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A regret-theory-based three-way decision method with a priori probability tolerance dominance relation in fuzzy incomplete information systems

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ABSTRACT

In real world, decision-makers' regret psychology often affects decision outcomes due to uncertain risks. Moreover, decision information may be missing in the process of data acquisitions or data storages. Three-way decision has been widely explored in the risky decision-making area by providing effective strategies to divide objects into three mutually disjoint regions. Existing three-way decision methods in fuzzy incomplete information systems rarely consider the influence of decision-makers' psychological states on decision outcomes. In the current paper, we primarily study a new decision-making method that combines regret theory with three-way decision in fuzzy incomplete information systems. First, a prior probability tolerance dominance relation in a fuzzy incomplete information system is defined to handle a binary relation among evaluation values, and a method to calculate objective weights is designed as well. When an incomplete information system does not contain a fuzzy decision attribute value, we put forward a new method to calculate the decision attribute value of each object in the incomplete information system. Then, integrated utility perception values are obtained by combining with regret theory. Further, a regret theory-based three-way decision method with a priori probability tolerance dominance relation is proposed for fuzzy incomplete information systems. At last, the stability and validity of the presented method are verified via corresponding experimental and comparative analysis of realistic cases.

1. Introduction

With the rapid advancement of digital society, the types of data and information have dramatically increased. In practice, incomplete information is rich due to the complexity of the problem itself and lack of expertise. Thus, decision-making problems in incomplete information systems (IISs) have been explored by many scholars during the past years. Kryszkiewicz [1] pioneered a rough set inference method that argues incomplete data can be replaced by evaluated values of arbitrary attributes. Chen et al. [2] developed rough set models in IISs by filling in missing values and proposed an incomplete probabilistic linguistic decision-making method. Liu et al. [3] introduced support matrix, precision matrix and coverage matrix in a dynamic IIS, and proposed a matrix-based incremental method to discover knowledge. Moreover, several scholars have studied decision-making problems in IISs from the perspective of similarities as well. For instance, Liu et al. [4] put forward a novel relation to depict the similarity in incomplete hybrid information systems. Luo et al. [5] introduced a new method for measuring object similarities and investigated a decision-making method based on similarities under incomplete information. Liu et al. [6] designed a fuzzy α-similarity relation in light of the similarity degree among interval values, and studied an attribute reduction method based on conditional entropies. When dealing with binary relations between objects, the similarity relations are too strict in dealing with real-valued decision-making problems that may lead to limitations in the use of these methods. The dominance relation can reflect the relation among the magnitude of evaluated values for each object, and thus the idea of dominance relations is developed in decision-making problems. In multi-criteria selection and ranking

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problems, Greco et al. [7] established an approximate preference relation by using the gradient dominance relation and then proposed a decision rule derived from the approximation of preference relations. For multi-criteria classification problems, Greco et al. [8] put forward a rough set method in light of dominance relations by considering preference orders. In view of the above studies, Yang et al. [9] considered the extent to which one type of interval-valued information is dominating other types of interval-valued information, and designed a rough set approach by incorporating the α-dominance. Szlapczynski et al. [10] explored a preference evolution multi-objective optimization method based on a tradeoff-inspired dominance relation. Li et al. [11] explored the approximate dynamics maintenance of dominance-based rough set methods with the change of objective sets. Huang et al. [12] presented a new rough set approach in light of compound dominance relations and studied an approximate updating method based on matrix increments. From the perspective of incremental feature selections, Sang et al. [13] investigated a robust conditional entropy and proposed a new neighborhood rough set model based on the fuzzy dominance. In sum, these studies in the above IISs are mainly concerned with attribute reductions and information processing. However, they do not consider the classification of objects.

With the support of three regions obtained by the rough set theory to approximate various concepts, three-way decision (3WD) [14] is a reasonable decision-making tool with rich semantic interpretations, i.e., positive regions, boundary regions and negative regions are relevant to the acceptance decisions, the delayed decisions and the rejection decisions. Yao [15] identified the trichotomy and action method as two basic tasks, and proposed the trichotomous decision-making model of the trichotomy and action method. On this basis, there are a large number of studies related to the narrow sense of 3WD. For instance, Jia and Liu [16] defined two notions of inverse and relative loss functions in a multi-criteria context, and put forward a decision-making rule in light of multi-attribute decision-making (MADM) and 3WD. In an intuitionistic fuzzy information system (IFIS), Liu et al. [17] explored a new 3WD approach to acquire relative loss functions, and proposed a new 3WD method in an IFIS. Long et al. [18] constructed a three-way granular concept and studied a dynamic changing strategy of 3WD-based granular notions. In the case of conflicts, Sun et al. [19] proposed an updated rough conflict analysis approach via incorporating 3WD-based probabilistic rough sets. It is worth noting that the studies are available for loss functions in 3WD methods below. Deng et al. [20] transformed multi-scale assessment information systems into quantitative assessment counterparts by using fuzzy affiliation functions, and a multi-scale three-way MADM method was explored. Wang et al. [21] constructed hesitant fuzzy probabilistic dominance similarity relations and proposed a new hesitant fuzzy three-way MADM method. Zhang et al. [22] established a novel q-rung orthopair fuzzy multi-granularity 3WD approach via using relative loss functions. In incomplete multi-scale information systems, Luo et al. [23] combined the dynamic trend of the conditional probability into the incremental updating process of 3WD. Xu and Wang [24] constructed an aggregation rule in light of reasonable granularity principles, and put forward a 3WD model in a mixed incomplete information table. Zhan et al. [25] introduced the concept of pre-decisions and relative utility functions to study 3WD models in an IIS. In sum, the above narrow sense of 3WD methods primarily focus on the semantics of 3WD in actual decision-making processes.

For the wide sense of 3WD, the generalized 3WD can be explained by trisecting-acting-outcome models initiated by Yao [26] in 2018, which provides the connotation and extension of 3WD. Then, Yao [27] observed the principle of tripartite thinking digital thinking, and textual thinking and visual thinking are fused together to investigated the geometric of 3WD. Given the advantage of a broader application scope for the generalized 3WD, Xu et al. [28] introduced a series of thresholds and proposed a generalized 3WD model with ranked and referenced tuples. The latest researches show that Yao [29] examined the introduction of three-step conceptual models with data sciences by regarding 3WD as the three-way thinking. In sum, the related 3WD studies mentioned above do not mention the influence of decision-makers' regretful emotions on decision outcomes.

Regret theory (RT) theory, proposed by Bell [30] in 1982, can explain situations in which an actual behavior is contrary to the desired utility. RT suggests that an individual assesses expected responses to a future event or situation, which can be described as the emotion that arises from comparing the outcome or state of a given event. Recently, there has been an increasing body of RT studies. Zhang et al. [31] presented a group consistency index and a group inconsistency index with perceived utility values, and discussed fuzzy group decision-making problems in light of RT. Peng et al. [32] integrated ELECTRE III, RT with Z-numbers to propose a comprehensive decision support model. Tian et al. [33] considered regret–rejoice sentiments of individuals and studied a consensus group decision-making model with probabilistic linguistic information. Recently, some scholars have combined 3WD with RT to explore three-way behavior decision-making (3WBDM). For instance, Liang et al. [34] used RT to describe decision-makers' risk behaviors on the loss functions and constructed a new interval-value-at-risk 3WD model. Liang et al. [35] explored the risk appetite of individuals and used TODIM as a valid tool to deal with risk preference characteristics, and constructed a risk preference two-hesitation fuzzy 3WD model in a two-hesitation fuzzy environment. Huang and Zhan [36] investigated a RT-based 3WD method in multi-scale decision information systems. Wang et al. [37] developed a regret-based 3WD model in an interval type-2 fuzzy context by using RT. In light of the above statement, we can find that the data processing of IISs is not addressed in behavioral decision-making studies, and existing studies of the 3WBDM method fails to take into account missing information and the existence of a dominance relation among object evaluation values.

As the dominance relation progresses, the 3WD method based on dominance relations has received increasing attention as well. For instance, Yang et al. [38] put forward the notion of variable precision overall dominance relations and studied the partial–overall dominance 3WD model with interval-valued data. Wang et al. [39] used probability dominance relations to obtain objective state sets, and proposed a three-way MADM method. Wang et al. [40] put forth a method to obtain conditional probabilities and constructed a 3WD method in light of probabilistic dominance relations in IFISs.

From the above literature analysis, it is necessary to explore a regret theory-based three-way decision method with a priori probability tolerance dominance relation (RT-3WD-PPTDR) method in a fuzzy incomplete information system (FIIS). In view of the above literature review, the motivations of the work are summed up below:

(1) Among existing methods for handling decision-making issues where data may be missing, Zhan et al.'s method [25] requires the presence of fuzzy decision attributes in original information tables, and Liu et al.'s method [4] requires the presence of interval loss functions in original information tables, leading to limitations in the use of these methods. In addition, Yang et al.'s method [41] can only rank all objects instead of classifying all objects. Thus, this paper will explore a novel 3WD method in FIISs.

(2) Among existing RT methods, Bell [30] proposed RT by quantifying psychological responses of decision-makers, which allows calculating the regret and elation of decision-makers. This method allows ranking all objects but does not consider categorical situations. The outcome matrix is subjectively given in Wang et al.'s method [37]. Thus, this paper will construct a decision-making method that combines 3WD with RT in FIISs.

(3) Similarity relations [4,25] are commonly used in an IIS to deal with binary relations between objects. Since similarity relations are too strict in dealing with real-valued decision-making problems, this paper will define a priori probabilistic dominance

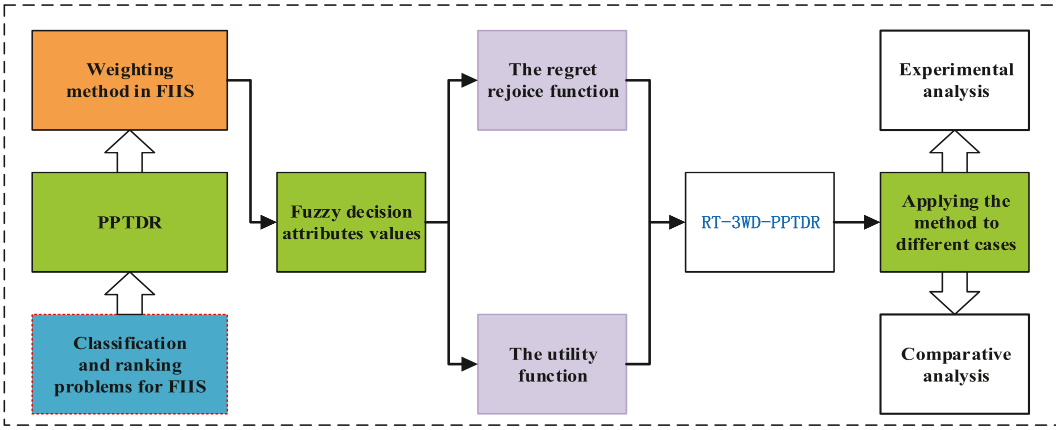

Fig. 1. The framework of the paper.

relation with a broader application scope. Thus, this paper will propose a RT-3WD-PPTDR in FIISs.

