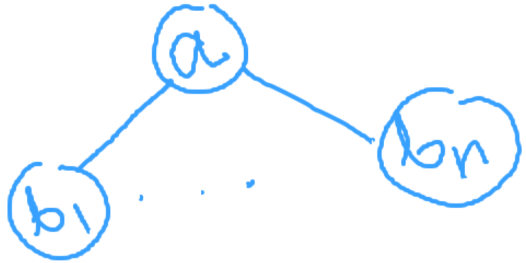
Trees with even no. of nodes: $\{0,1\}$

odd,



$\delta(a, p_1, \ldots, p_n) = (p_1 + \ldots + p_n + 1) \pmod 2$

$1 \text{ xor } 0 \text{ xor } 0 \text{ xor } 1 = 0$

Q_1

$x \to \{\cdot\}$

$\subseteq (Q \times \Sigma) \to \ldots \quad Q^0 = \{\cdot\}$

Trees:

Leav: alphabet $\to 0, 1$

internal n $\to \wedge, \vee$

$L = \{$ trees which evaluate to true $\}$

$Q = \{T, F\}$ $\quad$ $R = \{T\}$

$\delta \cancel{(=}$, $\delta(0) = F$

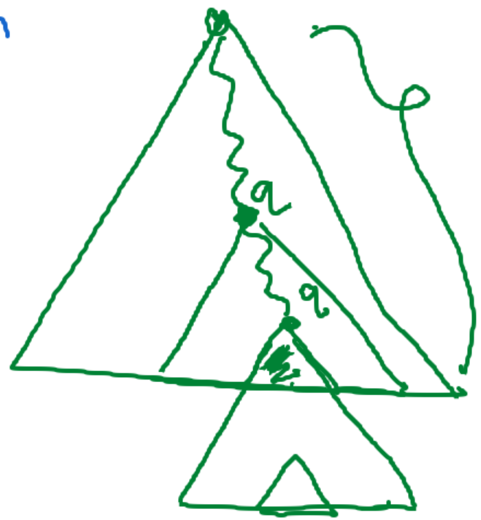
$\delta(1) = T$

$\delta(\wedge, T, T) = T$

$\vdots$

L = {balanced trees} ← Not regular

every leaf is at the same depth



Top-down }
$\parallel$ } non-det
Bottom-up }

What about det.?

$(Q, R \subseteq Q, \delta$

$a \in \Sigma$. arity $: \Sigma \to \mathbb{N}$

$\delta_a \subseteq Q \times Q^{arity(a)}$ (top-down)

$\delta_a \subseteq Q^{arity(a)} \times Q$ (bottom-up)

$\cancel{R} = Q = \{Y, N\}$

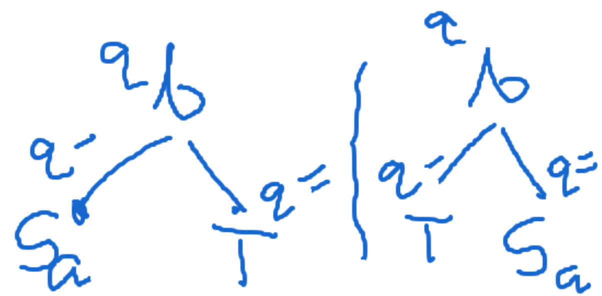
$R = \{Y\}$

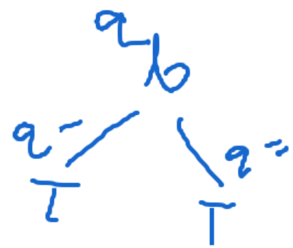
$\delta_a(-, \ldots) = Y$

$\delta_b(p_1, \ldots, p_n) = \begin{cases} Y & \text{if } \exists i\; p_i = Y \\ N & \text{if } \forall i\; p_i = N \end{cases}$

Trees with at least one "a"-labelled node

L


ab
a⁻
a⁼
S_a
T
a
b
a⁻
a⁼
T
S_a


ab
a⁻
a⁼
T
T

Show that for every n, there is a formula of MSO which has size polynomial in n and is true in a unique word which has length

$$a^{\text{tower}(n-1)}$$

$$\varphi_n(x,y)$$

$$\varphi_{n+1}(x,y)$$

$$\neg\exists$$

$$\forall$$

$$\underbrace{O\left(2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}\Big\}n\right)}_{\text{it}}$$

$$\varphi_{\text{len}_n} = (0, 0, \ldots, 0, 1)$$

$$\varphi_{1st} = \underbrace{(1, 0, 0, \ldots)}_{n}$$

$$\varphi \xrightarrow{(0 \ldots 0, -1, 2, \ldots 0)} \varphi'$$

$$\| \quad \overbrace{0 \ldots\ldots 0}^{\varphi}\#$$

$$0 \ldots\ldots 1\#$$

$$0 \ldots 10$$

$$0 \to 2^{\text{tower}(n-1)} - 1$$

$|\varphi_{n+1}| = \cancel{2}|\varphi_n| + c^n = 2^n$

Show that for every n, there is a formula of MSO which has size polynomial in n and is true in a unique word which has length

$\varphi_{01}(x,y)$

$x \sqcup y$

$\exists z, w$

$x < z < w <$

$\begin{cases} succ(x,z) \\ \quad " \quad (z,w) \\ \quad " \quad (w,y) \end{cases}$

$\begin{cases} 0..01 \; 1 \\ \quad +1 \\ \hline 10..0 \end{cases}$

$\left.2^{2^{\cdot^{\cdot^{2}}}}\right\}$ tower(n) $=$

$\varphi_n \qquad \varphi_{n+1}$

X   Y

$\# \underbrace{0\,1 \ldots 0}_{\text{tower}(n)} \# 0 1 \ldots 1 \# \ldots \# 1 \ldots 1 \#$

$\forall X, Y,$ $\begin{cases} \text{Start with } 0..0 \\ \text{end } \quad " \quad 1 \ldots 1 \\ \text{If } X, Y \text{ are conseq} \Rightarrow X+1=Y \end{cases}$

$\forall x, y, z$

start(x, x)
end(x, y) $\Rightarrow \varphi_n(x,y)$

$\varphi_n(x,y) \Rightarrow$