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Research paper

Behavioral three-way decision based multi-attribute decision-making for credible hesitant fuzzy information systems

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ABSTRACT

Multi-attribute decision-making, as a core of decision analysis, plays important roles in knowledge discovery for artificial intelligence, and finds extensive applications in medical assessments and assisted diagnosis. Assessment information systems are fundamental models of describing properties of objects or patients, which are composed of evaluation values of objects regarding multiple attributes, captured from such as biochemical tests, physical and instrument examinations, and statistic inferences. Due to existence of uncertainty and imprecision in collected these data at different time or from different hospitals, hesitant fuzzy sets are an effective tool of quantizing the evaluation values. In view of experiences, techniques or instrument sensitivity, the quantitative hesitant membership values may be not completely credible. The derived information systems are referred to as credible hesitant fuzzy information systems. To tackle the challenge of multi-attribute decision-making in the scenarios of credible hesitant fuzzy information systems, a behavioral three-way decision model is proposed. Firstly, the notions of dominance degree between credible hesitant fuzzy elements and dominance relation between objects are introduced in credible hesitant fuzzy information systems. With these notions, the weights of attributes are quantitatively assigned to discriminate impact of attributes on decision-making. Secondly, the regret and rejoice functions in regret theory are improved to reflect human psychological emotions for decision-making. A set of payoff functions are then constructed and decision rules are subsequently extracted by introducing the behavior three-way decision model. Finally, three medical assessment cases are comparatively analyzed using representative decision-making methods. The experimental results demonstrate that the proposed method reduces the error rate by at least 50% and promotes the $F1$ score by 1.4% in comparison to the best results among representative decision-making methods on the benchmark baseline cases.

1. Introduction

Artificial Intelligence demonstrates powerful capabilities in data processing and pattern recognition by simulating human intelligence processes (Jin et al., 2025; Guo et al., 2024; Yuan et al., 2024). However, it still faces many difficulties in complex decision-making domains such as risk assessment and medical engineering. In risk assessment domains, decision-makers must balance multiple conflicting objectives (Badhon et al., 2025; Jairi et al., 2025). In medical engineering, a key difficulty is handling the inherent ambiguity and uncertainty associated with clinical diagnosis (Hwang et al., 2025). Furthermore, a pervasive issue across both fields is the "black box" problem, which leads to non-interpretability of decisions results (Xu and Shuttleworth, 2024; Gul et al., 2024). To systematically address these difficulties, multi-attribute decision-making provides a structured analytical framework capable of integrating multidimensional information and handling data with uncertainty. Multi-attribute decision-making is realized in information systems, which are composed of multiple objects or alternatives, each being assessed under multiple attributes. Medical assisted diagnosis and assessment systems are typical scenarios of information systems for multi-attribute decision-making (Karthik et al., 2025; Qin et al., 2024a), where evaluation values of attributes (criteria or indices) are observed from laboratory tests, imaging inspection, clinical records, and so on. Due to uncertainty in describing symptoms or diseases, objects are usually quantified by using real numbers or fuzzy sets (Demir et al., 2024; Zia et al., 2024).

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To address semantic fuzziness of objects on multiple attributes more rationally, fuzzy information systems are usually extended to such as hesitant fuzzy information systems (Chen et al., 2025a), interval-valued information systems (Pei et al., 2024a; Yang et al., 2020), intuitionistic fuzzy information systems (Hussain et al., 2024; Xin et al., 2024), or linguistic-valued information systems (Pei et al., 2024b; Wang et al., 2025). In medical assessment scenarios, the health status of a patient or the severity of a disease is usually assessed based on historical data of necessary attributes. This data may originate from different time periods or distinct healthcare institutions. Hesitant fuzzy set is an appropriate notion of quantizing this situation and hesitant fuzzy information system is applied to formulate the decision-making problem (Qin et al., 2024b; Zhan et al., 2023). Chen and Xu et al. developed two models, HF-ELECTRE I (Chen et al., 2015) and HF-ELECTRE II (Chen and Xu, 2015), for decision problems in hesitant fuzzy information system. Mishra et al. (2021) proposed an additive ratio assessment method (HF-ARAS) to screen out the optimal medications for COVID-19 treatment.

In classic decision-making analysis, the decisions on objects are made with either acceptance or rejection options. To achieve reliable decision-making for multi-attribute decision-making problem in complex scenarios, Yao (2010) initialed the notion of three-way decision by extending the classic binary decision-making paradigm to acceptance, rejection or delayed decisions by combining rough set theory with Bayesian principle of minimum risk. The innovative idea of three-way decision meets human intuition of maximizing gains and minimizing losses. The three-way decision paradigm has been paid wide attention to in decision analysis, knowledge discovery and data mining (Chen et al., 2025b; Yuan et al., 2025; Yan et al., 2024b; Guo et al., 2023). To eliminate subjectivity of the classic three-way decision model, Jia and Liu (2019) introduced the notion of relative loss functions and presented the three-way decision model based on aggregated relative loss functions (TWD-EV). Following that, Jiang and Hu (2021) proposed a data-driven loss function technique and developed a decision-theoretic fuzzy rough set based three-way decision model in the scenario of hesitant fuzzy information systems (DTFRS-HFIS). To address the failure of classic models in asymmetric affiliations, Yi et al. (2022) presented the Pythagorean fuzzy three-way decision method based on S-shaped utility (PFTWD-SSU). In response to complex decision-making problems, Qian et al. (2024) proposed a three-way multi-attribute decision making method in a multiscale decision information system (TWD-MSDIS), which solved the problem of cross-level analysis in the traditional framework. Yan et al. (2024a) took into account both benefit and cost attributes and developed a novel three-way classification and ranking model based on regret theory and TOPSIS (TWD-RT-TOPSIS). The model achieved multi-objective equilibrium optimization in fuzzy information systems. At the semantic modeling level, Wang et al. (2023) introduced a new distance formula to compute the similarity of probabilistic linguistic term sets, then proposed a three-way decision method via incorporating prospect theory and probabilistic linguistic term sets (PL-PT-TWD). Zhang and Yu (2025) combined the prospect theory with three-way decision and proposed the PT-TWD-PPTDR model for fuzzy incomplete information systems, which reduces cognitive bias in fuzzy incomplete information systems.

Usually, decision-makers may regret the options they did not choose. In order to avoid that puzzle, the regret theory proposed by Bell (1982) was introduced into the three-way decision framework. A regret-based three-way decision model (RTWD-IN2FN) was proposed in interval type-2 fuzzy environments by Wang et al. (2022). Meanwhile, Mondal et al. (2023) combined prospect and regret theory to develop a three way model for q-type fuzzy information systems (TWD-qROFSs). Furthermore, Zhu et al. (2022) developed a relative outcome and regret-based utility function to enhance decision modeling. Building on this, they constructed a three-way multi-attribute decision-making method (3W-MADM-R) under regret theory, tailored for medical diagnosis in fuzzy environments.

In order to quantify uncertainty of a hesitant fuzzy element in hesitant fuzzy information system, Zhu (2014) introduced a probability measure for each membership value of a hesitant fuzzy element in hesitant fuzzy set and addressed the decision-making in probabilistic hesitant fuzzy information systems. Inspired by that work, Gao et al. (2022) studied weighted integral operators for a probabilistic hesitant fuzzy set. In the case of incomplete and hesitant evaluation data, Zhu et al. (2024) built an optimization model for probabilistic interval hesitant fuzzy information systems to solve the multi-attribute decision-making problem.

However, the probability of a membership value in a hesitant fuzzy element is not easily determined. Probabilistic hesitant fuzzy information systems also fail to reflect and measure uncertainty in the hesitant fuzzy element. For example, in medical assessment systems, the evaluation values of certain indicators (attributes or criteria) captured from patients across different times or hospitals often exhibit fluctuations. These variations may arise from temporal changes, environmental differences, or disparities in equipment sensitivity. In other words, the hesitant fuzzy element captured may have just a certain credibility, which can be given by experts based on experience or obtained by the assessment systems. The notion of credible hesitant fuzzy set is appropriate to describe and measure patients' health status or symptoms of diseases. An information system consisting of credible hesitant fuzzy sets is referred to as a credible hesitant fuzzy information system. In this paper, we attempt to establish a three-way decision model based on regret theory for multi-attribute decision-making in credible hesitant fuzzy information system. The proposed model is nominated as CHF-BTWDM. A flowchart of proposed CHF-BTWDM is summarized as Fig. 1.

The notations used in this text are listed in Table 1.

The contributions in this work include

- The dominance degree between two credible hesitant fuzzy elements is quantized and the attribute weights are assessed based on the dominance degree.
- The dominance relation on credible hesitant fuzzy information system is established. The overall conditional probability of an object belonging to a state set is assessed according to the dominance relation.
- A set of payoff functions are built by improving the regret and rejoice functions to reflect psychological emotion of decision-makers.
- A behavioral three-way decision model, called CHF-BTWDM, is developed to make decisions in credible hesitant fuzzy information system.

The rest of the paper is structured as follows. Section 2 provides a review on necessary notions related to hesitant fuzzy set, three-way decision, and regret theory. Section 3 introduces the notion of credible hesitant fuzzy set and credible hesitant fuzzy information system. Section 4 presents the definition of a dominance relation in credible hesitant fuzzy information systems. Its properties are investigated. Section 5 develops a behavioral three-way decision model for the credible hesitant fuzzy information system. Section 6 applies the proposed CHF-BTWDM to three medical cases and presents a comprehensive comparison with representative decision-making methods. Section 7 follows the conclusions and provides future considerations on this work.

2. Preliminaries

In this section, the fundamentals from three-way decision (Yao, 2010), regret theory (Bell, 1982) and hesitant fuzzy sets (Torra and Narukawa, 2009) are reviewed.

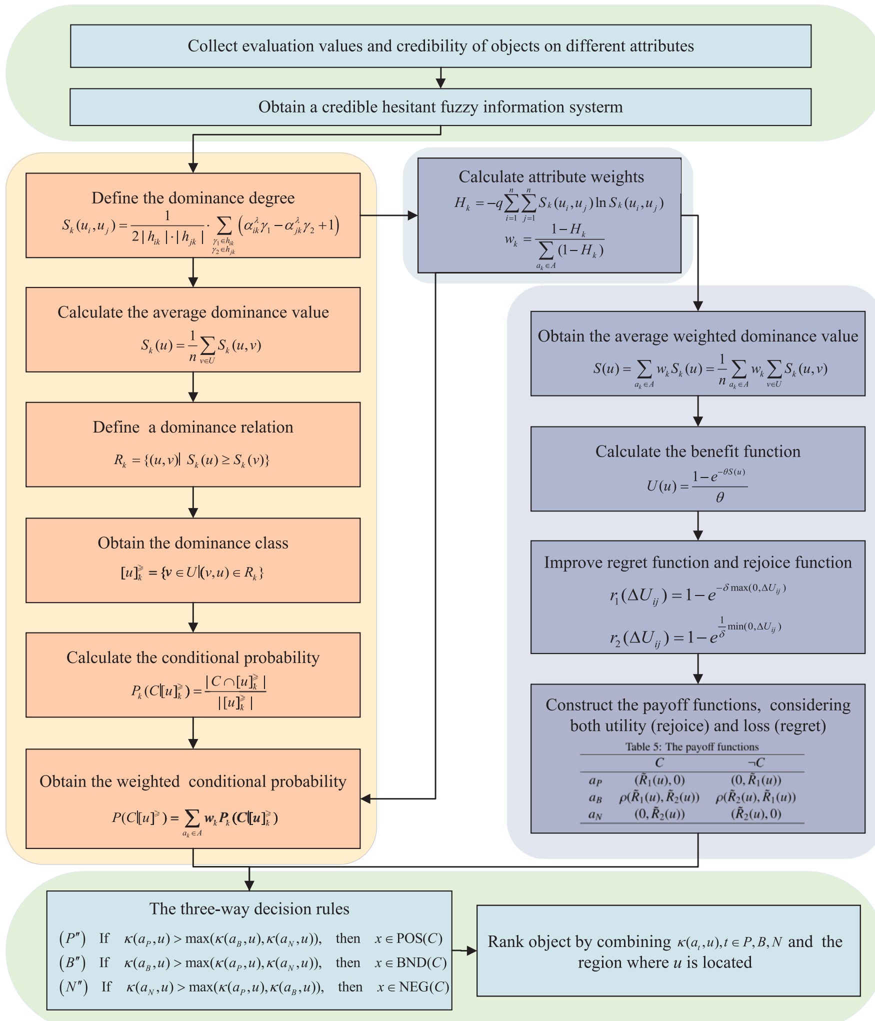


Fig. 1. The flowchart of the proposed CHF-BTWDM.

Table 1
List of notations.