(4) The acquisition of attribute weights in MADM methods is also an issue worth investigating. In the case of complete data, the attribute weight method in Wang et al.'s method [39] is subjectively provided by experts according to their experiences. In an IIS, Zhan et al.'s method [25] gives entropy weights based on similarity relations, however similarity relations have limitations when dealing with real-valued decision-making problems. Thus, this paper designs a novel approach for obtaining objective weights in FIISs.

The innovations of this work are described as follows:

(1) For the first time, this paper defines a PPTDR in FIISs, which provides a more generalized calculation way to deal with binary relations between objects.
(2) In this paper, a method for determining attribute weights in FIISs is proposed for the first time, which can be used to estimate the importance of all attributes.
(3) By combining RT and 3WD in FIISs, this paper proposes a generalized three-way decision model in FIISs to classify and sort all objects.

The layout of the paper is structured below. In Section 2, we revisit fundamental notions of IFISs, RT and 3WD. In Section 3, we explore a new dominance relation, a maximum deviation weight and a new RT-3WD-PPTDR method. In Section 4, we apply the presented RT-3WD-PPTDR method to address realistic applications. In Sections 5 and 6, comparative and experimental studies are conducted for showing the validity of the presented algorithm. Section 7 illustrates some primary research contents and presents directions for future studies. In addition, an overall diagram of the paper is given in Fig. 1.

2. FIISs, RT and generalized 3WD models

In this section, we review several fundamental notions and results on FIISs, RT and generalized 3WD models.

2.1. FIISs

Definition 2.1 ([41]). Suppose that a fuzzy information system (FIS) is expressed as $I = \{U, C \bigcup \{d\}, V, F\}$, where $U = \{o_1, o_2, \ldots, o_n\}$ denotes a non-empty finite set of n objects and $C = \{c_1, c_2, \ldots, c_m\}$ denotes a non-empty finite set of m conditional attributes. $d \in F(U)$ and $d(x_i) \in [0,1]$, where $F(U)$ denotes a power set of fuzzy sets on $V = \bigcup_{c_j \in C} V_{c_j}$ and V_{c_j} is a value region of the conditional attribute c_j. $F : U \times C \to V$ is an information function such that $F(o_i, c_j) = o_{ij} \in V_{c_j}$ for each $i \in N = \{1, 2, \ldots, n\}$ and $j \in M = \{1, 2, \ldots, m\}$. If a number of evaluation values for conditional attributes are unknown in an FIS, unknown values are denoted by the symbol "$*$". In other words, there exist some $i \in N$ and $j \in M$ such that $o_{ij} = *$. Then such an FIS is updated to an FIIS and shown in Table 1. In the current paper, an FIIS is denoted by $I^* = \{U, C \bigcup D, V, F\}$, where $V = \bigcup_{c_j \in C} V_{c_j} \bigcup \{*\}$.

Table 1
An FIIS $I^* = \{U, C, V, F\}$.

U/C	c_1	c_2	$\cdots$	c_m	$\{d\}$
o_1	o_{11}	o_{12}	$\cdots$	o_{1m}	d_1
o_2	o_{21}	$*$	$\cdots$	o_{2m}	d_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
o_n	o_{n1}	o_{n2}	$\cdots$	o_{nm}	d_n

Table 2
An information evaluation table of candidates.

	c_1	c_2	c_3	c_4	d
o_1	0.12	0.23	0.54	0.71	0.34
o_2	0.3	$*$	0.91	0.82	0.42
o_3	$*$	0.33	0.12	0.11	0.51
o_4	0.27	0.97	0.44	$*$	0.63
o_5	0.56	$*$	0.24	0.56	0.37

Example 2.1 will show specific information about an FIIS.

Example 2.1. Table 2 is an FIIS, in which $U = \{o_1, o_2, \ldots, o_5\}$ represents the objects, $C = \{c_1, c_2, c_3, c_4\}$ represents the set of conditional attributes and d is a decision attribute.

In Table 2, we can find that the evaluation information of the object o_2 on the attribute c_1 is 0.3, but the evaluation information on the attribute c_2 is unknown.

2.2. RT

RT was proposed by Bell [30], which describes regrets as the emotion that arises from comparing the outcome or state of a given event. For instance, when consumers choose between a familiar brand and an unfamiliar one, they may rate the regret of choosing an unfamiliar one resulting in a bad outcome as greater than the regret of choosing a familiar one. Thus, consumers rarely choose an unfamiliar brand. In actual decision-making processes, decision-makers usually worry about the regret of not choosing the best option by the risk-averse psychology.

Based on RT, the formula for the regret–rejoice functions of the object o_i relative to the object o_l in terms of the attribute c_j is shown as follows:

$$\mathrm{O}(o_i, o_l) = \begin{cases} 1 - e^{-\gamma(o_{ij} - o_{lj})} & o_{ij} < o_{lj}; \\ 0 & o_{ij} \geq o_{lj}, \end{cases} \tag{1}$$

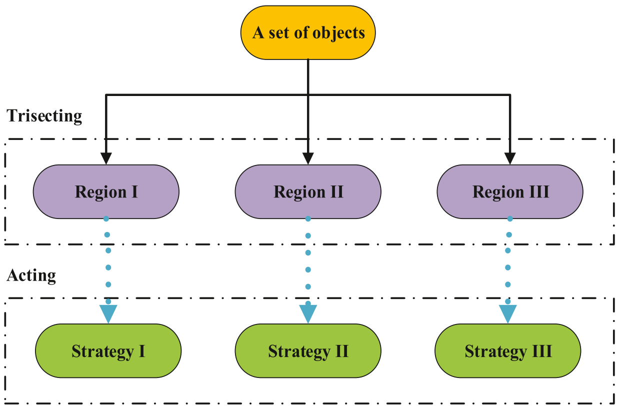

Fig. 2. Generalized 3WD models of "trisecting and acting".

$$\mathrm{G}(o_i, o_l) = \begin{cases} 0 & o_{ij} < o_{lj}; \\ 1 - e^{-\gamma(o_{ij} - o_{lj})} & o_{ij} \geq o_{lj}, \end{cases} \tag{2}$$

where γ stands for a regret–rejoice aversion coefficient and $\gamma \in [0, +\infty)$. Moreover, the utility function is represented as follows:

$$U(o_i, o_l) = \frac{1 - e^{-\theta o_{ij}}}{\theta} - \frac{1 - e^{-\theta o_{lj}}}{\theta}, \tag{3}$$

where θ stands for a risk aversion coefficient and $\theta \in (0, 1)$.

2.3. Generalized 3WD models

3WD [14] is consistent with human cognitions and the core idea of 3WD is to separate a unified set into three disjoint regions, and to develop a corresponding decision strategy for each region. The generalized 3WD model of "trisecting and acting" model [15] is shown in Fig. 2.

Definition 2.2. Assume that $IS = \{U, C\}$ is an information system, where U is the set of all objects and C is the set of all attributes. The function F can divide all objects into three regions that do not intersect with each other, and its formula is shown as follows:

$$F : U \rightarrow \{D_1, D_2, D_3\}, \tag{4}$$

where $D_1, D_2, D_3 \subseteq U$, $D_1 \cap D_2 = \emptyset$, $D_1 \cap D_3 = \emptyset$, $D_2 \cap D_3 = \emptyset$ and $D_1 \cup D_2 \cup D_3 = U$.

Remark 2.1. Fig. 2 clearly shows the two important components of the generalized 3WD model, namely trisecting and acting. The specific semantic explanations are as follows:

Trisecting: Generally speaking, decision-makers need to establish a reasonable and effective trisecting rule, the purpose of which is to reasonably allocate all objects to three different decision-making regions. Note that these three regions are independent of each other and the union is the entire object universe. In particular, when solving sorting problems, researchers generally require that these three regions have a certain order relationship.

Acting: For different decision-making regions, decision-makers need to set up corresponding decision-making strategies. Note that the decision-making strategies in these three regions can be the same or different. When solving the sorting problem, researchers generally set a sorting rule in the three decision-making regions to obtain the sorting of objects in the regions.

3. RT-3WD-PPTDR

Since most of existing methods can only solve the decision-making problem with complete information. Thus, we define a RT-3WD-PPTDR method for solving MADM problems in this section.

3.1. PPTDR and the maximum deviation weight

For the reason that classic equivalence relations and dominance relations are usually not able to handle binary relations between objects with missing evaluation values, then we will define a PPTDR, and a maximum deviation weight is provided as well.

Definition 3.1 ([42]). According to the probability distribution theory, an FIIS $I^* = \{U, C, V, F\}$ contains some prior knowledge in anticipation. Consequently, we can effectively describe the unknown evaluation values via the probabilities of all known values in the evaluation range. Based on the prior knowledge, the probability of obtaining each evaluation value in $\}\}c_j\varepsilon$ for an unknown evaluation value $\}\} * \varepsilon$ in an attribute V_{c_j} is represented as $P(*= v_1) = P_1, P(*= v_2) = P_2, \ldots, P(*= v_k) = P_k$.

Suppose that $I^* = \{U, C, V, F\}$ is an FIIS. For $\forall c_j \in C$, the range of evaluation values $V_{c_j} = \{v_1, \ldots, v_f, \ldots, v_k\}$ for c_j is a finite set, and $v_1 \leq \cdots \leq v_f \leq \cdots \leq v_k$. The vector consisting of the probabilities for all evaluation values in V_{c_j} is $P_{c_j} = \{P_1, P_2, \ldots, P_k\}$. The probability P_f $(f \in \{1, 2, \ldots, k\})$ is called the prior probability, and its basic semantics is based on the attribute c_j, the ratio of the number of times the evaluation value v_f appears to the total number of objects n. For example, in Table 2, based on the attribute c_1, the prior probability of $P(*= 0.12)$ is $\frac{1}{5}$. In general, the unknown evaluation value $\}\} * \varepsilon$ takes the value with higher occurrence probability, which is represented with the probability by $P_1 \geq P_2 \Longrightarrow P(*= v_1 | p(v_1) = P_1) \geq P(*= v_2 | p(v_2) = P_2)$.

