Tutorial 7

JAIO - II

Question 1 (*toolbox* problem 95)**.** Which of the following ordered sets are WQO's?

1. $\mathbb{N}^2$ with lexicographic order;
2. $\{a, b\}^*$ with lexicographical order;
3. $\mathbb{N}$ with divisibility order, i.e. x smaller than y if $x|y$;
4. Σ^* with prefix order;
5. Σ^* with infix order;
6. line segments with an order: $[a, b]$ smaller than $[c, d]$ if $b < c$ or, $a = c$ and $b < d$;
7. graphs with subgraph order (remove some edges and some vertices);

Question 2 (*toolbox* problem 96)**.** Let $(X, \leq_X)$ and $(Y, \leq_Y)$ be two WQO's. Define $\leq$ on $X \times Y$ as: $(x_1, y_1) \leq (x_2, y_2)$ iff $x_1 \leq_X x_2$ and $y_1 \leq_Y y_2$. Show that $(X \times Y, \leq)$ is also a WQO.

Question 3 (*toolbox* problem 98)**.** Let $(X, \leq)$ be a WQO. Show that there is no infinite growing sequence of upward-closed subsets X, i.e. no sequence $U_1 \subsetneq U_2 \subsetneq \dots$ s.t. for all $i \in \mathbb{N}$ the set $U_i \subseteq X$ is upward-closed wrt $\leq$. Is it equivalent to saying $(X, \leq)$ is a WQO?

Question 4 (*toolbox* problem 99)**.** Show that given a d-dimensional VAS and $s \in \mathbb{N}^d$, one can compute the set of all configurations from which s is coverable. (Hint: use the previous problem).

Question 5 (*toolbox* problem 100)**.** Show that given a vector addition system with a distinguished source configuration, one can decide if the set of configurations reachable from the source is finite.

Question 6. Define a *vector addition system with states* to be a finite set of states Q, a dimension d, and a finite set $\delta \subseteq Q \times \mathbb{Z}^d \times Q$. A *configuration* is an element of $\mathbb{Q} \times \mathbb{N}^d$, and a transition is a pair $(q, x) \to (p, y)$ such that $(q, y - x, p) \in \delta$. Show a translation from vector addition system with states to vector addition systems, which preserves reachability and coverability.

Question 7 (*toolbox* problem 104)**.** Show that the following problem is decidable: given states p, q decide if there is a run from the configuration $(p, \bar{0})$ to some configuration with state q.