International Journal of Approximate Reasoning 132 (2021) 88–106

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

# Conflict analysis based on three-way decision for triangular fuzzy information systems

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ARTICLE INFO

*Article history:*
Received 6 July 2020
Received in revised form 5 October 2020
Accepted 2 December 2020
Available online 19 December 2020

*Keywords:*
Three-way decision
Conflict analysis
Triangular fuzzy information systems

ABSTRACT

Triangular fuzzy numbers (TFNs) can not only provide the range of fuzzy points, but contain the three most representative fuzzy points, which play an essential role in describing fuzzy information. Combining with conflict situations, the agents' caution in the decision-making process can be well depicted by TFNs. However, little effort has been paid to conflict analysis on triangular fuzzy information systems (TFISs). In this paper, we study three-way conflict analysis based on TFISs. First, we propose the concept of TFISs and then establish a TFIS for conflict analysis. Further, relative area $\Delta S$ is defined to describe concrete attitudes of agents about issues. Second, since the importance of the same issue may vary for agents, we attach fuzzy weights to issues according to triangular fuzzy symmetric judgment matrices (TFSJMs) of agents. Third, we determine the total attitude of each agent to all issues. Then we illustrate how to compute thresholds $\alpha$ and $\beta$ using the theory of triangular fuzzy decision-theoretic rough sets. Finally, a tri-partition of agents is obtained based on the thresholds.



## 1. Introduction

Conflicts exist in many aspects of our real life, thus the study of which is very necessary. Especially, conflict analysis provides some guidance for conflict resolution, which has recently attracted increasing attention. Many scholars succeeded in seeking effective methods to promote conflict resolution [10–12,27–29,31,32,40,44]. For example, Deja [2] explained the nature of conflict when we formally defined the conflict situation model. Lang [10] studied a general conflict analysis model based on three-way decision. Pawlak [26] proposed the auxiliary functions and distance functions and offered deeper insight into the structure of conflicts. Sun et al. [31] studied three-way decision making approach to conflict analysis and resolution using probabilistic rough set over two universes. Yang et al. [35] presented a novel information fusion method based on Dempster-Shafer evidence theory for conflict resolution. Yao [40] investigated three-way conflict analysis: reformulations and extensions of the Pawlak model. Zhi et al. [44] focused on conflict analysis under one-vote veto based on approximate three-way concept lattice.

Three-way decision theory, proposed by Yao [36], promotes thinking, problem solving and information processing in threes, such as three regions, three components, three views and so on. Actually, in many decision-making processes, we tend to make a decision based on existing information and evidence. When the evidence is insufficient or weak, it seems

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https://doi.org/10.1016/j.ijar.2020.12.004

to be unreasonable to make either a positive or a negative decision. At this time, we may be apt to choose an alternative decision that is neither yes nor no. In other words, we move from yes/no to yes/maybe/no with three-way decision, which provides the necessary flexibility and reduces decision risks in making decisions. At present, there are considerable researches on three-way decision theory [4,7,8,14,20–24,30,37–39,43]. For instance, Hu and Yao [4] focused on structured approximations as a basis for three-way decisions in rough set theory. Jiao et al. [6] studied three-way decision based on decision-theoretic rough sets with single-valued neutrosophic information. Li et al. [15] introduced the model of three-way decision based on subset-evaluation, and defined three-way fuzzy matroids via the subset-evaluation model [16]. It was interesting that three-way fuzzy matroids presented specific mathematical models of three elements of granular structures: granules, levels, and hierarchies [17]. Liang et al. [20] considered risk appetite dual hesitant fuzzy three-way decisions with TODIM. Yao [39] presented tri-level thinking: models of three-way decision. Zhang et al. [43] investigated three-way class-specific attribute reducts from the information viewpoint. And it is worth mentioning that three-way decision theory is closely related to conflict analysis. On the one hand, in conflict problems, an agent's attitude about an issue can only be positive, central or negative, which happens to coincide with the thinking of three-way decision theory. On the other hand, the relationship among agents is nothing more than positive, central or negative alliance, which is also a reflection of the idea of three-way decision theory. Therefore, it is natural and significant to combine three-way decision theory with conflict analysis.

Fuzzy set theory, founded by Zadeh [41], has influenced quite a bit of researchers all over the world since 1965. Fuzzy sets are characterized by membership functions, which assign a membership, ranging from zero to one, to each object. Fuzzy numbers are fuzzy subsets of the real line $R$ with the membership functions possessing many superior properties [3,5]. And a fuzzy number becomes a TFN when the membership function is presented as a triangle in the figure. TFNs, denoted by $(l, m, u)$, are capable of showing fuzzy points from $l$ to $u$, where $l, m$ and $u$ correspond to the three most representative fuzzy points, respectively. In terms of conflict situations, it naturally occurs to us that TFNs can be used to represent the attitudes of agents about issues so as to depict the agents' caution in the decision-making processes. Therefore, we find it reasonable to study conflict analysis, three-way decision theory and TFNs together.

