Homework on sets with atoms

Deadline: 22 January 2017

All (strongly) homogeneous structures mentioned below are implicitly assumed to be infinite countable ones over a finite relational vocabulary. By '(strongly) homogeneous graphs' we mean infinite countable (strongly) homogeneous structures with one binary irreflexive and symmetric relation.

**Exercise 1.** Consider the following condition: for every atoms $a \neq b$, every atom automorphism $\pi$ equals to a finite composition

$$\pi = \pi_1 \circ \ldots \circ \pi_n,$$

where every $\pi_i$ is either an $a$-automorphism or a $b$-automorphism. Find a structure of atoms where the condition holds, but $n \geq 3$ is necessary.

**Exercise 1*.** As above, except that $n \geq 4$.

**Exercise 2.** Show that languages recognized by deterministic Turing machines over equality atoms are closed under orbit-finite equivariant intersections. In other words, for an indexed equivariant family of deterministic Turing machines $\{M_i\}$, where $I$ is an orbit-finite set, the language

$$\bigcap_{i \in I} L(M_i)$$

is recognized by a deterministic Turing machine.

**Exercise 3.** Consider the set $\Sigma = \text{Atoms}^{(T)}/G$ over equality atoms, where $G \leq S_7$ is generated by the following two permutations:

$$ (123), \quad (1234567). $$

Prove that there is a nondeterministic Turing machine over the input alphabet $\Sigma$, such that there is no equivalent deterministic Turing machine.

**Exercise 4.** A structure of atoms is called *transitive* when its set of elements is one equivariant orbit. Prove that every transitive homogeneous structure of atoms that admits least supports is strongly homogeneous.

**Exercise 5.** Let $K'_4$ denote the clique $K_4$ with one edge missing. Consider strongly homogeneous graphs $G$ such that neither $G$, nor its complement $G^c$ is a disjoint union of (possibly infinite) cliques. For every such graph show that either $G$, or $G^c$ embeds $K'_4$.

**Exercise 5*.** Prove the same for homogeneous graphs, instead of strongly homogeneous ones.