is fixed and the upper threshold changes from  $M_2$  to  $M_2 + 1$ , we have the following inequality:

$$\sum_{N_C=M_2}^N C_{N_g-1}^{N_C} x^{N_C} (1-x)^{N_g-N_C} \left(\frac{N_C}{M_2} - 1\right) > \sum_{N_C=M_2+1}^{N_g} C_{N_g-1}^{N_C} x^{N_C} (1-x)^{N_g-N_C} \left(\frac{N_C}{M_2+1} - 1\right). \tag{3.8}$$

Therefore  $b/c$  is monotone-decreasing function of  $M_2$ , which means larger  $M_2$  always facilitates cooperation (see Fig. 2(b)).

Sometimes a collective task requires at least  $M_1$  cooperators to bear the total cost  $c$ , and meanwhile each cooperator makes a constant contribution  $c/M_1$ . This corresponds to the special case  $M_1 = M_2 = M$  and we have the following identity

$$\pi_C - \pi_D = bC_{N_g-1}^{M-1} x^{M-1} (1-x)^{N_g-M} - \left(\frac{c}{M}\right) \sum_{N_C=0}^{N_g-1} C_{N_g-1}^{N_C} x^{N_C} (1-x)^{N_g-N_C}. \tag{3.9}$$

Let  $\pi_C - \pi_D = 0$ , we get the value of  $b/c$  as  $1/(MC_{N_g-1}^{M-1} x^{M-1} (1-x)^{N_g-M})$ , and let

$$\left(\frac{b}{c}\right)'_x = -\frac{MC_{N_g-1}^{M-1} x^{M-2} (1-x)^{N_g-M-2} [M-1-(N_g-1)x]}{[MC_{N_g-1}^{M-1} x^{M-1} (1-x)^{N_g-M}]^2} = 0, \tag{3.10}$$

we derive that one zero point of the function is  $x = 0$ , and the other zero point is  $x = (M-1)/(N_g-1)$ . Hence the critical value of  $b/c$  associated with the equilibrium point  $x$  decreases in the interval  $(0, (M-1)/(N_g-1))$ , reaches the minimum value at  $(M-1)/(N_g-1)$ , and increases in the interval  $((M-1)/(N_g-1), 1)$ .

### 3.1.2. Finite population

In a well-mixed finite population of size  $N$  with  $N_C$  individuals of type C and  $N - N_C$  individuals of type D, groups of size  $N_g$  are assembled randomly, and thus the probability of selecting  $j$  individuals of type C and  $N_g - j$  individuals of type D obeys a hyper-geometric distribution [45]. The payoffs of type C and D are respectively:

$$\pi_C(N_C) = \frac{\sum_{j=0}^{N_g-1} C_{N_C-1}^j C_{N-N_C}^{N_g-j-1} \Pi_C(j+1)}{C_{N-1}^{N_g-1}} \tag{3.11}$$

$$\pi_D(N_C) = \frac{\sum_{j=0}^{N_g-1} C_{N_C}^j C_{N-N_C-1}^{N_g-j-1} \Pi_D(j)}{C_{N-1}^{N_g-1}} \tag{3.12}$$

where  $\Pi_C(j+1)$  and  $\Pi_D(j)$  are defined in Eq. (3.4) and Eq. (3.5). Strategy is updated following a pairwise comparison process [46], and the strategy C replaces D with a probability given by Fermi function [47,48]

$$p = \frac{1}{1 + \exp(-\beta(\pi_C(N_C) - \pi_D(N_C)))} \tag{3.13}$$

Then the transition probability that the number of cooperators changes from  $N_C$  to  $N_C + 1$  (or from  $N_C$  to  $N_C - 1$ ) is

$$T_{N_C}^{\pm} = \frac{N_C}{N} \frac{N - N_C}{N} \frac{1}{1 + \exp(\mp\beta(\pi_C(N_C) - \pi_D(N_C)))} \tag{3.14}$$

For large populations, the evolution process can be approximated by stochastic differential equation

$$dX = (T_{N_C}^+ - T_{N_C}^-)dt + \sqrt{\frac{T_{N_C}^+ + T_{N_C}^-}{N}} dW, \tag{3.15}$$

with the drift term  $T_{N_C}^+ - T_{N_C}^-$  and diffusion term  $\sqrt{(T_{N_C}^+ + T_{N_C}^-)/N}$  [48]. For the pairwise comparison process, we have

$$\dot{x} = x(1-x) \tanh\left(\frac{\beta}{2}(\pi_C(x) - \pi_D(x))\right) + \sqrt{\frac{x(1-x)}{N}} \xi. \tag{3.16}$$