Symbol	Description
$U = \{u_1, u_2, \dots, u_n\}$	The universe of discourse with n objects
$A = \{a_1, a_2, \dots, a_m\}$	Attribute set with m attributes
$S = (U, A, V, f)$	An information system
C, $\neg C$	The state set and its complement
R_k	The dominance relation under attribute a_k
$[u]_k$ ($[u]$), $[u]_k^{\succcurlyeq}$ ($[u]^{\succcurlyeq}$)	The equivalence class and dominance class of object u under attribute a_k
$P_{C,u} \triangleq P(C \mid [u])$ or $P(C \mid [u]^{\succcurlyeq})$	Conditional probability of object u belonging to C
$P_{\neg C,u} \triangleq P(\neg C \mid [u])$ or $P(\neg C \mid [u]^{\succcurlyeq})$	Conditional probability of object u belonging to $\neg C$
$POS(C)$	Positive region in three-way decision
$BND(C)$	Boundary region in three-way decision
$NEG(C)$	Negative region in three-way decision
$h_H(u)$	The hesitant fuzzy element in hesitant fuzzy set H related to u
α_i	Credibility degree of hesitant fuzzy element $h_H(u_i)$
s_H	The score function (score vector) of a credible hesitant fuzzy set H
$S_k(u, v)$	The dominance degree of object u over v with respect to attribute a_k
w_k	Weight of attribute a_k
$S(u, v)$	Overall dominance degree of object u over v
$U(u)$	Benefit function of object u
$r_1(\Delta U_{ij})$	Rejoice function based on utility difference between u_i and u_j
$r_2(\Delta U_{ij})$	Regret function based on utility difference between u_i and u_j
$R_1(u)$	Rejoice value of object u
$R_2(u)$	Regret value of object u
$\tilde{R}_1(u)$, $\tilde{R}_2(u)$	Normalized rejoice and regret values of object u
$R_{P,u}$, $R_{B,u}$, $R_{N,u}$	Payoff functions for actions a_P, a_B, a_N
$\kappa(a_t, u)$	Evaluation function of object u for action a_t ($t = P, B, N$)
OD	Ordinal degree-index reflecting misclassification ranking influence

2.1. Fundamentals of three-way decision

Let $U = \{u_1, u_2, \dots, u_n\}$ be a non-empty finite set, called the universe of discourse, composed of n objects, $A = \{a_1, a_2, \dots, a_m\}$ be a non-empty finite set, containing m attributes. For each attribute $a \in A$, V_a is the set composed of values of a on U, $V = \bigcup_{a \in A} V_a$ forms the global value space. If a function $f : U \times A \to V$ satisfies $f(u,a) \in V_a$ for every $u \in U$ and $a \in A$, then the quadruple $S = (U, A, V, f)$ is referred to as an information system.

If A is partitioned into two sets $D_0 \cup D_1$ with $D_0 \cap D_1 = \emptyset$, then S is called a decision information system, D_0 is called the conditional attribute set and D_1 is referred to the decision attribute set.

For any $B \subseteq A$, let R_B be a binary equivalence (fuzzy similarity) relation on U under the conditional attribute set B, for any $u \in U$, $[u]_B$, briefly $[u]$, is referred as the equivalence (fuzzy similarity) class of u under B and

$$P(C \mid [u]) = \frac{|[u] \cap C|}{|[u]|} \tag{1}$$

is designated as the conditional probability of u being affiliated to the given subset C of U.

With the defined conditional probability (1), the lower approximation, upper approximation and boundary of D_0 with respect to R_B can be derived. Meanwhile, the positive, negative and boundary regions are followed. The induced rough sets are called probabilistic rough sets or decision theoretical rough sets provided that the given subset D_0 is taken as one of the decision classes of objects in U with respect to the decision attributes in D_1.

A given reference set C together with its complementary $\neg C$, namely $\Omega = \{C, \neg C\}$, is referred to as two states. Three actions a_P, a_B and a_N represent an object in U being classified into the acceptance region (positive region) $POS(C)$, rejection region (negative region) $NEG(C)$ and delaying decision region (boundary region) $BND(C)$, respectively. λ_{PP}, λ_{BP} and λ_{NP} denote the losses of taking actions a_P, a_B and a_N. Similarly, λ_{PN}, λ_{BN} and λ_{NN} denote the losses of taking actions a_P, a_B and a_N.

The expected losses for an object $u \in U$ taking actions a_P, a_B, a_N are expressed by

$$R\left(a_P \mid [u]\right) = \lambda_{PP} P(C \mid [u]) + \lambda_{PN} P(\neg C \mid [u])$$
$$R\left(a_B \mid [u]\right) = \lambda_{BP} P(C \mid [u]) + \lambda_{BN} P(\neg C \mid [u])$$
$$R\left(a_N \mid [u]\right) = \lambda_{NP} P(C \mid [u]) + \lambda_{NN} P(\neg C \mid [u])$$

Following the Bayesian theory, the minimum risk decision rules can be derived as

(P) If $R(a_P \mid [u]) \le R(a_B \mid [u])$ and $R(a_P \mid [u]) \le R(a_N \mid [u])$, then $u \in \text{POS}(C)$

(B) If $R(a_B \mid [u]) \le R(a_P \mid [u])$ and $R(a_B \mid [u]) \le R(a_N \mid [u])$, then $u \in \text{BND}(C)$

(N) If $R(a_N \mid [u]) \le R(a_P \mid [u])$ and $R(a_N \mid [u]) \le R(a_B \mid [u])$, then $u \in \text{NEG}(C)$

2.2. Essential functions in regret theory

Regret theory was proposed by Bell (1982) to quantitatively describe human psychological emotion. It suggests that when decision-makers compare the real situation they find themselves with the situation they could have been, they may regret their decisions if they realize they could have gotten a better result by choosing another option. Conversely, they may rejoice.

For two objects $u_i, u_j \in U$, their evaluation values on a reference attribute a_k are denoted by $f(u_i, a_k)$ and $f(u_j, a_k)$, briefly $f(u_i)$ and $f(u_j)$, respectively, then the expected utility for u_i is

$$V(u_i) = U(u_i) + \sum_{j=1}^{n} R(\Delta U_{ij}) \tag{2}$$

where

$$U(u_i) = \frac{1 - e^{-\theta f(u_i)}}{\theta} \tag{3}$$

is the utility value of u_i on a and

$$R(\Delta U_{ij}) = 1 - e^{-\delta \Delta U_{ij}} \tag{4}$$

is the regret-rejoice value of $\Delta U_{ij} = U(u_i) - U(u_j)$, the difference between two utility values of u_i and u_j on a_k, the risk aversion parameter is $\theta \in (0,1)$ and the regret aversion parameter is $\delta \in [0, +\infty)$.

2.3. Notations on hesitant fuzzy set

The concept of hesitant fuzzy set proposed by Torra and Narukawa (2009) is an extension of classic fuzzy sets to describe objects with uncertainty.

Definition 2.1. Let U be a universe of discourse, a hesitant fuzzy set A on U can be represented by a multi-valued function from U to a subset of $[0,1]$, which can be modeled as:

$$A = \{\langle u, h_A(u) \rangle \mid u \in U\} \tag{5}$$

where $h_A(u)$ is a set of some values in $[0,1]$, called a hesitant fuzzy element related to $u \in U$, representing the possible membership degrees of u being affiliated to A.

Definition 2.2. Let h be a hesitant fuzzy element, then $s(h) = \frac{1}{l_h} \sum_{\gamma \in h} \gamma$ is called the score of h, where l_h is the number of elements in h.

The score function $s(h)$ is usually used to rank hesitant fuzzy elements. The higher the score, the larger the hesitant fuzzy element.

The classic hesitant fuzzy set, although it records all the evaluation values, does not reflect the degree of difference between different evaluation values. To address the fuzzy information in complex decisions, Zhu (2014) developed the probabilistic hesitant fuzzy sets to quantify relationship between membership values in a hesitant fuzzy element.

Definition 2.3. Let U be the universe of discourse, a probabilistic hesitant fuzzy set A on U is a hesitant fuzzy set along with a probability distribution for each hesitant element, denoted by

$$A = \{(u, h_A(u,p)) \mid u \in U\} \tag{6}$$

where $h_A(u,p) = \left\{ \gamma_i\left(p_i\right) \mid i = 1, 2, \dots, l_u, \sum_{i=1}^{l_u} p_i = 1 \right\}$ is referred to as a probabilistic hesitant fuzzy element, $\gamma_i \in [0,1]$ indicates one of the membership degree of u being affiliated to A and p_i is the probability of γ_i, and l_u is the number of membership degrees in $h_A(u,p)$.

Probabilistic hesitant fuzzy set is a useful concept of describing objects with uncertainty. In general, the membership degrees in a hesitant fuzzy element show a strong fuzziness, rather than the randomness. On the other hand, the probability distribution of membership degrees is not easily determined.

3. Credible hesitant fuzzy information systems

In this section, the notion of credible hesitant fuzzy information system is firstly introduced. Fundamental operations are provided for aggregating credible hesitant fuzzy sets. An example of multi-attribute decision-making on a credible hesitant fuzzy information system is provided.

Definition 3.1. Let $U = \{u_1, u_2, \dots, u_n\}$ be the universe of discourse, a credible hesitant fuzzy set H on U is a hesitant fuzzy set along with a credible degree for each hesitant element, denoted by

$$H = \{(u_i, \langle h_H(u_i), \alpha_i \rangle) \mid u_i \in U\} \tag{7}$$

abbreviated as

$$H = \{\langle h_H(u_i), \alpha_i \rangle \mid u_i \in U\} \tag{8}$$

Table 2
A CHFIS $S = (U, A)$.

U/A	a_1	a_2	$\cdots$	a_m
u_1	H_{11}	H_{12}	$\cdots$	H_{1m}
u_2	H_{21}	H_{22}	$\cdots$	H_{2m}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
u_n	H_{n1}	H_{n2}	$\cdots$	H_{nm}

where $h_H(u_i)$ is a finite discrete subset of $[0,1]$, representing all possible membership values of u_i being affiliated to the fuzzy concept H, $\alpha_i \in [0,1]$ indicates the credible degree of hesitant element $h_H(u_i)$, the pair $\langle h_H(u_i), \alpha_i\rangle$ is called a credible hesitant fuzzy element.

It is worth noting that the credible hesitant fuzzy sets can be regarded as the generalization of hesitant fuzzy sets and the credible hesitant fuzzy information system is an extension of hesitant fuzzy information system. A credible hesitant fuzzy set degenerates to a classic hesitant fuzzy set if $\alpha_i = 1$ for all $i = 1, 2, \ldots, n$.

In comparison to a probabilistic hesitant fuzzy element, each credible hesitant fuzzy element possesses solely one credible degree. Specifically, if a credible hesitant fuzzy element contains only one value, e.g., the hesitant fuzzy element is exactly a definite membership degree of the object being affiliated to a fuzzy concept, there is no any hesitant information in it. In this case, the credible degree is 1. Conversely, a larger deviation of membership degrees in a hesitant fuzzy element implies a lower credible degree of this hesitant fuzzy element.

Definition 3.2. Let $U = \{u_1, u_2, \ldots, u_n\}$ be the universe of discourse and $A = \{a_1, a_2, \ldots, a_m\}$ be the set of attributes, if the values of objects in U on A are credible hesitant fuzzy elements, then $S = (U, A)$ is called a credible hesitant fuzzy information system or a credible hesitant fuzzy information table.

Table 2 shows a scenario of credible hesitant fuzzy information system, where H_{ik} is the evaluation value of the object u_i under conditional attributes a_k, which is a credible hesitant fuzzy element. For each attribute a_k, $H_k = \{\langle h_{H_k}(u_i), \alpha_{ik}\rangle \mid u_i \in U\}$ is a credible hesitant fuzzy set on U.

The algebraic operations of credible hesitant fuzzy sets can be defined following the operations of classic fuzzy sets.

Definition 3.3. Let $H = \{\langle h_H(u_i), \alpha_i\rangle \mid u_i \in U\}$ and $H_k = \{\langle h_{H_k}(u_i), \alpha_{ik}\rangle \mid u_i \in U\}$ be CHFSs on U, $k = 1, 2, \ldots, p$,

(1) The complement of H, denoted by H^c, is defined as $H^c = \{\langle\{1-\gamma | \gamma \in h_H(u_i)\}, \alpha_i\rangle \mid u_i \in U\}$;
(2) The power of H, denoted by H^λ, is defined as $H^\lambda = \{\langle\{\gamma^\lambda | \gamma \in h_H(u_i)\}, \alpha_i\rangle \mid u_i \in U\}$ for any $\lambda > 0$;
(3) The scalar multiplication of H, denoted by λH: $\lambda H = \{\langle\{1-(1-\gamma)^\lambda | \gamma \in h_H(u_i)\}, \alpha_i\rangle \mid u_i \in U\}$ for any $\lambda > 0$;
(4) Addition: $H_1 \oplus H_2 \oplus \cdots \oplus H_p = \oplus(H_1, H_2, \cdots, H_p) = \{\langle\{1-\prod_{k=1}^{p}(1-\gamma_k) \mid \gamma_k \in h_{H_k}(u_i), k = 1, 2, \cdots, p\}, \frac{1}{p}\sum_{k=1}^{p}\alpha_{ik}\rangle \mid u_i \in U\}$;
(5) Multiplication: $H_1 \otimes H_2 \otimes \cdots \otimes H_p = \otimes(H_1, H_2, \cdots, H_p) = \{\langle\{\prod_{k=1}^{p}\gamma_k \mid \gamma_k \in h_{H_k}(u_i), k = 1, 2, \cdots, p\}, \frac{1}{p}\sum_{k=1}^{p}\alpha_{ik}\rangle \mid u_i \in U\}$.

Given a credible hesitant fuzzy set $H = \{\langle h_H(u_i), \alpha_i\rangle \mid u_i \in U\}$ on U, all objects in U can be ranked based on H according to the so called the score function defined as follows.

Definition 3.4. Let $H = \{\langle h_H(u_i), \alpha_i\rangle \mid u_i \in U\}$ be a credible hesitant fuzzy set, then $s_H = \{\alpha_i \cdot \frac{1}{|h_H(u_i)|}\sum_{\gamma \in h_H(u_i)}\gamma \mid u_i \in U\}$ is called the score function (score vector) of H, where $|h_H(u_i)|$ denotes the number of elements in $h_H(u_i)$.

According to the score vector s_H, the objects in U can be ranked. A large component in $s(H)$ indicates the corresponding object possessing a prior or dominant ranking. For example, if $s_H(u_i) > s_H(u_j)$, then u_i is prior to u_j, denoted by $u_i \succ u_j$.

