Tutorial 2

JAIO - II

Question 1 (Problem 5 in *toolbox*)**.** Let L be the language of words w which contain at least one b. Can there be a Büchi automata M accepting L such that all states of M are accepting (but some transitions over some letters from some states can be missing)?

Question 2. If we define the accepting condition of a Müller automata using transitions instead of states, how does it affect the expressiveness of the automata?

Question 3. Can we define ω-regular expressions, i.e regular expressions which expresses all ω-regular languages and only ω-regular languages?

Question 4 (Problem 9 in *toolbox*)**.** Define a metric d on ω-words as:

$$d(u, v) := \frac{1}{\mathrm{diff}(u, v)}$$

where $\mathrm{diff}(u, v)$ is the smallest position where u and v differs. A language $L \subseteq \Sigma^\omega$ is called *open* if it is open in the metric d. Prove the following are equivalent for any $L \subseteq \Sigma^\omega$:

1. L is open;
2. L is of the form $K\Sigma^\omega$ for some $K \subseteq \Sigma^*$;
3. L is of the form $K\Sigma^\omega$ for some regular $K \subseteq \Sigma^*$;

Question 5. Which of the following equivalence relations have the property that, L is ω-regular iff $\sim_L$ has finite index.

1. $\sim_L$ defined on Σ^* as: $u \sim_L v$, iff
$$uw \in L \iff vw \in L \text{ for all } w \in \Sigma^\omega$$
2. $\sim_L$ defined on Σ^ω as: $u \sim_L v$, iff
$$wu \in L \iff wv \in L \text{ for all } w \in \Sigma^*$$
3. $\sim_L$ defined on Σ^* as: $u \sim_L v$, iff
$$uw \in L \iff vw \in L \text{ for all } w \in \Sigma^\omega$$
and,
$$s(ut)^\omega \in L \iff s(vt)^\omega \in L \text{ for all } s, t \in \Sigma^*$$