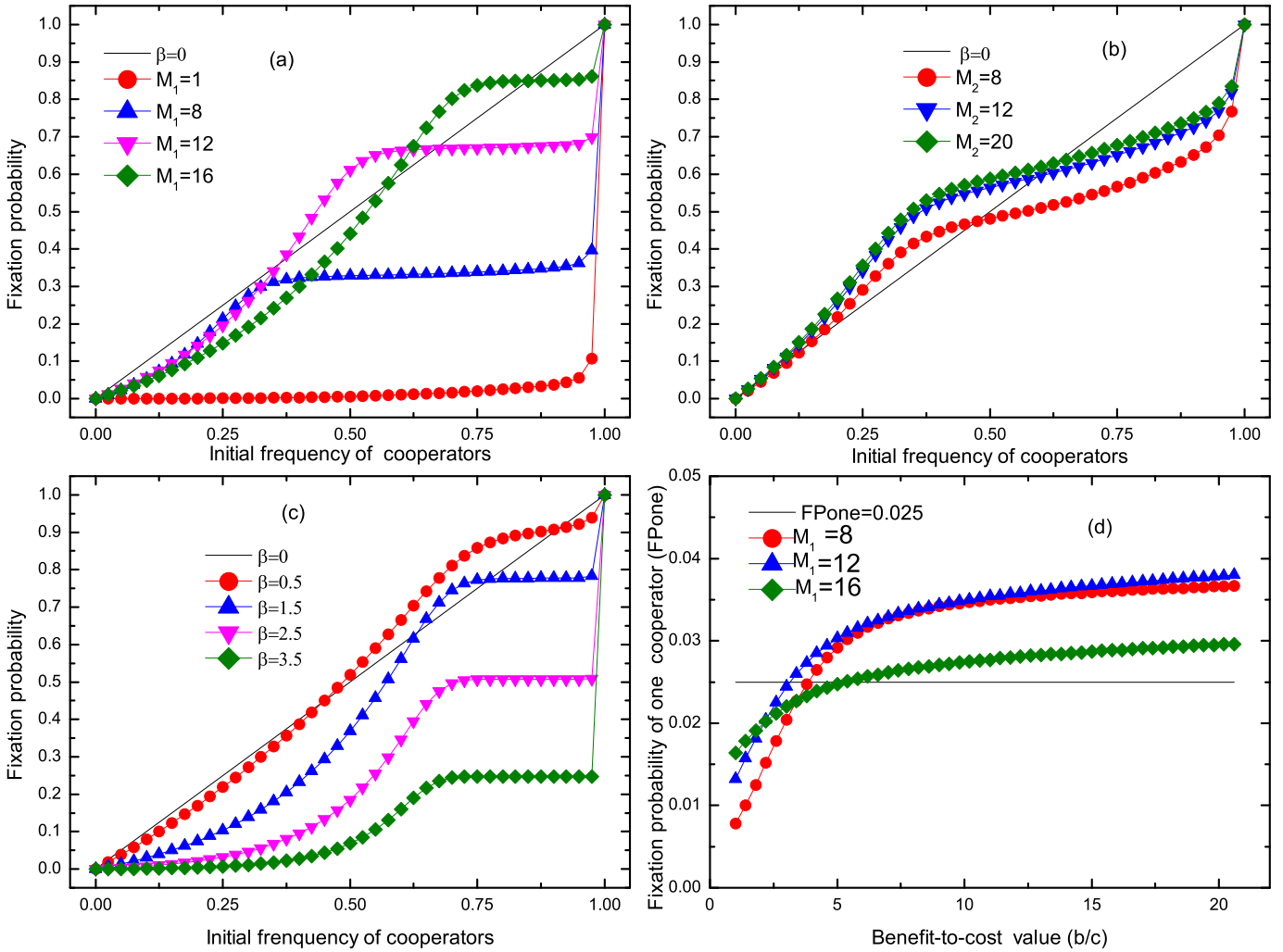
For  $N \rightarrow \infty$ , Gaussian white noise  $\xi$  vanishes, and the equation becomes a deterministic one with regard to variable  $x = N_C/N$ . The gradient of selection [46] in finite population is defined as follows

$$g(N_C) = T_{N_C}^+ - T_{N_C}^- = \frac{N_C}{N} \frac{N - N_C}{N} \tanh\left(\frac{\beta}{2}[\pi_C(N_C) - \pi_D(N_C)]\right) \tag{3.17}$$

For  $1 \leq M_1 \leq M_2 \leq N_g < N$ , we explore how the upper and lower thresholds affect the critical value of  $b/c$  when  $T_{N_C}^+ = T_{N_C}^-$ . As illustrated in Fig. 2(c) and Fig. 2(d), critical value of  $b/c$  displays similar traits as those in infinite population.

In finite populations, another quantity of interest is the fixation probability of the cooperators, i.e., the probability for cooperators to take over the whole population. The fixation probability of  $N_C$  cooperators [49] is given by

$$\phi_{N_C} = \frac{1 + \sum_{i=1}^{N_C-1} \prod_{j=1}^i \alpha_j}{1 + \sum_{i=1}^{N-1} \prod_{j=1}^i \alpha_j}, \tag{3.18}$$



