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Multiple attribute group decision making based on multigranulation probabilistic models, MULTIMOORA and TPOP in incomplete q-rung orthopair fuzzy information systems

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A B S T R A C T

The concept of q-rung orthopair fuzzy sets (q-ROFSs) severs as an extended form of orthopair fuzzy sets, which excels in flexibly depicting imprecise information existed in multiple attribute group decision making (MAGDM) via allowing a greater space in terms of membership degrees (MD) and non-membership degrees (ND). Moreover, incomplete information systems play a significant role in depicting incomplete information existed in MAGDM. In this paper, for the sake of exploring MAGDM with imprecise and incomplete information, a new MAGDM method based on multigranulation probabilistic models, MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus the full MULTIplicative form) and the technique of precise order preference (TPOP) method in incomplete q-rung orthopair fuzzy (q-ROF) information systems is systematically investigated. First, the concept of multigranulation (MG) incomplete q-ROF information systems is established. Then, a completion technique is put forward to obtain MG q-ROF information systems via patching lost values. Moreover, three versions of MG q-ROF probabilistic rough sets (PRSs) are put forward in light of MG q-ROF information systems. Afterwards, a new MAGDM method is constructed by means of MG q-ROF PRSs, MULTIMOORA and the TPOP method. At last, two experimental studies from open datasets are performed to present the applicability of the constructed MAGDM method, and corresponding sensitivity analysis and comparative analysis are further conducted to demonstrate the validity of the presented methodology.

1. Introduction

As an interdisciplinary study direction that combines group decision making [1] with multiple attribute decision making [2], MAGDM usually consists of three fundamental problem solving processes, i.e., (a) Information description [3,4]. This process demands to invite a group of decision makers, and each individual needs to provide a decision matrix in terms of the assessment for all alternatives under multiple attributes. (b) Information fusion [5–7]. Viable information fusion strategies

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are scheduled to design for acquiring a unified consensus. (c) Information analysis [8,9]. According to intermediate results of previous two processes, it is necessary to explore an efficient information analysis method for accomplishing the overall MAGDM goal. In light of the above statements, information description serves as a vital prerequisite for information fusion and information analysis. Ever since Zadeh [10] pioneered the concept of fuzzy sets in 1965, plenty of generalized fuzzy sets have been subsequently put forward by scholars and practitioners during the past decades for keeping in line with various requirements of application scenarios [11], such as medical diagnosis [12,13], person-job fit [15,16], unconventional emergency evaluations [17,18], etc. Thus, the research on MAGDM under generalized fuzzy environments has actually emerged as a focal topic in intelligent decision making.

Nowadays, it is widely known that uncertainties [19] are ubiquitous in all walks of life, hence how to cope with various uncertainties existed in MAGDM constitutes a major issue in information description of MAGDM. In the current paper, we primarily focus on MAGDM with imprecise and incomplete information, and specific processing methods are elaborated on as follows.

(1) Among these extended notions of classic fuzzy sets, the notion of intuitionistic fuzzy sets (IFSs) [20] is regarded as a representative tool for describing imprecise information, and IFSs are featured by each element with MD and ND that satisfy the sum of MD and ND is less than or equal to 1. Later, Yager [21] initiated the form of Pythagorean fuzzy sets (PFSs) via IFSs and Pythagorean complements. Similar with IFSs, PFSs are also featured by MD and ND of elements, however the square sum of MD and ND is less than or equal to 1. At last, Yager [22] generalized IFSs and PFSs to a more general form, which is referred to as q-ROFSs. In a similar manner, a q-ROFS is still featured by MD and ND of elements, however the sum of the q-th power of MD and the q-th power of ND is less than or equal to 1, where $q \geq 1$. Compared with IFSs and PFSs, it is evident to see q-ROFSs are immune to the restricted limitation of MD and ND, and the space of acceptable MD and ND will increase when q increases [23]. Thus, it is widely recognized that q-ROFSs excel in depicting the preference of individuals in a more convenient way [24–32].
(2) For the sake of efficiently processing incomplete information, it is widely recognized that incomplete information systems [33] serve as a major way for depicting various incomplete information in numerous realistic scenarios. In specific, there usually exist three common rules for processing incomplete information systems [34–36], i.e., the substitution that aims to utilize special symbols to substitute for unknown values; the object deletion that aims to directly delete the unknown values; the completion that aims to utilize specific values to substitute for unknown values.

In light of the above statements in term of MAGDM with imprecise and incomplete information, it is worth noticing that the exploration on MAGDM approaches under the framework of incomplete q-ROF information systems is scarce. For the sake of efficiently handling incomplete q-ROF MAGDM scenarios, the current paper aims to first explore MG incomplete q-ROF information systems, and propose the completion technique to transform MG incomplete q-ROF information systems into corresponding complete counterparts. Then, we primarily focus on proposing information fusion strategies for MAGDM based on multigranulation probabilistic models, and specific processing methods are elaborated on as follows.

(1) Under the umbrella of granular computing [37], MG PRSs [38] serve as the most common multigranulation probabilistic model, which simultaneously take advantage of multigranulation rough sets (MGRSs) [39] and PRSs [40]. In MGRSs, two merits can be clearly found, i.e., the one is accelerating the information fusion efficiency by using the idea of parallel computing, another one is addressing realistic scenarios via incorporating the processing scheme of decision risks. Ever since the establishment of classic MGRSs, plenty of generalized MGRSs have been subsequently put forward to adjust to varieties of theoretical models and applications, such as double-quantitative MGRSs [42], adjustable MGRSs [43], covering-based MGRSs [45,46], local MGRSs [47], etc.
(2) Since the structure of original rough sets is fairly strict that hinders effective applications of rough set communities, PRSs excel in relaxing rigorous rough approximations and providing suitable thresholds from the perspective of fault-tolerant mechanisms. Thus, the integration of MGRSs with PRSs can not only provide an efficient information fusion tactic, but also provide relatively robust information analysis rules for complicated MAGDM problems [48].

At the stage of information analysis, different types of decision analysis tools usually concentrate on specific aspects of decision goals, thus resulting in different versions of ranking or classification results for all alternatives. Hence, it is meaningful to explore robust decision analysis methods that are immune to different decision standards. In order to address the above challenge, Brauers et al. [49,50] initiated the MULTIMOORA method, which is featured by providing final ranking results via three affiliated counterparts, i.e., ratio systems, reference point methods and full multiplicative forms. Recently, several scholars have explored plenty of decision-making-driven MULTIMOORA methods in diverse contexts, such as healthcare industries [51,52], the site selection of car sharing stations [53], person-job fit [15], the evaluation of solid waste management techniques [54], etc

Based on research results from information description, fusion and analysis of MAGDM, we aim to explore a new MAGDM method based on MG q-ROF PRSs and MULTIMOORA in incomplete q-ROF information systems for addressing MAGDM with imprecise and incomplete information. In what follows, several key research motivations of this paper are summed up.

- The majority of q-ROF MAGDM methods are investigated in complete information systems, whereas the exploration on q-ROF MAGDM methods in incomplete information systems is scarce. For the sake of reasonably describing varieties of realistic imprecise and incomplete MAGDM information, we propose the concept of MG incomplete q-ROF information systems for depicting complicated information in MAGDM situations.
- A completion technique is put forward for patching lost values in MG incomplete q-ROF information systems and corresponding complete counterparts are further obtained. Then in light of the above statements, three versions of MG q-ROF PRSs are proposed for fusing complicated information in MAGDM situations.
- In order to explore a stable q-ROF MAGDM method that is immune to different decision standards, the MULTIMOORA method is utilized for providing semantic interpretations of MG q-ROF PRSs. Moreover, since providing a final MULTIMOORA ranking according to three subordinate rankings is vital, the TPOP method [55,56] is developed to generate the MULTIMOORA ranking by means of the final scores in terms of each MG q-ROF PRS. Thus, it is necessary to explore a new MAGDM method via MG q-ROF PRSs, MULTIMOORA and the TPOP method.

In light of the above research motivations, we further list primary contributions of the paper below.

- The concept of MG incomplete q-ROF information systems is established for effectively describing complicated MAGDM problems with imprecise and incomplete information.
- A completion technique for transforming MG incomplete q-ROF information systems into corresponding complete counterparts is developed. Then three versions of MG q-ROF PRSs are proposed for effectively fusing multi-source q-ROF MAGDM information.
- Multigranulation probabilistic models are integrated with MULTIMOORA to provide a stable decision analysis method via ranking alternatives. The TPOP method is developed for providing the final MULTIMOORA ranking. In light of the above steps, a new MAGDM method is eventually constructed by means of MG q-ROF PRSs, MULTIMOORA and the TPOP method.

The arrangement of the work is presented below. Section 2 revisits several fundamental notions on q-ROFSs, MGRSs, PRSs and MULTIMOORA. Section 3 constructs MG incomplete q-ROF information systems and puts forth a completion technique to obtain corresponding complete counterparts, then three versions of multigranulation probabilistic models are proposed. In Section 4, a new MAGDM method is explored by means of MG q-ROF PRSs, MULTIMOORA and the TPOP method. In Section 5, we perform a case study within the background of financial quality matching for showing the validity of the presented MAGDM method, then an experiment analysis with a UCI dataset is further made to reveal the validity of the proposed methodology from diverse perspectives. In Section 6, main works of the paper are summed up and several potential future research directions are listed as well.

2. Preliminaries

In this section, several related fundamental notions for later theoretical analysis will be revisited.

2.1. q-ROFSs

The notion of q-ROFSs excels in depicting realistic uncertain information in a more flexible way compared with other typical orthopair fuzzy sets. In what follows, we revisit the definition of q-ROFSs.

Definition 2.1. [22] A q-ROFS B in terms of a finite universal set U is given as:

$$B = \{\langle x, (\mu_b(x), \nu_b(x))\rangle \,|x \in U\}, \tag{1}$$

where μ_b and ν_b denote MD and ND, respectively, under the following restriction: $(\mu_b(x))^q + (\nu_b(x))^q \leqslant 1$ and $q \geqslant 1$. $\pi_b(x) = \sqrt[q]{1 - (\mu_b(x))^q - (\nu_b(x))^q}$ denotes the indeterminacy degree. In addition, the set which includes all q-ROFSs is given as $q - ROF(U)$ and a q-ROF number (q-ROFN) is given as $b = (\mu, \nu)$.

The above q-ROFSs are defined over one universe, since the framework of two universes is beneficial to depicting more complicated uncertain information, the following definition of q-ROF relations (q-ROFRs) over two universes will be presented.

Definition 2.2. [44] A q-ROFR R in terms of two finite universal sets U and V is given as:

$$R = \{\langle (x, y), (\mu_R(x, y), \nu_R(x, y))\rangle \,|\, (x, y) \in U \times V\}, \tag{2}$$

where μ_R and ν_R represent MD and ND, respectively, under the similar restriction: $(\mu_R(x, y))^q + (\nu_R(x, y))^q \leqslant 1$ and $q \geqslant 1$. In addition, the set which includes all q-ROFRs is given as $q - ROFR(U \times V)$.