In conflict situations, it seems that we are interested in the relationships among agents. In other words, it is particularly important to determine whether agents are in positive alliance, central alliance, or negative alliance. In this study, we investigate how to obtain positive, central and negative alliances of agents based on TFISs. The motivations and contributions of this study are as follows.

(1) Why we study conflict analysis based on TFISs? Pawlak's conflict analysis model only used $-1$, 0 and $+1$ to represent negative, central and positive attitudes of agents about issues. However, the degrees of the three attitudes were not depicted. In [13], Lang et al. studied conflict analysis based on Pythagorean fuzzy set theory, which described the degrees of these three attitudes from two aspects. For example, in Table 2 of [13], we have $c_2(x_1) = P(0.9, 0.3)$, where 0.9 and 0.3 denote the positive and negative degrees of the agent $x_1$ about the issue $c_2$. In fact, the positive (negative) degree of an agent's attitude towards an issue is not enough to be represented by a real number, sometimes a range is needed. TFNs can not only provide a range of fuzzy points, but show the three most representative fuzzy points. Thus, to better describe fuzzy information in conflict situations, we need to study conflict analysis based on TFISs.

(2) Why we investigate conflict analysis with three-way decision theory? There are two reasons as follows. One reason is that three-way decision theory promotes problem solving using three views. And for an agent of a specific conflict problem, it is nothing more than a negative, a central or a positive attitude to an issue, which exactly fits with three-way decision theory. Another is that we can divide all agents into negative, central and positive alliances according to the idea of three-way decision theory, which is also consistent with the thinking of conflict analysis.

(3) What are contributions of this paper? At present, little research has been done on conflict analysis based on TFISs, in which agents' attitudes are TFNs. The contributions of this study are as follows: 1) propose the concept of TFISs and then establish a TFIS for conflict analysis, which improves Lang et al.'s conflict analysis model; 2) provide the concepts of positive, central and negative alliances with thresholds $\alpha$ and $\beta$; 3) present an approach to thresholds by using the theory of triangular fuzzy decision-theoretic rough sets; 4) employ many examples to illustrate how to conduct three-way conflict analysis based on TFISs.

The remainder of this paper is organized as follows. In section 2, we briefly compile some related concepts of fuzzy sets, and review two existing models of conflict analysis. To improve Lang et al.'s model, we propose the concept of TFISs and further establish a TFIS for conflict analysis in section 3. In section 4, we research a model of three-way conflict analysis on TFISs. Finally, the conclusion and future work are given in section 5.

## 2. Preliminaries

In this section, we review some related concepts of fuzzy sets and two existing models of conflict analysis.

### 2.1. Fuzzy sets and triangular fuzzy numbers

**Definition 2.1.** [45] A *fuzzy set* $A$ on a nonempty set $X$ is a mapping of the form $f_A : X \to [0, 1]$, for $x \in X$. Moreover, $f_A$ is called the membership function and $f_A(x)$ represents the membership of $x$ with respect to $A$.

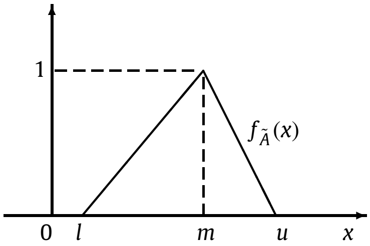


**Fig. 1.** A positive TFN.

With the concept of fuzzy sets, now we review the definition of fuzzy numbers.

**Definition 2.2.** A *fuzzy number* $\tilde{A}$ is a fuzzy subset of real line $R$, whose membership function $f_{\tilde{A}}$ is defined as

$$f_{\tilde{A}}(x) = \begin{cases} f^L_{\tilde{A}}(x), & a \le x < b; \\ 1, & b \le x \le c; \\ f^R_{\tilde{A}}(x), & c < x \le d; \\ 0, & \text{otherwise}, \end{cases}$$

where $a, b, c, d \in R$, $f^L_{\tilde{A}}$ is continuous and increasing on $[a, b)$, and $f^R_{\tilde{A}}$ is continuous and decreasing on $(c, d]$. Furthermore, $f^L_{\tilde{A}}$ and $f^R_{\tilde{A}}$ are called the left and right membership functions, respectively.

Note that fuzzy numbers have different definitions, and a fuzzy number in Definition 2.2 sometimes is called *a fuzzy interval* ([45]). We now review a special kind of fuzzy numbers, which is effective in describing fuzzy information.

