Tutorial 5

JAIO - II

Question 1. Let A be a distance automaton with input alphabet S. We say A is limited on a regular language L, if there exists $n \in \mathbb{N}$ such that every word $x \in L$ has an accepting run of A of cost $\leq n$. Show that this problem is decidable.

Question 2. We say that a regular language L has the finite power property if there exists an n such that $L^* = L^0 \cup L^1 \cup \cdots \cup L^n$. Show that one can decide if a regular language has the finite power property.

Question 3. We say that languages K and L are *separated by language* S if $K \subseteq S$ and $L \cap S = \emptyset$. For words u, v we say that $u = a_1 \dots a_k$ is a subsequence of v, denoted $u \preceq v$, if $v \in \Sigma^* a_1 \Sigma^* \dots \Sigma^* a_k \Sigma^*$. A language M is called *upward closed* if for any words $u \preceq v$, $u \in M$ implies $v \in M$. Show that deciding whether two given regular languages K and L are separated by some upward closed language is decidable.

Question 4. Let $\mathcal{F}$ be the class of finite unions of languages of the form $\Sigma^* w_1 \Sigma^* \dots \Sigma^* w_k \Sigma^*$ where $w_1, \dots, w_k \in \Sigma^*$. Show that for given regular languages K and L it is decidable whether they are separated by a set from $\mathcal{F}$.

Question 5. Show that limitedness remains decidable when distance automata are equipped with two counters with reset operations. (the cost of a run is the biggest number of costly transitions between some two consecutive resets).

Question 6. Show that the problem of deciding whether a regular language has star height one is decidable.