Given a credible hesitant fuzzy information system (U, A), all of the m credible hesitant fuzzy sets with respect to the m attributes are firstly aggregated in terms of Definitions 3.3 and 3.4. After which, the score s_H is applied to the aggregated credible hesitant fuzzy sets and all of the n objects can be ranked to achieve the multi-attribute decision-making results.

In what follows, a real-world medical case under the credible hesitant fuzzy information circumstance is provided to show the multi-attribute decision-making process, where the data is extracted from the local Medical Security Bureau.

Example 3.1. Suppose there are six objects $U = \{u_1, u_2, \ldots, u_6\}$ with COVID-19 that need to be assessed for the severity of their conditions. Each object is tested with seven biochemical indicators (conditional attributes) $A = \{a_1, a_2, \ldots, a_7\}$, viral load, lung CT, blood oxygen saturation (SpO_2), lymphocyte percentage, white blood cell count, D-dimer and C-reactive protein (CRP), separately. To accurately assess diseases, seven indicators are measured across three rounds of testing for each object. Due to instrumental sensitivity and change of disease, the recorded values are not exactly the same. Table 3 presents the normalized values of inspection results, where the credible degrees are given by experts or clinical doctors.

In Table 3, the viral load results $\{0.11, 0.21, 0.26\}$ for the object u_1 indicate a high degree of variability among the three inspections, with a credible degree $\alpha = 0.51$ reflecting moderate reliability. This is consistent with the clinical characterization of viral loads as potentially fluctuating depending on sampling site, time, and stage of illness. In contrast, the lung CT severity test results $\{0.69, 0.78, 0.92\}$ for object u_5 show consistent measurements across the three tests, with a credibility $\alpha = 0.88$, which corresponds to the high reliability of CT imaging in assessing lung parenchymal injury.

To determine objects with serious diseases based on their testing results is a typical multi-attribute decision-making, which can be achieved by aggregating all observed values for each object and then ranking the aggregated values. The observed values for each object can be simply aggregated according to Definition 3.3 (4). After that, the score function introduced in Definition 3.4 can be applied and the scores of all objects as:

$$s(u_1) = 0.5498, s(u_2) = 0.8498, s(u_3) = 0.3668,$$
$$s(u_4) = 0.0924, s(u_5) = 0.8041, s(u_6) = 0.3862$$

All objects can be prioritized as the following ranking.

$$u_2 \succ u_5 \succ u_1 \succ u_6 \succ u_3 \succ u_4$$

The ranking results above show that the multi-attribute decision-making can rationally identify high-risk objects and provide guidance of priority processing. However, aggregation of credible hesitant fuzzy sets needs exponential computational cost based on Definition 3.3. The high computational complexity of aggregation makes it inapplicability to decision-making information systems with multiple attributes. Additionally, the equal aggregation of attributes is a serious shortcoming, as it results in indiscernibility of attributes in decision-making.

In what follows, a three-way decision based multi-attribute decision-making method is investigated with the guidance of regret theory.

4. Dominance relation on credible hesitant fuzzy information systems

A dominance relation is a fundamental but important notion for tackling decision-making problem. In this study, a new definition of fuzzy dominance relation is proposed in a credible hesitant fuzzy information system. The properties of the proposed fuzzy dominance relation is discussed.

Table 3
An example of CFHIS $S = (U, A)$.

$U \backslash A$	a_1	a_2	a_3	a_4
u_1	$\langle\{0.11, 0.21, 0.26\},\ 0.51\rangle$	$\langle\{0.45, 0.55, 0.65\},\ 0.55\rangle$	$\langle\{0.19, 0.20, 0.21\},\ 0.53\rangle$	$\langle\{0, 0.2\},\ 0.72\rangle$
u_2	$\langle\{0.34, 0.364, 0.45\},\ 0.94\rangle$	$\langle\{0.31, 0.66, 0.77\},\ 0.75\rangle$	$\langle\{0.29, 0.36, 0.54\},\ 0.87\rangle$	$\langle\{0.19, 0.31, 0.49\},\ 0.84\rangle$
u_3	$\langle\{0.16, 0.27, 0.32\},\ 0.42\rangle$	$\langle\{0.15, 0.19, 0.25\},\ 0.45\rangle$	$\langle\{0.08, 0.15, 0.24\},\ 0.62\rangle$	$\langle\{0.18, 0.25, 0.29\},\ 0.24\rangle$
u_4	$\langle\{0.08, 0.10, 0.13\},\ 0.47\rangle$	$\langle\{0, 0.07\},\ 0.46\rangle$	$\langle\{0.02, 0.02, 0.03\},\ 0.49\rangle$	$\langle\{0, 0.005\},\ 0.50\rangle$
u_5	$\langle\{0.76, 0.79, 1\},\ 0.87\rangle$	$\langle\{0.69, 0.78, 0.92\},\ 0.88\rangle$	$\langle\{0.80, 0.81, 1\},\ 0.89\rangle$	$\langle\{0.30, 0.55, 0.79\},\ 0.74\rangle$
u_6	$\langle\{0.14, 0.16, 0.25\},\ 0.44\rangle$	$\langle\{0.31, 0.66, 0.76\},\ 0.26\rangle$	$\langle\{0.29, 0.36, 0.44\},\ 0.42\rangle$	$\langle\{0.19, 0.31, 0.49\},\ 0.34\rangle$
	a_5	a_6	a_7	
u_1	$\langle\{0.13\},\ 0.64\rangle$	$\langle\{0.65, 0.74\},\ 0.55\rangle$	$\langle\{0.19, 0.22, 0.24\},\ 0.57\rangle$	
u_2	$\langle\{0.12, 0.25, 0.37\},\ 0.87\rangle$	$\langle\{0.60, 0.74, 1\},\ 0.79\rangle$	$\langle\{0.26, 0.35, 0.35\},\ 0.95\rangle$	
u_3	$\langle\{0.10, 0.16, 0.23\},\ 0.53\rangle$	$\langle\{0.09, 0.22, 0.23\},\ 0.43\rangle$	$\langle\{0.08, 0.15, 0.23\},\ 0.62\rangle$	
u_4	$\langle\{0, 0.003\},\ 0.50\rangle$	$\langle\{0.022\},\ 0.50\rangle$	$\langle\{0.010, 0.017, 0.017\},\ 0.50\rangle$	
u_5	$\langle\{0.42, 0.78, 1\},\ 0.69\rangle$	$\langle\{0.54, 0.87, 1\},\ 0.75\rangle$	$\langle\{0.6, 0.8, 0.97\},\ 0.80\rangle$	
u_6	$\langle\{0.02, 0.05, 0.10\},\ 0.46\rangle$	$\langle\{0.60, 0.72\},\ 0.44\rangle$	$\langle\{0.16, 0.22, 0.27\},\ 0.44\rangle$	

Definition 4.1. Support $S = (U, A)$ be a credible hesitant fuzzy information system, given an attribute $a_k \in A$ and two objects $u_i, u_j \in U$, assume that the evaluation values of u_i and u_j on a_k are $\langle h_{ik}, \alpha_{ik}\rangle$ and $\langle h_{jk}, \alpha_{jk}\rangle$, respectively, the degree of u_i dominating u_j is defined as:

$$S_k(u_i, u_j) = \frac{1}{2|h_{ik}| \cdot |h_{jk}|} \cdot \sum_{\substack{\gamma_1 \in h_{ik} \\ \gamma_2 \in h_{jk}}} \left(\alpha_{ik}^{\lambda}\gamma_1 - \alpha_{jk}^{\lambda}\gamma_2 + 1\right) \tag{9}$$

where $|h_{ik}|$ denotes the number of elements in h_{ik} and $\lambda \in \{0, 1\}$ is a regulation parameter. At this time, S_k is referred to as the fuzzy dominance relation under the attribute $a_k \in A$.

It is evident that if $\lambda = 0$, then for any $u_i, u_j \in U$,

$$S_k(u_i, u_j) = \frac{1}{2|h_{ik}| \cdot |h_{jk}|} \cdot \sum_{\substack{\gamma_1 \in h_{ik} \\ \gamma_2 \in h_{jk}}} (\gamma_1 - \gamma_2 + 1)$$

which can be regarded as the dominance degree of u_i against u_j in the circumstance of classic hesitant fuzzy information system. On the contrary, if $\lambda = 1$, $S_k(u_i, u_j)$ is the degree of dominance of u_i over u_j in the case of the credible hesitant fuzzy information system.

Proposition 4.1. *Given a credible hesitant fuzzy information system $S = (U, A)$, for any $a_k \in A$ and any objects $u_i, u_j, u_l \in U$,*

(1) $0 \le S_k(u_i, u_j) \le 1$ *and* $S_k(u_i, u_i) = 0.5$;
(2) $S_k(u_i, u_j) + S_k(u_j, u_i) = 1$;
(3) $S_k(u_i, u_l) = S_k(u_i, u_j) + S_k(u_j, u_l) - 0.5$;
(4) If α_{ik} *is fixed,* $S_k(u_i, u_j)$ *is decreasing as* α_{jk} *increases;*
(5) If α_{jk} *is fixed,* $S_k(u_i, u_j)$ *is increasing as* α_{ik} *increases.*

The detailed proof of Proposition 4.1 is provided in Appendix A.

Definition 4.2. Given a credible hesitant fuzzy information system $S = (U, A)$, an attribute $a_k \in A$ and any objects $u, v \in U$ then

$$R_k = \{(u, v) \mid S_k(u) \ge S_k(v)\} \tag{10}$$

is a dominance relation on S under the given attribute a_k, where $S_k(u) = \frac{1}{n}\sum_{v \in U} S_k(u, v)$ is the average dominance degree of $u \in U$.

Obviously, R_k satisfies the reflexivity and transitivity, but does not satisfy the antisymmetry. For any $u \in U$,

$$[u]_k^{\geqslant} = \{v \in U \mid (v, u) \in R_k\} \tag{11}$$

is referred to as the dominance class of u.

5. Behavioral three-way decision model in credible hesitant fuzzy information systems

In this section, a three-way decision model based multi-attribute decision-making on credible hesitant fuzzy information system is developed by improving regret-rejoice functions in regret theory.

5.1. Evaluation of attribute weights

Given a state set $C \subseteq U$, the conditional probability of $u \in U$ belonging to C can be measured as:

$$P_k(C \mid [u]_k^{\geqslant}) = \frac{|C \cap [u]_k^{\geqslant}|}{|[u]_k^{\geqslant}|} \tag{12}$$

In the credible hesitant fuzzy information system, each attribute plays different roles or possesses difference significance in multi-attribute decision-making.

The information entropy of an attribute $a_k \in A$ in S can be defined by

$$H_k = -q \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{S}_k(u_i, u_j) \ln \tilde{S}_k(u_i, u_j) \tag{13}$$

where $\tilde{S}_k(u_i, u_j) = S_k(u_i, u_j) / \sum_{i=1}^{n}\sum_{j=1}^{n} S_k(u_i, u_j)$ and $q = 1/(2\ln n)$.

The weight of an attribute $a_k \in A$ is assessed via

$$w_k = \frac{1 - H_k}{\sum_{a_k \in A}(1 - H_k)} \tag{14}$$

It is evident that the more consistent the elements in S_k, the significance of $a_k \in A$ is smaller. With the obtained weights of attributes, the overall conditional probability of $u \in U$ belonging to C as:

$$P(C \mid [u]^{\geqslant}) = \sum_{a_k \in A} w_k P_k(C \mid [u]_k^{\geqslant}) \tag{15}$$

It is evident that for any $u \in U$, the conditional probability satisfies $P(C \mid [u]^{\geqslant}) + P(\neg C \mid [u]^{\geqslant}) = 1$. For simplicity, the conditional probabilities $P(C \mid [u]^{\geqslant})$ and $P(\neg C \mid [u]^{\geqslant})$ are briefly denoted by $P_{C,u}$ and $P_{\neg C,u}$, respectively.

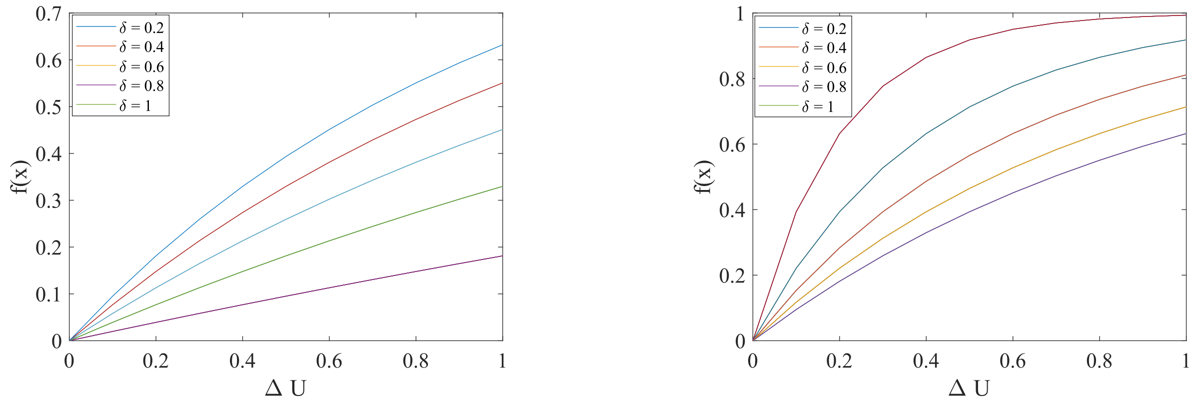
5.2. Three-way decision based on improved regret-rejoice functions

As stated in Section 2.2, the regret theory has been often applied to describe the psychological emotion of decision-makers who make their decisions. Due to limits of classic regret theory in collaboratively describing the regret degree and rejoice degree under a given choice, a pair of improved regret function and rejoice function is proposed to meet the demand of payoff functions in three-way decision.