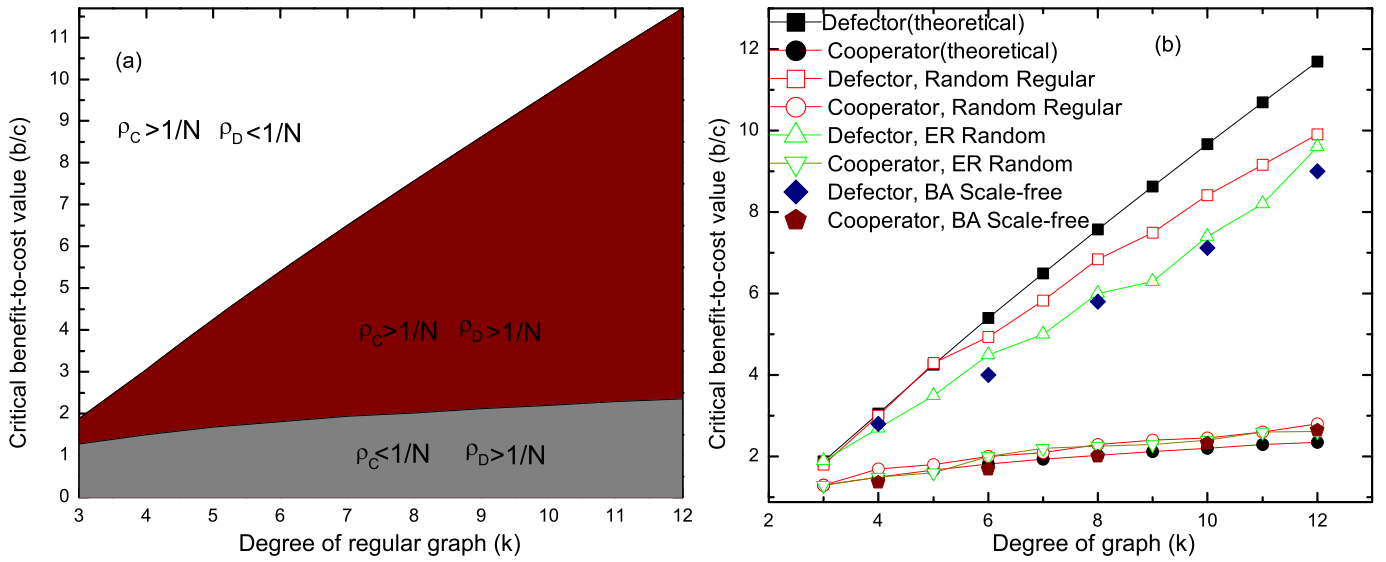
**Fig. 3.** Fixation probabilities for cooperators in multi-player snowdrift game with double thresholds. (a)–(c) show the influences on fixation probability of lower threshold  $M_1$ , upper threshold  $M_2$ , and the selection intensity  $\beta$  respectively, starting from initial frequency of cooperators indicated by horizontal axis. (d) shows the fixation probability of a single cooperator as a function of  $b/c$  for  $\beta = 0.2$  and  $M_2 = 20$ . Selection intensity is  $\beta = 1$  in (a) and  $\beta = 0.5$  in (b).  $b/c = 5$  in (a)–(c).  $M_2 = 20$  in (a),  $M_1 = 8$  in (b),  $M_1 = 16$  and  $M_2 = 20$  in (c). Population size is  $N = 40$  and group size is  $N_g = 20$  for all panels.

where

$$\alpha_j = \frac{T_j^-}{T_j^+} = \exp(-\alpha(\pi_C(j) - \pi_D(j))), \quad (3.19)$$

for the updating rule we considered.

For  $\beta \rightarrow 0$ , the fixation probability increases linearly from zero to one with the initial fraction of cooperators  $x$ , as indicated by the diagonal in Fig. 3(a)–(c). As  $\beta$  gets larger, sudden rise in fixation probability level can be more clearly observed at two points of  $x$ . The former point is dependent on specific game parameters and the latter point is  $x = 1$  (see Fig. 3(c)). Between these two points, a plateau is developed where the fixation probability is insensitive to the increment of  $x$ . A larger lower threshold  $M_1$  will delay the appearance of this plateau, but makes it higher, as illustrated in Fig. 3(a). A larger upper threshold  $M_2$  will not qualitatively change the scenario but boost fixation probability for the whole range of  $x$ , as shown in Fig. 3(b). It should be noted that transitions at these two points display the characteristics of both stag-hunt game and snowdrift game. The first point indicates that a certain number of cooperators are required to complete the task, just like the coordination behavior in two-person stag-hunt game (equivalent to a threshold  $M_1 = 2$ ). And the coordination point coincides with the unstable equilibrium point in replicator dynamics in infinite populations [46]. Transition across this point depends sensitively on the intensity of selection, becoming sharper for larger intensity of selection. After reaching the lower threshold  $M_1$ , the fixation probability exhibits the trait of the snowdrift game. It is shown that further increment of  $x$  does not help much, since extra cooperators do not add to collective benefit, which is a typical snowdrift situation. Fixation probability slowly increases and remains less than the neutral baseline except for the final jump to full cooperators ( $x = 1$ ), just like the 2-person snowdrift game [46] and multi-person snowdrift game without threshold (see Fig. 3(a) for  $M_1 = 1$ ). We have further inspected the fixation probability of one cooperator in a population of defectors, as shown in Fig. 3(d). It can be observed that the fixation probability does not monotonously increase with threshold except for very small  $b/c$ .



**Fig. 4.** Critical benefit-to-cost ratio  $b/c$  as a function of network average degree  $k$  for natural selection to favor cooperation ( $f_1(k)$ ) or defection ( $f_2(k)$ ). Panel (a) shows theoretical results derived by pair approximation. The plane is divided into three regions with the upper one  $\rho_C > 1/N$ , the lower one  $\rho_D > 1/N$ , and the medium one  $\rho_C > 1/N$  and  $\rho_D > 1/N$ . Panel (b) shows the comparison of theoretical results to those obtained by individual-based simulations. The population starts with only one cooperator (defector), and the fixation probability of cooperator (defector) under given  $b/c$  is calculated by averaging over 20000 runs for a period of  $10^5$  time steps. Other simulation parameters: population size  $N = 100$ , selection intensity  $w = 0.01$ .

