Star problems - the third series (deadline 7.02.2021)

1. A language $L \subseteq \Sigma^*$ we call *weighted-recognizable* if there is a weighted automaton $\mathcal{A}$ over the rational field with the property that $\mathcal{A}(w) = 1$ for $w \in L$ and $\mathcal{A}(w) = 0$ for $w \notin L$. Is the class of weighted-recognizable languages equal to the class of regular languages?

2. Consider fixed input and output alphabets $\Sigma = \{a, b\}$ and $\Gamma = \{c\}$. An *NFA with output and ε-steps* is a nondeterministic finite automaton $\mathcal{A}$ with transitions labeled by $w|v$, where $w \in \Sigma^*$ is an input and $v \in \Gamma^*$ is an output. Every accepting run of $\mathcal{A}$ defines a pair $\langle w, v \rangle \in \Sigma^* \times \Gamma^*$ obtained by concatenation of inputs and outputs of all transitions along the run. The set of all such pairs defines the relation computed by $\mathcal{A}$.

   Is the following problem decidable?

   Input: An NFA $\mathcal{A}$ with output and ε-steps.
   Question: Does $\mathcal{A}$ compute the relation $\Sigma^* \times \Gamma^*$?