According to Eq. (3), the improved utility function, rejoice function and regret function can be derived from based on the dominant degrees of objects. To achieve these, the overall fuzzy dominance relation is expressed as

$$S(u, v) = \sum_{a_k \in A} w_k S_k(u, v) \tag{16}$$

for any $u, v \in U$. The overall dominant degree of each object $u \in U$ is computed by $S(u) = \frac{1}{n}\sum_{v \in U} S(u, v)$.



(a) Classic regret function

(b) Improved regret function

Fig. 2. The classic regret function and improved regret function.

Definition 5.1. The utility function related to each object is defined based on the dominant degree as

$$U(u) = \frac{1 - e^{-\theta S(u)}}{\theta} \tag{17}$$

Meanwhile, the rejoice function and regret function are defined, respectively, as:

$$r_1(\Delta U_{ij}) = 1 - e^{-\delta \max(0, \Delta U_{ij})} \tag{18}$$

$$r_2(\Delta U_{ij}) = 1 - e^{\frac{1}{\delta} \min(0, \Delta U_{ij})} \tag{19}$$

where $\Delta U_{ij} = U(u_i) - U(u_j)$ is the difference between the utility values of u_i and u_j, the risk aversion parameter is $\theta \in (0, 1)$ and regret aversion parameter is $\delta \in (0, 1)$.

Fig. 2 shows the comparison of classic regret function (Fig. 2(a)) and improved ones (Fig. 2(b)), where curves with different colors express the functions when the parameter δ takes different values.

From the figures, at the same ΔU, the variation of function in Fig. 2(a) is too small to accurately describe psychological regret degrees. Instead, the improved regret function is more sensitive to ΔU, specifically when ΔU is less than 0.5.

According to the improved rejoice function and regret function, the rejoice value and regret value of an object u_i is evaluated by

$$R_1(u_i) = \sum_{j=1}^{n} r_1(\Delta U_{ij}) \tag{20}$$

$$R_2(u_i) = \sum_{j=1}^{n} r_2(\Delta U_{ij}) \tag{21}$$

The normalized rejoice value and normalized regret value of u_i is expressed as:

$$\tilde{R}_1(u_i) = \frac{R_1(u_i)}{\max_{1 \le j \le n} R_1(u_j)} \tag{22}$$

$$\tilde{R}_2(u_i) = \frac{R_2(u_i)}{\max_{1 \le j \le n} R_2(u_j)} \tag{23}$$

The improved rejoice function and regret function are employed to evaluate the utility of objects taking their actions.

The rejoice value of an object $u \in C$ taking the action a_P is considered as the utility of classifying this object into the acceptance region. In this case, there will be no loss and the regret value can be neglected. The utility of U taking the action a_N is set to be 0 and the loss is evaluated by the regret degree regarding this object. Furthermore, the utility of u taking the action a_B should be a value between 0 and the rejoice value. The loss is between 0 and the regret value. To quantize them, a parameter $\rho \in [0, 1]$ is introduced to balance the utility and loss of classifying the object to boundary region. On the contrary, the utility of an object $u \in \neg C$ taking the action a_P should be 0 and the loss is the rejoice value of this object, while its utility of taking the action a_N is the regret value and the loss is 0. The utility of this u taking the action a_B is between 0 and the regret value. The loss is between 0 and the rejoice value. These declarations can be summarized in Table 4.

Table 4
The payoff functions.

	C	$\neg C$
a_P	$(\tilde{R}_1(u), 0)$	$(0, \tilde{R}_1(u))$
a_B	$\rho(\tilde{R}_1(u), \tilde{R}_2(u))$	$\rho(\tilde{R}_2(u), \tilde{R}_1(u))$
a_N	$(0, \tilde{R}_2(u))$	$(\tilde{R}_2(u), 0)$

The basic operations on the payoff functions in Table 4 are defined as follows.

Definition 5.2. For any $u \in U$ and two payoff functions $(\tilde{R}_1(u), \tilde{R}_2(u))$ and $(\tilde{R}_3(u), \tilde{R}_4(u))$ in Table 4, the scalar multiplication and addition operations are defined as

(1) $\rho(\tilde{R}_1(u), \tilde{R}_2(u)) \triangleq (\rho\tilde{R}_1(u), \rho\tilde{R}_2(u))$ for any $\rho \in [0, 1]$;
(2) $(\tilde{R}_1(u), \tilde{R}_2(u)) \oplus (\tilde{R}_3(u), \tilde{R}_4(u)) \triangleq (\tilde{R}_1(u) + \tilde{R}_3(u) - \tilde{R}_1(u) \cdot \tilde{R}_3(u), \tilde{R}_2(u) \cdot \tilde{R}_4(u))$.

According to Definition 5.2, the utility values of $u \in U$ for actions a_P, a_B, a_N are evaluated, respectively, by

$$\begin{aligned} R_{P,u} &= P_{C,u}(\tilde{R}_1(u), 0) \oplus P_{\neg C,u}(0, \tilde{R}_1(u)) \\ R_{B,u} &= \rho P_{C,u}(\tilde{R}_1(u), \tilde{R}_2(u)) \oplus \rho P_{\neg C,u}(\tilde{R}_2(u), \tilde{R}_1(u)) \\ R_{N,u} &= P_{C,u}(0, \tilde{R}_2(u)) \oplus P_{\neg C,u}(\tilde{R}_2(u), 0) \end{aligned} \tag{24}$$

which can be simplified as:

$$\begin{aligned} R_{P,u} &= (P_{C,u}\tilde{R}_1(u), 0) \\ R_{B,u} &= (\rho P_{C,u}\tilde{R}_1(u) + \rho P_{\neg C,u}\tilde{R}_2(u) \\ &\quad - \rho^2 P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u), \rho^2 P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u)) \\ R_{N,u} &= (P_{\neg C,u}\tilde{R}_2(u), 0) \end{aligned} \tag{25}$$

Definition 5.3. Let $R_{t,u} = (E_1, E_2)$ be the payoff function of an object $u \in U$ being classified into the tth region. Then the evaluation function is defined as $\kappa(a_t, u) = E_1 - E_2$, where E_1 denotes the utility value and E_2 is the loss value, $t = P, B, N$.

According to the Bayesian risk theory, it is essential to select the best action that brings out a large utility value and a small loss value.

Therefore, three decision rules can be formulated as follows.

$$
\begin{aligned}
&(P'')\ \text{If } \kappa(a_P,u) > \max(\kappa(a_B,u),\kappa(a_N,u)), \text{then } x \in \mathrm{POS}(C)\\
&(B'')\ \text{If } \kappa(a_B,u) > \max(\kappa(a_P,u),\kappa(a_N,u)), \text{then } x \in \mathrm{BND}(C)\\
&(N'')\ \text{If } \kappa(a_N,u) > \max(\kappa(a_P,u),\kappa(a_B,u)), \text{then } x \in \mathrm{NEG}(C)
\end{aligned} \tag{26}
$$

Based on the extracted rules (26), the decisions for all objects can be derived. For convenience, the notions $POS(C), BND(C), NEG(C)$ are briefly notated as POS, BND, NEG, respectively.

Proposition 5.1. *When $\rho < 0.5$, $BND = \emptyset$.*

The proof is Proposition 5.1 is provided in Appendix B. Note that when $\rho \geq 0.5$, the objects may be classified into the boundary region. As ρ increases, more objects may be assigned to the boundary region. This trend can be verified by subsequent experiments.

Ranking objects is an important issue on multi-attribute decision-making. According to the relation between utility values of objects and their decisions, the ranking of objects in different regions should obey the rules that $POS \succ BND \succ NEG$, meaning that all objects in POS should be ranked before the objects in BND, and then objects in NEG follow.

As for objects in each of POS, BND and NEG, the evaluation function $\kappa(a_t,u)$ given by Definition 5.3 is applied to every region, respectively. For each u in POS, the larger the value of $\kappa(a_P,u)$, the higher its ranking. Conversely, for u in NEG, the larger the value of $\kappa(a_N,u)$, the lower the ranking. For each object in BND, both $\kappa(a_P,u)$ and $\kappa(a_N,u)$ are applied to rank it, separately. The average ranking result is regarded as the ranking result of this object. When the results of the average ranking are the same, $\kappa(a_P,u)$ is considered preferentially.

An algorithm is designed to conduct the CHF-BTWDM, shown in Algorithm 1.

Algorithm 1: Three-way decision process on a credible hesitant fuzzy information system

```
Input: A credible hesitant fuzzy information system S = (U, A), a state
       set C, four parameters λ, θ, δ, and ρ.
Output: Partition and ranking of all object.
begin
    Given a state set C, all parameters λ ∈ {0,1}, θ ∈ (0,1), δ ∈ (0,1),
    ρ ∈ [0,1].
    for i = 1 to n, j = 1 to n, k = 1 to m do
        calculate the fuzzy dominance degree S_k(u_i, u_j) and dominance
        relation R_k under each attribute a_k by (9) and (10).
    end
    for i = 1 to n, k = 1 to m do
        calculate the conditional probability P_k(C|[u_i]_k^≽) by (12).
    end
    for k = 1 to m do
        calculate the weight of attribute w_k by (14).
    end
    for i = 1 to n do
        calculate the overall conditional probability P(C | [u_i]^≽) by Eq.
        (15).
    end
    for i = 1 to n do
        calculate the score κ(a_t, u_i) of u_i (t = P, B, N) according to
        Definitions 5.3.
    end
    for i = 1 to n do
        determine the region to which each object belongs to by rules
        (P''), (B'') and (N'').
    end
    Obtain three regions POS, BND and NEG.
    for i = 1 to n do
        rank all objects by priority of POS ≻ BND ≻ NEG.
    end
    return the partition and ranking of all objects.
end
```

6. Case analysis on medical data

In this section, the effectiveness of the proposed CHF-BTWDM method is tested on three cases, including COVID-19 severity assessment, breast cancer diagnosis, and heart disease diagnosis. To further confirm the rationality of the proposed method, eleven representative methods are introduced for comparison, as summarized in Table 5. The evaluation metrics used in this study are detailed in Section 6.2. Furthermore, an ablation study and a sensitivity to parameters are carried out to further investigate the influence of each module and parameter on the models performance.

6.1. Comparison methods

This subsection provides a detailed analysis of the representative decision-making methods listed in Table 5. In this table, "Data type" indicates a method can deal with the data type, where "CHFS" means the credible hesitant fuzzy sets, "HFS" hesitant fuzzy sets,"FS" fuzzy sets,"IT2FNS" interval type-2 fuzzy numbers,"PFNS" Pythagorean fuzzy sets, "qFS" is q-rung fuzzy sets. "Weights" indicates whether attribute weights are concerned. "Psychosocial" indicates whether psychological behaviors are considered. "Functions" indicates whether the loss functions L, utility functions U, loss or utility functions L/U, and loss and utility functions $L\&U$ are involved in the decision-making models. "Ranking" and "Partition" indicate whether the method is responsible for ranking and/or partition task. "Incomplete" and "Large-scale" indicate whether the method can process incomplete or large-scale data.

HF-ELECTRE II, an extension of HF-ELECTRE, is designed to handle hesitant fuzzy decision-making problems. It ranks objects by constructing level outranking relations, but its application is limited to small-scale cases. Similarly, HF-ASAS is capable of processing hesitant fuzzy data; however, unlike HF-ELECTRE II, it takes attribute weights into consideration. Nevertheless, both methods share common shortcomings: they are designed just for ranking and fail to characterize the psychological behaviors of decision-makers.

TWD-EV introduces the concept of relative loss functions to handle classic fuzzy sets, while DTFRS-HFIS extends this functions to hesitant fuzzy three-way decision problems. Meanwhile, PFTWD-SSU utilizes S-shaped utility functions to solve three-way decision problems for Pythagorean fuzzy sets. Although all three methods construct complete three-way decision models and are capable of performing partition and ranking tasks, they share common limitations: they fail to take attribute weights into account, nor do they characterize the psychological factors of decision-makers.

RTWD-IN2FN integrates psychological behavior theory into the interval type-2 fuzzy environment, while TWD-MSDIS utilizes regret theory to construct relative loss functions. Furthermore, both 3W-MADM-R and PT-TWD-PPTDR address three-way decision problems on fuzzy sets by constructing utility functions. Although all these methods incorporate psychological behavior into the decision-making process, they establish decision rules only in terms of utility or loss, failing to assess both comprehensively.

TWD-qROFSs and TWD-RT-TOPSIS are more comprehensive methods, but their applicability is limited to classic fuzzy sets or q-rung orthopair fuzzy sets. Although both loss function and utility function are calculated, they are not considered simultaneously in decision-making. This leads to an insufficient utilization of information, as it fails to effectively integrate both loss and utility.

The parameter settings in the proposed method and comparison methods for case study are displayed in Table 6.

Table 5
The details in comparison methods.

Method	Data type	Weights	Psychosocial	Functions	Ranking	Partition
CHF-BTWDM	CHFS	✓	✓	L&U	✓	✓
HF-ELECTRE II (Chen and Xu, 2015)	HFS	×	×	\	✓	×
DTFRS-HFIS (Jiang and Hu, 2021)	HFS	×	×	L	✓	✓
HF-ARAS (Mishra et al., 2021)	HFS	✓	×	\	✓	×
TWD-EV (Jia and Liu, 2019)	FS	×	×	L	✓	✓
RTWD-IN2FN (Wang et al., 2022)	IT2FNS	×	✓	L	✓	✓
3W-MADM-R (Zhu et al., 2022)	FS	✓	✓	U	✓	✓
PFTWD-SSU (Yi et al., 2022)	PFNS	×	×	U	✓	✓
TWD-qROFSs (Mondal et al., 2023)	qFS	✓	✓	L/U	✓	✓
TWD-RT-TOPSIS (Yan et al., 2024a)	FS	✓	✓	L/U	✓	✓
TWD-MSDIS (Qian et al., 2024)	FS	×	✓	L	✓	✓
PT-TWD-PPTDR (Zhang and Yu, 2025)	FS	✓	✓	U	✓	✓

Table 6
The parameter settings in comparison methods.

