Star problems - the first series (deadline 15.12.2020)

1. The shuffle of two words w and v is the set of all words obtained by interleaving w and v. For instance, the shuffle of jao and oh contains 9 words:

$$\{jaooh, joaoh, ojaoh, jaoho, joaho, ojaho, johao, ojhao, ohjao\}.$$

   The shuffle of two languages L and K is the union of shuffles of all pairs of words, one from L and one from K.

   Let's interpret regular expression in an interleaving manner: let's interpret the concatenation operation as the shuffle of two languages, and accordingly let's interpret the star operation. Thus regular expressions describe the least class of languages that contains all signleton languages $\{a\}$ for an alphabet letter a, and is closed under shuffle, shuffle star, and union.

   Under the interleaving interpretation, show the collapse: every language defined by a regular expression has star height at most 1.

2. Two languages L, K are *separated* by a third language S if S includes one of them and is disjoint from the other, say $L \subseteq S$ and $S \cap K = \emptyset$. We use the same terminology for recognising devices and say that two automata are separated by a third one.

   Show decidability of the following decision problem:

   Input: Two nondeterministic Büchi automata.
   Question: Are they separated by some deterministic Büchi automaton?

3. Consider distance automata with increment and reset operations on *two* counters. The counters are initially 0 and each transition either increments one of them by 1, or resets one of them to 0, or preserves the values of counters. Prove decidability of the limitedness problem for this model (does there exist a bound $m \in \mathbb{N}$ such that any input word admits an accepting run with values of both counters bounded by m?).

   Hint: You may try to reduce to *nested* distance automata (as defined in Problem 5 last year https://www.mimuw.edu.pl/~wczerwin/stars2019.pdf).