Definition 3.2. Assume that $I^* = \{U, C, V, F\}$ is an FIIS. For any $o_i, o_l \in U (i, l \in \{1, 2, \ldots, n\})$, $c_j \in C (j \in \{1, 2, \ldots, m\})$ and $v_f \in V_{c_j} (f \in \{1, 2, \ldots, k\})$, the probability that the object o_i is worse than the object o_l is:

$$R_{c_j}(o_i, o_l) = \begin{cases} 0 & o_{ij} \geq o_{lj} \quad \wedge o_{ij} \neq * \wedge o_{lj} \neq *; \\ \frac{o_{lj} - o_{ij}}{o_{lj}} & o_{ij} < o_{lj} \quad \wedge o_{ij} \neq * \wedge w_{lj} \neq *; \\ \sum_{f=1}^{k} P_f^2 & o_{ij} = * \wedge o_{lj} = *; \\ \sum_{1 \leq h \leq f} P_f & o_{ij} = * \wedge o_{lj} = v_f; \\ \sum_{f \leq h \leq k} P_f & o_{ij} = v_f \wedge o_{lj} = * . \end{cases} \tag{5}$$

Definition 3.3. Given an FIIS $I^* = \{U, C, V, F\}$, for all $B \subseteq C$, $\forall o_i, o_l \in U$, the priori probability tolerance dominance degree with respect to the attribute set B is represented below:

$$R_B(o_i, o_l) = \sum_{c_j \in B} R_{c_j}(o_i, o_l) / |B|, \tag{6}$$

where $|\cdot|$ denotes the number of attribute sets.

Afterwards, the priori probability tolerance dominance of I^* under the attribute set B is represented as:

$$R_B^{\leq * \beta} = \{(o_i, o_l) \in U \times U | R_B(o_i, o_l) \leq \beta\}, \tag{7}$$

where β is called a tolerance rate for the attribute set B, which satisfies $0 \leq \beta \leq 1$. Here, $R_B^{\leq * \beta}$ is called a PPTDR.

Moreover, we can obtain the priori probability tolerance dominance class in terms of the conditional attribute set B:

$$[o_i]_B^{\leq * \beta} = \{o_l \in U | R_B(o_l, o_i) \in R_B^{\leq * \beta}\}. \tag{8}$$

For the sake of obtaining the best order within a certain range to promote the best common benefits, we normalize the data in an information table. The calculation formula is expressed as:

$$h_{ij} = \frac{o_{ij}}{\sqrt{\sum_{i=1}^{n} o_{ij}^2}} \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, m. \tag{9}$$

Table 3
An information evaluation table of candidates.

	c_1	c_2	c_3	c_4
o_1	2	2	4	1
o_2	3	*	2	2
o_3	3	4	2	1
o_4	1	3	2	*
o_5	2	*	3	3
o_6	3	4	*	3
o_7	3	2	2	1
o_8	3	*	1	3
o_9	*	2	4	2
o_{10}	3	3	2	2

Table 4
The weight value of each conditional attribute c_j.

	c_1	c_2	c_3	c_4
w_j	0.1377	0.2295	0.3140	0.3188

Example 3.1. Table 3 is an information evaluation table of candidates when a company recruits, in which $U = \{o_1, o_2, \dots, o_{10}\}$ represents the collection of candidates, $C = \{c_1, c_2, c_3, c_4\}$ represents the set of assessment indicators, c_1 denotes the business ability, c_2 denotes the professional knowledge, c_3 denotes the work experience and c_4 denotes the work attitude.

We normalize the data in Table 3 by using Formula (8). With the above Definitions 3.1–3.3, we can obtain the priori probability tolerance dominance class of each candidate o_i via calculation, and the results are shown below:

$[o_1]_O^{\leq *0.1} = \{o_1, o_5, o_6, o_9\}$,
$[o_2]_O^{\leq *0.1} = \{o_2, o_6, o_9, o_{10}\}$,
$[o_3]_O^{\leq *0.1} = \{o_2, o_3, o_5, o_6, o_{10}\}$,
$[o_4]_O^{\leq *0.1} = \{o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_9, o_{10}\}$,
$[o_5]_O^{\leq *0.1} = \{o_5, o_6, o_9\}$,
$[o_6]_O^{\leq *0.1} = \{o_2, o_5, o_6, o_8\}$,
$[o_7]_O^{\leq *0.1} = \{o_1, o_2, o_3, o_5, o_6, o_7, o_9, o_{10}\}$,
$[o_8]_O^{\leq *0.1} = \{o_6, o_8, o_9, o_{10}\}$,
$[o_9]_O^{\leq *0.1} = \{o_5, o_6, o_9\}$,
$[o_{10}]_O^{\leq *0.1} = \{o_2, o_5, o_6, o_9, o_{10}\}$.

Subsequently, we will study the method for obtaining attribute weights in an FIIS.

Definition 3.4. Given an FIIS $I^* = \{U, C, V, F\}$, the equation for the maximum deviation weight w_j based on the priori probability tolerance dominance class for the jth attribute is expressed as:

$$w_j = \frac{\sum_{i=1}^{n} \sum_{k=1}^{n} |H_{ij} - H_{kj}|}{\sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{n} |H_{ij} - H_{kj}|}, \tag{10}$$

where $H_{ij} = 1 - \frac{|[o_i]_{c_j}^{\leq *\beta}|}{n}$ for $i \in \{1, 2, \dots, n\}$, $j \in \{1, 2, \dots, m\}$.

Obviously, $0 \leq w_j \leq 1$ and $\sum_j^m w_j = 1$.

Example 3.2 (*Continued with Example 3.1*). We calculate the weight value of each conditional attribute c_j by Definition 3.4, and the calculation results are shown in Table 4.

3.2. A RT-3WD-PPTDR method

The classic RT for decision-makers is considered to be presented in Section 2. Regret is the emotion that arises when comparing the outcome or state of something with the state that would be chosen. For example, when consumers choose a familiar brand and an unfamiliar brand, they may think that the regret of choosing an unfamiliar brand for the adverse effects is greater than the regret of choosing a familiar brand, so consumers rarely choose an unfamiliar brand. RT can reflect the psychological state of regret that often occurs in the decision-making process. In this section, we describe an RT model based on dominance relations in an FIIS.

Definition 3.5. The actual values of each object are compared with the standard values and then aggregated using linear synthesis to obtain a composite score for each object, a method called the composite index method. We use the composite index method to calculate the fuzzy decision attribute values for each object, and show them as follows:

$$d_i = \sum_{j=1}^{m} \frac{h_i^j}{h_\epsilon^j} W_j, \tag{11}$$

where h_ϵ^j is the standard value, it can be set flexibly according to the practical necessity. The above formula shows that the evaluation value h_i^j of the object o_i under the attribute c_j is compared with the standard value h_ϵ^j under the attribute, and then the composite score d_i of the object o_i under different attributes is obtained by using linear comprehensive summary. In this paper, we make the standard value h_ϵ^j to be the maximum of the known values under each conditional attribute c_j.

Firstly, we use the fuzzy decision attribute function of each object to obtain the satisfaction of each object o_i:

$$Q_i = \frac{d_i}{d_{\max}}, \tag{12}$$

where $d^{\max}$ is the maximum value among the decision attribute values of all objects, i.e., $d^{\max} = \max_{i \in \{1,2,\dots,n\}} d_i$. In Formula (12), the fuzzy decision attribute value of each object divided by the largest fuzzy decision attribute value among all objects can represent the relative size of the object o_i. The closer d_i is to the maximum value of $\max d_i$ in all objects, the more satisfied the decision maker is with the fuzzy decision attribute value of the object. Therefore, Formula (12) can represent the satisfaction of each object.

Secondly, we calculate the regret–rejoice value of object o_i relative to object o_l:

$$\mathrm{O}(o_i, o_l) = \begin{cases} 1 - e^{-\gamma(Q_i - Q_l)} & Q_i < Q_l; \\ 0 & Q_i \geq Q_l. \end{cases} \tag{13}$$

$$\mathrm{G}(o_i, o_l) = \begin{cases} 0 & Q_i < Q_l; \\ 1 - e^{-\gamma(Q_i - Q_l)} & Q_i \geq Q_l, \end{cases} \tag{14}$$

where $\gamma \in [0, +\infty)$ is a regret–rejoice aversion coefficient, $O(o_i, o_l)$ represents a regret value of object o_i relative to object o_l, $G(o_i, o_l)$ represents a rejoice value of object o_i relative to object o_l. For the regret–rejoice values $O(o_i, o_l)$ and $G(o_i, o_l)$ in Formulas (13) and (14), the regret–rejoice values are also functions that increases monotonically with the increase of $Q_i - Q_l$.

Thirdly, the utility function is shown:

$$U(o_i, o_l) = \frac{1 - e^{\theta Q_i}}{\theta} - \frac{1 - e^{\theta Q_l}}{\theta}, \tag{15}$$

where $\theta \in (0, 1)$ is a risk aversion coefficient, $U(o_i, o_l)$ represents a utility function of object o_i relative to object o_l.

Fourthly, the utility perception function based on regret–rejoice of object o_i relative to object o_l is calculated as follows:

$$F_{il} = O(o_i, o_l) + G(o_i, o_l) + U(o_i, o_l). \tag{16}$$

For Formula (16), the utility perception function of the object o_i relative to the object o_l is obtained by integrating the regret value, the rejoice value and the utility value of the object o_i relative to the object o_l.

Fifthly, the combined utility perception function of each object o_i is calculated as follows:

$$F(o_i) = \sum_{l=1}^{n} F_{il}. \tag{17}$$

Since Formula (16) shows the perceived utility value of object o_i relative to object o_l, in Formula (17), the summation method is used to obtain the combined perceived utility value of object o_i for other objects.

Further, we consider the distance between the object and the positive ideal solution F_i^+ and the negative ideal solution F_i^- to calculate the degree of similar proximity for each object o_i to the positive ideal solution, and the formula is shown as follows:

$$\widetilde{F}(o_i) = \frac{F_i^-}{F_i^+ + F_i^-}, \tag{18}$$

where the positive ideal solution $F_i^+ = F_{max} - F(o_i)$ and the negative ideal solution $F_i^- = F(o_i) - F_{min}$. In Formula (18), the smaller the value of the combined perceived utility value of object o_i is from the largest combined perceived utility value of the objects, and the larger the value of the combined perceived utility value of object o_i is from the smallest combined perceived utility value of the objects, the value of $\widetilde{F}(o_i)$ is bigger.