Method	Parameters
CHF-BTWDM	$\lambda = 1,\ \theta = 0.3,\ \delta = 0.35,\ \rho = 0.8$
HF-ELECTRE II	$(c^-, c^0, c^*) = (0.3, 0.4, 0.5),\ d^0 = 0.35, d^* = 0.4$
DTFRS-HFIS	$\upsilon = 0.4$
HF-ARAS	$\upsilon = 0.5$
TWD-EV	$\sigma = 0.4$
RTWD-IN2FN	$\theta = 0.3,\ \delta = 0.35$
3W-MADM-R	$\theta = 0.3,\ \delta = 0.35,\ \zeta = 0.5$
PFTWD-SSU	$\delta_{PP} = 7.0,\ \delta_{BP} = 6.3,\ \delta_{NP} = 2.6,\ \delta_{NN} = 5.0,\ \delta_{BN} = 4.5,\ \delta_{PN} = 1.0$
TWD-qROFSs	$\theta = 0.3,\ \eta = 0.6,\ \phi = 0.9,\ \zeta_1 = 0.4$, and $\zeta_2 = 0.6$
TWD-RT-TOPSIS	$\delta = 0.3,\ \zeta = 0.35,\ \mu_1 = 0.55$
TWD-MSDIS	$\theta = 0.3,\ \partial = 0.35,\ \upsilon = 0.1$
PT-TWD-PPTDR	$\alpha = 0.2,\ \phi = 0.61,\ \theta = 0.69,\ \omega = 0.88,\ \mu = 0.6$

6.2. Evaluation metrics

The error rate (ER), precision rate (PR), recall rate (RR), and $F1$ score are typical indices of assessing performance on classification. For three-way decision based classification, The error rate (ER) and its improved version are presented as

$$ER = \frac{|C \to NEG(C)| + |\neg C \to POS(C)|}{|U|}$$

and

$$IER = \sigma \cdot \frac{|C \to NEG(C)| + |\neg C \to POS(C)|}{|U|} + (1-\sigma) \cdot \frac{|BND(C)|}{|POS(C)| + |BND(C)|}$$

where σ is a parameter to adjust the effect of misclassification objects.

IER concerns objects in boundary region in the decision process. It balances the influences of objects being correctly partitioned and the objects with delayed decisions. In this article, we set $\sigma = 0.8$.

The PR, RR and $F1$ score are expressed as

$$PR = \frac{|C \to POS(C)|}{|C \to POS(C)| + |\neg C \to POS(C)|}$$

$$RR = \frac{|C \to POS(C)|}{|C \to NEG(C)| + |C \to POS(C)|}$$

$$F1 = 2 \times \frac{PR \times RR}{PR + RR}$$

For the five metrics, ER, IER, PR, RR, and the $F1$ score, the lower the first two metrics, the better performance the model possesses, and the higher the last three metrics.

The Spearman's rank correlation coefficient

$$SRCC = 1 - \frac{6\sum_{i=1}^{n}(x_i - y_i)^2}{n(n^2-1)} \tag{27}$$

is a commonly used metric of evaluating the correlation degree between two variables. It can be used to assess the consistency between ranking results for objects achieved by different methods. Besides, a novel ranking index is constructed to assess the effectiveness of ranking objects.

Let the objects $u_1, u_2, \ldots, u_n$ possess the ranking positions $o_1, o_2, \ldots, o_n$, $P \to N$ denote the set of objects in the state set being incorrectly classified to the negative region. $N \to P$ be the set of objects in the complement of state set being incorrectly classified to the positive region, $J_1 = \{j | u_j \in P \to N\}$ and $J_2 = \{j | u_j \in N \to P\}$, o_{J_N} and o_{J_P} represent the position index of an object ranked first in the complement of state set and the index of an object ranked last in the state set, respectively, the ordinal degree (OD) is defined by

$$OD = \frac{|J_1| + |J_2|}{n} \cdot (\max(\frac{1}{|J_1|}\sum_{j \in J_1} o_j, o_{J_N}) - \min(\frac{1}{|J_2|}\sum_{j \in J_2} o_j, o_{J_P})) \tag{28}$$

In Eq. (28), both $\frac{1}{|J_1|}\sum_{j \in J_1} o_j$ and $\frac{1}{|J_2|}\sum_{j \in J_2} o_j$ are the average ranking position of objects that are incorrectly classified. If $J_1 = \emptyset$, indicating that no objects in the state set are incorrectly classified to the negative region, we set $\frac{1}{|J_1|}\sum_{j \in J_1} o_j = 0$. Similarly, if $J_2 = \emptyset$, indicating that no objects in the complement of state set are incorrectly classified to the positive region, we set $\frac{1}{|J_2|}\sum_{j \in J_2} o_j = 0$.

The indicator OD accounts for both the number and ranking position of incorrectly classified objects. A lower OD indicates a less influence of misclassifications of objects on the overall ranking of objects, and a better ranking result.

6.3. The multi-attribute decision-making for COVID-19 severity assessment

In Example 3.1, the aggregation-based multi-attribute decision-making method is implemented on an excerpt real-world medical case. In this subsection, we continued with Example 3.1 to investigate the multi-attribute decision-making problem with the proposed CHF-BTWDM.

Let $C = \{u_1, u_2, u_5\}$ and $\neg C = \{u_3, u_4, u_6\}$ denote the sets of objects with severe and non-severe conditions, respectively, called two state sets, take $\lambda = 1$, $\theta = 0.3$, $\delta = 0.35$, and $\rho = 0.65$ in the proposed CHF-BTWDM. (Algorithm 1), the partition result of U can be derived as follows.

$$POS = \{u_2, u_5\}$$

$$BND = \{u_1\}$$

Table 7
Comparison of different methods.

	Partition results	Ranking results
CHF-BTWDM	$POS = \{u_2, u_5\}, BND = \{u_1\}, NEG = \{u_3, u_4, u_6\}$	$u_5 \succ u_2 \succ u_1 \succ u_6 \succ u_3 \succ u_4$
HF-ELECTRE II	\	$u_5 \succ u_2 \succ u_1 \succ u_6 \succ u_3 \succ u_4$
DTFRS-HFIS	$POS = \{u_2, u_5\}, BND = \{u_1, u_6\}, NEG = \{u_3, u_4\}$	$u_5 \succ u_2 \succ u_1 \succ u_6 \succ u_4 \succ u_3$
HF-ARAS	\	$u_5 \succ u_2 \succ u_6 \succ u_1 \succ u_3 \succ u_4$
TWD-MSDIS	$POS = \{u_2, u_5\}, BND = \{u_1, u_3, u_6\}, NEG = \{u_4\}$	$u_5 \succ u_2 \succ u_6 \succ u_1 \succ u_3 \succ u_4$

$NEG = \{u_3, u_4, u_6\}$

According to Definitions 5.3 and the principle of region priority, we set $POS \succ BND \succ NEG$, e.g., any objects in POS are prioritized over those BND, and prioritized over those NEG. The ranking result is

$u_5 \succ u_2 \succ u_1 \succ u_6 \succ u_3 \succ u_4$

If we neglect the credible factor in Table 3, that is, we set the regulation parameter λ in Eq. (9) as $\lambda = 0$, the partition result can be induced.

$POS = \{u_1, u_2, u_5, u_6\}$

$BND = \emptyset$

$NEG = \{u_3, u_4\}$

Meanwhile, the ranking result can be also obtained as follows.

$u_5 \succ u_2 \succ u_6 \succ u_1 \succ u_3 \succ u_4$

It is evident that there is a little difference between both two partition results and between two ranking results. Due to the fact that viral load test values of u_1 are relatively high but exhibit significant fluctuation across three round tests, the credibility of collecting values is low, and the credibility of the lymphocyte count is moderate, it is rational for u_1 being classified into the delayed region and needing further inspection. If the credibility of evaluation values is not involved, u_1 is classified to the positive region (high-risk or emergency group). This could lead to the allocation of the highest-level medical resources to a object whose condition is still unclear and whose indicators are unstable.

Correspondingly, the patient u_6 has relatively low assessment values of all attribute except for the D-dimer and all evaluation values have low credibility. It is correctly classified into the negative region. When the credibility is overlooked, u_6 is falsely classified into positive region due to its D-dimer value, which could lead to unnecessary anticoagulant therapy or the consumption of medical resources.

The ranking results above not only aligns with clinical assessment but also enable more rational allocation of medical resources, avoiding misclassification and unnecessary interventions. These results demonstrate that the proposed model can tackle such a scenario very well, but the method without credibility cannot characterize such difference.

To further verify the validity of CHF-BTWDM, we compare CHF-BTWDM with four classic multi-attribute decision-making methods (HF-ELECTRE II, DTFRS-HFIS, HF-ARAS, and TWD-MSDIS), selected from Table 6, the comparison results are listed in Table 7.

It is observed from the results obtained that CHF-BTWDM brings out the same ranking result as HF-ELECTRE II, the same result as DTFRS-HFIS except for the ranking concerning u_3 and u_4. In terms of partitioning, only the proposed CHF-BTWDM assigns u_6 to the negative region. This result is consistent with the previous analysis and helps prevent unnecessary medical treatments.

In comparison to the decision-making method as addressed in Example 3.1 by straightforwardly aggregating evaluation values of objects across all attributes, the proposed CHF-BTWDM takes the weights of attributes into account and integrates psychological emotions of decision-makers into the decision-making processing, and therefore achieves interpretable decision results. Moreover, the computational complexity has been greatly reduced.

In the following section, the applications of proposed CHF-BTWDM are investigated on two benchmark real-world medical cases to further verify the performance of CHF-BTWDM in partitioning and ranking objects so as to make fully use limited medical resource to treat emergency patients.

6.4. Case study on two sets of benchmark medical data

Breast cancer is a common female disease. Early diagnosis and prevention of breast cancer is the first choice. The benchmark breast cancer case[1] (Breast Cancers) is cited from open repositories to test the performance of proposed model on decision-making. In this case, there are 699 individuals, in which 16 objects with missing attribute values are removed. Each object is with 9 conditional attributes and 1 decision attribute. The state set is taken as the family of individuals who are all labeled as symptoms of breast cancers. It involves 239 objects. The Heart Disease[2] is another benchmark case. In this case, there are 303 individuals, in which 6 objects with missing attribute values are removed. Each object is with 13 conditional attributes and 1 decision attribute. The state set is taken as the family of individuals who are all labeled as symptoms of heart disease, which involves 137 objects.

To construct a credible hesitant fuzzy information system, we normalize the values of each attribute of objects in these cases and transform each real number into a hesitant fuzzy element, which contains three hesitant membership values. For each normalized real number $v \in (0, 1)$, two random numbers are drawn from the intervals $[40\%v, v)$ and $(v, 160\%v]$, respectively, which regarded as the three hesitant membership values together with v. If $v = 0$ or $v = 1$, the corresponding hesitant fuzzy elements are $\{0, 0, v_1\}$ and $\{v_2, 1, 1\}$, where v_1 and v_2 are random numbers drawn from $[40\%v, v)$ and $(v, 160\%v]$, respectively. The hesitant fuzzy decision systems are therefore generated. The three values in each hesitant fuzzy element can be regarded as the inspection results of three round tests at different time or from different hospitals.

The credibility of a hesitant fuzzy element is determined based on the distribution of evaluation values of each objects on each attributes. The higher the credibility, the more similar the three evaluation values are, or the smaller the deviation of the hesitant fuzzy membership values in a hesitant fuzzy element. Following this idea, the credible degree of a hesitant fuzzy element h_{ik} is achieved based on the standard deviation of values in h_{ik} together with the state value C_i of the ith object through

$$\alpha_{ik} = (1 - \mathrm{SD}(h_{ik})) * \max(s(h_{ik}), C_i) \tag{29}$$

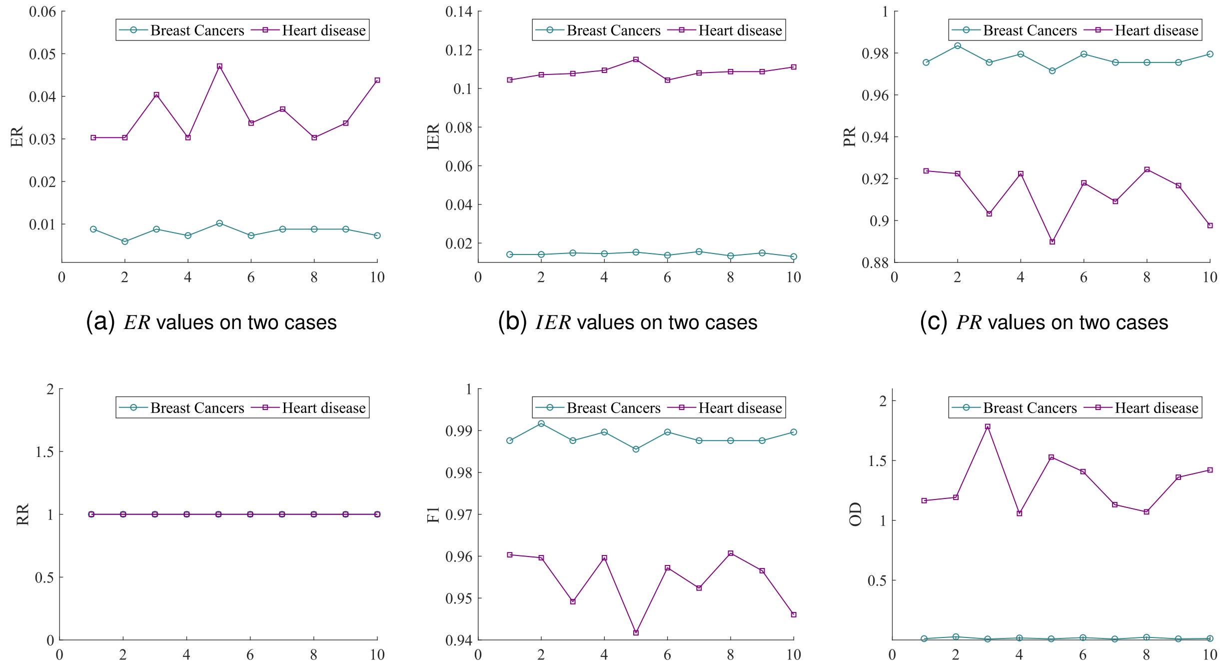
where $\mathrm{SD}(h_{ik})$ is the standard deviation of values in h_{ik} and $s(h_{ik})$ is the score of h_{ik}. The state value C_i is set as 1 if the ith object belongs to C, and 0 otherwise.

