Tutorial 4

JAIO - II

Question 1. A *Büchi* game is a game with the winning condition being a Büchi condition on the game arena. Are Büchi games positionally determined? (Hint: Use the result on parity games)

Question 2 (Problem 15 in *toolbox*)**.** Consider the following finite arena game with V as the vertices of the arena. There is function $rank : V \to \mathbb{N}$. The game continues until there is a loop in the play. The winner of the game is the parity of the smallest rank on the loop. Prove that player i wins this game iff player i wins the parity game on the same arena.

Question 3 (Problem 25 in *toolbox*)**.** Consider a game played on a finite game graph with vertices V, equipped with a function $rank : V \to \mathbb{N}$. Consider the following *bisimilarity* game. Two players, *Spoiler* and *Duplicator* start from a position $(u, v) \in V \times V$. The play proceeds in rounds. If at the beginning of a round $rank(u) \neq rank(v)$ or u and v belong to different players then Spoiler immediately wins. Otherwise Spoiler makes a move to (u_1, v) or (u, v_1) such that $u \to u_1$ or $v \to v_1$, respectively. Then Duplicator makes a move to (u_1, v_2) or (u_2, v_1) respectively such that $v \to v_2$ or $u \to u_2$, respectively. Next round starts from (u_1, v_2) or (u_2, v_1) respectively. If play continues infinitely long then Duplicator wins. Show that if Duplicator has a winning strategy from position (u, v) and one of the player has a winning strategy in the parity game from u, then the same player has a winning strategy in the parity game starting from v.

Constructing a game which is not determined

Question 4 (Problem 21 in *toolbox*)**.** Consider the following game. There are (countable)infinitely many dwarfs. Every dwarf is given a hat, which is either red or green. Every dwarf can see the color of every hat other than his own. Every dwarf has to tell what is the color of his hat, such that only finitely many dwarfs make a mistake. They can fix a strategy in advance, before getting their hats. But they cannot communicate after getting their hats. Is there a winning strategy for dwarfs?

Question 5 (Problem 22 in *toolbox*)**.** Define a function inf-xor : $\{0, 1\}^\omega \to \{0, 1\}$, such that changing one bit of an argument always changes the result.

Question 6 (Problem 23 in *toolbox*)**.** Consider the following two player game, called *Chomp*. There is a rectangular chocolate in a shape of $n \times k$ grid. The right upper corner piece is rotten. Players move in an alternating manner, the first one moves first. Any player in his move picks square of the chocolate that is not yet eaten, and eats all pieces that are to the left and to the bottom from the picked piece. The player who eats the rotten piece loses. Does any of the players have a winning strategy?

Question 7 (Problem 24 in *toolbox*)**.** Show a game which is not determined.