Star problems - the first series (deadline 19.12.2021)

1. Two languages L, K are *separated* by a third language S if S includes the first one and is disjoint from the other one: $L \subseteq S$ and $S \cap K = \emptyset$.

   Show decidability of the following decision problem:

   Input: Two nondeterministic Büchi automata.
   Question: Are their languages separated by the language of some *deterministic* Büchi automaton?

   Hint: Use decidability and finite-memory determinacy of infinite duration games with ω-regular winning conditions.

2. A Büchi automaton is *reverse-deterministic* if for every state q and input letter a there is exactly one state p such that $p \xrightarrow{a} q$ is a transition of the automaton. Is every ω-regular language recognised by a reverse-deterministic Büchi automaton?

3. A *probabilistic automaton* $\mathcal{A}$ is like a finite automaton, except that the transition function assigns to each state q and input letter $a \in \Sigma$ a *probability distribution* over states. For instance, in state p, when a is read, the automaton goes with probability $\frac{3}{4}$ to state q_1 and with probability $\frac{1}{4}$ to state q_2, which could be written as $p \xrightarrow{a} \frac{3}{4} \cdot q_1 + \frac{1}{4} \cdot q_2$. Likewise, instead of initial states a probabilistic automaton has an initial probability distribution over states, e.g. $\frac{1}{3} \cdot p + \frac{2}{3} \cdot q_1 + 0 \cdot q_2$. Hence, after reading a word $w \in \Sigma^*$, the automaton is in a probability distribution over states. The probability $\mathcal{A}(w)$ of acceptance of w by $\mathcal{A}$ is the sum of probabilities of accepting states after reading w.

   Show decidability of the following decision problem:

   Input: Two probabilistic automata $\mathcal{A}$ and $\mathcal{B}$.
   Question: Do they accept all words with the same probability: $\mathcal{A}(w) = \mathcal{B}(w)$ for every $w \in \Sigma^*$?

4. Consider tree automata over a ranked alphabet as recognisers of languages of finite trees. Show decidability of the following decision problem:

   Input: A nondeterministic tree automaton.
   Question: Is the automaton equivalent to a *top-down deterministic* one?

5. Let MSO′ be the extension of MSO over finite words with one additional predicate $P(X, Y)$, for subsets X, Y of positions in a word, which is true when X and Y have the same size. An MSO′ sentence φ over the alphabet $\Sigma = \{a, b\}$ *defines* a subset $[\varphi] \subseteq \mathbb{N}$:

   $$[\varphi] \; = \; \{|w|_a \mid w \in \Sigma^*, w \models \varphi\}$$

   where $|w|_a$ is the number of occurrences of a in w. Is every recursively enumerable subset of $\mathbb{N}$ definable by an MSO′ sentence?