

Tutorial 1

JAIO - II

Question 1. Construct a deterministic Büchi automata for the language $L \subseteq \{a, b\}^\omega$.

$x \in L \iff$ both a and b appears infinitely often in x , and between any two consecutive a there are even number of b .

Question 2 (Problem 1 and 4 in *toolbox*). Which of the following languages are ω -regular?

1. ω -words which have infinitely many prefixes in a fixed regular language of finite words $L \subseteq \Sigma^*$.
2. ω -words with infinitely many infixes of the form $ab^p a$, where p is prime.
3. ω -words with infinitely many infixes of the form $ab^n a$, where n is even.
4. ω -words with arbitrarily long infixes belonging to a fixed regular language of finite words L .
5. ω -words which have infinitely many prefixes in a fixed language of finite words $L \subseteq \Sigma^*$ (not necessarily regular).

Question 3. Show that ω -regular languages are closed under union and intersection.

Question 4. Show that every co-Büchi language is a Büchi language.

Question 5 (Problem 2 in *toolbox*). An ω -word is called *ultimately periodic* if it is of the form uv^ω for some finite words u, v . Show that if an ω -regular language is nonempty, then it contains an ultimately periodic word.

Question 6 (Problem 3 in *toolbox*). Let UP be the set of ultimately periodic words. Let K and L be ω -regular languages. Show that if $K \cap UP = L \cap UP$ then $K = L$.

Question 7. Show that every Muller automata is equivalent to a Büchi automata.

Question 8 (Problem 8 in *toolbox*). Show that nonemptiness is decidable for automata with the Muller acceptance condition.