### 3.2. Multi-player snowdrift game on networks

We derive the fixation probability of both cooperators ( $\rho_C$ ) and defectors ( $\rho_D$ ) in multi-person snowdrift game played on networks by pair approximation and diffusion approximation (see details in Appendix A). For large population size  $N$  and weak selection density  $w$ , we have the following rules: Selection favors the fixation of cooperators, i.e.,  $\rho_C > 1/N$  if

$$\frac{b}{c} > \frac{\sum_{i=0}^k \frac{c_i}{(i+1)(i+2)}}{\sum_{i=0}^k \frac{b_i}{(i+1)(i+2)}} (\equiv f_1(k)), \quad (3.20)$$

and selection favors the fixation of defectors, i.e.,  $\rho_D > 1/N$  if

$$\frac{b}{c} < \frac{\sum_{i=0}^k \frac{c'_i}{(i+1)(i+2)}}{\sum_{i=0}^k \frac{b'_i}{(i+1)(i+2)}} (\equiv f_2(k)). \quad (3.21)$$

The notations  $c_i$  and  $b_i$  in Eq. (3.20) and  $c'_i$  and  $b'_i$  in Eq. (3.21) are the functions of the degree of the graph  $k$  (see their detailed expressions in Appendix B). Consistent with the simple rule  $b/c > k$  proposed by [34], it is more difficult for cooperation to evolve when there are more neighbors for individuals playing multi-player snowdrift game. That is to say, high connectivity should reduce cooperation. Fig. 4(a) presents the two critical benefit-to-cost ratios as a function of degree  $k$  (i.e.  $f_1(k)$  and  $f_2(k)$ ), and the plane can be divided into 3 regions ( $f_2(k) > f_1(k)$  for any  $k$ ). Selection favors cooperation and opposes defection ( $\rho_C > 1/N$  and  $\rho_D < 1/N$ ), if  $b/c > f_2(k)$ ; Selection favors defection and opposes cooperation ( $\rho_C < 1/N$  and  $\rho_D > 1/N$ ), if  $b/c < f_1(k)$ ; Selection favors both cooperation and defection ( $\rho_C > 1/N$  and  $\rho_D > 1/N$ ), if  $f_1(k) < b/c < f_2(k)$ .

It should be noted that the medium region where both cooperation and defection are favored by natural selection observed in multi-person snowdrift game does not exist in traditional 2-person prisoner's dilemma game, but exists in 2-person snowdrift game. For 2-person PDG, the range of  $b/c$  favors cooperation and that favors defection are mutually exclusive, and thus  $\rho_C > 1/N$  also means  $\rho_D < 1/N$  under the assumption of weak selection and large population. However, due to the nonlinearity of payoff in snowdrift game, if the mixed equilibrium  $x^* \in (1/3, 2/3)$  in infinite populations, both  $\rho_C > 1/N$  and  $\rho_D > 1/N$  hold in finite populations [50].

Monte Carlo simulations are carried out on a variety of graphs, and the results are in agreement with those obtained by theoretical approximations, as illustrated in Fig. 4(b). It is not surprising that the largest difference between numerical and theoretical prediction can be observed at high degree of graphs. Besides pair-approximation, diffusion approximation is also utilized to derive the fixation probability, and increasing average degree  $k$  will lead to larger deviation due to diffusion approximation [34]. We can also see that the obtained theoretical approximations provide effective predictions for heterogeneous networks like Erdős-Renyi random network and BA scale-free network.

## 4. Conclusion

In this letter, we have investigated the evolutionary dynamics of a multi-player snowdrift game with double thresholds in well-mixed populations. The lower threshold  $M_1$  describes the minimum number of cooperators to create the benefit. The higher threshold  $M_2$  represents the critical mass of cooperators, beyond which new cooperators produce no more net benefit. In infinite populations, larger  $M_1$  leads to larger equilibrium cooperation level, but requires larger initial cooperation level for this equilibrium to be reached; larger

$M_2$  boosts equilibrium cooperation level, yet leaves the attraction basin of this equilibrium point unaffected. Influences of these two thresholds display similar traits when it comes to the fixation probability in finite populations. Thus the fixation probability curve for cooperators presents the characteristics of both the stag-hunt game and the snowdrift game. The model with only one higher threshold  $M_2$  is equivalent to that the benefits are smaller compared to the classical N-person snowdrift game. It can also be regarded as the effect of discounting, whose instances can be found in biology [51]. When the enzymes are eventually saturated for the resource with the increase of cooperators, the cooperating cells joining the group later only contribute diminishing small benefits to the group [52].

For the multi-player snowdrift game on networks, we use the pair approximation and diffusion approximation to derive the critical benefit-to-cost ratio that favors cooperation or defection theoretically under the assumption of weak selection and large populations. Network can oppose the appearance of cooperation as the degree of the network  $k$  increases. We have also conducted individual-based simulations on regular networks and heterogeneous networks, the results are in agreement with theoretical predictions.

Recently, by using economic experiments of prisoner’s dilemma game, Rand et al. first provided evidence that static network leads to higher level of cooperation than well-mixed populations [53]. In the future, we expect to draw supports from behavior experiments and explore the conditions for cooperation to be favored over defection in real-life situations of N-person snowdrift game.