Noting that operational laws of q-ROFNs serve as the backbone when fusing and analyzing q-ROF MAGDM information, thus we revisit some common operational laws of q-ROFNs below.

Definition 2.3. [22,44] For any two q-ROFNs $b=(\mu,\nu)$ and $b'=(\mu',\nu')$,

(1) $b\oplus b'=(\sqrt[q]{(\mu)^q+(\mu')^q-(\mu)^q(\mu')^q},\nu\nu')$;
(2) $b\otimes b'=(\mu\mu',\sqrt[q]{(\nu)^q+(\nu')^q-(\nu)^q(\nu')^q})$;
(3) $b\ominus b'=(\max(0,\sqrt[q]{\frac{(\mu)^q-(\mu')^q}{1-(\mu')^q}}),\min(1,\frac{\nu}{\nu'}))$;
(4) $b\oslash b'=(\min(1,\frac{\mu}{\mu'}),\max(0,\sqrt[q]{\frac{(\nu)^q-(\nu')^q}{1-(\nu')^q}}))$;
(5) $b^\lambda=(\mu^\lambda,\sqrt[q]{1-(1-\nu^q)^\lambda}),\lambda>0$;
(6) $\lambda b=(\sqrt[q]{1-(1-\mu^q)^\lambda},\nu^\lambda),\lambda>0$.

In what follows, we revisit the comparing law of q-ROFNs.

Definition 2.4. [27] For any q-ROFN $b=(\mu,\nu)$, the score function and the accuracy function are given as:

$$s(b)=\mu_b^q-\nu_b^q; \tag{3}$$
$$a(b)=\mu_b^q+\nu_b^q, \tag{4}$$

where $s(b)\in[0,1]$ and $a(b)\in[0,1]$.

2.2. MGRSs

MGRSs act as a typical representative in rough set communities, especially owing a powerful performance when fusing multi-source uncertain information. In specific, MGRSs can not only integrate multiple binary relations at the same time to improve computing efficiencies, but also deal with decision risks by providing diverse versions in light of decision groups' risk preferences. In what follows, the basic definition of MGRSs is presented.

Definition 2.5. [39] Suppose $R_k\,(k=1,2,\cdots,r)$ is a binary relation over a finite universal set U. Then for any $X\subseteq U$, the optimistic version of multigranulation rough approximations is given as:

$$\underline{\sum_{k=1}^{r}R_k}^{O}(X)=\left\{[x]_{R_1}\subseteq X\vee[x]_{R_2}\subseteq X\vee\cdots\vee[x]_{R_r}\subseteq X\,|x\in U\right\}; \tag{5}$$
$$\overline{\sum_{k=1}^{r}R_k}^{O}=(\underline{\sum_{k=1}^{r}R_k}^{O}(X^c))^c. \tag{6}$$

Similarly, corresponding pessimistic counterparts with respect to the optimistic version are given as:

$$\underline{\sum_{k=1}^{r}R_k}^{P}(X)=\left\{[x]_{R_1}\subseteq X\wedge[x]_{R_2}\subseteq X\wedge\cdots\wedge[x]_{R_r}\subseteq X\,|x\in U\right\}; \tag{7}$$
$$\overline{\sum_{k=1}^{r}R_k}^{P}=(\underline{\sum_{k=1}^{r}R_k}^{P}(X^c))^c, \tag{8}$$

based on the above assumptions, $(\underline{\sum_{k=1}^{r}R_k}^{O}(X),\overline{\sum_{k=1}^{r}R_k}^{O}(X))$ and $(\underline{\sum_{k=1}^{r}R_k}^{P}(X),\overline{\sum_{k=1}^{r}R_k}^{P}(X))$ are titled optimistic and pessimistic versions of MGRSs, respectively.

2.3. PRSs

One of the major drawbacks for original rough sets lies in the stringent requirement for constructing rough approximations, in order to handle possible misclassifications in realistic applications, PRSs have been subsequently put forward by introducing a pair of thresholds and loosening stringent rough approximations [40,41].

Definition 2.6. [40] Suppose R is an equivalence relation over a finite universal set U. Then for any $X \subseteq U$ with $0 \leqslant \beta < \alpha \leqslant 1$, the lower and upper probabilistic rough approximations of X are given as:

$$\underline{R}_{\alpha}(X) = \left\{ P\left(X | [x]_R\right) \geqslant \alpha | x \in U \right\}; \tag{9}$$

$$\overline{R}_{\beta}(X) = \left\{ P\left(X | [x]_R\right) > \beta | x \in U \right\}, \tag{10}$$

in light of the above assumptions, $\left(\underline{R}_{\alpha}(X), \overline{R}_{\beta}(X)\right)$ is titled a PRS.

2.4. MULTIMOORA

MULTIMOORA [14] is a method for obtaining stable decision-making results which is usually consisted of ratio systems, reference points and full multiplication forms. In specific, x_{ij} denotes the corresponding value of the alternative $j(j = 1, 2, \cdots, m)$ for the target $i(i = 1, 2, \cdots, n)$ in the ratio system. Taking each corresponding value of a target to alternatives as the numerator, and the square root of the sum with respect to the squares of all alternatives related to the target can be seen in the denominator, i.e., $x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^m x_{ij}^2}}$, where x_{ij}^* is a dimensionless number, indicating the standardized value of j to i, and the value range is $[0, 1]$. However in some cases, the value of x_{ij}^* may not meet this requirement and needs to be further optimized. The optimization method is to increase the corresponding value when maximizing and subtracting the corresponding value, i.e., $y_j^* = \sum\limits_{i=1}^{i=g} x_{ij}^* - \sum\limits_{i=g+1}^{i=n} x_{ij}^*$, where g is the goal to be maximized, $i = g+1, g+2, \cdots, n$ is the goal to be minimized, y_j^* is the normalized evaluation of all targets, and the ranking result is the final preference. Moreover, the reference point is to find the minimum and maximum metric between the reference point and the corresponding point, i.e., $\min\limits_{(j)}\{\max\limits_{(i)} |r_i - x_{ij}^*|\}$. In addition, the full multiplicative form for multiple objectives is $U_j = \prod\limits_{i=1}^{n} x_{ij}$ with $j = 1, 2, \cdots, m$, where m is the number of alternatives, and with $i = 1, 2, \cdots, n$, where n is the number of objectives. x_{ij} is the response of alternative j on objective i, U_j is the whole utility of alternative j. Then the entire utilities, determined by the multiplication of diverse units, become dimensionless.

Compared with other decision-making tools, MULTIMOORA owns the following merits:

(1) By combining multiple fundamental decision conclusions, the accuracy of the final decision conclusions is higher;
(2) MULTIMOORA can effectively address complicated decision-making problems with multiple alternatives;
(3) The calculation process is easy to understand and the calculation speed is relatively fast.

3. MG q-ROF PRSs based on MULTIMOORA

This section first proposes the concept of MG q-ROF incomplete information systems to reasonably depict multi-source imprecise and incomplete decision making information. Then, we design a completion method to transform MG q-ROF incomplete information systems into corresponding complete counterparts. At last, three versions of MG q-ROF PRSs are subsequently investigated in accordance to the main theme of MULTIMOORA.

3.1. MG q-ROF incomplete information systems

In real world, there may exist many lost values in typical information systems due to various reasons, such as man-made ones and natural ones. Thus, it is necessary to depict these kinds of information systems in an efficient way. Prior to giving the definition of MG q-ROF incomplete information systems, the following concept of incomplete information systems is given at first.

Definition 3.1. [33] An information system can be regarded as a quadruple form $S = (U, V, R, B)$, where $U = \{x_1, x_2, \cdots, x_m\}$ denotes a set of objects, $V = \{y_1, y_2, \cdots, y_n\}$ denotes a set of attributes, R denotes the relationship set over $U \times V$, and B denotes the standard evaluation set on V. If certain evaluation values are unknown in an information system, such an information system can be named an incomplete information system. In the current paper, the symbol "*" is used to record the unknown values. In light of the above assumption, we can construct an incomplete information system (U, V, R^*, B).

Noting that the significance of q-ROFSs in information depictions and MGRSs in information fusion, exploring incomplete information systems with q-ROFNs from multiple sources is imperative. Thus, we intend to propose the following notion of MG q-ROF incomplete information systems.

Definition 3.2. Suppose $U = \{x_1, x_2, \cdots, x_m\}$ is a set of alternatives, $V = \{y_1, y_2, \cdots, y_n\}$ is a set of attributes, $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}^{\boldsymbol{T}}$ is a set of attribute weights with $\omega_j \in [0, 1]$ and $\sum\limits_{j=1}^{n} \omega_j = 1 (j = 1, 2, \cdots, n)$, $w = \{w_1, w_2, \cdots, w_r\}^{\boldsymbol{T}}$

Table 1
An MG incomplete q-ROF information system.

		R_1^*						R_2^*			$\cdots$				R_r^*		
U/V	y_1	$\cdots$	y_j	$\cdots$	y_n	U/V	y_1	$\cdots$	y_j	$\cdots$	y_n	$\cdots$	U/V	y_1	$\cdots$	y_j	$\cdots$	y_n
x_1	t_{11}	$\cdots$	t_{1j}	$\cdots$	$*$	x_1	$*$	$\cdots$	t_{1j}	$\cdots$	t_{1n}	$\vdots$	x_1	t_{11}	$\cdots$	t_{1j}	$\cdots$	$*$
x_2	$*$	$\cdots$	t_{2j}	$\cdots$	t_{2n}	x_2	t_{21}	$\cdots$	$*$	$\cdots$	t_{2n}	$\vdots$	x_2	t_{21}	$\cdots$	$*$	$\cdots$	t_{2n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
x_m	$*$	$\cdots$	t_{mj}	$\cdots$	t_{mn}	x_m	$*$	$\cdots$	t_{mj}	$\cdots$	t_{mn}	$\vdots$	x_m	$*$	$\cdots$	t_{mj}	$\cdots$	t_{mn}

is a set of expert weights with $w_k \in [0,1]$ and $\sum_{k=1}^{r} w_k = 1 (k = 1, 2, \cdots, r)$. Based on the above statements, each decision maker evaluates all alternatives with the support of q-ROFNs. In specific, suppose $t_{ij} = (\mu_{ij}, \nu_{ij})$ is the evaluation value of x_i in terms of y_i, and the symbol "*" is used to record lost values, then R_k^* denotes the q-ROFR set over $U \times V$. In addition, B denotes the standard evaluation set on V. According to the above assumptions, (U, V, R_k^*, B) can be constructed in the following Table 1.

3.2. Completion methods

For the sake of efficiently processing lost values in MG incomplete q-ROF information systems, an completion method will be proposed by filling in lost values based on the average of other determined values under a certain attribute. In what follows, we present the specific completion method.

