Star problems - the second series (deadline 15.01.2021)

1. Consider lossy counter machines as language recognisers. To this aim we assume that transitions are additionally labeled by letters of a finite alphabet Σ, and acceptance is determined by reaching the halting state. Show that every two languages L_1, L_2 of such machines which are complements, i.e. $L_1 \cap L_2 = \emptyset$ and $L_1 \cup L_2 = \Sigma^*$, are actually regular languages.

2. Let's define a quasi-order over $\mathbb{N}^*$ (finite sequences of natural numbers) as follows: $w \preceq v$ if there is a permutation $\pi : \mathbb{N} \to \mathbb{N}$ such that $\pi(w)$ is a subsequence of v, where $\pi(w)$ denotes the result of point-wise application of π to w. Is this ordering a WQO?

3. Is the following problem decidable?

   Input: A timed automaton $\mathcal{A}$.
   Question: Is $\mathcal{A}$ equivalent to a timed automaton with 1 clock?