Tutorial 12

JAIO - II

Question 1 (*toolbox* problem 107)**.** Show that adding the function $sin(x)$ to the real numbers yields an undecidable theory.

Question 2 (*toolbox* problem 108)**.** Show that the following structure has a decidable first-order theory: the universe consists of subsets of the Euclidean plane $\mathbb{R}^2$ that are points, lines or circles, and there is a binary predicate for inclusion.

Question 3 (*toolbox* problem 109)**.** Show that every $X \subseteq R$ definable by a first-order formula with one free variable is a finite union of points and open intervals.

Question 4. Show that any real number r definable by first-order logic is *algebraic* (i.e. there is a polynomial $p(x)$ with integer coefficients such that $p(r) = 0$).