Definition 3.3. Given an MG incomplete q-ROF information system (U, V, R_k^*, B). Then $V_j = \{t_{1j}, \cdots, t_{ij}, \cdots, t_{mj}\} (1 \leqslant i \leqslant m, t_{ij} \neq *)$ denotes all known values under y_j, and the lost value $*$ under y_j is provided as follows:

$$t_j^* = \frac{\sum V_j}{n(V_j)}, \tag{11}$$

where $\sum V_j$ denotes the sum of all known values under y_j, $n(V_j)$ denotes the number of known values under y_j. According to the above operation, (U, V, R_k^*, B) can be transformed into an MG q-ROF information system (U, V, R_k, B).

3.3. Type-I MG q-ROF PRSs

Based on the above completion processes for lost values in MG incomplete q-ROF information systems, corresponding complete counterparts can be obtained for proposing subsequent multigranulation probabilistic models. In this part, we aim to integrate different single q-ROF membership degrees to obtain various multiple q-ROF membership degrees, and further construct diverse versions of MG q-ROF PRSs. Since original MGRSs own a special information fusion strategy, i.e., the maximum operator is used when constructing the optimistic version, whereas the minimum operator is used when constructing the pessimistic version. In order to overcome the limitation of original MGRSs when fusing multi-source decision information, new information fusion strategies need to be employed when integrating diverse single q-ROF membership degrees. Taking advantages of MULTIMOORA, the current paper aims to obtain a stable decision result via the proposed theoretical models, thus three versions of MG q-ROF PRSs can be put forward to keep in line with the central idea of MULTIMOORA, i.e., ratio systems, reference points and full multiplication forms. Before presenting the notion of type-I MG q-ROF PRSs, we first present the form of single q-ROF membership degrees below.

Definition 3.4. Given an MG q-ROF information system (U, V, R_k, B). For any $x_i \in U$ and $y_j \in V$, the single q-ROF membership degree of x_i with regard to (U, V, R_k, B) is given as:

$$\theta_B^{R_k}(x_i) = \frac{\sum_{y_j \in V} \omega_j R_k(x_i, y_j) B(y_j)}{\sum_{y_j \in V} \omega_j R_k(x_i, y_j)}. \tag{12}$$

In what follows, multiple q-ROF membership degrees via q-ROF weighted arithmetic average (q-ROFWA) operators are put forward by integrating single q-ROF membership degrees.

Definition 3.5. Let w_k be the weight of R_k, $\theta_B^{R_k}(x_i)$ be the single q-ROF membership degrees of x_i. For any $x_i \in U (i = 1, 2, \cdots, m)$, the notion of multiple q-ROF membership degrees by means of q-ROFWA operators of x_i in B with regard to R_k, represented by $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$, is given as:

$$\xi_B^{\sum_{k=1}^{r} R_k}(x_i) = q - ROFWA(\theta_B^{R_1}(x_i), \theta_B^{R_2}(x_i), \cdots, \theta_B^{R_r}(x_i)) = (\sqrt[q]{1 - \prod_{k=1}^{r}(1-\mu_k(x_i))^{w_k}}, \prod_{k=1}^{r}(\nu_k(x_i))^{w_k}). \tag{13}$$

Definition 3.6. Let α and β be thresholds that are characterized by q-ROFNs. For any $x_i \in U$ and $\beta < \alpha$, type-I MG q-ROF PRSs of B with regard to R_k, denoted by $\underline{\sum_{k=1}^{r} R_k}^{\xi,\alpha}(B)$ and $\overline{\sum_{k=1}^{r} R_k}^{\xi,\beta}(B)$, are defined as follows:

$$\underline{\sum_{k=1}^{r} R_k}^{\xi,\alpha}(B) = \{\xi_B^{\sum_{k=1}^{r} R_k}(x_i) \geqslant \alpha | x_i \in U\}; \tag{14}$$

$$\overline{\sum_{k=1}^{r} R_k}^{\xi,\beta}(B) = \{\xi_B^{\sum_{k=1}^{r} R_k}(x_i) > \beta | x_i \in U\}, \tag{15}$$

then we name the pair $(\underline{\sum_{k=1}^{r} R_k}^{\xi,\alpha}(B), \overline{\sum_{k=1}^{r} R_k}^{\xi,\beta}(B))$ a type-I MG q-ROF PRS.

3.4. Type-II MG q-ROF PRSs

Noting that q-ROF distance measures act as another widely-used method for processing complicated MAGDM problems, thus it is necessary to study the following multiple q-ROF membership degrees via distance measures.

Definition 3.7. Let w_k be the weight of R_k, $\theta_B^{R_k}(x_i)$ be the single q-ROF membership degrees of x_i. For any $x_i \in U (i = 1, 2, \cdots, m)$, the notion of multiple q-ROF membership degrees by means of q-ROF weighted Euclidean distances of x_i in B with regard to R_k, represented by $\zeta_B^{\sum_{k=1}^{r} R_k}(x_i)$, is provided as follows:

$$\zeta_B^{\sum_{k=1}^{r} R_k}(x_i) = \sqrt[q]{\sum_{k=1}^{r} w_k(\frac{1}{q}(|\mu_k(x_i) - 1|^q + |\nu_k(x_i)|^q))}. \tag{16}$$

Definition 3.8. Let α and β be thresholds that are characterized by q-ROFNs. For any $x_i \in U$ and $\beta < \alpha$, type-II MG q-ROF PRSs of B with regard to R_k, represented by $\underline{\sum_{k=1}^{r} R_k}^{\zeta,\alpha}(B)$ and $\overline{\sum_{k=1}^{r} R_k}^{\zeta,\beta}(B)$, are defined as follows:

$$\underline{\sum_{k=1}^{r} R_k}^{\zeta,\alpha}(B) = \{\zeta_B^{\sum_{k=1}^{r} R_k}(x_i) \geqslant \alpha | x_i \in U\}; \tag{17}$$

$$\overline{\sum_{k=1}^{r} R_k}^{\zeta,\beta}(B) = \{\zeta_B^{\sum_{k=1}^{r} R_k}(x_i) > \beta | x_i \in U\}, \tag{18}$$

then we name the pair $(\underline{\sum_{k=1}^{r} R_k}^{\zeta,\alpha}(B), \overline{\sum_{k=1}^{r} R_k}^{\zeta,\beta}(B))$ a type-II MG q-ROF PRS.

3.5. Type-III MG q-ROF PRSs

The above presented type-I MG q-ROF PRSs are constructed based on q-ROFWA operators, which can show the appetite of decision groups. Since q-ROFWG operators can show the appetite of each individual, which can be regarded as a beneficial supplement to q-ROF multigranulation probabilistic models. Following the above guidance, the definition of type-III MG q-ROF PRSs based on q-ROFWG operators is presented below.

Definition 3.9. Let w_k be the weight of R_k, $\theta_B^{R_k}(x_i)$ be the single q-ROF membership degrees of x_i. For any $x_i \in U(i = 1, 2, \cdots, m)$, the notion of multiple q-ROF membership degrees by means of q-ROF weighted Euclidean distances of x_i in B with regard to R_k, represented by $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$, is provided as follows:

$$\begin{aligned}\psi_B^{\sum_{k=1}^{r} R_k}(x_i) &= q-ROFWG(\theta_B^{R_1}(x_i), \theta_B^{R_2}(x_i), \cdots, \theta_B^{R_r}(x_i)) \\ &= (\prod_{k=1}^{r}(\mu_k(x_i))^{w_k}, \sqrt[q]{1-\prod_{k=1}^{r}(1-\nu_k(x_i))^{w_k}}).\end{aligned} \tag{19}$$

Definition 3.10. Let α and β be the thresholds that are characterized by q-ROFNs. For any $x_i \in U$ and $\beta < \alpha$, type-III MG q-ROF PRSs of B with regard to R_k, represented by $\underline{\sum_{k=1}^{r} R_k}^{\psi,\alpha}(B)$ and $\overline{\sum_{k=1}^{r} R_k}^{\psi,\beta}(B)$, are defined as follows:

$$\underline{\sum_{k=1}^{r} R_k}^{\psi,\alpha}(B) = \{\psi_B^{\sum_{k=1}^{r} R_k}(x_i) \geqslant \alpha | x_i \in U\}; \tag{20}$$

$$\overline{\sum_{k=1}^{r} R_k}^{\psi,\beta}(B) = \{\psi_B^{\sum_{k=1}^{r} R_k}(x_i) > \beta | x_i \in U\}, \tag{21}$$

then we name the pair $(\underline{\sum_{k=1}^{r} R_k}^{\psi,\alpha}(B), \overline{\sum_{k=1}^{r} R_k}^{\psi,\beta}(B))$ a type-III MG q-ROF PRS.

4. MAGDM based on MG q-ROF PRSs, MULTIMOORA and TPOP

In what follows, a new MAGDM method via MG q-ROF PRSs and MULTIMOORA in incomplete q-ROF information systems is constructed at first, and then the TPOP method is introduced to obtain a precise ranking order of alternatives.

4.1. MAGDM based on MG q-ROF PRSs and MULTIMOORA

Given an MG q-ROF incomplete information systems (U, V, R_k^*, B), we complete the information to make it (U, V, R_k, B). Then we develop a new method for acquiring the weight value of attributes and decision makers in light of the TOPSIS method. In specific, the following formula is presented.

$$Y_j = \sum_{j=1}^{m}\sum_{k=1}^{m}\sqrt[q]{\frac{1}{qr}\sum_{\lambda=1}^{r}(|R_\lambda \mu_{mj}^q - R_\lambda \mu_{mk}^q|^q + |R_\lambda \nu_{mj}^q - R_\lambda \nu_{mk}^q|^q) + |\pi(R_\lambda x_j)^q - \pi(R_\lambda x_k)^q|}, \tag{22}$$

then the weight value of attributes is

$$\omega_j = \frac{Y_j}{\sum_{j=1}^{n} Y_j}. \tag{23}$$

In a similar manner in terms of the formula (22), the following formula can be presented below.

$$\Re_k = \sum_{i=1}^{m}\sum_{j=1}^{m}\sqrt[q]{\frac{1}{qn}\sum_{\lambda=1}^{n}(|R_k \mu_{\lambda i}^q - R_k \mu_{\lambda j}^q|^q + |R_k \nu_{\lambda i}^q - R_k \nu_{\lambda j}^q|^q) + |\pi(R_k x_i)^q - \pi(R_k x_j)^q|}, \tag{24}$$

then the weight value of decision makers is

$$w_k = \frac{\Re_k}{\sum_{k=1}^{r}\Re_k}. \tag{25}$$

According to the idea of MULTIMOORA, it mainly adopts three subordinate ranks to fuse multi-source information, i.e., ratio systems, reference point approaches and full multiplicative forms. In specific, noting that q-ROF membership degrees in terms of q-ROFWA operators remain consistent with the notion of ratio systems, and q-ROF weighted Euclidean distances and q-ROFWG operators also keep in line with the notion of reference point approaches and full multiplicative forms, then it is meaningful to explore q-ROF MAGDM by means of MG q-ROF PRSs and MULTIMOORA. Given a transformed MG q-ROF information system (U, V, R_k, B), for any alternative $x_i \in U(i = 1, 2, \cdots, m)$, we can obtain multiple q-ROF membership degrees via q-ROFWA operators of x_i in B in term of R_i according to Definition 3.5, which is denoted by $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$. Thus, we can obtain the type-I ranking result of these alternatives via the value of $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$, and when the value of $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$ is larger, x_i is more in line with the optimal alternative.