6.4.1. Randomness tests

The process of constructing a credible hesitant fuzzy information system involves in a certain degree of randomness, as stated aforementioned. To examine the impact of randomization on classifying objects, the proposed method are conducted on ten randomized generations

[1] https://archive.ics.uci.edu/dataset/15

[2] https://archive.ics.uci.edu/dataset/45/heart+disease



(a) ER values on two cases (b) IER values on two cases (c) PR values on two cases

(d) RR values on two cases (e) $F1$ values on two cases (f) OD values on two cases

Fig. 3. Randomized test results of proposed model on two cases.

of the credible hesitant fuzzy information system based on the cases aforementioned. The outcomes are presented in Fig. 3.

Fig. 3 indicates that there are small fluctuations in the six metrics on the credible hesitant fuzzy information system. Specifically, the fluctuation ranges of ER is 0.0059 ∼ 0.102 on the Breast Cancers. The fluctuation ranges of IER is 0.013 ∼ 0.0156. The fluctuation ranges of PR is 0.9715 ∼ 0.9835, and there is no fluctuation in RR for the Breast Cancers. The fluctuation ranges of $F1$ is 0.9855 ∼ 0.9917. The fluctuation ranges of OD is 0.0088 ∼ 0.278. On the Heart disease, the fluctuation ranges of ER is 0.0303 ∼ 0.0471, The fluctuation ranges of IER is 0.1043 ∼ 0.115. The fluctuation ranges of PR is 0.8898 ∼ 0.9244, and there is no fluctuation in RR. The fluctuation ranges of $F1$ is 0.9417 ∼ 0.9607. The fluctuation ranges of OD is 1.0572 ∼ 1.7845. It is concluded that the volatility of each indicator is small and the proposed CHF-BTWDM possesses strong stability against randomness.

In the following experiments, the results are selected once for comparison to ensure consistency of comparative experiments.

6.4.2. Analysis on partition performance

Based on the parameter settings aforementioned, the proposed method brings out 245 objects as breast cancer patients and 9 objects to be diagnosed. Among them, all 239 true breast cancer patients (all objects in the state set) have been successfully and accurately diagnosed. Unfortunately, there are 6 healthy persons who are misdiagnosed as patients. In addition, the proposed method treats 118 objects as heart disease patients and 79 objects need to be further diagnosed. Among them, the 137 true heart disease patients (all objects in the state set) have been diagnosed or require to be further inspected. These results imply that the proposed CHF-BTWDM is able to identify all patients. It is a conservative decision-making method and consistent with the requirements of clinical medicine.

To further verify the rationality and superiority of CHF-BTWDM, we benchmark the proposed model with state-of-the-art methods listed in Table 6. Due to the fact that HF-ARAS and HF-ELECTRE II in Table 6 are not designed for partition, and DTFRS-HFIS cannot be applied to processing of moderately large data. Only the rest eight methods in Table 6 are comparatively tested. The partition results of all nine methods for the both cases are quantitatively displayed in Tables 8 and 9.

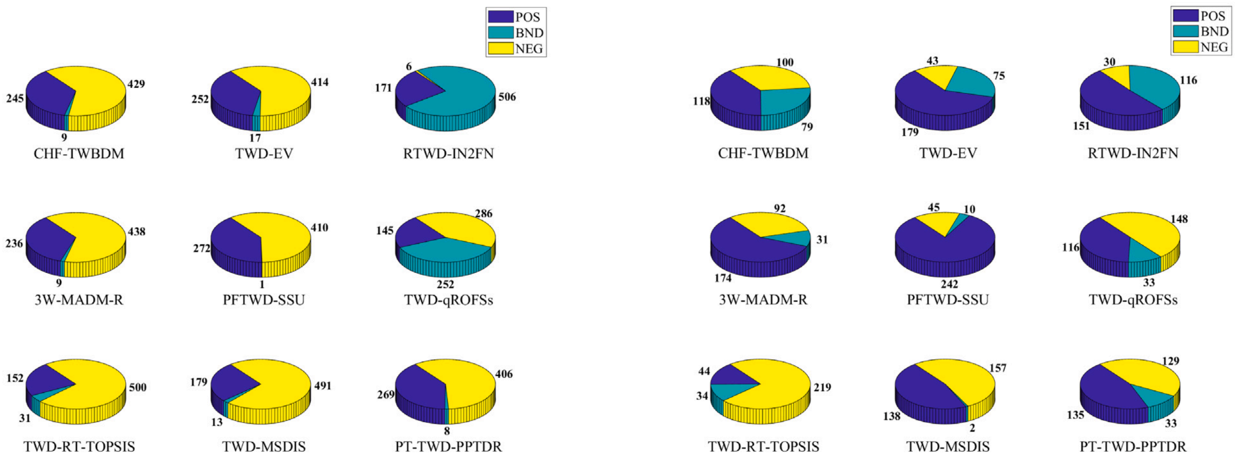
Table 8
Partition results by different methods on Breast cancers.

	ER	IER	PR	RR	$F1$
CHF-BTWDM	**0.0088**	**0.0141**	0.9755	**1**	**0.9876**
TWD-EV	0.0190	0.0279	0.9484	**1**	0.9735
RTWD-IN2FN	0.0176	0.1635	0.9649	0.9649	0.9649
3W-MADM-R	0.0249	0.0273	0.9576	0.970	0.9638
PFTWD-SSU	0.0937	0.0757	0.8199	0.937	0.8745
TWD-qROFSs	0.0966	0.2042	0.600	0.9158	0.7250
TWD-RT-TOPSIS	0.0864	0.1030	**0.9934**	0.7225	0.8366
TWD-MSDIS	0.0893	0.0850	0.9888	0.7500	0.8530
PT-TWD-PPTDR	0.0469	0.0433	0.8848	0.9958	0.9370

Table 9
Partition results by different methods on Heart disease.

	ER	IER	PR	RR	$F1$
CHF-BTWDM	**0.0303**	**0.1044**	**0.9237**	**1**	**0.9603**
TWD-EV	0.1684	0.1937	0.7207	**1**	0.8377
RTWD-IN2FN	0.2088	0.2539	0.6225	0.9495	0.7520
3W-MADM-R	0.2458	0.2269	0.6494	0.904	0.7558
PFTWD-SSU	0.3636	0.2988	0.5579	0.9926	0.7143
TWD-qROFSs	0.229	0.2275	0.7586	0.6875	0.7213
TWD-RT-TOPSIS	0.2424	0.2811	0.9091	0.3704	0.5263
TWD-MSDIS	0.2054	0.1672	0.7754	0.781	0.7782
PT-TWD-PPTDR	0.165	0.1713	0.7778	0.8468	0.8108

It is shown from Table 8 that the CHF-BTWDM achieves the best performance on all metrics except for PR on the breast cancer case. Specifically, the RR reaches 1.0, demonstrating that all true malignant cases are correctly identified. Although the resulted PR is not the largest, it is far larger than that derived by other six methods. The $F1$ score is 0.9876, ranking first, showing that the medical diagnosis results are very accurate. Meanwhile, The ER value is 0.0088 and the IER value is 0.0141, which are the lowest among all other methods, indicating the lowest misdiagnosis risk. As a whole, it is simultaneously satisfied that almost all breast cancer patients are correctly identified



(a) Results of nine methods on Breast Cancers

(b) Results of nine methods on Heart Disease

Fig. 4. The visualization of partition results.

and fewer patients are misdiagnosed as breast cancer. Meanwhile, it is shown from Table 9 that the CHF-BTWDM achieves the best performance on all metrics on heart disease case. Specifically, the RR reaches 1.0, the PR is 0.9237 and the $F1$ score is 0.9603, indicating that the classification results are highly accurate in comparison with the results derived from existing methods. Meanwhile, The ER value is 0.0303 and the IER value is 0.1044, indicating that the proposed CHF-BTWDM achieves a lower misdiagnosis risk than other methods. These facts demonstrate the proposed CHF-BTWDM possesses higher accuracy and reliability than all other representative methods.

To highlight the performance of all methods for classifying objects on two cases, the visualization of partition results is illustrated in Fig. 4.

It is evident from this figure that, on the breast cancer case, the proposed CHF-BTWDM classifies the objects into three regions, where the positive region and negative region are approximately consistent to the state set and its complementary, respectively. Similar results are derived from TWD-EV and 3W-MADM-R.

Unfortunately, TWD-RT-TOPSIS and TWD-MSDIS mistakenly classify objects in the state set into negative region, leading to missed diagnosis. Instead, RTWD-IN2FN and TWD-qROFSs classify most objects into their boundary regions, possibly causing exacerbation, while a few objects are mistakenly classified into positive region by PFTWD-SSU and PT-TWD-PPTDR, and expensive medical resources are squeezed out. On the heart disease case, the proposed CHF-BTWDM classifies the objects into three regions, where the positive region is approximately consistent to the state set. Similarly, TWD-EV, 3W-MADM-R, TWD-qROFSs and PT-TWD-PPTDR also classify the results into three regions. Similar to the results observed in the breast cancer case, TWD-RT-TOPSIS mistakenly classifies most objects in the state set into negative region. This misclassification is mainly attributed to the methods independent consideration of loss or utility, which leads to an excessive number of objects in the negative or positive region. In contrast, PFTWD-SSU mistakenly classifies most objects in the complement of state set into positive region. This result demonstrates that the method suffers from poor robustness across different cases. Meanwhile, TWD-MSDIS classifies only few objects to the boundary region. Although the numbers of objects in positive region and negative region are similar to those in the state set and the complement of state set, separately, Table 9 shows that TWD-MSDIS has a relatively high error rate and low precision rate, indicating that the partitioning results are less accurate. Consistent with the breast cancer case, RTWD-IN2FN also classifies most objects into the boundary region. This can be attributed to the methods failure to consider attribute weights.

6.4.3. Analysis on ranking performance

To substantiate the rationality and superiority of CHF-BTWDM in decision-making (ranking objects), six methods from Table 6 are randomly selected and tested for comparison. $SRCC$ and the constructed OD are used to evaluate the performance of comparison method on decision-making. The correlation coefficient matrix is shown in Fig. 5.

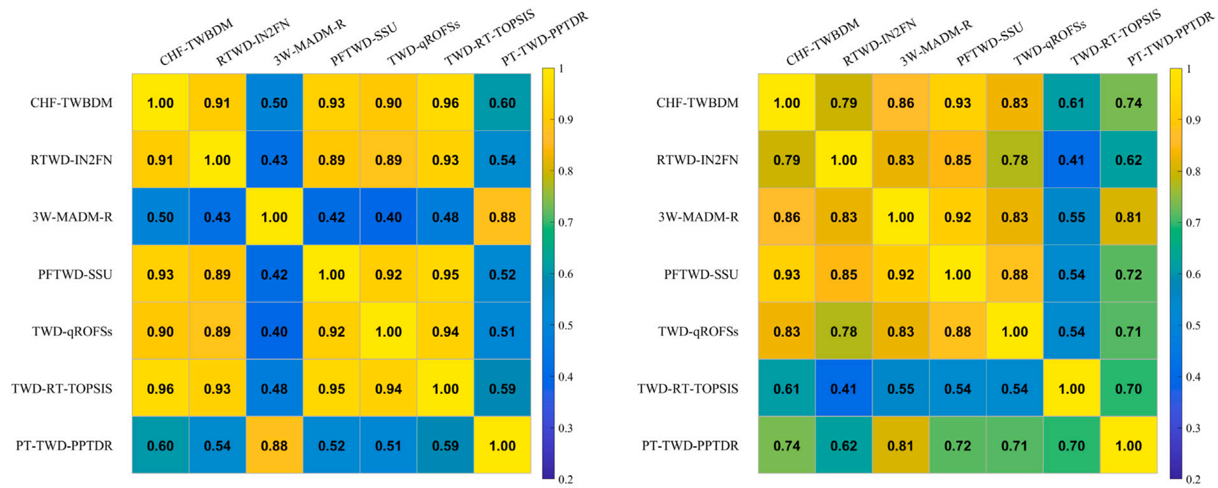
Each element in the matrix shown in Fig. 5 expresses the correlation value between two ranking results obtained from two methods. According to the critical value table of SRCC, for $n > 100$, a two-tailed Spearman's rank correlation test at $\alpha = 0.01$ yields a critical value of 0.257. If the observed SRCC exceeds this threshold, it indicates a statistically significant positive correlation between the two methods. It is observed in Fig. 5 that, on the breast cancer case, the proposed CHF-BTWDM has high correlations with all methods except for 3W-MADM-R and PT-TWD-PPTDR. Specifically, CHF-BTWDM possesses at least 0.90 correlation values with most methods, achieving the greatest correlation 0.96 with TWD-RT-TOPSIS. On the heart disease case, the proposed CHF-BTWDM shows high correlations with all methods except for TWD-RT-TOPSIS and PT-TWD-PPTDR. Specifically, it achieves correlation values above 0.80 with most methods, peaking at 0.93 with PFTWD-SSU. CHF-BTWDM achieves low correlations with PT-TWD-PPTDR on both cases, The main reason is that they rank objects in negative regions with the same rule as rank objects in positive regions, and therefore lead to large ranking differences of objects in negative regions.