Next, we introduce the classical 3WD method in Section 2, then we describe the classification rule of 3WD method in FIIS. The rules that determine the region in which each object is located, and they are shown below:

(P) Decide $o_i \in POS(\mathrm{C})$, if $\widetilde{F}(o_i) \in (\widetilde{F}^{\max}(o_i) - \delta, \widetilde{F}^{\max}(o_i)]$,

(B) Decide $o_i \in BND(\mathrm{C})$, if $\widetilde{F}(o_i) \in [\widetilde{F}^{\min}(o_i) + \delta, \widetilde{F}^{\max}(o_i) - \delta]$,

(N) Decide $o_i \in NEG(\mathrm{C})$, if $\widetilde{F}(o_i) \in [\widetilde{F}^{\min}(o_i), \widetilde{F}^{\min}(o_i) + \delta)$,

where $\delta \in [0, 0.5]$.

Finally, we rank all objects with $\widetilde{F}(o_i)$ of objects in each region and the priority of $POS(\mathrm{C}) \succ BND(\mathrm{C}) \succ NEG(\mathrm{C})$.

3.3. Steps of the RT-3WD-PPTDR method

For the sake showing the decision-making procedure of the RT-3WD-PPTDR method in an FIIS in a detailed way, we summarize the detailed steps of the method along with the algorithm of the method below.

Input: An FIIS $I^* = \{U, C, V, F\}$ on a 3WBDM problem, and four parameters β, γ, θ and δ.

Output: The ranking and classification of all objects.

Step 1: Compute the normalized data for each object o_i by Formula (9).

Step 2: Obtain the priori probability tolerance dominance class of each object o_i by Definitions 3.1–3.3.

Step 3: Calculate the maximum deviation weight w_j based on the priori probability tolerance dominance class for each attribute c_j by Definition 3.4.

Step 4: Compute the value of the fuzzy decision attribute for each object o_i by Definition 3.5.

Step 5: Calculate the combined utility perception function of each object o_i by Formula (17).

Step 6: Get the degree of similar proximity $\widetilde{F}(o_i)$ for each object to the positive ideal solution F_i^+ by Formula (18).

Step 7: Determine the region in which each object is located by the rules (P)–(N).

Step 8: Rank all objects with $\widetilde{F}(o_i)$ of objects in each region and the priority of $POS(\mathrm{C}) \succ BND(\mathrm{C}) \succ NEG(\mathrm{C})$.

Remark 3.1. Definition 3.5 shows a method to obtain fuzzy decision attribute values by using the conditional attribute values of each object. If fuzzy decision attribute values exist in an original FIIS, the method presented in this section can be directly applied without computing fuzzy decision attribute values. In other words, if the fuzzy decision attribute values exist in the original FIISs, the decision step can directly go from Step 3 to Step 5.

Remark 3.2. In order to present the efficiency of our method, we describe the time complexity of the presented method below. In specific, n stands for the number of elements in the object set, m stands for the number of elements in the attribute set. In Step 1, we compute the normalized data for each object by Formula (9) and the complexity is 1. In Step 2, the priori probability tolerance dominance class of each object by Definitions 3.1–3.3, and the maximum time complexity is $O(n^3m)$. Step 3 computes the maximum deviation weight w_j based on the priori probability tolerance dominance class for each attribute c_j by Definition 3.4, and the complexity is $O(n^2m)$. Step 4 computes the value of the fuzzy decision attribute for each object o_i by Definition 3.5, and the complexity is $O(m)$. Step 5 calculates the combined utility perception function of each object o_i by Formula (17), and the complexity is $O(n^2)$. In Step 6, the degrees of similar proximity $\widetilde{F}(o_i)$ are calculated for each object to the positive ideal solution F_i^+ by Formula (18), and the complexity is $O(1)$. Step 7 determines the region in which each object is located by the rules (P)–(N) and the complexity is $O(n)$. Step 8 ranks all objects with $\widetilde{F}(o_i)$ of objects in each region and the priority of $POS(\mathrm{C}) \succ BND(\mathrm{C}) \succ NEG(\mathrm{C})$, and the complexity is $O(n)$. In sum, the total time complexity of the presented algorithm is $O(n^3m)$.

Remark 3.3. In this paper, we propose a 3WBDM method based on RT with a PPTDR in FIISs. From Definition 3.2, it can be found that our method can also be applied to IISs, incomplete ordered information systems, complete information systems (CIS), fuzzy information systems, ordered information systems, etc.

In what follows, we describe Algorithm 1 for the presented method in this paper.

4. Application cases

In Section 3, we have presented a 3WBDM method based on the dominance relations by combining 3WD with RT. It is found from Definition 3.2 that our method can not only solve decision-making problems in FIISs, but also can effectively address decision-making problems in CISs. Consequently, this section applies our method to two different aspects of application cases.

Heart disease [43] includes rheumatic heart disease, congenital heart disease, hypertensive heart disease, coronary heart disease, myocarditis, etc. Nowadays, heart disease is the number one killer of human health. According to the statistical data released by the World Health Organization, 1/3 of the world's population deaths are caused by heart disease, and hundreds of thousands of people die from heart disease each year in China. In turn, regular medical examinations for heart disease patients are vital. The echocardiogram [44] is one of the most important items that need to be checked during the physical examination of heart disease patients. An echocardiogram is commonly known as an ultrasound of hearts. Echocardiography uses the radar scanning technology and the characteristics of sound wave reflections via various layers of hearts, so as to observe the morphology and structure of hearts, large blood vessels and beating states, and to further understand the pattern of atrial systoles, diastoles and valve closures. Thus, the early detection and treatment of heart disease act as challenging issues for the following three reasons: First, the situation in which the results of patient-examined data may be lost during data storages or migrations, hence incomplete information in medical diagnosis emerges. Second, doctors usually classify patients as treated, under observation or not treated according to indicators of test results. If necessary, physicians need to select the most severely ill patients for immediate treatment as well as rank all patients according to medical examinations. Third, decision-making results may be influenced by the emotion of decision-makers because making decisions may own certain risks.

Algorithm 1: The RT-3WD-PPTDR model in an FIIS

Input: A 3WBDM problem, and four parameters β, γ, θ and δ.
Output: The classification and ranking of all objects.

begin
  Given the parameters $\beta \in [0,1]$, $\gamma \in [0,+\infty)$, $\theta \in (0,1)$ and $\delta \in [0,0.5]$.
  for $i=1$ to n, $j=1$ to m **do**
    calculate: the normalized data for each object. // by Formula (9)
  end
  for $j=1$ to m, $i=1$ to n, $l=1$ to n and $k=1$ to length $(\text{unique}\{o_{ij}\})$ $(i=1,2,\cdots,n)$ **do**
    calculate: the priori probability tolerance dominance class of each object. // by Definitions 3.1–3.3
  end
  for $i=1$ to n, $j=1$ to m, $k=1$ to n, **do**
    calculate: the maximum deviation weight W_j based on the priori probability tolerance dominance class for each attribute c_j. // by Definition 3.4
  end
  for $i=1$ to m **do**
    calculate: the value of the fuzzy decision attribute for each object o_i. // by Definition 3.5
  end
  for $i=1$ to n, $k=1$ to n **do**
    calculate: the combined utility perception function of each object o_i. // by Formula (17)
  end
  for $i=1$ to n **do**
    calculate: the degree of similar proximity $\widetilde{F}(o_i)$ for each object to the positive ideal solution F_i^+. // by Formula (18)
  end
  Obtain three regions: $POS(\mathrm{C})$, $BND(\mathrm{C})$ and $NEG(\mathrm{C})$.
  for $i=1$ to n **do**
    determine: the region in which each object is located by the rules (P)-(N).
    if $\widetilde{F}(o_i) \in (\widetilde{F}^{\max}(o_i)-\delta, \widetilde{F}^{\max}(o_i)]$ **then** $(o_i) \in POS(\mathrm{C})$;
    // the decision rule (P)
    if $\widetilde{F}(o_i) \in [\widetilde{F}^{\min}(o_i)+\delta, \widetilde{F}^{\max}(o_i)-\delta]$ **then** $(o_i) \in BND(\mathrm{C})$;
    // the decision rule (B)
    if $\widetilde{F}(o_i) \in [\widetilde{F}^{\min}(o_i), \widetilde{F}^{\min}(o_i)+\delta)$ **then** $(o_i) \in NEG(\mathrm{C})$.
    // the decision rule (N)
  end
  for $i=1$ to n **do**
    rank: all objects with $\widetilde{F}(o_i)$ of objects in each region and the priority of $POS(\mathrm{C}) \succ BND(\mathrm{C}) \succ NEG(\mathrm{C})$.
  end
  return: the classification and ranking of all objects.
end

4.1. The case of incomplete information: Echocardiogram

In this section, the presented method is applied to an incomplete dataset in UCI databases, and the problem background is stated below (http://archive.ics.uci.edu/ml/datasets/Echocardiogram). A hospital intends to use Echocardiography to diagnose whether a patient has heart disease or not. For the purpose of experimental and comparative analyses, we select objects without missing decision attributes and no more than two missing values per object. The following example consists of 73 objects o_i and five conditional attributes. Then, we address such decision-making problems via utilizing the method[1] presented in this paper and process this dataset with the ranking results shown in Fig. 3 and the classification results shown in Fig. 4.

[1] β=0.1, γ=0.3, θ=0.3 and δ=0.36.

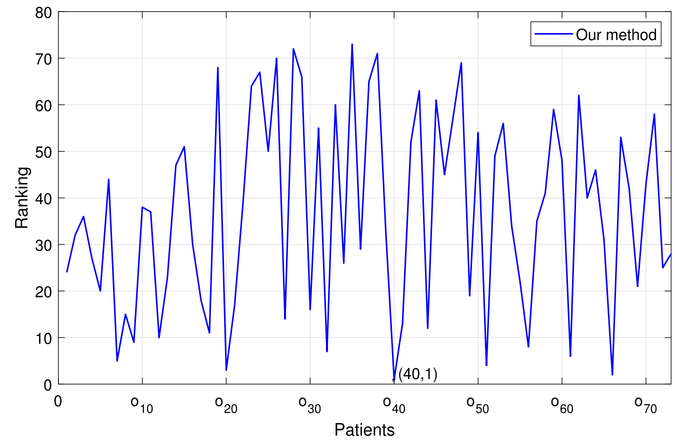

Fig. 3. The ranking result of Echocardiography.