Similarly, we can obtain multiple q-ROF membership degrees via q-ROF weighted Euclidean distances and q-ROFWG operators of x_i in B in terms of R_k according to Definition 3.7 and Definition 3.9, which can be denoted by $\zeta_B^{\sum_{k=1}^{r} R_k}(x_i)$ and $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$, respectively. Thus, we can obtain the type-II ranking result of these alternatives via the value of $\zeta_B^{\sum_{k=1}^{r} R_k}(x_i)$, and when the value of $\zeta_B^{\sum_{k=1}^{r} R_k}(x_i)$ is smaller, the x_i is more in line with the optimal alternative. In addition, the type-III ranking result of these alternatives via the value of $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$ can be further obtained, and when the value of $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$ is larger, the x_i is more in line with the optimal alternative.

4.2. MAGDM with TPOP

According to the previous section, we can obtain the traditional MULTIMOORA sorting results. Then how to effectively utilize the above three kinds of sorting results is vital in the q-ROF information analysis procedure. In the following, we develop a new sorting scheme via the TPOP method for obtaining a precise ranking order of all alternatives.

The TPOP method [55,56] is highly related to the reference point. On one hand, the TPOP method assumes that the alternative that has the minimum distance from the reference point is provided the top priority and be chosen as the optimal alternative. On the other hand, the alternative that has the maximum distance from the reference point is provided the least priority and be chosen as the worst alternative. Moreover, the detailed steps of the developed TPOP approach in terms of MG q-ROF information systems are described below.

Suppose (U, V, R_k, B) includes m alternatives and n attributes that is preliminary processed by three versions of MG q-ROF PRSs by virtue of MULTIMOORA, and the above procedure gives $3m$ assessment values for obtaining sorting results for all alternatives. If the ranking order of alternatives is inconsistent, then the TPOP method can be employed via the steps below.

(1) Establish a decision matrix that includes assessment values obtained by three versions of MG q-ROF PRSs.

$$S = \begin{matrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_m \end{matrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ \vdots & \vdots & \vdots \\ f_{i1} & f_{i2} & f_{i3} \\ \vdots & \vdots & \vdots \\ f_{m1} & f_{m2} & f_{m3} \end{bmatrix}, \tag{26}$$

where $f_{il}(l = 1, 2, 3)$ is the assessment value of the i-th alternative x_i obtained by the l-th type MG q-ROF PRSs, that is, f_{i1}, f_{i2} and f_{i3} represent the value of $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$, $\zeta_B^{\sum_{k=1}^{r} R_k}(x_i)$ and $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$, respectively.

(2) Normalize assessment values for acquiring their weights. The magnitude of assessment values determined by various MG q-ROF PRSs may vary over a wide range. Thus, assessment values of alternatives require a normalization procedure for maintaining compatibility and removing bias. The following formula is utilized for normalization for assessment values:

$$\tau_{il} = \frac{|f_{il}|}{\sum_{i=1}^{m} |f_{il}|}, \tag{27}$$

where $0 \leqslant \tau_{il} \leqslant 1, i = 1, 2 \ldots m$. l denotes the index of various MG q-ROF PRSs and $|f_{il}|$ denotes the absolute value of f_{il}.

(3) Compute the entropy of assessment values by the following formula:

$$e_l = \frac{1}{\ln m}\sum_{i=1}^{m} |\tau_{il} \ln \tau_{il}|, \tag{28}$$

where e_l represents the entropy of assessment values computed by l-th approach. The entropy method is used as a measure for the degree of disorderliness of assessment values.

(4) Obtain the weight of the l-th approach by the following formula:

$$s_l = \frac{1-e_l}{\sum_{j=1}^{t}(1-e_l)}, \tag{29}$$

where $(1-e_l)$ can be considered as a complement of the entropy for assessment values determined by the l-th approach, whereas $\sum_{l=1}^{t}(1-e_l)$ is regarded as the sum of the complements of entropies of all assessment values.

(5) In the classic entropy method, the value of s_l is considered as the weight. Then, we introduce an advanced version in terms of the classic entropy method by using the following formula:

$$s'_l = (1+\sqrt{s_l}), \tag{30}$$

where $l = 1, 2, 3, \ldots, t$ and $1 \leqslant s'_l \leqslant 2$. The objective for developing the above formula is to decrease the percentage of max (s'_l) to min (s'_l).

(6) Compute the precise weight of assessment values determined by the l-th approach by using the following formula:

$$w_l = \frac{s'_l}{S'_l}, \tag{31}$$

where S'_l represents the sum of all s'_l values and $\sum_{l=1}^{t} w_l = 1$ holds.

(7) Normalize assessment values for all alternatives by using the following formula:

$$g_{il} = \begin{cases} \frac{(f_l)_{\max}-f_{il}}{(f_l)_{\max}-(f_l)_{\min}}, f_{il} \in H \\ \frac{f_{il}-(f_l)_{\min}}{(f_l)_{\max}-(f_l)_{\min}}, f_{il} \in L \end{cases}, \tag{32}$$

where $0 \leqslant g_{il} \leqslant 1$ represents the normalized value of f_{il}, $(f_l)_{\min}$ and $(f_l)_{\max}$ are the minimum and the maximum assessment values determined by the l-th approach. $f_{il} \in L$ indicates that the assessment is cost, a lower value of f_{il} is optimal and $f_{il} \in H$ indicates that the assessment is benefit, a higher value of f_{il} is optimal. Moreover, a lower value of g_{il} shows that the corresponding alternative is closer to the best solution. Thus, a lower value of g_{il} is optimal.

(8) Calculate the exponentially weighted normalized assessment value by using the following formula:

$$h_{il} = \exp(w_l + g_{il}). \tag{33}$$

(9) Calculate the precise selection index (PSI) for all alternatives. The TPOP method gauges PSI that finds the precise order preference for all alternatives. The PSI for the i-th alternative can be obtained by utilizing the following formula:

$$PSI_i = \sum_{l=1}^{t} h_{il}. \tag{34}$$

(10) Arrange all alternatives with an increasing order of different PSI values. PSI denotes the relative distance of an alternative from the ideal reference point. Thus, the top priority is allocated to the alternative that has the minimum PSI, the second priority to the next alternative that has PSI just higher to the minimum (i.e., the second minimum PSI), then the same scheme continues for the next priority. In other word, we need to choose the best alternative with the minimum PSI value and the worst alternative with the maximum PSI value.

4.3. Simplified steps of q-ROF MAGDM in incomplete q-ROF information systems

In this section, we summarize the presented MAGDM approach stated in Section 4.1 and Section 4.2 in a more concise way and simplify it into the following steps:

Input: An MG q-ROF incomplete information system (U, V, R_k^*, B).

Output: The sorting results of available alternatives.

Step 1: Set the minimal value of q in light of each value in (U, V, R_k^*, B).

Step 2: Complete lost information to obtain (U, V, R_k, B).

Step 3: Calculate the weight of attributes $\boldsymbol{w}$ and the weight of decision makers $\boldsymbol{\omega}$.

Step 4: Calculate single q-ROF membership degrees $\theta_B^{R_k}(x_i)$ of x_i with respect to R_k and B.

Step 5: Calculate MG q-ROF membership degrees $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$, $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$ and $\zeta_B^{\sum_{k=1}^{r} R_k}(x_i)$.

Step 6: Obtain the type-I, type-II and type-III ranking results.

Step 7: Construct the decision matrix that includes assessment values obtained by three versions of MG q-ROF PRSs and calculate PSI_i of each x_i.

Step 8: Rank all alternatives by means of PSI values and to select the best alternative with the minimum PSI value.

Remark 4.1. In the above simplified steps of q-ROF MAGDM in incomplete q-ROF information systems, let m denote the number of all objects, n denote the number of all attributes and r denote the number of all decision makers. In Steps 1-2, the complexities are $O(mn)$. In Step 3, the complexities of calculating the weight of attributes and decision makers are $O\left(m^2 r\right)$ and $O\left(m^2 n\right)$, respectively. In Step 4, the time complexity of calculating single q-ROF membership degrees is $O(mnr)$. In Step 5, the time complexity of calculating MG q-ROF membership degrees is $O\left(r^2\right)$. The remaining Steps 6-8 cause the time complexity of $O(m)$. Thus, the overall time complexity of the above steps is $O(m^2 n)$.

Compared with other q-ROF MAGDM methods, the main difference in terms of the time complexity lies in Steps 4-8. In specific, if we use q-ROFWA operators, q-ROF Euclidean distances and q-ROFWG operators to solve q-ROF MAGDM problems, the overall time complexity is $O(mnr)$; if we use the q-ROF TOPSIS method, the q-ROF VIKOR method, the q-ROF 3WD method and the q-ROF MULTIMOORA method to solve q-ROF MAGDM problems, the time complexity is consistent with the presented method. Thus, the time complexity of the presented method is not worse than other counterparts. Moreover, the presented method considers decision risks and stable decision conclusions at the same time, which shows several merits when making q-ROF MAGDM.

5. Illustrative case studies

The current section shows two detailed case studies to illustrate the applicability of the presented MAGDM approach established in Section 4, and corresponding sensitivity analysis and comparative analysis are further performed for showing the validity of the presented approach.

