Homework on sets with atoms

Deadline: 22 January 2017

All (strongly) homogeneous structures mentioned below are implicitly assumed to be infinite countable ones over a finite relational vocabulary. By '(strongly) homogeneous graphs' we mean infinite countable (strongly) homogeneous structures with one binary *irreflexive and symmetric* relation.

Exercise 1. Consider the following condition: for every atoms $a \neq b$, every atom automorphism π equals to a finite composition

$$\pi \;=\; \pi_1 \circ \ldots \circ \pi_n,$$

where every π_i is either an a-automorphism or a b-automorphism. Find a structure of atoms where the condition holds, but $n \geqslant 3$ is necessary.

Exercise 1*. As above, except that $n \geqslant 4$.

Exercise 2. Show that languages recognized by deterministic Turing machines over equality atoms are closed under orbit-finite equivariant intersections. In other words, for an indexed equivariant family of deterministic Turing machines $\{M_i\}$, where I is an orbit-finite set, the language

$$\bigcap_{i \in I} L(M_i)$$

is recognized by a deterministic Turing machine.

Exercise 3. Consider the set $\Sigma = \text{Atoms}^{(7)}/G$ over equality atoms, where $G \leqslant S_7$ is generated by the following two permutations:

$$(1\,2\,3) \qquad (1\,2\,3\,4\,5\,6\,7).$$

Prove that there is a nondeterministic Turing machine over the input alphabet Σ, such that there is no equivalent deterministic Turing machine.

Exercise 4. A structure of atoms is called *transitive* when its set of elements is one equivariant orbit. Prove that every transitive homogeneous structure of atoms that admits least supports is strongly homogeneous.

Exercise 5. Let K_4' denote the clique K_4 with one edge missing. Consider strongly homogeneous graphs G such that neither G, nor its complement G^c is a disjoint union of (possibly infinite) cliques. For every such graph show that either G, or G^c embeds K_4'.

Exercise 5*. Prove the same for homogeneous graphs, instead of strongly homogeneous ones.