For a method that provides a ranking result, if there are n_1 positive objects in the original case or there are n_1 objects in the state set C, the top n_1 ranked objects are considered positive, while the remaining objects are regarded as negative. Specifically, on the breast cancer case, the top 239 ranked objects are labeled as positive, and the remaining objects are labeled as negative. On the heart disease case, the top 137 ranked objects are labeled as positive, and the remaining objects are labeled as negative. Fig. 6 presents the OD results of seven methods regarding their rankings.

As shown in Fig. 6, the proposed CHF-TWBDW produces the lowest OD value on both cases, which is significantly smaller than those by the other six methods. The experimental results imply that CHF-TWBDW has the best ranking effect.

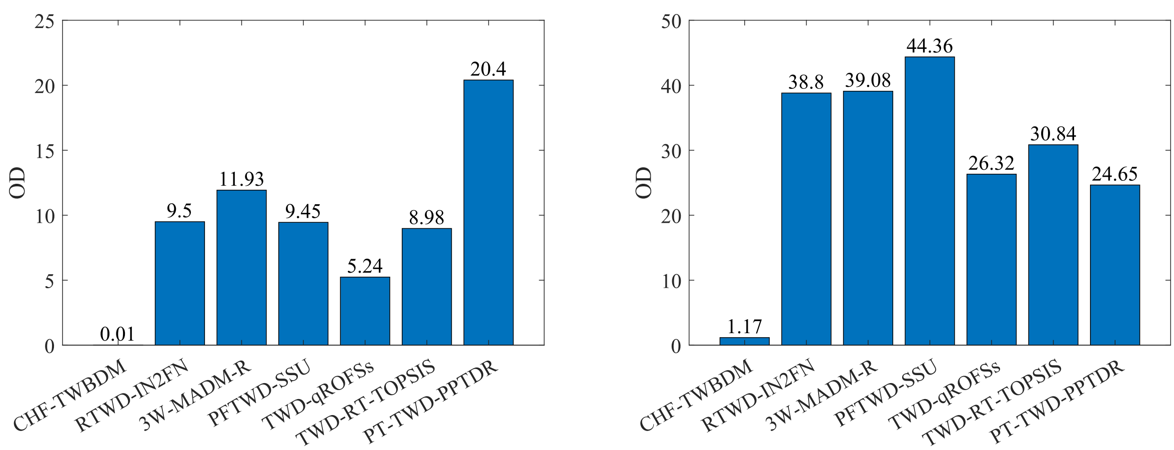
6.5. Ablation study

To verify the necessities of introducing the credibility degree and improving the regret function and rejoice function, two studies are designed in this subsection. Two variant models are constructed and applied to the breast cancer case.



(a) $SRCC$ values on Breast Cancers (b) $SRCC$ values on Heart Disease

Fig. 5. The SRCC values between rankings derived from seven methods.



(a) OD values on Breast Cancers (b) OD values on Heart Disease

Fig. 6. The OD values between rankings derived from seven methods.

6.5.1. Necessity of introducing credibility

To argue the necessity of introducing credibility degree for evaluation values, we comparatively analyze the decision-making results in the case whether the factor of credibility is involved in the proposed method. As stated before, if $\lambda = 0$, the credible hesitant fuzzy information system reduces to a classic hesitant fuzzy information system, i.e. the credibility is not concerned.

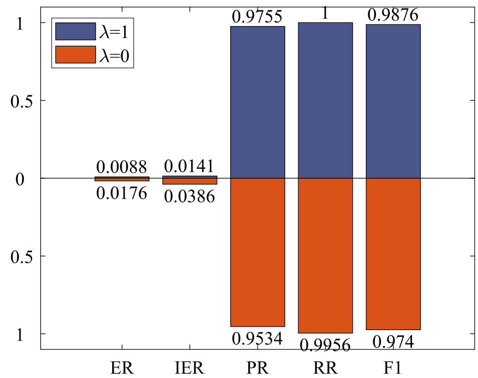
The parameters in both models (the proposed model with and without credibility) are set as $\theta = 0.3$, $\delta = 0.35$ and $\rho = 0.8$. The comparative results are displayed in Fig. 7.

It is shown from Fig. 7 that the results of ER and IER obtained using our proposed method with credibility are reduced by about half compared to that without credibility, while the OD value is less than one percent of that by the method without credibility. PR, RR and $F1$ are all significantly improved in comparison with the proposed model without credibility.

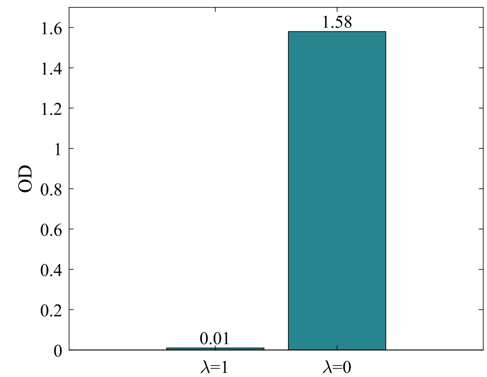
Upon looking through the given data, some objects (healthy persons), such as u_{227}, u_{252}, u_{286}, u_{339}, and u_{401} in the complement of state set, have large deviations on measured size, thickness, and cellular homogeneity of nodules, which lead to low credibility of collected evaluation values. The proposed CHF-BTWDM classifies them into the delayed region to be further inspected or treated. In contrast, if the credibility of evaluation values is not involved, those objects will be classified to the positive region, leading to misallocation of medical resources away from truly high-risk patients.

On the other hand, the breast cancer patients u_{42}, u_{49}, u_{50}, u_{56}, u_{58}, u_{64}, u_{100}, u_{102}, u_{226}, u_{335}, u_{343}, u_{441}, and u_{475} in the state set, possess large evaluation values on attributes such as nucleolus, edge adhesion, and mitosis of the cell, but with low overall fluctuation. That fact indicates the credibility of collected evaluation values is high, and thus all are prioritized for intervention. Compared to the case where the credibility is ignored, the 13 patients cannot be promptly and correctly diagnosed but need to be further inspected. The reason is that other patients with the same or similar evaluation values but with lower credibility have been assigned higher clinical priority than the 13 patients.

The experimental results demonstrate that the introduction of credibility can significantly reduce wrong decisions. The improvement of $F1$ score indicates that incorporating credibility can effectively eliminate the occurrence of both false positives and false negatives. A high credibility supports conclusive decision-making, while a low credibility prompts doctors to adopt more cautious strategies, thereby optimizing the allocation of medical resources. By integrating credibility of evaluation values, the ranking results align more closely with the patients' actual risk levels, ensuring that medical resources are prioritized for high-reliability, high-risk patients.

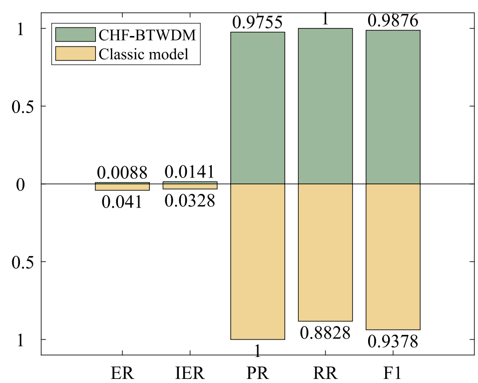


(a) The comparative results of five metrics

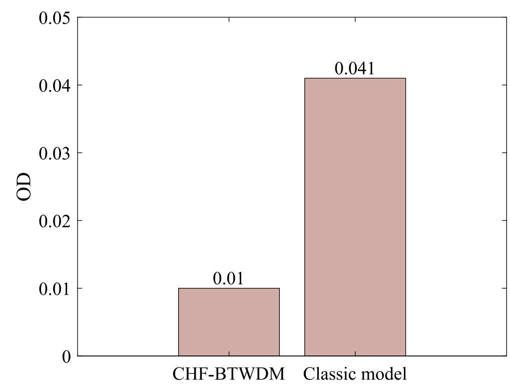


(b) The comparative results of OD

Fig. 7. Comparative results of proposed method with and without credibility. (Fig. 7(a) shows the results of five metrics ER, IER, PR, RR, and $F1$. The blue bars and the numbers over them denote the results achieved by the proposed method with credibility ($\lambda = 1$), while the red bars and the numbers below them express the metric results by the proposed method without credibility ($\lambda = 0$). Fig. 7(b) denotes the OD ranking values of CHF-BTWDW in the two cases.).



(a) The comparative results of five metrics



(b) The comparative results of OD

Fig. 8. Comparative results of proposed model with and without improved regret-rejoice functions. (Fig. 8(a) shows the results of five metrics ER, IER, PR, RR, and $F1$. The green bars and the numbers over them denote the metric values with the proposed payoff functions, while the yellow bars and the numbers below them express the results derived from the payoff functions incorporating classic regret theory. Fig. 7(b) denotes the OD ranking values derived from the two models.).

6.5.2. Comparison of regret-rejoice functions

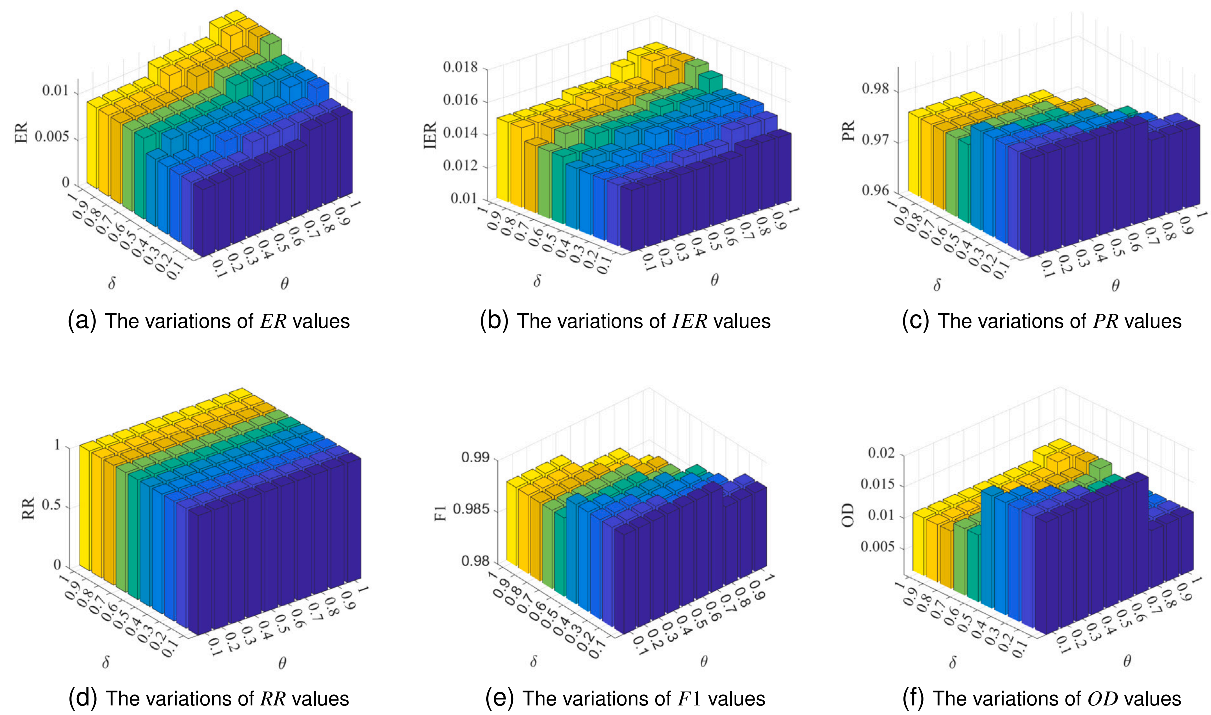
To test the effectiveness of improving regret function and rejoice function, we compare the proposed method (accompanying with the payoff functions shown in Table 4) against our method but with relative aggregation loss functions as payoff functions. The relative aggregation loss functions were introduced in the Qian et al. (2024), which are indeed the combination of the classic regret-rejoice function (Eqs. (2)–(4)) and relative loss functions. The decision-making processes by the proposed model with two sets of payoff functions are the same. The parameters are set as $\theta = 0.3$, $\delta = 0.35$ and $\rho = 0.8$. The comparative results are displayed in Fig. 8.

It is shown from Fig. 8 that the improved regret function and rejoice function can significantly reduce both ER and OD values by 80% in comparison with the classic regret-rejoice function together with relative loss functions. Additionally, IER is reduced to about 60%, RR and $F1$ are significantly improved. These facts demonstrate that the introduction of credible factor and improving regret-rejoice functions bring out distinguishable performance.

In-depth analysis of the data reveals that in models using the classic regret-rejoice function, 28 objects in state set, such as u_{13}, u_{16}, etc. are incorrectly classified into the negative region. This is regarded as a misdiagnosis in clinical diagnosis, which may cause patients to miss the best treatment time. The fundamental reason lies in the fact that the classic regret-rejoice function fails to sufficiently characterize the decision-maker's regret psychology is far greater than the rejoice psychology. This results in higher relative loss when taking action a_P and lower relative loss when taking action a_N, and thus the objects are misclassified into the negative region. In contrast, our improved regret function and rejoice function can more accurately reflect the psychology of decision makers. Compared to the classical model that calculates the loss function alone, we provide a more comprehensive assessment of each object by calculating the payoff function, which ensures that these 28 objects in state set are correctly classified into the positive region.