5.1. Case descriptions and decision making procedures

In what follows, we first present a case study in the context of financial quality matching [44]. In order to open up new markets, expand business scopes and enhance overall competitiveness, a construction investment company plans to select a enterprise as a partner to achieve mutual benefits and win-win results. After a preliminary investigation, there exist four enterprises that are fairly compatible to the construction investment company in terms of cooperation ideas, and the set of enterprise alternatives is denoted by $U = \{x_1, x_2, x_3, x_4\}$. For maximizing profits and minimizing risks, the company invited three financial experts to evaluate the financial situation of these four companies. The five main evaluated indicators are the main business profit rate, current percentage, accounts receivable cycling percentage, asset-debt percentage and return on net assets, and the financial indicator set is denoted by $V = \{y_1, y_2, y_3, y_4, y_5\}$. By referring to the above universes, three financial experts provided evaluation results that are recorded as q-ROFRs R_1^*, R_2^* and R_3^*. However, if the financial experts are hesitant to give the specific value due to lack of sufficient knowledge and experiences, the data can be given as a missing value, and the missing value is indicated by "*" in the following q-ROFRs. Considering the principle of complementarity in cooperative developments, the enterprises' standard set of assessments on the financial status is expressed as $B = \{\langle y_1, (0.8, 0.3)\rangle, \langle y_2, (0.5, 0.9)\rangle, \langle y_3, (0.2, 0.9)\rangle, \langle y_4, (0.4, 0.7)\rangle, \langle y_5, (0.7, 0.5)\rangle\}$. In light of the above statements, a typical MG q-ROF incomplete information system (U, V, R_k^*, B) with respect to financial quality matching can be conveniently constructed.

$$R_1^* = \begin{pmatrix} & y_1 & y_2 & y_3 & y_4 & y_5 \\ x_1 & (0.2, 0.6) & (0.4, 0.8) & (0.3, 0.9) & (0.6, 0.8) & (0.3, 0.7) \\ x_2 & (0.4, 0.8) & (0.5, 0.9) & (0.7, 0.4) & (0.3, 0.7) & (0.8, 0.4) \\ x_3 & (0.6, 0.2) & (*, *) & (0.3, 0.9) & (0.3, 0.7) & (0.8, 0.3) \\ x_4 & (0.7, 0.4) & (0.8, 0.3) & (0.2, 0.8) & (0.6, 0.9) & (0.5, *) \end{pmatrix};$$

$$R_2^* = \begin{pmatrix} & y_1 & y_2 & y_3 & y_4 & y_5 \\ x_1 & (0.3, 0.7) & (0.4, 0.9) & (0.2, 0.8) & (*, *) & (0.3, 0.8) \\ x_2 & (0.3, 0.8) & (0.3, 0.9) & (0.8, 0.4) & (0.6, 0.7) & (0.8, 0.5) \\ x_3 & (0.7, 0.1) & (0.3, 0.8) & (0.4, 0.8) & (0.2, 0.8) & (0.8, 0.4) \\ x_4 & (0.8, 0.4) & (0.9, 0.3) & (0.1, 0.8) & (*, *) & (0.6, 0.7) \end{pmatrix};$$

$$R_3^* = \begin{pmatrix} & y_1 & y_2 & y_3 & y_4 & y_5 \\ x_1 & (0.2, 0.9) & (0.6, 0.9) & (0.4, 0.8) & (0.7, 0.8) & (0.4, 0.8) \\ x_2 & (*, *) & (0.2, 0.9) & (0.8, 0.3) & (0.5, 0.9) & (0.9, 0.5) \\ x_3 & (0.7, 0.3) & (0.4, 0.8) & (0.4, 0.9) & (0.4, 0.8) & (0.9, 0.6) \\ x_4 & (0.8, 0.4) & (0.9, 0.1) & (0.4, 0.9) & (0.6, 0.9) & (0.5, 0.8) \end{pmatrix}.$$

Table 2
Single q-ROF membership degrees.

$\theta_B^{R_k}(x_i)$	R_1	R_2	R_3
x_1	(0.4800, 0.5861)	(0.5565, 0.6316)	(0.4891, 0.7070)
x_2	(0.6163, 0.6421)	(0.5668, 0.6684)	(0.6607, 0.6617)
x_3	(0.6907, 0.4120)	(0.7337, 0.3768)	(0.7220, 0.4730)
x_4	(0.6394, 0.6067)	(0.6596, 0.6282)	(0.6467, 0.6865)

In order to effectively address the above problem, we first determine the minimal value of q according to all values of incomplete q-ROF preference relations, thus we set $q = 3$. Then, we patch the lost information to further obtain (U, V, R_k, B), and the following complete q-ROFRs can be listed:

$$R_1 = \begin{pmatrix} & y_1 & y_2 & y_3 & y_4 & y_5 \\ x_1 & (0.2, 0.6) & (0.4, 0.8) & (0.3, 0.9) & (0.6, 0.8) & (0.3, 0.7) \\ x_2 & (0.4, 0.8) & (0.5, 0.9) & (0.7, 0.4) & (0.3, 0.7) & (0.8, 0.4) \\ x_3 & (0.6, 0.2) & (0.6159, 7505) & (0.3, 0.9) & (0.3, 0.7) & (0.8, 0.3) \\ x_4 & (0.7, 0.4) & (0.8, 0.3) & (0.2, 0.8) & (0.6, 0.9) & (0.5, 0.525) \end{pmatrix};$$

$$R_2 = \begin{pmatrix} & y_1 & y_2 & y_3 & y_4 & y_5 \\ x_1 & (0.3, 0.7) & (0.4, 0.9) & (0.2, 0.8) & (0.482, 0.7533) & (0.3, 0.8) \\ x_2 & (0.3, 0.8) & (0.3, 0.9) & (0.8, 0.4) & (0.6, 0.7) & (0.8, 0.5) \\ x_3 & (0.7, 0.1) & (0.3, 0.8) & (0.4, 0.8) & (0.2, 0.8) & (0.8, 0.4) \\ x_4 & (0.8, 0.4) & (0.9, 0.3) & (0.1, 0.8) & (0.482, 0.7533) & (0.6, 0.7) \end{pmatrix};$$

$$R_3 = \begin{pmatrix} & y_1 & y_2 & y_3 & y_4 & y_5 \\ x_1 & (0.2, 0.9) & (0.6, 0.9) & (0.4, 0.8) & (0.7, 0.8) & (0.4, 0.8) \\ x_2 & (0.6601, 0.649) & (0.2, 0.9) & (0.8, 0.3) & (0.5, 0.9) & (0.9, 0.5) \\ x_3 & (0.7, 0.3) & (0.4, 0.8) & (0.4, 0.9) & (0.4, 0.8) & (0.9, 0.6) \\ x_4 & (0.8, 0.4) & (0.9, 0.1) & (0.4, 0.9) & (0.6, 0.9) & (0.5, 0.8) \end{pmatrix}.$$

Afterwards, we calculate the weights of financial experts: $\omega_1 = \frac{Y_1}{\sum_{j=1}^{3} Y_j} = 0.3087$, $\omega_2 = 0.3189$, $\omega_3 = 0.3723$.

In a similar manner, the weights of financial indicators are $w_1 = \frac{\Re_1}{\sum_{k=1}^{5} \Re_k} = 0.2113$, $w_2 = 0.2424$, $w_3 = 0.1992$, $w_4 = 0.1531$, $w_5 = 0.1940$.

Then, the single q-ROF membership degree of x_i with regard to (U, V, R_k, B) can be obtained: $\theta_B^{R_1}(x_1) = \frac{\sum_{y_j \in V} \omega_j R_1(x_1, y_j) B(y_j)}{\sum_{y_j \in V} \omega_j R_1(x_1, y_j)} = (0.4800, 0.5861)$, and the other values are provided in the following Table 2.

According to the above results, multiple q-ROF membership degrees by means of q-ROFWA operators can be obtained: $\xi_B^{\sum_{k=1}^{3} R_k}(x_1) = q - ROFWA(\theta_B^{R_1}(x_i), \theta_B^{R_2}(x_i), \theta_B^{R_3}(x_i)) = (0.5106, 0.6519)$.

Similarly, the following results can be obtained: $\xi_B^{\sum_{k=1}^{3} R_k}(x_2) = (0.6206, 0.6581)$, $\xi_B^{\sum_{k=1}^{3} R_k}(x_3) = (0.7168, 0.4278)$, $\xi_B^{\sum_{k=1}^{3} R_k}(x_4) = (0.6487, 0.6462)$. Moreover, we further calculate the score function of $\xi_B^{\sum_{k=1}^{r} R_k}(x_i)$: $s(\xi_B^{\sum_{k=1}^{3} R_k}(x_1)) = -0.1439$, $s(\xi_B^{\sum_{k=1}^{3} R_k}(x_2)) = -0.0460$, $s(\xi_B^{\sum_{k=1}^{3} R_k}(x_3)) = 0.2900$, $s(\xi_B^{\sum_{k=1}^{3} R_k}(x_4)) = 0.0032$.

Therefore, we obtain the type-I ranking order: $\xi_3 \succ \xi_4 \succ \xi_2 \succ \xi_1$.

Afterwards, q-ROF weighted Euclidean distances can be obtained: $\zeta_B^{\sum_{k=1}^{3} R_k}(x_1) = 0.5138$, $\zeta_B^{\sum_{k=1}^{3} R_k}(x_2) = 0.5585$, $\zeta_B^{\sum_{k=1}^{3} R_k}(x_3) = 0.5297$, $\zeta_B^{\sum_{k=1}^{3} R_k}(x_4) = 0.5651$.

Therefore, we can obtain the type-II ranking order: $\zeta_1 \succ \zeta_3 \succ \zeta_2 \succ \zeta_4$.

In the next step, multiple q-ROF membership degrees by means of q-ROFWG operators can be obtained: $\psi_B^{\sum_{k=1}^{3} R_k}(x_1) = (0.5067, 0.6411)$, $\psi_B^{\sum_{k=1}^{3} R_k}(x_2) = (0.6158, 0.6576)$, $\psi_B^{\sum_{k=1}^{3} R_k}(x_3) = (0.7158, 0.4210)$, $\psi_B^{\sum_{k=1}^{3} R_k}(x_4) = (0.6485, 0.6412)$. Then the

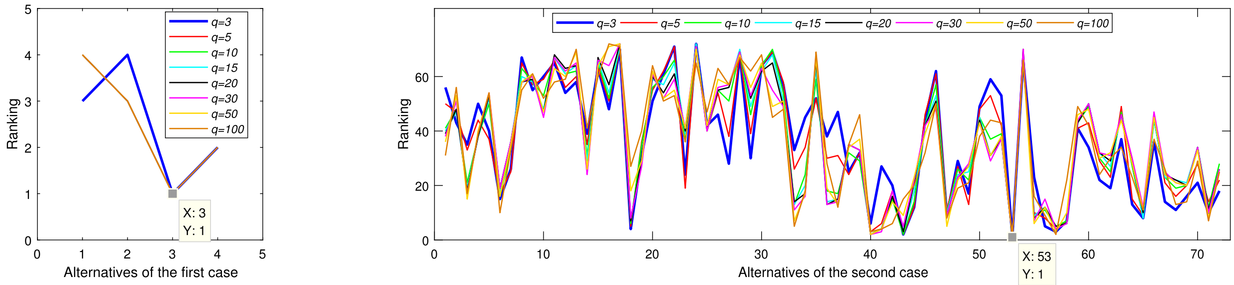

Fig. 1. The ranking of sensitivity analysis of two cases. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

score function of $\psi_B^{\sum_{k=1}^{r} R_k}(x_i)$ can be further obtained: $s(\psi_B^{\sum_{k=1}^{3} R_k}(x_1)) = -0.1333$, $s(\psi_B^{\sum_{k=1}^{3} R_k}(x_2)) = -0.0508$, $s(\psi_B^{\sum_{k=1}^{3} R_k}(x_3)) = 0.2921$, $s(\psi_B^{\sum_{k=1}^{3} R_k}(x_4)) = 0.0091$.

Therefore, we can obtain the type-III ranking order: $\psi_3 \succ \psi_4 \succ \psi_2 \succ \psi_1$.