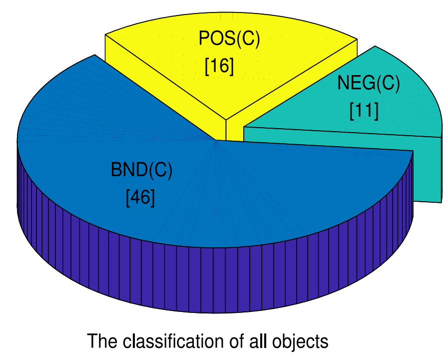

Fig. 4. The classification result of Echocardiography.

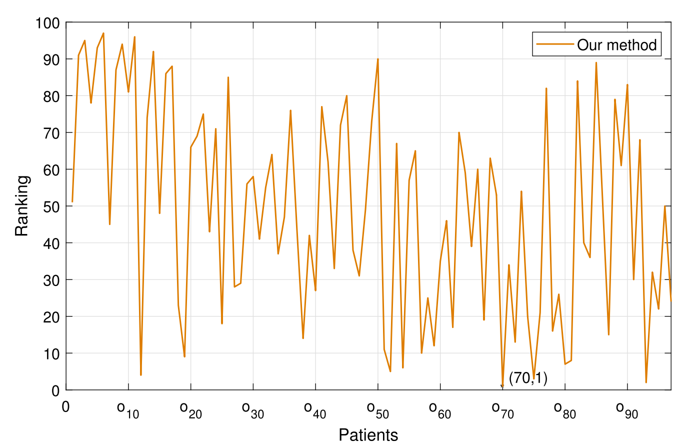

Fig. 5. The ranking result of Breast Cancer Coimbra.

Fig. 3 provides a visual observation of the ranking results of our method for medical diagnosis processing issues, and we can obtain the optimal result as o_{40}. In Fig. 4, we can see the number of objects contained in each region. Thus, our method can solve medical diagnosis problems in an FIIS.

4.2. The case of complete information: Breast Cancer Coimbra

In Section 4.1, our method can effectively use Echocardiography to diagnose the problem of whether a patient has heart disease in an FIIS. In addition, we know that our method can also address medical

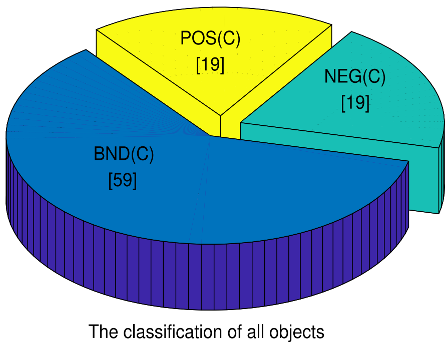

Fig. 6. The classification result of Breast Cancer Coimbra.

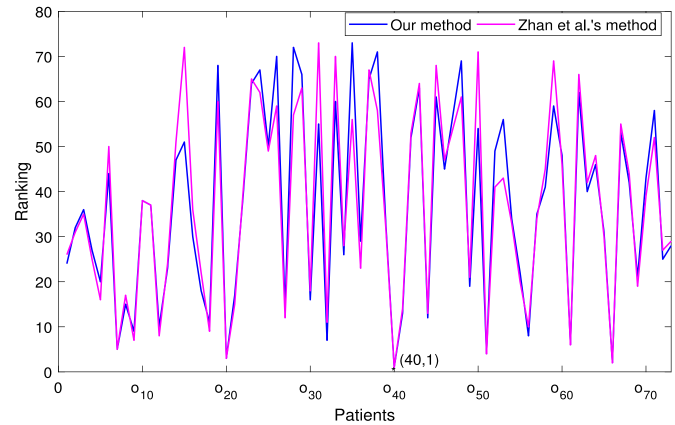

Fig. 7. Comparison of the ranking of different methods in FIISs.

diagnosis problems in a CIS. At present, most of existing decision-making methods in an FIIS can only solve classification problems, but classification problems need to be explored. For the sake of comparing the classification results of our method with existing methods, our method is applied to a dataset (http://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Coimbra) for diagnosing whether a patient has breast cancer or not. Due to the need of arithmetic processes, we keep the objects whose evaluation values are non-zero. Thus, the case contains 97 objects and seven attributes. Then, we address such decision-making problems by utilizing the method[2] presented in this paper and process this dataset with the ranking results shown in Fig. 5 and the classification results shown in Fig. 6.

Fig. 5 gives a visual observation of the ranking results of our method for medical diagnosis processing issues and the optimal result is o_{70}. In Fig. 6, we can see the number of objects contained in each region. Thus, our method can solve medical diagnosis problems in a CIS.

Remark 4.1. In this paper, a 3WD method in light of RT for a PPTDR in FIISs is proposed. First, the prior probability tolerance dominance relation of FIISs is defined and the method of calculating objective weights is given. Further, the decision attribute values of each object can be calculated in light of the conditional attribute values of each object, and the integrated utility perception values are obtained by combining RT. In this section, it can be found that our method can effectively solve medical decision-making problems.

5. Comparative analysis

In Section 4, the presented method is applied to FIIS and CIS where it is found that the method can consider the effect of decision-makers' emotional changes on decision outcomes and effectively address medical decision-making problems. This section is arranged by three subsections, which compare the method with decision-making methods in FIISs and CISs.

5.1. Comparative analysis for the case of Echocardiogram

In what follows, we intend to compare the ranking results obtained by several methods in FIISs. First, the ranking results obtained in Section 4.1 are compared with the ranking results obtained by Zhan et al.'s method [25],[3] as shown in Fig. 7.

It is clear from Fig. 7 that the trends of ranking results acquired by our method in FIISs are similar. In addition, the Spearman's rank correlation coefficient (SRCC) value of ranking results for both methods is 0.9557. Further, it can be quantitatively known that our method is close to the ranking results obtained by existing methods. Thus, it can be shown that our method owns validity.

In order to compare the sorting results obtained by different approaches, the notion of SRCC which shows the correlation among variables [45] is employed as follows:

$$SRCC = 1 - \frac{6\sum_{i=1}^{n}(x_i - y_i)_i^2}{n^3 - n}. \tag{19}$$

In what follows, we use the SRCC to demonstrate the consistency of the sorting performance of our method with some existing methods, and the SRCC conclusions are utilized in two ways: (1) The correlation between the two samples is illustrated by comparing the SRCC values obtained from the two samples through Spearman's rank correlation threshold table. (2) Given the superior ranking performance of the methods, the SRCC values obtained by both methods are compared with those obtained by the same method, and if the first method has a larger SRCC value than that obtained by the second one. Then, in the case of that method being compared, the first method owns superior ranking performances than the second counterpart.

Moreover, our method also classifies all objects into three regions. Thus, we compare the classification situations that are obtained for four methods in FIISs, including Zhan et al.'s method [25],[4] Yang et al.'s method [41][5] and Liu et al.'s method [4].[6] The classification of two datasets in different methods is shown in Fig. 8.

It is clear that only our method and Zhan et al.'s method can get three regions from Fig. 8. 3WD gives reasonable semantic interpretations of three regions in decision-theoretic rough sets. The 3WD method considers the boundary region, i.e., it also takes into account delayed decisions. Thus, our method has a certain error tolerance when handling practical decision-making issues and is more in line with practical situations. Then, we analyze the misclassification rates of several methods in Fig. 9.

It can be visualized from Fig. 9 that in the Echocardiogram dataset, since the loss function of Liu et al.'s method is subjectively provided as interval loss functions, it appears that the misclassification rate is 0. Then only the misclassification rate of the results obtained by our method is smaller than the misclassification rate of the results obtained by other methods. Moreover, in the case of the data set of Hepatitis, the classification error rate obtained by our method is minimal. The formula for calculating the misclassification rate is shown below [37]:

$$Misclassification \quad rate = \frac{n_{C\longrightarrow NEG(C)} + n_{\neg C\longrightarrow POS(C)}}{|U|}, \tag{20}$$

[2] β=0.1, γ=0.3, θ=0.3 and δ=0.36.
[3] The confidence level $\theta = 0.6$ and the utility pursuit coefficient $\kappa = 0.9$.
[4] The confidence level $\theta = 0.6$ and the utility pursuit coefficient $\kappa = 0.9$.
[5] The pair of thresholds $\alpha = 0.5, \beta = -0.5$.
[6] The given threshold L=0.6 and $\theta = 0.5$.

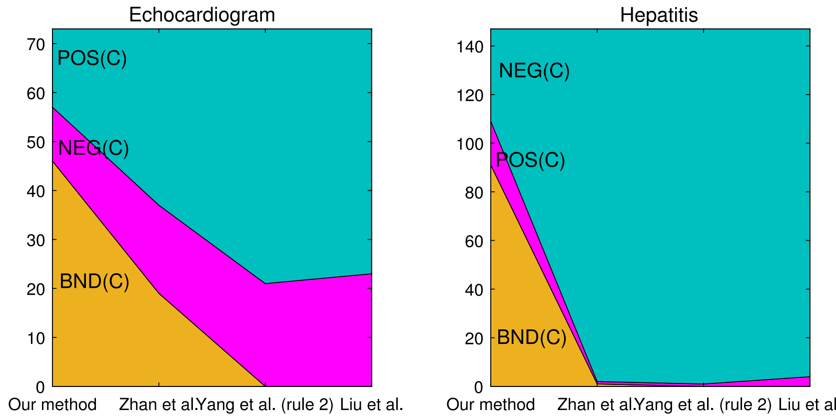

Fig. 8. Comparison of the classification of different methods in FIISs.

