

Factorisation systems

definition

Factorisation for $\mathbf{Alg}(\Sigma)$

Factorisation for $\mathbf{PAlg}(\Sigma)$

Factorisation for $\mathbf{CAlg}(\Sigma)$

Diagonal fill-in

“Theory of reachability”

in $\mathbf{Alg}(\Sigma)$

Def: A is reachable if it is generated by the empty set

Every algebra has a unique reachable subalgebra

A is reachable iff the unique morphism from the term algebra is surjective

If A is reachable then for any B there exists at most one homomorphism from A to B .

If A is reachable then any homomorphism to A is surjective

Reachability structure

definition

examples: $\mathbf{Alg}(\Sigma)$, $\mathbf{PAlg}(\Sigma)$, $\mathbf{CAlg}(\Sigma)$

“Theory of reachability”

- Every object has a unique (up to isomorphism) reachable subobject
- A is reachable iff the unique morphism from the initial object is factorisation epi
- If A is reachable then for any B there exists at most one morphism from A to B .
- If A is reachable then any morphisms to A is factorisation epi

Existence of initial objects

Every quasivariety has a reachable initial object.