In accordance with the above results, we can obtain the final index PSI_i by the TPOP method: $PSI_1 = 9.2437$, $PSI_2 = 9.4846$, $PSI_3 = 4.6559$, $PSI_4 = 9.1267$. Thus, the final ranking is $x_3 \succ x_4 \succ x_1 \succ x_2$ and the optimal alternative is x_3.

In the following part, we apply the q-ROF MAGDM approach in incomplete q-ROF information systems to obtain comprehensive results of air quality in a region, and the selected data set comes from the UCI database (http://archive.ics.uci.edu/ml/datasets/Air+Quality). The data records the air pollution indicators in Italy over a period of one year. Among the air pollutants, we select the common concentrations of CO, C_6H_6, NO_2 and O_3 to determine the air quality. Considering that air quality may be affected by different times within one day, this paper selects data at four time nodes, namely zero, six, twelve and eighteen. Moreover, we select the 5th, 10th, 15th, 20th, 25th, and 30th days of each month from April 2004 to March 2005 as alternatives (February uses the 28th day as the last day). Hence in light of the above dataset, let $U = \{x_1, x_2, \cdots, x_{72}\}$ be a set of 72 selected days and $V = \{y_1, y_2, y_3, y_4\}$ be a set of attributes, where y_1 denotes CO, y_2 denotes C_6H_6, y_3 denotes NO_2 and y_4 denotes O_3, and let R_1^*, R_3^*, R_3^* and R_4^* be the assessment data over $U \times V$, it is necessary to convert original information systems into MG q-ROF incomplete information systems. First, we normalize the raw data to the fuzzy type based on the normalization formula as follows:

$$x_{ij} = \begin{cases} \frac{t_{ij} - \min\limits_i t_{ij}}{\max\limits_i t_{ij} - \min\limits_i t_{ij}}, \text{if the attribute } t_j \text{ is benefit} \\ \frac{t_{ij} - \min\limits_i t_{ij}}{\max\limits_i t_{ij} - \min\limits_i t_{ij}}, \text{if the attribute } t_j \text{ is cost} \end{cases}.$$

Then, we regard x_{ij} as the MD and $\sqrt[3]{1 - x_{ij}}$ as the ND, thus we can obtain q-ROFNs from the raw data. In addition, the standard evaluation set B can be obtained by the following formula:

$$B = \{< y_1, [\frac{\sum_{i=1}^{m}(x_{i1})}{m}, \frac{\sum_{i=1}^{m}\sqrt[3]{1 - x_{i1}^3}}{m}] >, \cdots, < y_n, [\frac{\sum_{i=1}^{m}(x_{in})}{m}, \frac{\sum_{i=1}^{m}\sqrt[3]{1 - x_{in}^3}}{m}] >\}.$$

In light of the above statements, an MG q-ROF incomplete information system on the assessment of Italian city air qualities can be established. Finally, we can obtain the detailed sorting result by using the presented MAGDM approach: $x_{53} \succ x_{43} \succ x_{57} \succ x_{18} \succ x_{56} \succ x_{40} \succ x_{58} \succ x_{65} \succ x_{47} \succ x_{71} \succ x_{68} \succ x_{44} \succ x_{64} \succ x_{67} \succ x_6 \succ x_{69} \succ x_{49} \succ x_{72} \succ x_{62} \succ x_{42} \succ x_{70} \succ x_{61} \succ x_{55} \succ x_{23} \succ x_{38} \succ x_7 \succ x_{41} \succ x_{27} \succ x_{48} \succ x_{29} \succ x_{19} \succ x_{39} \succ x_{33} \succ x_{60} \succ x_3 \succ x_{66} \succ x_{63} \succ x_{36} \succ x_{14} \succ x_5 \succ x_{59} \succ x_{25} \succ x_2 \succ x_{45} \succ x_{34} \succ x_{26} \succ x_{37} \succ x_{16} \succ x_{50} \succ x_4 \succ x_{20} \succ x_{35} \succ x_{52} \succ x_{12} \succ x_9 \succ x_1 \succ x_{32} \succ x_{13} \succ x_{51} \succ x_{10} \succ x_{21} \succ x_{46} \succ x_{15} \succ x_{30} \succ x_{11} \succ x_{54} \succ x_8 \succ x_{28} \succ x_{31} \succ x_{17} \succ x_{22} \succ x_{24}$. Thus, the optimal day with the best air quality is the 44th day, i.e., the pollution index reaches the smallest level.

5.2. Sensitivity analysis

Sensitivity analysis can be used as a common experimental method to analyze the stability of some presented decision making methods, and its essence is to explain the law of how key indicators can affect the changes by changing the values of related variables one by one. This section observes the changes in decision-making results by changing the value of q in the above case studies. The sensitivity analysis results of two case studies are shown in Fig. 1, Table 3 and Table 4.

Table 3
The ranking results of sensitivity analysis of the first case.

q	Ranking results	q	Ranking results
$q=3$	$x_3 \succ x_4 \succ x_1 \succ x_2$	$q=20$	$x_3 \succ x_4 \succ x_2 \succ x_1$
$q=5$	$x_3 \succ x_4 \succ x_2 \succ x_1$	$q=30$	$x_3 \succ x_4 \succ x_2 \succ x_1$
$q=10$	$x_3 \succ x_4 \succ x_2 \succ x_1$	$q=50$	$x_3 \succ x_4 \succ x_2 \succ x_1$
$q=15$	$x_3 \succ x_4 \succ x_2 \succ x_1$	$q=100$	$x_3 \succ x_4 \succ x_2 \succ x_1$

Table 4
The ranking results of sensitivity analysis of the second case.

