

# Tutorial 6

## JAIO - II

**Question 1.** Adapt the proof for finite words to show that

MSO definable languages of infinite words =  $\omega$ -regular languages

**Question 2** (Problem 40 in *toolbox*). Which of the following (finite)tree languages are regular:

1. trees with an even number of nodes;
2. trees with an even number of  $a$ -labelled nodes;
3. trees over leaf alphabet  $0, 1$  and internal alphabet  $\vee, \wedge$  which evaluate to true when treated as boolean expressions;
4. balanced trees (every leaf is at the same depth)

**Question 3** (Problem 43 in *toolbox*). Can a deterministic top-down tree automata recognize the language: “some node has label  $a$ ”?

**Question 4** (Problem 42 in *toolbox*). Define the *yield* of a tree to be the word composed from labels of its leaves written in infix order. Show that for every  $L \subseteq \Sigma^*$  the following are equivalent

1.  $L$  is context-free;
2.  $L$  is the set of yields of some regular tree language.

**Question 5.** Is the top-down model weaker than the bottom-up model in case of non-deterministic tree automata?

**Question 6.** Show that for every  $n$ , there is a formula of MSO which has size polynomial in  $n$  and is true in a unique word which has length

$$\underbrace{2^{2^2 \dots 2^{2^2}}}_{n \text{ times}}$$

**Question 7.** Use the subset construction to determinise non-deterministic tree automata.