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**Appendix A. Pair approximation**

We adopt pair approximation Refs. [34,54] and diffusion approximation to derive the critical benefit-to-cost ratio  $b/c$  for  $\rho_C = 1/N$  and  $\rho_D = 1/N$ . The following method is similar to that for 2-person games in Ref. [34]. Let  $P_X$  and  $P_{XY}$  denote the frequencies of the strategy  $X$  and the pair  $XY$  respectively.  $P_{X|Y}$  denotes the conditional probability of finding a player of strategy of  $X$  in the neighborhood of a player of strategy  $Y$ . Here  $X$  and  $Y$  stand for  $C$  or  $D$ . Consider the four identities:

$$\begin{aligned}
 P_C + P_D &= 1 \\
 P_{C|X} + P_{D|X} &= 1 \\
 P_{XY} &= P_{X|Y}P_Y \\
 P_{CD} &= P_{DC}
 \end{aligned} \tag{A.1}$$

There are only two independent variables  $P_C$  and  $P_{CC}$ . For simplicity, we use the following five functions to express the payoff of multi-player snowdrift game on the network:

$$w(x) = \sum_{d=0}^{k-1} C_{k-1}^d x^d (1-x)^{k-1-d} d = (k-1)x \tag{A.2}$$

$$m(x) = \sum_{d=0}^{k-1} C_{k-1}^d x^d (1-x)^{k-1-d} \frac{1}{d+1} = \frac{1 + (1-x) + \dots + (1-x)^{k-1}}{k} \tag{A.3}$$

$$m^{(k-2)}(x) = \sum_{d=0}^{k-2} C_{k-2}^d x^d (1-x)^{k-2-d} \frac{1}{d+1} = \frac{1 + (1-x) + \dots + (1-x)^{k-2}}{k-1} \tag{A.4}$$

$$n(x) = \sum_{d=0}^{k-1} C_{k-1}^d x^d (1-x)^{k-1-d} \frac{1}{d+2} = \frac{k - (1-x)(1 + (1-x) + \dots + (1-x)^{k-1})}{k(k+1)x} \tag{A.5}$$

$$n^{(k-2)}(x) = \sum_{d=0}^{k-2} C_{k-2}^d x^d (1-x)^{k-2-d} \frac{1}{d+2} = \frac{k-1 - (1-x)(1 + (1-x) + \dots + (1-x)^{k-2})}{(k-1)kx}, \tag{A.6}$$

where  $x$  stands for conditional probability  $P_{C|C}$ ,  $P_{D|C}$ ,  $P_{C|D}$ , or  $P_{D|D}$ . The payoff functions for cooperators and defectors are respectively denoted by

$$P_C(k_C) = \begin{cases} 0 & (k_C = 0) \\ b - \frac{c}{k_C} & (k_C \geq 1) \end{cases} \tag{A.7}$$

and

$$P_D(k_C) = \begin{cases} 0 & (k_C = 0) \\ b & (k_C \geq 1) \end{cases}, \tag{A.8}$$

where  $k_C$  is the number of cooperators.

*A.1. Updating a D player*

A  $D$  player dies with probability  $P_D$ . Its  $k$  neighbors compete for the vacancy site. The frequency of such a configuration is  $C_k^{k_C} P_{C|D}^{k_C} P_{D|D}^{k_D}$  ( $k_C + k_D = k$ ), where  $k_C$  and  $k_D$  denote the numbers of  $C$  and  $D$  among the  $k$  neighbors. The fitness of each  $C$  player is



$$\begin{aligned}
f_C &= 1 - w + w \left\{ \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|C})^d (P_{D|C})^{k-1-d} \left( b - \frac{c}{d+1} \right) \right. \\
&\quad \left. + \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|C})^d (P_{D|C})^{k-1-d} [dE_C + (k-1-d)E_D] + P_C(k_C) \right\} \\
&= 1 - w + w \{ bk - c[m(P_{C|C}) + P_C(k_C) + (k-1)n(P_{C|C})P_{C|C} + (k-1)m(P_{C|D})P_{D|C}] \},
\end{aligned} \tag{A.9}$$

where

$$E_C = \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|C})^d (P_{D|C})^{k-1-d} \left( b - \frac{c}{d+2} \right) = b - cn(P_{C|C}) \tag{A.10}$$

and

$$E_D = \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|D})^d (P_{D|D})^{k-1-d} \left( b - \frac{c}{d+1} \right) = b - cm(P_{C|D}). \tag{A.11}$$

The fitness of each  $D$  player is

$$\begin{aligned}
f_D &= 1 - w + w \left\{ \sum_{d=1}^{k-1} C_{k-1}^d (P_{C|D})^d (P_{D|D})^{k-1-d} b \right. \\
&\quad \left. + \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|D})^d (P_{D|D})^{k-1-d} [dE'_C + (k-1-d)E'_D] + P_D(k_C) \right\} \\
&= 1 - w + w \{ bk - b(P_{D|D})^{k-1} - b(k-1)(P_{D|D})^k + P_D(k_C) \},
\end{aligned} \tag{A.12}$$

where  $E'_C = b$  and  $E'_D = (1 - P_{D|D}^{k-1})b$ .

The probability that a  $C$  replaces the vacancy is  $k_C f_C / (k_C f_C + k_D f_D)$ . The probability that the number of  $C$  increases by 1 is

$$\text{prob}(\Delta P_C = \frac{1}{N}) = P_D \sum_{k_C+k_D=k} C_k^{k_C} (P_{C|D})^{k_C} (P_{D|D})^{k_D} \frac{k_C f_C}{k_C f_C + k_D f_D}. \tag{A.13}$$

Meanwhile, the number of  $CC$  pairs will increase by  $k_C / (kN/2)$  after a defector is replaced by a cooperator. Therefore the probability that  $P_{CC}$  increases by  $k_C / (kN/2)$  is given by

$$\text{prob}(\Delta P_{CC} = \frac{2k_C}{kN}) = P_D C_k^{k_C} (P_{C|D})^{k_C} (P_{D|D})^{k_D} \frac{k_C f_C}{k_C f_C + k_D f_D}. \tag{A.14}$$