q	Ranking results
$q=3$	$x_{53} \succ x_{43} \succ x_{57} \succ x_{18} \succ x_{56} \succ x_{40} \succ x_{58} \succ x_{65} \succ x_{47} \succ x_{71} \succ x_{68} \succ x_{44} \succ x_{64} \succ x_{67} \succ x_6 \succ x_{69} \succ x_{49} \succ x_{72} \succ x_{62} \succ x_{42} \succ x_{70} \succ x_{61} \succ x_{55} \succ x_{23} \succ x_{38} \succ x_7 \succ x_{41} \succ x_{27} \succ x_{48} \succ x_{29} \succ x_{19} \succ x_{39} \succ x_{33} \succ x_{60} \succ x_3 \succ x_{66} \succ x_{63} \succ x_{36} \succ x_{14} \succ x_5 \succ x_{59} \succ x_{25} \succ x_2 \succ x_{45} \succ x_{34} \succ x_{26} \succ x_{37} \succ x_{16} \succ x_{50} \succ x_4 \succ x_{20} \succ x_{35} \succ x_{52} \succ x_{12} \succ x_9 \succ x_1 \succ x_{32} \succ x_{13} \succ x_{51} \succ x_{10} \succ x_{21} \succ x_{46} \succ x_{15} \succ x_{30} \succ x_{11} \succ x_{54} \succ x_8 \succ x_{28} \succ x_{31} \succ x_{17} \succ x_{22} \succ x_{24}$
$q=5$	$x_{53} \succ x_{43} \succ x_{40} \succ x_{57} \succ x_{18} \succ x_{41} \succ x_{58} \succ x_{56} \succ x_{65} \succ x_{55} \succ x_{47} \succ x_{44} \succ x_{49} \succ x_{71} \succ x_{64} \succ x_{68} \succ x_6 \succ x_{42} \succ x_{23} \succ x_{69} \succ x_{67} \succ x_{72} \succ x_{62} \succ x_{38} \succ x_7 \succ x_{33} \succ x_{48} \succ x_{70} \succ x_{61} \succ x_{36} \succ x_{37} \succ x_{39} \succ x_3 \succ x_{34} \succ x_{14} \succ x_{19} \succ x_5 \succ x_{27} \succ x_{29} \succ x_{25} \succ x_{59} \succ x_{52} \succ x_{60} \succ x_4 \succ x_{66} \succ x_{45} \succ x_2 \succ x_{50} \succ x_{63} \succ x_1 \succ x_{16} \succ x_{35} \succ x_{51} \succ x_{26} \succ x_{20} \succ x_{12} \succ x_9 \succ x_{32} \succ x_{10} \succ x_{13} \succ x_{46} \succ x_{21} \succ x_{15} \succ x_{30} \succ x_8 \succ x_{11} \succ x_{54} \succ x_{28} \succ x_{31} \succ x_{17} \succ x_{22} \succ x_{24}$
$q=10$	$x_{53} \succ x_{43} \succ x_{40} \succ x_{41} \succ x_{57} \succ x_{18} \succ x_{58} \succ x_{65} \succ x_{55} \succ x_{47} \succ x_{56} \succ x_{71} \succ x_{33} \succ x_{44} \succ x_{42} \succ x_6 \succ x_{37} \succ x_{36} \succ x_{68} \succ x_{69} \succ x_3 \succ x_{49} \succ x_{67} \succ x_{34} \succ x_{62} \succ x_{48} \succ x_{64} \succ x_{72} \succ x_{39} \succ x_{61} \succ x_{14} \succ x_{38} \succ x_7 \succ x_{70} \succ x_{19} \succ x_{23} \succ x_{51} \succ x_4 \succ x_{52} \succ x_{25} \succ x_1 \succ x_{45} \succ x_{59} \succ x_{66} \succ x_{50} \succ x_{29} \succ x_{63} \succ x_2 \succ x_{60} \succ x_5 \succ x_{27} \succ x_{16} \succ x_{10} \succ x_{32} \succ x_{26} \succ x_{20} \succ x_{46} \succ x_9 \succ x_{35} \succ x_{21} \succ x_{12} \succ x_{13} \succ x_8 \succ x_{30} \succ x_{15} \succ x_{22} \succ x_{11} \succ x_{54} \succ x_{28} \succ x_{31} \succ x_{17} \succ x_{24}$
$q=15$	$x_{53} \succ x_{40} \succ x_{43} \succ x_{41} \succ x_{57} \succ x_{58} \succ x_{18} \succ x_{65} \succ x_{55} \succ x_{47} \succ x_{71} \succ x_{56} \succ x_{33} \succ x_{36} \succ x_{37} \succ x_{42} \succ x_6 \succ x_{44} \succ x_3 \succ x_{34} \succ x_{69} \succ x_{68} \succ x_{67} \succ x_{48} \succ x_{49} \succ x_{72} \succ x_{62} \succ x_{64} \succ x_{14} \succ x_{19} \succ x_{51} \succ x_{61} \succ x_{39} \succ x_{70} \succ x_{38} \succ x_7 \succ x_{52} \succ x_4 \succ x_{23} \succ x_1 \succ x_{25} \succ x_{45} \succ x_{59} \succ x_{66} \succ x_{50} \succ x_{63} \succ x_2 \succ x_{10} \succ x_{29} \succ x_{60} \succ x_{32} \succ x_5 \succ x_{46} \succ x_{16} \succ x_{26} \succ x_{27} \succ x_{21} \succ x_9 \succ x_{20} \succ x_8 \succ x_{12} \succ x_{30} \succ x_{35} \succ x_{13} \succ x_{22} \succ x_{15} \succ x_{11} \succ x_{31} \succ x_{54} \succ x_{28} \succ x_{17} \succ x_{24}$
$q=20$	$x_{53} \succ x_{40} \succ x_{43} \succ x_{41} \succ x_{57} \succ x_{58} \succ x_{18} \succ x_{55} \succ x_{47} \succ x_{65} \succ x_{71} \succ x_{56} \succ x_{36} \succ x_{33} \succ x_{37} \succ x_{42} \succ x_{34} \succ x_3 \succ x_6 \succ x_{69} \succ x_{44} \succ x_{68} \succ x_{48} \succ x_{67} \succ x_{14} \succ x_{72} \succ x_{49} \succ x_{64} \succ x_{62} \succ x_{19} \succ x_{51} \succ x_{61} \succ x_{39} \succ x_{70} \succ x_{38} \succ x_7 \succ x_{52} \succ x_4 \succ x_1 \succ x_{23} \succ x_{25} \succ x_{45} \succ x_{59} \succ x_{50} \succ x_{66} \succ x_{63} \succ x_{10} \succ x_2 \succ x_{32} \succ x_{60} \succ x_{46} \succ x_{29} \succ x_5 \succ x_{21} \succ x_{26} \succ x_{27} \succ x_{16} \succ x_8 \succ x_9 \succ x_{20} \succ x_{22} \succ x_{30} \succ x_{12} \succ x_{13} \succ x_{31} \succ x_{35} \succ x_{15} \succ x_{11} \succ x_{28} \succ x_{54} \succ x_{24} \succ x_{17}$
$q=30$	$x_{53} \succ x_{40} \succ x_{41} \succ x_{57} \succ x_{43} \succ x_{58} \succ x_{55} \succ x_{18} \succ x_{47} \succ x_{71} \succ x_{33} \succ x_{65} \succ x_{36} \succ x_{37} \succ x_{56} \succ x_{34} \succ x_3 \succ x_{42} \succ x_6 \succ x_{69} \succ x_{68} \succ x_{44} \succ x_{48} \succ x_{14} \succ x_{67} \succ x_{72} \succ x_{19} \succ x_{49} \succ x_{51} \succ x_{64} \succ x_{62} \succ x_{61} \succ x_{39} \succ x_{70} \succ x_{38} \succ x_7 \succ x_{52} \succ x_1 \succ x_4 \succ x_{25} \succ x_{23} \succ x_{45} \succ x_{50} \succ x_{59} \succ x_{10} \succ x_{63} \succ x_{66} \succ x_{46} \succ x_{32} \succ x_{60} \succ x_2 \succ x_{21} \succ x_{29} \succ x_5 \succ x_{31} \succ x_{26} \succ x_{27} \succ x_8 \succ x_{22} \succ x_9 \succ x_{20} \succ x_{12} \succ x_{30} \succ x_{16} \succ x_{13} \succ x_{15} \succ x_{11} \succ x_{35} \succ x_{28} \succ x_{54} \succ x_{24} \succ x_{17}$
$q=50$	$x_{53} \succ x_{40} \succ x_{57} \succ x_{41} \succ x_{47} \succ x_{55} \succ x_{33} \succ x_{71} \succ x_{43} \succ x_{58} \succ x_{65} \succ x_{56} \succ x_{37} \succ x_{42} \succ x_3 \succ x_6 \succ x_{34} \succ x_{18} \succ x_{36} \succ x_{69} \succ x_{68} \succ x_{48} \succ x_{44} \succ x_{67} \succ x_{72} \succ x_{14} \succ x_{49} \succ x_{64} \succ x_{19} \succ x_{61} \succ x_{51} \succ x_{62} \succ x_{70} \succ x_{38} \succ x_7 \succ x_1 \succ x_{39} \succ x_{52} \succ x_{45} \succ x_4 \succ x_{23} \succ x_{25} \succ x_{50} \succ x_{63} \succ x_{66} \succ x_{59} \succ x_{10} \succ x_{60} \succ x_{31} \succ x_{46} \succ x_{32} \succ x_{21} \succ x_2 \succ x_5 \succ x_{22} \succ x_{29} \succ x_{27} \succ x_8 \succ x_{26} \succ x_{12} \succ x_9 \succ x_{20} \succ x_{11} \succ x_{15} \succ x_{30} \succ x_{54} \succ x_{35} \succ x_{28} \succ x_{13} \succ x_{24} \succ x_{16} \succ x_{17}$
$q=100$	$x_{53} \succ x_{57} \succ x_{40} \succ x_{41} \succ x_{33} \succ x_{42} \succ x_{71} \succ x_{47} \succ x_{56} \succ x_6 \succ x_{65} \succ x_{37} \succ x_{68} \succ x_{69} \succ x_{43} \succ x_{55} \succ x_3 \succ x_{34} \succ x_{48} \succ x_{44} \succ x_{49} \succ x_{58} \succ x_{64} \succ x_{61} \succ x_{72} \succ x_{67} \succ x_{18} \succ x_{36} \succ x_{70} \succ x_{62} \succ x_1 \succ x_{45} \succ x_7 \succ x_{63} \succ x_{38} \succ x_{23} \succ x_{66} \succ x_{50} \succ x_4 \succ x_{19} \succ x_{14} \succ x_{60} \succ x_{52} \succ x_{51} \succ x_{31} \succ x_{39} \succ x_{25} \succ x_{32} \succ x_{59} \succ x_{46} \succ x_{21} \succ x_{10} \succ x_{22} \succ x_5 \succ x_8 \succ x_2 \succ x_{27} \succ x_{11} \succ x_{12} \succ x_{15} \succ x_9 \succ x_{29} \succ x_{26} \succ x_{20} \succ x_{24} \succ x_{54} \succ x_{28} \succ x_{30} \succ x_{35} \succ x_{13} \succ x_{17} \succ x_{16}$

From the results of two cases, we can see that when q changes, the trend of two curves remains a similar shape, and the optimum decision result remains identical in spite of changes of q, indicating that the presented q-ROF MAGDM approach in incomplete q-ROF information systems owns a strong stability when dealing with the above two cases.

In the first case, it is obvious that when q increases, the ranking results only change in a small range between x_2 and x_1. The optimal alternative is always x_3. In the second case, it can be clearly seen that with the increase of q, although the ranking results are not completely consistent, the trend is stable. The optimal alternative is always x_{53}.

5.3. Comparative analysis

For the sake of effectively verifying the effectiveness of the developed method in the current paper, this section separately compares each component below.

5.3.1. Comparative analysis with classic q-ROF MAGDM methods

In what follows, since q-ROFWA operators, q-ROF Euclidean distances and q-ROFWG operators are utilized for constructing multiple q-ROF membership degrees, we separately use these methods to process the data presented in the above two case studies. Moreover, we further select two classic q-ROF MAGDM methods to make corresponding comparative analysis, i.e., the q-ROF TOPSIS method and the q-ROF VIKOR method. The q-ROF TOPSIS method aims to provide an order preference via the q-ROF similarities to ideal solutions, and the ranking of all alternatives is recorded by positive and negative ideal terms of settlements. Since the q-ROF TOPSIS method fails to consider the q-ROF distance between each alternative and the negative ideal settlement way, the q-ROF VIKOR method simultaneously considers the q-ROF distances between each alternative and the positive and the negative ideal solutions, which can be seen as an effective q-ROF decision making method. In what follows, we perform a comparative analysis with the above-stated five classic q-ROF MAGDM methods and the comparative results are shown in Fig. 2.

From Fig. 2, we can see that sorting results obtained by different methods are not completely consistent. However, the overall trend remains consistent, and the optimal alternative determined by sorting results is identical, which can show the effectiveness of the presented method in this paper.

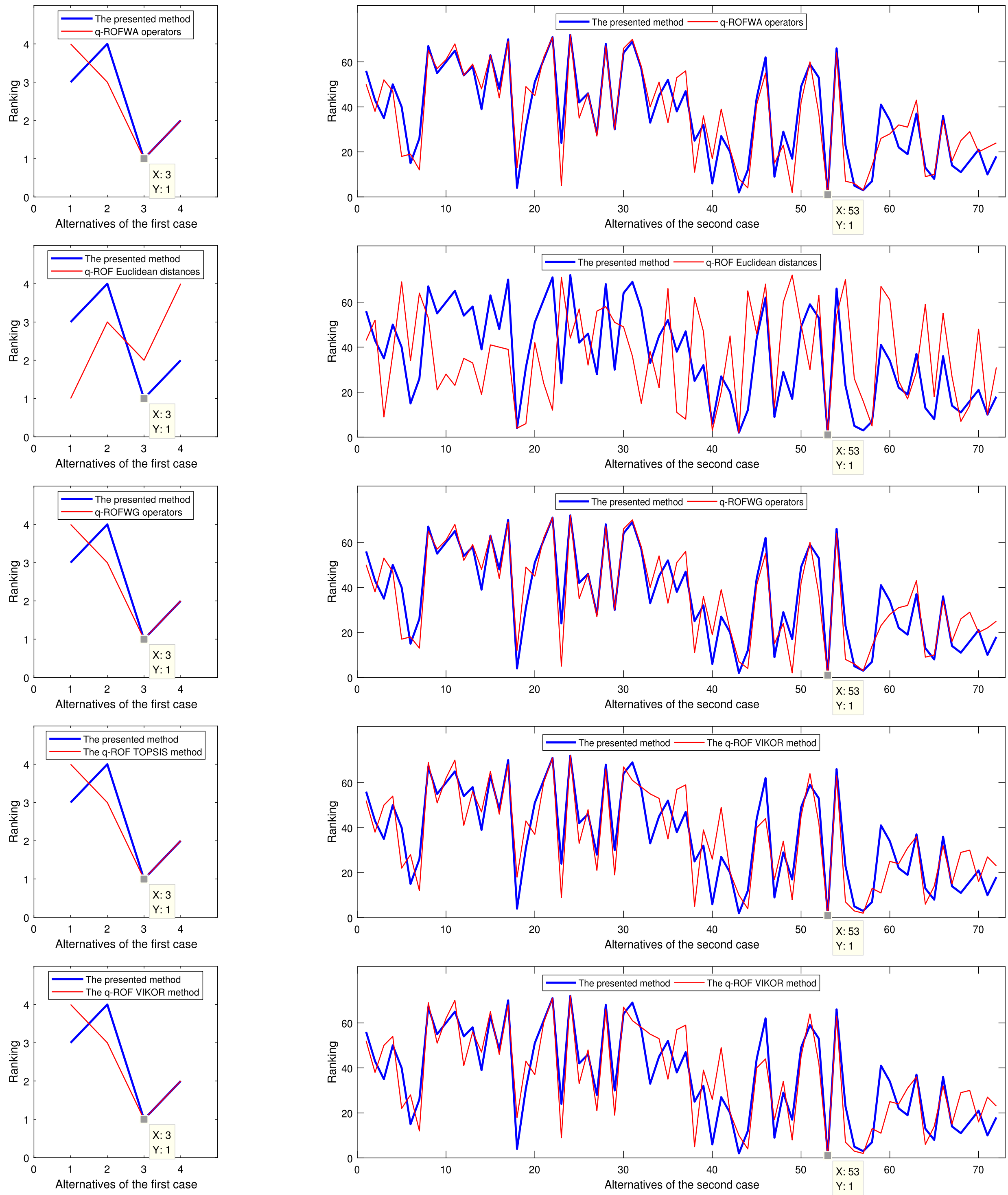

Fig. 2. Comparison of the ranking results obtained by classic q-ROF MAGDM methods.