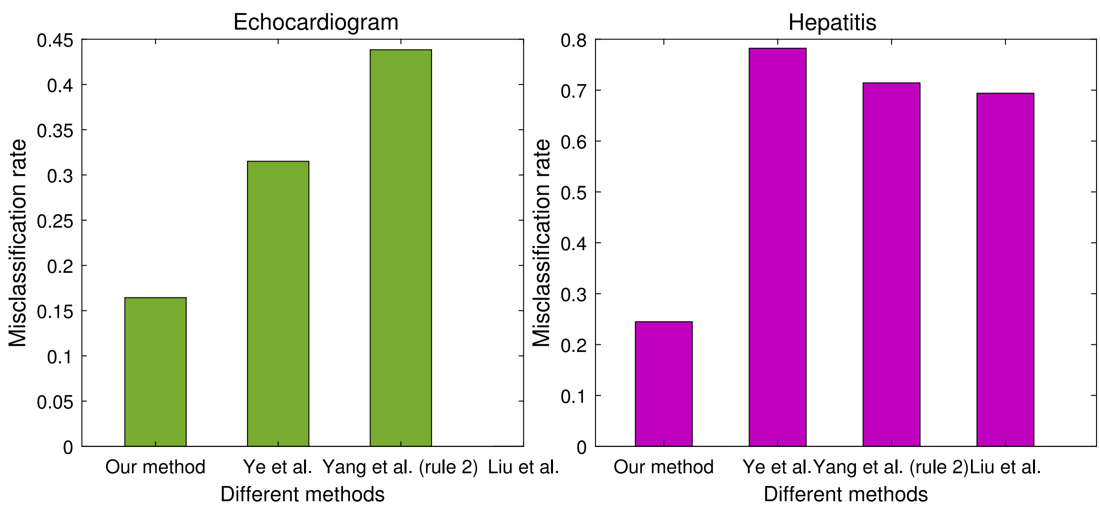

Fig. 9. Misclassification rate of different methods in FIISs.

where $n_{C\longrightarrow NEG(C)}$ denotes the number of objects classified into the negative region $NEG(C)$ belonging to C. $n_{\neg C\longrightarrow POS(C)}$ denotes the number of objects classified into the positive region $POS(C)$ belonging to ¬C, U is the overall number of objects. Obviously, the smaller the misclassification rate is, the better the performance of the presented 3WD model.

5.2. Comparative analysis for the case of Breast Cancer Coimbra

In what follows, we compare the ranking results obtained by several methods in CISs. First, the ranking results obtained in Section 4.2 are compared with the ranking results obtained by the WAA operator method [46], the PROMETHEE II method [47],[7] Bell's method [30],[8] Wang et al.'s method [37],[9] Jia and Liu's method [16][10] and Zhan et al.'s method [48],[11] as shown in Fig. 10.

It can be seen in Fig. 10 that our method is close to the ranking trend obtained by the existing methods in complete cases. Thus, it can be shown that our method has certain application values. Moreover, in order to verify the effectiveness of our method, the SRCC values of the ranking results of these methods are shown in Fig. 11.

With the Spearman rank correlation threshold table, we know that when the sample size is 90 and the significance level of the two-tailed test is 0.01, comparing two samples to obtain an SRCC value greater than 0.271 indicates that the two samples are highly correlated. As can be seen in Fig. 11, the SRCCs between the 3WBDM method and the six decision-making methods, including Bell's method [30], Wang et al.'s method [37], Jia and Liu's method [16], Zhan et al.'s method [48], the WAA operator method [46] and the PROMETHEE II method [47] are all higher than 0.9632, shows that the ranking performance of these methods is highly consistent. On the other hand, we can find that the SRCCs between the 3WBDM method and all six decision-making methods is higher than that between the five decision methods and Bell's method [30], Wang et al.'s method [37] and Zhan et al.'s method [48], indicating that our method outperforms Bell's method [30], Wang et al.'s method [37] and Zhan et al.'s method [48].

5.3. Discussion

Our method defines a PPTDR for FIISs and further constructs an objective method for obtaining weights. In addition, our method evaluates decision attribute values of each object in light of existing conditional attribute values of each object, considers the emotional changes of decision-makers and constructs a decision-making method that combines 3WD with RT. However, some methods can only solve decision-making problems for complete information systems and do not consider

[7] The parameters $p_1 = 120$, $p_2 = 80$, $p_3 = 10$, $p_4 = 15$, $p_5 = 50$, $p_6 = 90$, $p_7 = 20$ and $q_i = 0 (i = 1, 2, \dots, 7)$.

[8] The parameter $g = 0.1$ and the regret–rejoice aversion coefficient $h = 0.3$.

[9] The risk avoidance coefficient $\theta = 0.3$ and the regret–rejoice aversion coefficient $\delta = 0.3$.

[10] The risk avoidance coefficient $\theta = 0.4$ and the conditional probability is 0.55.

[11] The parameters in the ELECTRE-I method $\sigma = 0.4$ and $p = 0.55$. Choose a pessimistic strategy.

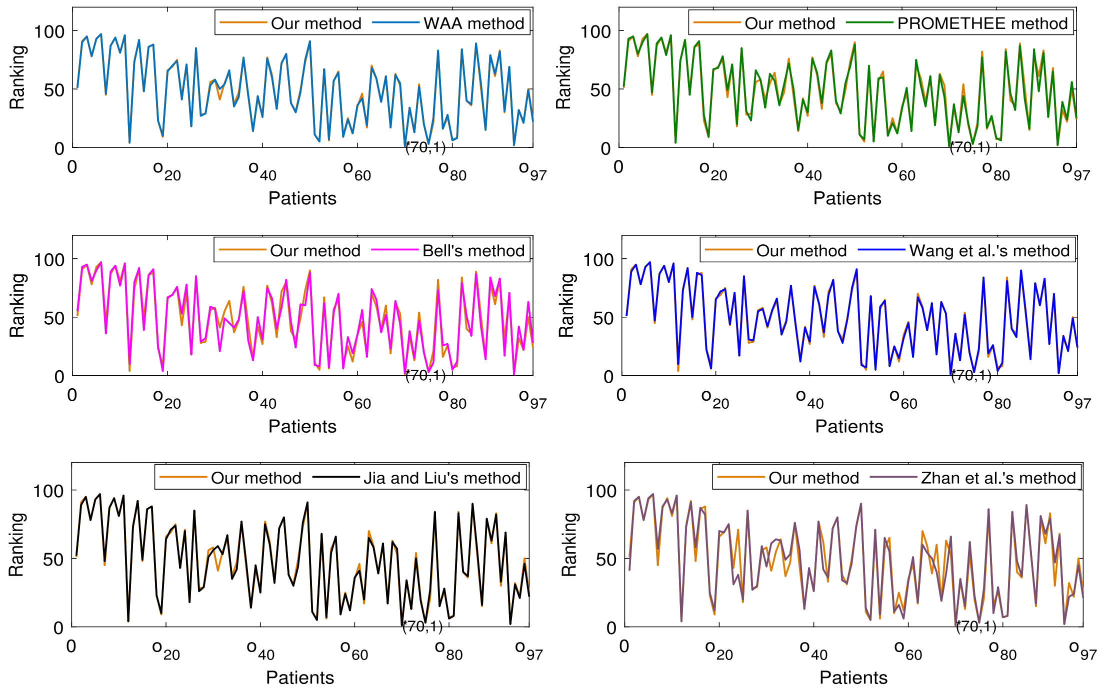

Fig. 10. Ranking obtained by different methods in CISs.

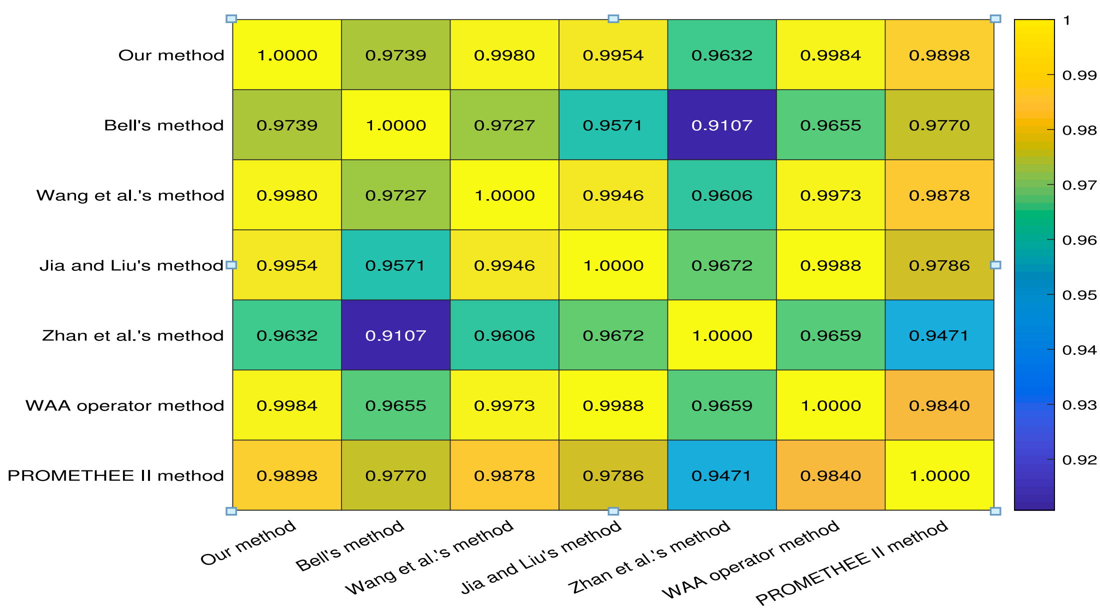

Fig. 11. SRCCs of the ranking results obtained from different methods in CISs.

the influence of decision-makers' emotions on the decision outcome. Therefore, we analyze the above compared methods in detail.

From Table 5, we summarize the features of our method in comparison analysis with nine different methods:

(1) Our method takes advantages of dominance relations by using RT and 3WD to explore a new method for 3WBDM of FFIISs. Thus, our method can well solve the decision-making problem of FIISs. Moreover, it is worth noticing that our method can also solve decision-making problems for CISs. In Table 5, only our method and Liu et al.'s method [4], Zhan et al.'s method [25] and Yang et al.'s method [41] are used to solve the problem in FIISs. The other methods can only be applied to CISs.

(2) In Table 5, only four methods can be applied when there are missing values in the information table. Further, we analyze these four methods: Yang et al.'s method [41] and Liu et al.'s method [4] only classify objects, but do not consider the ranking of objects. Notwithstanding Zhan et al.'s method [25] can classify and rank all objects, it requires the use of known fuzzy decision attribute values to calculate the utility function. There are few known fuzzy decision attribute values in the data in the database or in real-world cases, hence the application scope is limited. Nonetheless, our method can calculate fuzzy decision attribute values based on evaluation value information in FIISs, and then classify and rank all objects.

(3) Our method obtains objective state sets according to decision attributes of data sets, hence we can obtain objective state sets.