## A.2. Updating a $C$ player

A  $C$  player is eliminated with probability  $P_C$ . Its  $k$  neighbors compete for the vacancy. As above, the frequency of such a configuration is  $C_k^{k_C} P_{C|C}^{k_C} P_{D|C}^{k_D}$ . The fitness of each  $C$  player is

$$\begin{aligned}
g_C &= 1 - w + w \left\{ \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|C})^d (P_{D|C})^{k-1-d} \left( b - \frac{c}{d+2} \right) \right. \\
&\quad \left. + \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|C})^d (P_{D|C})^{k-1-d} [dE''_C + (k-1-d)E''_D] + P_C(k_C + 1) \right\} \\
&= 1 - w + w \{ bk - c[n(P_{C|C}) + P_C(k_C + 1) + (k-1)m(P_{C|D})P_{D|C} + (k-1)n(P_{C|C})P_{C|C}] \},
\end{aligned} \tag{A.15}$$

where

$$E''_C = \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|C})^d (P_{D|C})^{k-1-d} \left( b - \frac{c}{d+2} \right) = b - cn(P_{C|C}) \tag{A.16}$$

and

$$E''_D = \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|D})^d (P_{D|D})^{k-1-d} \left( b - \frac{c}{d+1} \right) = b - Cm(P_{C|D}). \tag{A.17}$$

The fitness of each  $D$  player is

$$\begin{aligned}
g_D &= 1 - w + w \left\{ \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|D})^d (P_{D|D})^{k-1-d} b \right. \\
&\quad \left. + \sum_{d=0}^{k-1} C_{k-1}^d (P_{C|D})^d (P_{D|D})^{k-1-d} [dE'''_C + (k-1-d)E'''_D] + P_D(k_C + 1) \right\} \\
&= 1 - w + w \{ bk - b(k-1)(P_{D|D})^k \},
\end{aligned} \tag{A.18}$$

where  $E'''_C = b$  and  $E'''_D = (1 - (P_{D|D})^{k-1})b$ .

The probability that a  $D$  replaces the vacancy is  $k_D g_D / (k_C g_C + k_D g_D)$ . The probability that the number of  $C$  decreases by 1 is

$$\text{prob}(\Delta P_C = -\frac{1}{N}) = P_C \sum_{k_C+k_D=k} C_k^{k_C} (P_{C|C})^{k_C} (P_{D|C})^{k_D} \frac{k_D g_D}{k_C g_C + k_D g_D}. \tag{A.19}$$

Meanwhile, the number of  $CC$  pairs will decrease by  $k_C/(kN/2)$  after a cooperator is replaced by a defector. Therefore the probability that the pairs  $P_{CC}$  decreases by  $k_C/(kN/2)$  is given by

$$prob(\Delta P_{CC} = -\frac{2k_C}{kN}) = P_C C_k^{k_C} (P_{C|C})^{k_C} (P_{D|C})^{k_D} \frac{k_D g_D}{k_C g_C + k_D g_D}. \tag{A.20}$$

**Appendix B. Diffusion approximation**

Suppose that the death-birth updating occurs in one unit of time. The derivatives of  $P_C$  and  $P_{CC}$  with respect to time are given by

$$\dot{P}_C = \frac{1}{N} prob(\Delta P_C = \frac{1}{N}) + (-\frac{1}{N}) prob(\Delta P_C = -\frac{1}{N}) = w \frac{k-1}{kN} P_{CD} (bI_b - cI_c) + o(w^2) \tag{B.1}$$

and

$$\dot{P}_{CC} = \sum_{k_C=0}^N prob(\Delta P_{CC} = \frac{2k_C}{kN}) + \sum_{k_C=0}^N prob(\Delta P_{CC} = -\frac{2k_C}{kN}) = \frac{2}{kN} P_{CD} [1 + (k-1)(P_{C|D} - P_{C|C})] + o(w), \tag{B.2}$$

where

$$\begin{aligned} I_b &= P_{D|D}^k + (k-1)(P_{C|C} + P_{D|D})P_{D|D}^k, \\ I_c &= P_{D|D} [m(P_{C|C}) + m^{(k-2)}(P_{C|D})] + [n(P_{C|C}) + n^{(k-2)}(P_{C|C})]P_{C|C} + (k-1) \\ &\quad \times (P_{C|C} + P_{D|D})n(P_{C|C})P_{C|C} + (k-1)(P_{C|C} + P_{D|D})m(P_{C|D})P_{D|C}. \end{aligned} \tag{B.3}$$

From Eq. (B.2), we can derive that

$$\dot{P}_{C|C} = \frac{d}{dt} \frac{P_{CC}}{P_C} = \frac{2}{kN} \frac{P_{CD}}{P_C} [1 + (k-1)(P_{C|D} - P_{C|C})] + o(w). \tag{B.4}$$