5.3.2. Comparative analysis with the q-ROF MULTIMOORA method

In what follows, since the MULTIMOORA method is utilized for seeking stable decision results by integrating various MG q-ROF PRSs, comparing the presented method with the MULTIMOORA method is necessary. The TPOP method excels in determining precise sorting results for alternatives, it can integrate the calculation results of diverse methods, and the

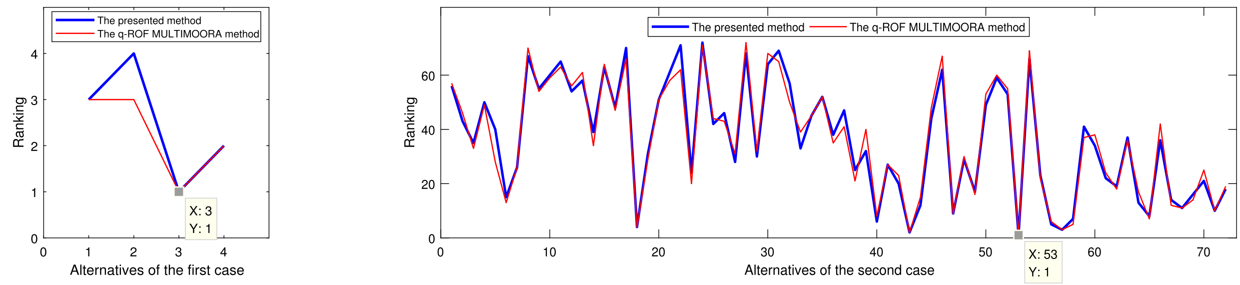

Fig. 3. Comparison of the ranking results obtained by the q-ROF MULTIMOORA method.

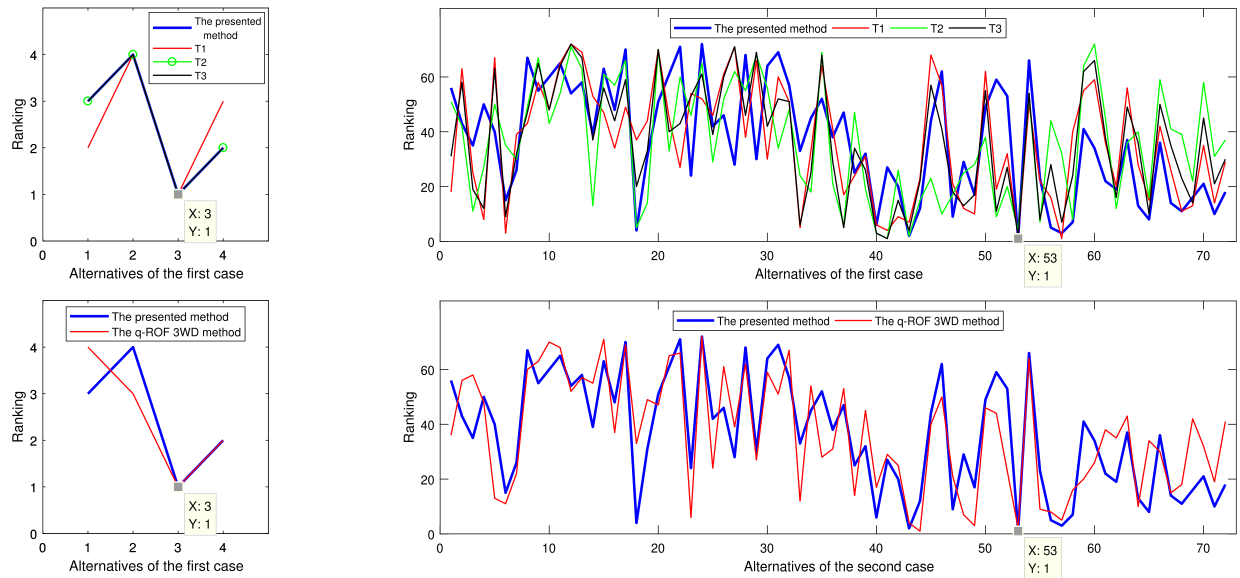

Fig. 4. Comparison of the ranking results obtained by q-ROF multigranulation models.

current paper utilizes the TPOP method to provide final sorting results in virtue of q-ROF PRSs and MULTIMOORA. The comparative results are shown in Fig. 3.

From Fig. 3, we can see that both the presented method and the MULTIMOORA method remain an identical changing trend, and the optimal alternative determined by sorting results is identical as well. Moreover, it is noted that the presented method simultaneously integrates q-ROF PRSs, MULTIMOORA with TPOP, which gives more accurate decision results than the q-ROF MULTIMOORA method via considering diverse models in terms of information fusion and analysis.

5.3.3. Comparative analysis with q-ROF multigranulation models

MGRSs act as a typical representative in multigranulation models. By using the decision making rule presented in Zhang et al. [16], we apply the corresponding models in q-ROF information systems to conduct the comparative analysis below. In specific, the index T_1 indicates the optimistic information fusion result, T_2 indicates the pessimistic information fusion result, and T_3 indicates the neutral information fusion result. Moreover, it is noted that Zhang et al. [44] proposed another q-ROF multigranulation models from the perspective of three-way decisions (3WD) in complete q-ROF information systems, which consider the risks of decision makers by taking advantages of the Bayesian theory. In light of the above-stated methods, we perform a comparative analysis with q-ROF multigranulation models for showing the validity of the presented method, and the comparative results are shown in Fig. 4.

From Fig. 4, both two methods indicate the optimal decision results are identical and the changing trends are similar. Our method uses q-ROF PRSs to fuse multi-source q-ROF information, which can take advantages of MGRSs and PRSs with the support of MULTIMOORA. Thus, the above comparative analysis also reflects the effectiveness of the presented method.

5.4. Discussions

In order to better compare the differences between various q-ROF MAGDM methods, we select the q-ROF MULTIMOORA method as the ground truth and compare the results obtained by the methods mentioned in the above comparative analysis.

Table 5
Comparison between the sorting of objects and the ground truth value.

Different methods	The first case study	The second case study
The presented method	1	184
q-ROFWA operators	1	548
q-ROF Euclidean distances	5	1396
q-ROFWG operators	1	554
The q-ROF TOPSIS method	1	500
The q-ROF VIKOR method	1	600
The q-ROF T_1	3	1130
The q-ROF T_2	1	1234
The q-ROF T_3	1	1098
The q-ROF 3WD method	1	810

Table 6
Comparison of different methods.

Different methods	High-order fuzzy group decisions	Stable results	Ranking schemes	Weight vectors	Incomplete information
q-ROFWA operators	√	×	√	√	×
q-ROF Euclidean distances	√	×	√	√	×
q-ROFWG operators	√	×	√	√	×
The q-ROF TOPSIS method	×	×	√	√	×
The q-ROF VIKOR method	×	×	√	√	×
The q-ROF 3WD method	√	√	√	√	×
Zhang et al. [15]	√	√	√	√	×
Dorfeshan et al. [56]	√	√	√	√	×
The presented method	√	√	√	√	√

First, we use the function $Ind\,(x_i)$ to denote the ranking position number, i.e., the optimal alternative in terms of $Ind\,(x_i)$ is 1 and the worst alternative in terms of $Ind\,(x_i)$ is m. Moreover, suppose the ranking position number in terms of the ground truth is denoted by $Ind\,(X_i)$. Then, the difference between the ranking results of each comparative method and the ground truth can be expressed as $\sum_{i=1}^{m} |Ind\,(x_i) - Ind\,(X_i)|$. In what follows, we list the overall comparison between the sorting of objects and the ground truth value, and the specific results are shown in Table 5.

In light of the above comparative analysis between the sorting of objects and the ground truth value, we can find that the presented method owns the largest consistence degree of all alternatives when comparing with other counterparts, thus the stable performance of the presented q-ROF MAGDM method can be verified. In what follows, we further summarize the superiorities of the presented q-ROF MAGDM method that are listed in the following Table 6.

From the above table, we can see that the presented MAGDM method in incomplete q-ROF information systems can be seen as a comprehensive decision making approach when dealing with high-order fuzzy MAGDM problems in incomplete information systems and providing stable ranking results by including objective weight vectors. Thus, the presented decision making method takes advantages of multigranulation probabilistic models and MULTIMOORA in incomplete q-ROF information systems and excels in providing stable decision making results. Compared with similar decision making techniques, the presented decision making method can not only effectively depict incomplete MAGDM situations via MG incomplete q-ROF information systems, but also reasonably integrate multi-source information and analyze decision results by taking advantages of MG q-ROF PRSs, MULTIMOORA and TPOP.

6. Conclusions

In this paper, we primarily concentrate on exploring a robust MAGDM method with imprecise and incomplete information, that is, q-ROF MAGDM in incomplete information systems with the support of MULTIMOORA. In specific, we have first constructed the notion of MG incomplete q-ROF information systems. Then, we have developed a completion technique to transform MG incomplete q-ROF information systems into corresponding complete counterparts, and we have further proposed three versions of MG q-ROF PRSs according to MG q-ROF information systems. Afterwards, the presented theoretical models are incorporated to the framework of MULTIMOORA. Moreover, we have further explored the TPOP method for obtaining the MULTIMOORA ranking, and a new MAGDM method is eventually constructed by means of multigranulation probabilistic models, MULTIMOORA and TPOP. Finally, we have proved the applicability of the constructed MAGDM method from two perspectives, the one is a case study in the background of financial quality matching, another one is an experiment analysis with a UCI dataset, and both two experiments have verified the significance of the developed theoretical models and the MAGDM method.

For the further research, the current work also owns some room for further improvement, thus plenty of meaningful study issues are worthy of exploration in depth. First, this paper concentrates on MAGDM problems via the proposed multigranulation probabilistic models, it is also meaningful to explore other theoretical aspects of MG q-ROF PRSs, such as uncertainty measures [57], formal concept analysis [58], attribute reductions [59], etc. Second, the primary information depiction context of the paper is imprecise and incomplete information, exploring valid MAGDM methods under other significant information depiction contexts is necessary, such as the interaction between MD and ND [60], shadowed data [61], partially labeled data [62], multi-scale data [63], etc. Third, we plan to update the proposed theoretical models with the support of 3WD [64–67] and present more realistic applications via robust three-way MAGDM methods.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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