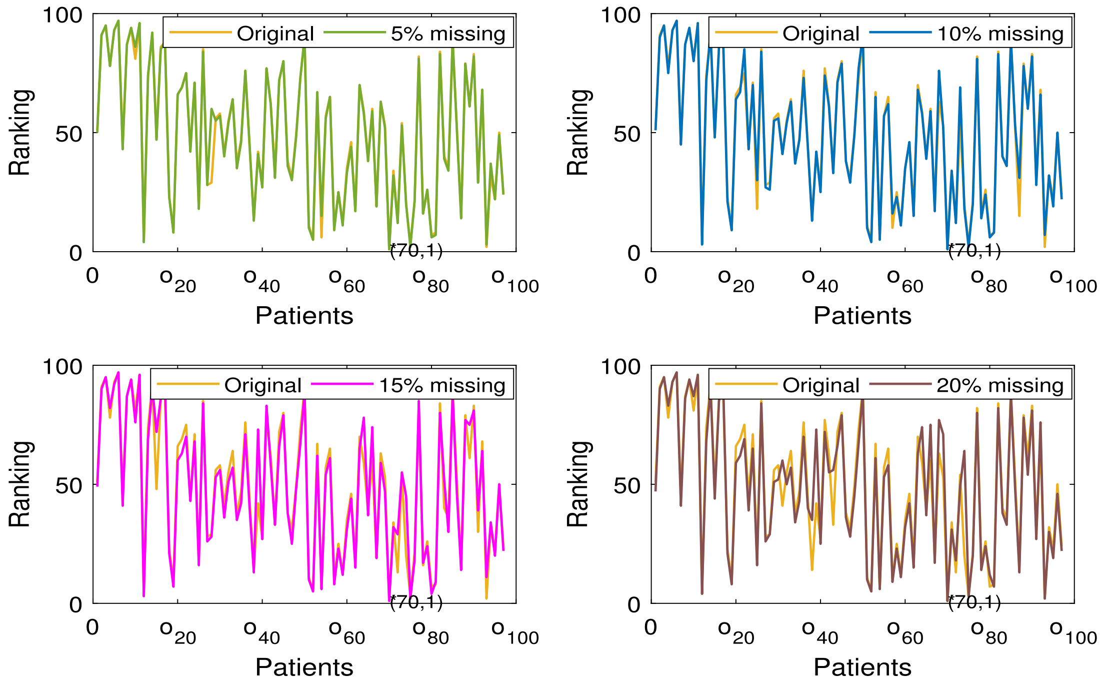

Fig. 12. Ranking results of a set with different degrees of missing evaluation values.

Table 5
Comparison of different methods.

Different methods	Incomplete information	Decision attributes	Objective weights	Loss/Utility /Value functions	Behavior decisions	Ranking	Classification
Our method	✓	✓	✓	✓	✓	✓	✓
Bell's method [30]	×	×	×	×	✓	✓	×
Wang et al.'s method [37]	×	×	×	✓	✓	✓	✓
Liu et al.'s method [4]	✓	×	×	✓	×	×	✓
Zhan et al.'s method [25]	✓	✓	✓	✓	×	✓	✓
Yang et al.'s method [41]	✓	✓	×	×	×	×	✓
Jia and Liu's method [16]	×	×	×	✓	×	✓	✓
Zhan et al.'s method [48]	×	×	×	✓	×	✓	✓
WAA operator method [46]	×	×	×	×	×	✓	×
PROMETHEE II method [47]	×	×	×	×	×	✓	×

At this time, we can obtain the conditional probability value according to classic conditional probability formulas, and then compute weight functions. In this calculation process, because the conditional probability is objectively obtained, there is less influence of subjective preference on decision-making results.

(4) In Table 5, among the above decision-making methods, only Bell's method [30], Wang et al.'s method [37], and our method consider the emotions of decision-makers arising from comparing the outcome or state of a given event with the state to be chosen. In other words, only these methods consider RT and are more in line with the actual decision-making situation.

(5) Our method calculates the objective weights of each object o_i based on the PPTDR, whereas considering the utility function of each object o_i.

By the above comparative analysis between our method and other nine methods, it is not difficult to find the advantages of our method and show them as follows:

(1) In the big data era, missing data in information systems are common due to the possibility of missing data in the process of data acquisitions or storages. At this time, our method can not only effectively handle the data in CISs, but also solve decision-making problems in FIISs.

(2) Among the above methods that can handle decision-making problems in FIISs, Liu et al.'s method [4] and Yang et al.'s method [41] are only able to classify all objects, whereas our method can classify and rank all objects.

(3) In other words, some methods do not consider the emotions of decision-makers arising from comparing the outcome or state of a given event with the state to be chosen. Our method combines RT with 3WD models, considers the influence of people's emotions on decision-making choices, and has more practical application values.

(4) Our method can obtain objective weight vectors based on dominance relations, and in addition the fuzzy decision attribute values for each object can be calculated based on evaluation values in information tables.

Table 6
The SRCC of ranking results with different degrees of missing evaluation values relative to the original results.

Different degrees	5%	10%	15%	20%
Group 1	0.9711	0.9764	0.9709	0.9576
Group 2	0.9794	0.9671	0.9669	0.9734
Group 3	0.9924	0.9926	0.9712	0.9630
Average	0.9810	0.9787	0.9697	0.9647

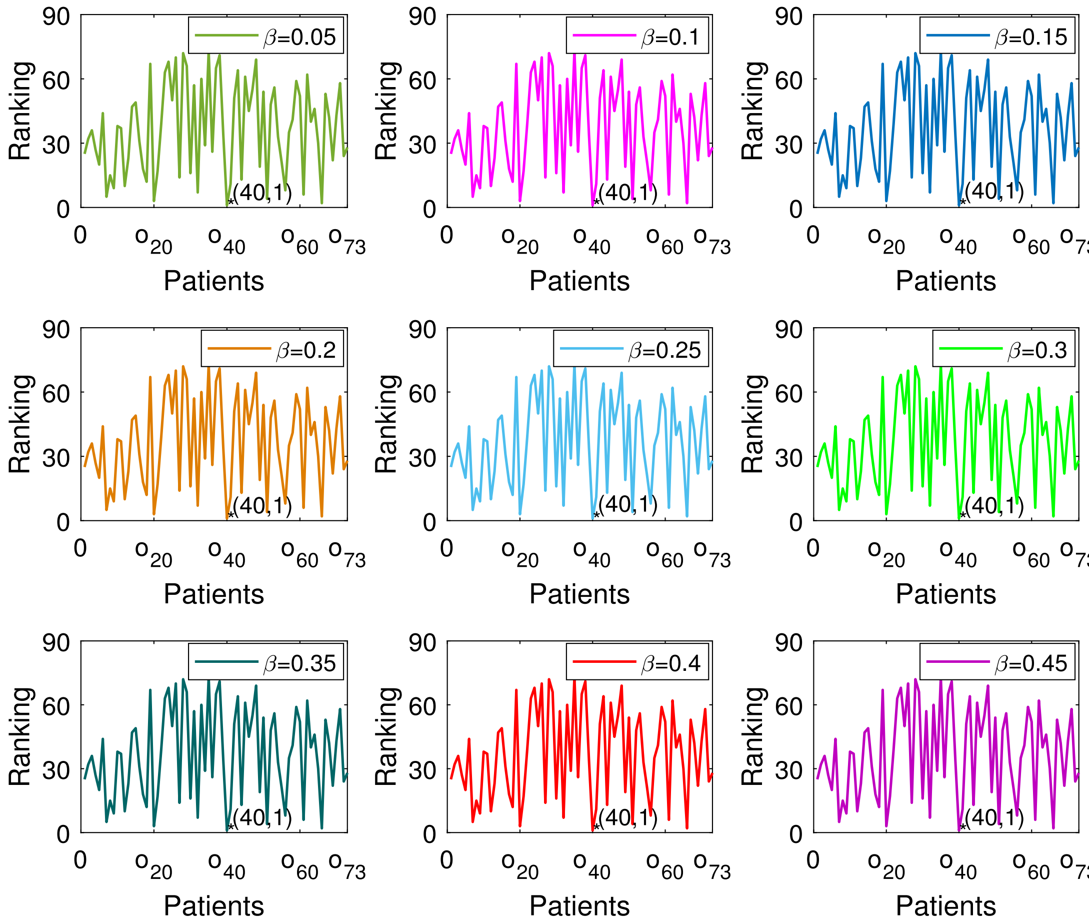

Fig. 13. Ranking of all objects when the values of the parameter β change.

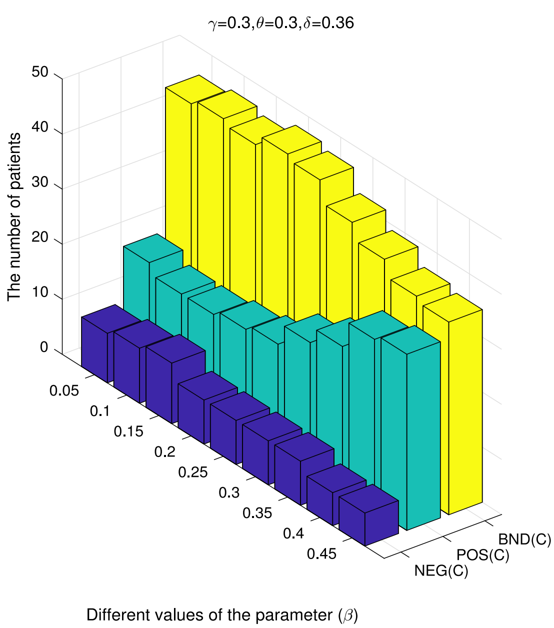

Fig. 14. Classification of all objects when the values of the parameter β change.

6. Experimental analysis

In this section, we verify the stability of our method in terms of both missing data and parameters.

6.1. Missing degree analysis

Based on the Breast Cancer Coimbra cases under the CIS in Section 4.2, we randomly remove different levels of evaluation values to the FIIS. Then we use the method proposed in this paper to deal with decision-making problems under incomplete cases. Fig. 12 shows the ranking results of a set with different degrees of missing evaluation values. Table 6 shows the SRCC values of three sets of ranking results with different degrees of missing evaluation values relative to the original results in Section 4.2.

From Fig. 12, it can be visualized that the most available objects of the ranking results obtained by different degrees of evaluation value missing are consistent, and the trends of ranking are also close. Meanwhile, it can be found from Table 6 that the SRCC values obtained from the ranking results with different degrees of missing evaluation values and the original ranking results are all greater than 0.9. This indicates that the correlation between the ranking results obtained with different degrees of missing evaluation values and the original ranking results is significant.

6.2. Sensitivity analysis with the RT-3WD-PPTDR method

Since our method is mainly designed to solve decision-making problems in FIISs, we take the Echocardiography problem in Section 5.1 as an example and perform a sensitivity analysis of our method by parametric analysis. In this paper, four parameters β, δ, γ and θ are involved, we will describe them in three aspects.