$P_C$  and  $P_{C|C}$  are two independent variables to describe the system. Rewrite Eq. (B.1) and Eq. (B.4) as the functions of  $P_C$  and  $P_{C|C}$ :

$$\dot{P}_C = wF_1(P_C, P_{C|C}) + o(w^2), \tag{B.5}$$

$$\dot{P}_{C|C} = F_2(P_C, P_{C|C}) + o(w). \tag{B.6}$$

For weak selection,  $w \ll 1$ , global frequency  $P_C$  changes at an order of  $w$ , but local frequency  $P_{C|C}$  changes at an order of 1. Hence  $\dot{P}_{C|C}$  converges to the stationary state more quickly than  $\dot{P}_C$ , thus we derive local frequencies at equilibrium. Let  $F_2(P_C, P_{C|C}) = 0$ , we obtain

$$P_{C|C} - P_{C|D} = \frac{1}{k-1}. \tag{B.7}$$

Furthermore, we can obtain the following relationships from Eq. (B.7):

$$\begin{aligned} P_{C|C} &= \frac{1}{k-1} + \frac{k-2}{k-1} P_C \\ P_{D|C} &= \frac{k-2}{k-1} - \frac{k-2}{k-1} P_C \\ P_{D|D} &= 1 - \frac{k-2}{k-1} P_C \\ P_{C|D} &= \frac{k-2}{k-1} P_C \\ P_{CD} &= \frac{k-2}{k-1} P_C (1 - P_C). \end{aligned} \tag{B.8}$$

Then, by virtue of Eqs. (B.8),  $I_b$  and  $I_c$  can be expressed as follows

$$\begin{aligned} I_b &= b_0 + b_1 P_C^1 + \dots + b_k P_C^k, \\ I_c &= c_0 + c_1 P_C^1 + \dots + c_k P_C^k. \end{aligned} \tag{B.9}$$

Then we have

$$bI_b - cI_c = \alpha_0 + \alpha_1 x^1 + \dots + \alpha_k P_C^k, \tag{B.10}$$

where

$$\alpha_i = bb_i - cc_i, \quad 0 \leq i \leq k. \tag{B.11}$$

Suppose that the relation given by Eqs. (B.8) always hold, we study the one-dimensional diffusion process of the random variable  $P_C$  as an approximation of the evolving process of strategy  $C$ . Within the short time  $\Delta t$ , we have

$$E[\Delta P_C] = w \frac{k-2}{kN} P_C(1-P_C)(\alpha_0 + \alpha_1 x^1 + \dots + \alpha_k P_C^k) \Delta t (\equiv m(P_C) \Delta t),$$

$$v[\Delta P_C] = \frac{2}{N^2} \frac{k-2}{k-1} P_C(1-P_C) \Delta t (\equiv v(P_C) \Delta t). \quad (\text{B.12})$$

The fixation probability [55]  $u(p)$  with initial frequency  $u(t=0) = p$  satisfies the following equation

$$0 = m(p) \frac{\partial u}{\partial p} + \frac{v(p)}{2} \frac{\partial^2 u}{\partial p^2}. \quad (\text{B.13})$$

The absorbing states of the one-dimensional diffusion process are two endpoints, and the diffusion equation satisfies the following boundary conditions

$$u(0) = 0, \quad u(1) = 1. \quad (\text{B.14})$$

The solution to the fixation probability Eq. (B.13) is

$$u(p) = \frac{\int_0^p D(s) ds}{\int_0^1 D(s) ds}, \quad (\text{B.15})$$

where

$$D(s) = \exp\left(-\int_0^s \frac{2m(x)}{v(x)} dx\right) \quad (\text{B.16})$$

and

$$\frac{2m(x)}{v(x)} = w \frac{N(k-1)}{k} (\alpha_0 + \alpha_1 x^1 + \dots + \alpha_k x^k). \quad (\text{B.17})$$

The derivative of  $u(p)$  with respect to the selection intensity  $w$  is

$$\frac{\partial u(p)}{\partial w} = \frac{\int_0^p \frac{\partial D(s)}{\partial w} ds \int_0^1 D(s) ds - \int_0^p D(s) ds \int_0^1 \frac{\partial D(s)}{\partial w} ds}{(\int_0^1 D(s) ds)^2}, \quad (\text{B.18})$$

where

$$\frac{\partial D(s)}{\partial w} = D(s) \left[-\frac{N(k-1)}{k} (\alpha_0 s^1 + \alpha_1 \frac{s^2}{2} + \dots + \alpha_k \frac{s^{k+1}}{k+1})\right]. \quad (\text{B.19})$$

Substituting  $w=0$  into Eq. (B.16) and Eq. (B.19), we have

$$D(s)|_{w=0} = 1 \quad (\text{B.20})$$

and

$$\frac{\partial D(s)}{\partial w}|_{w=0} = -\frac{N(k-1)}{k} (\alpha_0 s^1 + \alpha_1 \frac{s^2}{2} + \dots + \alpha_k \frac{s^{k+1}}{k+1}). \quad (\text{B.21})$$

Substituting Eq. (B.20) and Eq. (B.21) into Eq. (B.18), we have

$$\begin{aligned} \frac{\partial u}{\partial w}|_{w=0} &= \frac{N(k-1)}{k} p(1-p) \left[ \frac{\alpha_0}{2} + \frac{\alpha_1}{6} (1+p) + \dots + \frac{\alpha_k}{(k+1)(k+2)} (1+p+p^2+\dots+p^k) \right] \\ &= \frac{N(k-1)}{k} p(1-p) \left[ \sum_{i=0}^k \frac{\alpha_i}{(i+1)(i+2)} + f(p)p \right] \end{aligned} \quad (\text{B.22})$$

where

$$f(p) = \sum_{j=1}^k p^{j-1} \left( \sum_{i=j}^k \frac{\alpha_i}{(i+1)(i+2)} \right) = \frac{\alpha_1}{2 \times 3} + \frac{1-p^2}{1-p} \frac{\alpha_2}{3 \times 4} + \dots + \frac{1-p^k}{1-p} \frac{\alpha_k}{(k+1)(k+2)}. \quad (\text{B.23})$$