On the other hand, for the nine objects u_{145}, u_{227}, u_{252}, u_{286}, u_{339}, u_{401}, u_{480}, u_{606} and u_{642} in the complement of state set, our model classifies them into boundary region, whereas the classic model assigns them all to the negative region. It seems to achieve a more "accurate" partition on the surface. However, a deeper analysis of the data reveals that these healthy objects have similar assessed values to the patients on some key indicators (e.g. nucleolus and edge adhesion). It is necessary to focus on their loss aversion psychology. Compared to the classic regret-rejoice functions, our improved regret function and rejoice function accurately describe the psychological behavior of decision makers. Consequently, our model partitions these uncertain cases to the 'delayed decision'. This approach embodies a more responsible clinical decision-making.

As the heart disease case yields results similar to the breast cancer case, the heart disease case is not further presented in this section.



(a) The variations of ER values (b) The variations of IER values (c) The variations of PR values

(d) The variations of RR values (e) The variations of $F1$ values (f) The variations of OD values

Fig. 9. The variations of the six metric values with θ and δ.

6.6. Sensitivity to parameters

There are three parameters in CHF-BTWDM, the risk aversion parameter θ, regret aversion parameter δ and regulating parameter ρ. In this subsection, the sensitivity of these parameters to decision-making is investigated by conducting experiments. Since the parameter results are similar across cases, the parameter analysis is conducted on the breast cancer case.

The θ and δ are used to adjust the degree of regret aversion in regret theory. To test the influence of these parameters on decision-making, we set $\rho = 0.8$ and let θ and δ change in the range $[0, 1]$ with a step 0.1. The results are presented in Fig. 9. In each subfigure, the horizontal and vertical coordinates indicate the change of parameters and the vertical coordinates indicate the values of different metrics.

When δ and θ change in their range, ER varies in $0.0073 \sim 0.0117$ and IER is in $0.0137 \sim 0.0164$, which are of slight fluctuations. The values of PR and $F1$ vary in $0.9676 \sim 0.9795$ and in $0.9835 \sim 0.9896$ and the RR value is 1 and remains unchanged, while OD varies in $0.019 \sim 0.0117$. Specifically, as the values of parameters increase, both error rates slightly increase but approximate to stable except for both parameters taking their maximum 1, while the metric OD becomes better and better. However, when δ and θ takes smaller values, PR and $F1$ receive their better values. As a whole, the proposed model is robust to parameters. Therefore, we set $\theta = 0.3$ and $\delta = 0.35$ in the simulated experiments.

As for the impact of tradeoff parameter ρ on decision-making, according to Formula (25) and Definition 5.3, when $\rho \leq 0.5$, $\kappa(a_B)$ is smaller than at least one of $\kappa(a_P)$ and $\kappa(a_N)$, so no objects are classified to the boundary region. Therefore, the metrics for decision-making remain unchanged. To verify this fact, we set $\theta = 0.3$ and $\delta = 0.35$ unchangeable let ρ vary in the range $[0, 1]$ with a step 0.1, and conduct the experiment for the proposed model on Breast Cancers. The experimental results are presented in Fig. 10.

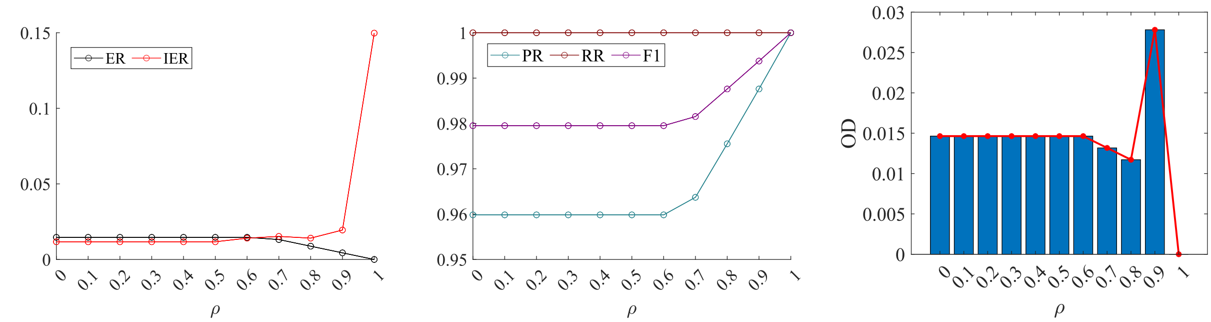
It is shown from Fig. 10 that the experimental results are consistent to the theoretical analysis if $\rho \leq 0.5$. When $\rho > 0.5$, the number of objects in the boundary region increases gradually with ρ increases, and thus PR, RR and $F1$ are increasing, while ER and OD decrease. However, the goal of true partition cannot be fulfilled if the majority of objects are classified into the boundary region, which is verified by the trends of IER and OD.

In practice, it is recommended to set the range of ρ as $(0.6, 0.9)$.

7. Conclusions and further considerations

In this paper, a notion of credible hesitant fuzzy information system is introduced to tackle multi-attribute decision-making for objects with credible uncertainty. Basic operations on credible hesitant fuzzy sets are discussed to aggregate credible hesitant fuzzy sets to guide the arrival of optimal decisions. To enhance performance of aggregation strategy based multi-attribute decision-making, a dominance degree between two credible hesitant fuzzy elements is quantized and a dominance relation on a credible hesitant fuzzy information system is derived. The weights of attributes are assessed based on the dominance relation and information theory. A pair of regret-rejoice functions in regret theory are improved to profoundly reveal decision-makers' psychological emotion in decision process. The enhanced regret-rejoice functions are integrated into the construction of loss functions and the behavior three-way decision model (CHF-BTWDM) is deduced for multi-attribute decision-making in the circumstance of credible hesitant fuzzy sets. Case studies on medical diagnosis have been conducted and experimental results verify the superior performance of proposed model compared to many state-of-the-art baseline methods.

The proposed method shows strong robustness on multi-attribute decision-making in complex circumstances with uncertainty. To address this shortcoming of the proposed model in handling incomplete data, future research directions will focus on developing improved models to handle incomplete credible hesitant fuzzy information system data.



(a) The variations of ER, IER values   (b) The variations of PR, RR, $F1$ values   (c) The variations of OD values

Fig. 10. The variations of the six metric values with ρ.

$$
\begin{aligned}
&S_k(u_i,u_j)+S_k(u_j,u_i)\\
=&\frac{1}{|h_{ik}|\cdot|h_{jk}|}\frac{\left(|h_{jk}|\alpha_{ik}^{\lambda}\sum_{\gamma_1\in h_{ik}}\gamma_1-|h_{ik}|\alpha_{jk}^{\lambda}\sum_{\gamma_2\in h_{jk}}\gamma_2+|h_{ik}|\alpha_{jk}^{\lambda}\sum_{\gamma_2\in h_{jk}}\gamma_2-|h_{jk}|\alpha_{ik}^{\lambda}\sum_{\gamma_1\in h_{ik}}\gamma_1\right)+2\cdot|h_{ik}|\cdot|h_{jk}|}{2}\\
=&1
\end{aligned}
$$

Box I.

In the circumstances of large-scale data and high-dimensional spaces, an efficient solution is to introduce and integrate technologies such as clustering, feature selection or extraction in machine learning, which is our further considerations.

CRediT authorship contribution statement

Tingquan Deng: Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization. **Chaoyue Wang:** Writing – original draft, Validation, Methodology, Investigation, Formal analysis. **Wenjie Wang:** Writing – original draft, Validation, Methodology, Investigation. **Jianming Zhan:** Visualization, Validation, Software, Resources, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

The proof of Proposition 4.1 is presented as follows.

Proof. According to Definition 4.1, the results in Item (1) are obvious.

For (2), given $\lambda > 0$, let $a_k \in A$, $u_i, u_j \in U$, assume $\langle h_{ik}, \alpha_{ik}\rangle$ and $\langle h_{jk}, \alpha_{jk}\rangle$ are two credible hesitant fuzzy elements related to u_i and u_j with respect to a_k, then we get the equation given in Box I.

For (3), see equation given in Box II.

For (4), if α_{ik} is fixed, $S_k(u_i,u_j)$ is an decreasing function with respect to α_{jk}, so $S_k(u_i,u_j)$ is decreasing as α_{jk} increases;

For (5), if α_{jk} is fixed, $S_k(u_i,u_j)$ is an increasing function with respect to α_{ik}, so $S_k(u_i,u_j)$ is increasing as α_{ik} increases. □

Appendix B

The proof of Proposition is provided as follow.

Proof. According to Eq. (25) and Definitions 5.3, we get

$$
\begin{aligned}
&\kappa(a_P,u)=P_{C,u}\tilde{R}_1(u)\\
&\kappa(a_B,u)=\rho P_{C,u}\tilde{R}_1(u)+\rho P_{\neg C,u}\tilde{R}_2(u)-2\rho^2P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u)\\
&\kappa(a_N,u)=P_{\neg C,u}\tilde{R}_2(u)
\end{aligned}
$$

then

$$
\begin{aligned}
\kappa(a_P,u)-\kappa(a_B,u)=&\,2P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u)\rho^2\\
&-(P_{C,u}\tilde{R}_1(u)+P_{\neg C,u}\tilde{R}_2(u))\rho+P_{C,u}\tilde{R}_1(u)
\end{aligned}
$$

Since $0 < P_{C,u}\tilde{R}_1(u) < 1$ and $0 < P_{\neg C,u}\tilde{R}_2(u) < 1$, we have that

$$
\frac{P_{C,u}\tilde{R}_1(u)+P_{\neg C,u}\tilde{R}_2(u)}{4P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u)}>\frac{1}{2}
$$

i.e., $\kappa(a_P,u)-\kappa(a_B,u)$ decreases as ρ increases when $\rho < 0.5$.

Therefore,

$$
\begin{aligned}
(\kappa(a_P,u)-\kappa(a_B,u))_{\min}=&\,0.25P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u)\\
&+0.5P_{C,u}\tilde{R}_1(u)-0.5P_{\neg C,u}\tilde{R}_2(u)
\end{aligned}
$$

The same reasoning gives

$$
\begin{aligned}
(\kappa(a_N,u)-\kappa(a_B,u))_{\min}=&\,0.25P_{C,u}P_{\neg C,u}\tilde{R}_1(u)\tilde{R}_2(u)\\
&-0.5P_{C,u}\tilde{R}_1(u)+0.5P_{\neg C,u}\tilde{R}_2(u)
\end{aligned}
$$

From the above equations, it can be seen that at least one of $(\kappa(a_P,u)-\kappa(a_B,u))_{\min}$ and $(\kappa(a_N,u)-\kappa(a_B,u))_{\min}$ is greater than 0. That is, $\kappa(a_P,u)-\kappa(a_B,u) > 0$ or $\kappa(a_N,u)-\kappa(a_B,u) > 0$ holds when $\rho < 0.5$, i.e., either $\kappa(a_P,u) > \kappa(a_B,u)$ or $\kappa(a_N,u) > \kappa(a_B,u)$ is true. These facts imply that each object is surely assigned to either the positive region or the negative region.

Therefore, when $\rho < 0.5$, the boundary region $BND=\emptyset$. □

Data availability

Data will be made available on request.

$$
\begin{aligned}
& S_k(u_i,u_j)+S_k(u_j,u_l) \\
=& \frac{1}{|h_{ik}|\cdot|h_{jk}|}\cdot\frac{|h_{jk}|\alpha_{ik}^{\lambda}\sum_{\gamma_1\in h_{ik}}\gamma_1-|h_{ik}|\alpha_{jk}^{\lambda}\sum_{\gamma_2\in h_{jk}}\gamma_2+|h_{ik}|\cdot|h_{jk}|}{2}+\frac{1}{|h_{jk}|\cdot|h_{lk}|}\cdot\frac{|h_{lk}|\alpha_{jk}^{\lambda}\sum_{\gamma_2\in h_{jk}}\gamma_2-|h_{jk}|\alpha_{lk}^{\lambda}\sum_{\gamma_3\in h_{lk}}\gamma_3+|h_{jk}|\cdot|h_{lk}|}{2} \\
=& \frac{1}{2|h_{ik}|\cdot|h_{jk}|\cdot|h_{lk}|}\left(|h_{lk}|\cdot|h_{jk}|\alpha_{ik}^{\lambda}\sum_{\gamma_1\in h_{ik}}\gamma_1-|h_{lk}|\cdot|h_{ik}|\alpha_{jk}^{\lambda}\sum_{\gamma_2\in h_{jk}}\gamma_2+|h_{ik}|\cdot|h_{lk}|\alpha_{jk}^{\lambda}\sum_{\gamma_2\in h_{jk}}\gamma_2-|h_{ik}|\cdot|h_{jk}|\alpha^{\lambda}\sum_{\gamma_3\in h_{lk}}\gamma_3\right)+ \\
& \frac{1}{2|h_{ik}|\cdot|h_{jk}|\cdot|h_{lk}|}\left(2\cdot|h_{ik}|\cdot|h_{jk}|\cdot|h_{lk}|\right) \\
=& \frac{1}{|h_{ik}|\cdot|h_{lk}|}\cdot\frac{|h_{lk}|\cdot\alpha_{ik}^{\lambda}\sum_{\gamma_1\in h_{ik}}\gamma_1-|h_{ik}|\cdot\alpha_{lk}^{\lambda}\sum_{\gamma_3\in h_{lk}}\gamma_3+|h_{ik}|\cdot|h_{lk}|}{2}+\frac{1}{2} \\
=& S_k(u_i,u_l)+0.5
\end{aligned}
$$

Box II.

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