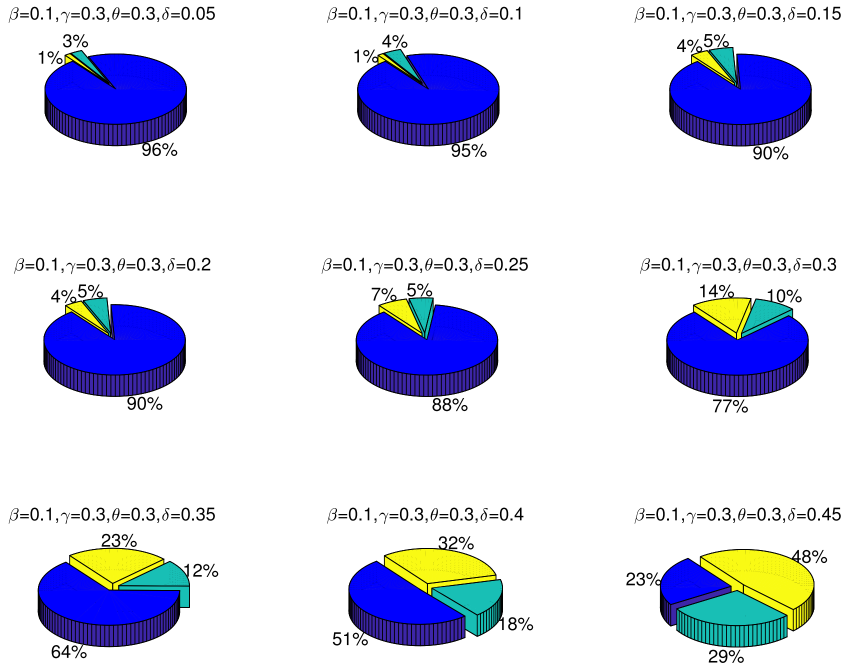

Fig. 15. Classification of all objects when the values of the parameter δ change.

6.2.1. Parameter analysis (β)

The values of the parameters γ, θ and δ are fixed at 0.3, 0.3 and 0.36, respectively. We observe the changes in the ranking and classification of all objects when changing the value of the parameter β, as shown in Figs. 13 and 14.

As can be seen in Fig. 13, when our method is applied to the case of Echocardiogram by fixing the values of the three parameters and changing the value of the parameter β, the ranking of all patients changes and the overall trend of the ranking results obtained is very similar. In addition, it can be seen that the most severely ill patient is o_{40}. This indicates that our method has some stability. The number of objects in the positive region increases when the value of the parameter β becomes larger, whereas the number of patients in the boundary and negative regions decreases in Fig. 14. It shows that the larger the value of the parameter β is, the higher the fault tolerance of the method.

6.2.2. Parameter analysis (δ)

The values of the parameters β, γ and θ and are fixed at 0.1, 0.3 and 0.3, respectively. We observe the changes in the classification of all objects when changing the value of the parameter δ, as shown in Fig. 15.

The ranking of all patients is not affected by fixing the values of the three parameters and changing the value of the parameter δ when our method is applied to the case of Echocardiography. Thus, we only analyze the effect of the parameter δ on the change of the classification of all patients. As can be seen in Fig. 15, when the value of the parameter δ becomes larger, the number of patients in the boundary region increases, whereas the number of patients in the positive and negative regions decreases.

6.2.3. Parameter analysis (γ, θ)

The values of the parameters β and δ are fixed at 0.1 and 0.36, respectively. We observe that the classification of all patients changes when the values of the parameters γ and θ change simultaneously, as shown in Fig. 16.

It can be seen from Fig. 16 that the number of objects in $POS(\mathrm{C})$ gradually increases when the parameters γ and θ increase simultaneously, whereas the number of objects in $BND(\mathrm{C})$ and $NEG(\mathrm{C})$ gradually decreases. Therefore, we can know that the larger the value of the parameters γ and θ is, the more patients are in the positive region. At this point, the doctor can help the patient to make a quick diagnosis and treat them in time.

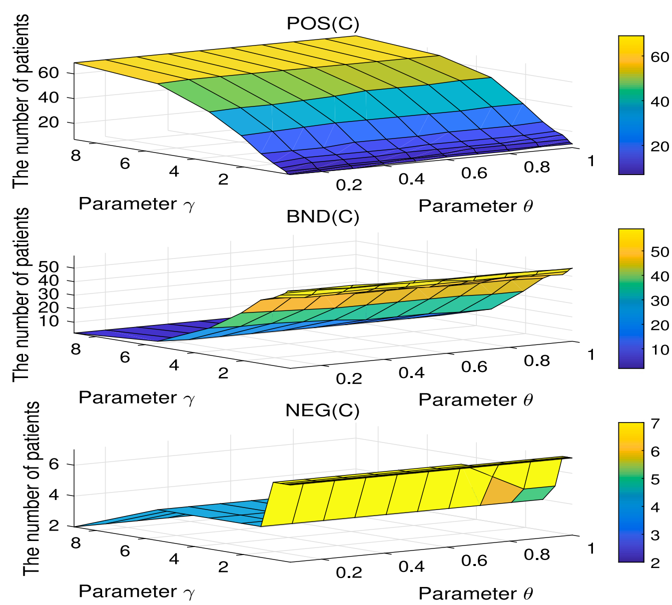

Fig. 16. The classification of all objects when the values of the parameters γ and θ change simultaneously.

The changes in the misclassification rate when the values of the parameters γ and θ are simultaneously varied, as shown in Fig. 15. We can see that the change in the value of the parameter θ has a small effect on the misclassification rate. The larger the value of the parameter γ is, the larger the misclassification rate. In practical decision-making problems, the greater the regret–rejoice aversion coefficient is and the smaller the number of objects in the boundary region is, the greater the possible misclassification rate. It can be found from Fig. 17 that the values of the parameters γ and θ simultaneously change with the value of the misclassification rate in (0,0.32).

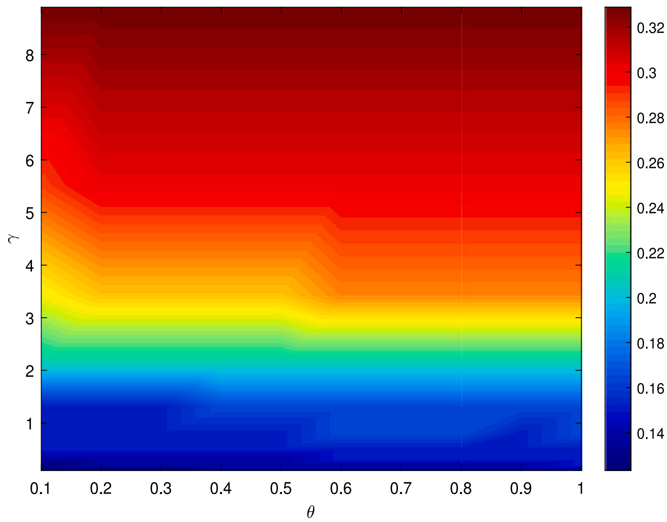

Fig. 17. The change in fault tolerance when the values of the parameters γ and θ change simultaneously.

7. Conclusions

Nowadays, with the rapid development of advanced technologies and information circulations, big data is emerged as the product of this era. At this time, the type of data and information is dramatically increasing, and data may be lost or omitted in the process of acquiring or storing. Thus, the problem of dealing with decision-making issues in FIISs is of great concern to many scholars. In addition, behavioral decision-making is also a significant study topic in the decision science. Thus, we have studied the RT-3WD-PPTDR method in FIISs. The primary contributions of this paper are summed up as follows:

(1) Since there may be missing data in the process of data acquisitions or data storages in actual decision-making problems, we propose a RT-3WD-PPTDR method to effectively handle decision-making problems with missing information. According to the description of the method in this paper, we know that the method can solve both decision-making problems with incomplete data and complete data.
(2) Existing 3WD methods of FIISs usually use some relations obtained by generalizing the equivalence relations when dealing with binary relations between objects, such as similarity relations and tolerance relations. Since the equivalence relation is too strict in dealing with real-valued decision-making problems, this paper defines the PPTDR. Obviously, this dominance relation can more extensively deal with binary relations between objects.
(3) The acquisition of attribute weights is also an issue of interest in classic MADM problems. In general, the weights of attributes are subjectively given by decision-makers or existing attribute weighting methods are only able to process the complete data information. Thus, this paper investigates the method of obtaining attribute weights in FIISs.
(4) 3WD usually includes two states and three behaviors, and classic 3WD methods mainly involve conditional probabilities and loss functions as two key issues. Since the loss function in existing methods is generally subjectively given by decision-makers or calculate the relative loss function. The existence of the relative value 0 in the calculation of relative loss function may appear to be restricted to use. Thus, this paper combines 3WD with RT and considers the emotion that decision-makers may regret taking a certain behavior and investigates the RT-3WD-PPTDR method. This method can separate objects into three disjoint regions according to the combined utility perception function. The comparative and experimental analysis can demonstrate the validity and superiority of the presented method.

For future study options, we can consider a few meaningful topics:

(1) This study combines 3WD with RT to consider the effect of an individual's expected response to a future event or situation on the decision outcome. We can then consider more behavioral decisions, such as the prospect theory. In addition, we can combine RT with three-way conflict analysis [19].
(2) We will consider extending 3WD model based on RT to the group decision-making environments. At the same time, we will try to consider how to further consider the corresponding decision when there are individual attributes of the decision-maker in complete or incomplete cases [49,50].
(3) The method established in this paper can effectively solve decision-making problems in FIISs. From the perspective of management sciences, the decision-making problem can be divided into single decision-making and group decision-making according to the number of individuals. Thus, the extension of our method to group decision-making information systems [51–54] is also worth studying. In addition, it is necessary to explore studies related to linguistic decision-making contexts [55].
(4) With the continuous development of society and economy, information and data dramatically increase and dynamically change nowadays. There are a large number of decision-making problems in dynamic information systems [56] in practical applications, thus we can generalize the presented 3WD method based on dominance relations to dynamic information systems.

CRediT authorship contribution statement

Wenjie Wang: Investigation, Conceptualization, Methodology, Writing – original draft. **Jianming Zhan:** Methodology, Investigation, Writing – original draft. **Chao Zhang:** Methodology, Writing – review & editing. **Enrique Herrera-Viedma:** Methodology, Writing – review & editing. **Gang Kou:** Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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