When  $p$  tends to 0,  $f(p)p$  is infinitesimals of higher order compared to  $\sum_{i=0}^k \alpha_i / ((i+1)(i+2))$ , then by Eq. (B.22), it holds that

$$\frac{\partial u}{\partial w}|_{w=0} \approx \frac{N(k-1)}{k} p(1-p) \left( \sum_{i=0}^k \frac{\alpha_i}{(i+1)(i+2)} \right). \quad (\text{B.24})$$

Since  $w$  is small, the Taylor expansion of  $u(p)$  about  $w=0$  is

$$u(p) = u(p)|_{w=0} + \frac{\partial u}{\partial w}|_{w=0} w + o(w^2) = p + \frac{\partial u}{\partial w}|_{w=0} w + o(w^2). \quad (\text{B.25})$$

For sufficiently large  $N$ ,  $p(= 1/N)$  is very small. According to Eq. (B.24) and Eq. (B.25),  $u(1/N) > 1/N$  is equivalent to

$$\sum_{i=0}^k \frac{\alpha_i}{(i+1)(i+2)} > 0. \tag{B.26}$$

Since  $\alpha_i = bb_i - cc_i$ , condition (B.26) is equivalent to the following inequality

$$\frac{b}{c} > \frac{\sum_{i=0}^k \frac{c_i}{(i+1)(i+2)}}{\sum_{i=0}^k \frac{b_i}{(i+1)(i+2)}}. \tag{B.27}$$

From Eq. (B.9), we can derive

$$\begin{aligned} c_0 &= 1 - \frac{k-2}{k} [1 - (\frac{k-2}{k-1})^k] + \frac{1}{k} - \frac{k-2}{(k-1)k} [1 - (\frac{k-2}{k-1})^{k-1}] + \frac{k(k-2)}{k-1} + \frac{k-1}{k} [1 - (\frac{k-2}{k-1})^k] + 1 \\ c_i &= -\frac{1}{k} \sum_{j=i}^k (\frac{k-2}{k-1})^j C_j^i (-1)^i - \frac{1}{(k-1)k} \sum_{j=i}^{k-1} (\frac{k-2}{k-1})^j C_j^i (-1)^i \\ &\quad - \frac{k-2}{k-1} [\sum_{j=i-1}^{k-1} C_j^{i-1} (-\frac{k-2}{k-1})^{i-1}] + \frac{k-2}{k-1} [\sum_{j=i}^{k-1} C_j^i (-\frac{k-2}{k-1})^i] \\ &\quad + \frac{1}{k} [\sum_{j=i}^{k-1} (\frac{k-2}{k-1})^j C_j^i (-1)^i] - \frac{k-2}{(k-1)k} [\sum_{j=i-1}^{k-1} (\frac{k-2}{k-1})^j C_j^{i-1} (-1)^{i-1}] + \frac{1}{k-1} \sum_{j=i}^{k-1} C_j^i (-\frac{k-2}{k-1})^i \quad (0 < i < k) \\ c_k &= (-\frac{k-2}{k-1})^k \end{aligned} \tag{B.28}$$

and

$$b_i = (k+1) C_k^i (-\frac{k-2}{k-1})^i. \tag{B.29}$$

Furthermore,

$$\sum_{i=0}^k \frac{b_i}{(i+1)(i+2)} = \frac{1}{(k+2)(\frac{k-2}{k-1})^2} [(1 - \frac{k-2}{k-1})^{k+2} - 1 + (k+2) \frac{k-2}{k-1}]. \tag{B.30}$$

Combining Eqs. (B.28) and Eq. (B.30), we can obtain the critical benefit-to-cost ratio for cooperation by the condition (B.27). Similarly, the condition  $P_D > 1/N$  is equivalent to the following inequality

$$\frac{b}{c} < \frac{\sum_{i=0}^k \frac{c'_i}{(i+1)(i+2)}}{\sum_{i=0}^k \frac{b'_i}{(i+1)(i+2)}}, \tag{B.31}$$

where

$$\begin{aligned} c'_0 &= 1 + \frac{1}{k} + \frac{1}{(k-1)k} + \frac{1}{(k-1)(k-2)} [1 - (\frac{1}{k-1})^{k-1}] \\ c'_i &= -\frac{1}{k} (\frac{k-2}{k-1})^i - \frac{1}{(k-1)k} (\frac{k-2}{k-1})^i + \frac{k-2}{k-1} [\sum_{j=i-1}^{k-1} C_j^{i-1} (\frac{k-2}{k-1})^{i-1} (\frac{1}{k-1})^{j-i+1}] + \frac{1}{k-1} (\frac{k-2}{k-1})^i \\ &\quad + \frac{1}{k-1} \sum_{j=i}^{k-1} C_j^i (\frac{k-2}{k-1})^i (\frac{1}{k-1})^{j-i} \quad (0 < i < k) \\ c'_k &= (\frac{k-2}{k-1})^k \end{aligned} \tag{B.32}$$

and

$$b'_i = (k+1) C_k^i (\frac{k-2}{k-1})^i (\frac{1}{k-1})^{k-i}. \tag{B.33}$$

Furthermore,

$$\sum_{i=0}^k \frac{b'_i}{(i+1)(i+2)} = \frac{1}{(k-1)^k (k+2)(k-2)^2} [(k-1)^{k+2} - (k+2)(k-2) - 1]. \tag{B.34}$$

The above calculation holds only for  $k \geq 3$ . The special graph of circle with  $k = 2$  is not considered.

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