**Definition 2.3.** [45] A fuzzy number $\tilde{A}$ is called a *triangular fuzzy number* (TFN), denoted by $\tilde{A} = (l, m, u)$, if $f_{\tilde{A}}(x)$ is given by

$$f_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m; \\ \frac{x-u}{m-u}, & m \le x \le u; \\ 0, & \text{otherwise}, \end{cases}$$

where $l, m, u \in R$, and $l \le m \le u$. And $l$ and $u$ are called the lower and upper bounds.

**Remark 2.4.** (1) A fuzzy number $\tilde{A}$ is called positive if its membership function is such that $f_{\tilde{A}}(x) = 0, \forall x < 0$. A positive TFN is shown as Fig. 1. Obviously, each TFN corresponds to a triangle $T_{\tilde{A}}$, and we denote the area of $T_{\tilde{A}}$ by $S(T_{\tilde{A}})$.

(2) TFNs can provide fuzzy points between $[l, u]$ instead of just $l, m$ and $u$. And there exist differences in memberships of these fuzzy points. Note that $l, m$ and $u$ are the three most representative fuzzy points. Because $l$ and $u$ are the minimum and maximum fuzzy points whose memberships are not less than 0, respectively. $m$ is the fuzzy point with the highest membership. Due to the particularity of $l, m$ and $u$, we in this paper focus on the study of $l, m$ and $u$ when it comes to the study of TFNs.

The following operations of TFNs can be easily obtained by the extension principle of fuzzy sets.

**Definition 2.5.** [1,9] Let $\tilde{A_1} = (l_1, m_1, u_1)$, $\tilde{A_2} = (l_2, m_2, u_2)$ be two TFNs, $\lambda$ a real number. Then the addition, subtraction, multiplication, division, inverse and scalar multiplication operations of $\tilde{A_1}$ and $\tilde{A_2}$ are defined as follows:
(1) $\tilde{A_1} \oplus \tilde{A_2} = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$;
(2) $\tilde{A_1} \ominus \tilde{A_2} = (l_1, m_1, u_1) \ominus (l_2, m_2, u_2) = (l_1 - u_2, m_1 - m_2, u_1 - l_2)$;
(3) $\tilde{A_1} \otimes \tilde{A_2} = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) \approx (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)$;
(4) $\tilde{A_1} \oslash \tilde{A_2} = (l_1, m_1, u_1) \oslash (l_2, m_2, u_2) \approx (l_1 \div u_2, m_1 \div m_2, u_1 \div l_2)$;
(5) $(\tilde{A_1})^{-1} = (l_1, m_1, u_1)^{-1} \approx (\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1})$;
(6) $\lambda \otimes \tilde{A_1} = \lambda \otimes (l_1, m_1, u_1) \approx \begin{cases} (\lambda \times l_1, \lambda \times m_1, \lambda \times u_1), & \lambda \ge 0; \\ (\lambda \times u_1, \lambda \times m_1, \lambda \times l_1), & \lambda \le 0. \end{cases}$

*2.2. Two existing models of conflict analysis*

Pawlak [26] initially established an information system (IS) for Middle East conflict, which used −1, 0 and +1 to depict attitudes of agents about issues.

**Table 1**
The IS for the Middle East conflict.

| $U$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $x_1$ | −1 | +1 | +1 | +1 | +1 |
| $x_2$ | +1 | 0 | −1 | −1 | −1 |
| $x_3$ | +1 | −1 | −1 | −1 | 0 |
| $x_4$ | 0 | −1 | −1 | 0 | −1 |
| $x_5$ | +1 | −1 | −1 | −1 | −1 |
| $x_6$ | 0 | +1 | −1 | 0 | +1 |

**Definition 2.6.** [26] An IS is a quadruple $S = (U, A, V, f)$, where $U = \{x_1, x_2, \cdots, x_s\}$ is a finite set of objects, $A = \{c_1, c_2, \cdots, c_t\}$ is a finite set of attributes, $V = \bigcup\{V_{c_j} | c_j \in A\}$, where $V_{c_j}$ is the set of values of attribute $c_j$, and $|V_{c_j}| > 1$. Moreover, $f$ is a function from $U \times A$ into $V$.

In Definition 2.6, if $U$ and $A$ are finite sets of agents and issues, $V_{c_j}$ is the set of values of issue $c_j$, and the meaning of $f$ is interpreted as follows.

$$f(x_i, c_j) = \begin{cases} -1, & \text{the agent } x_i \text{ is negative about the issue } c_j; \\ \ \ 0, & \text{the agent } x_i \text{ is central about the issue } c_j; \\ +1, & \text{the agent } x_i \text{ is positive about the issue } c_j, \end{cases}$$

then the IS is called an IS for conflict analysis.

**Definition 2.7.** [12] Let $S = (U, A, V, f)$ be an IS for conflict analysis, and $0 \le \beta \le \alpha \le 1$. For any $x \in U$, the probabilistic conflict, neutral and allied sets $CO^{\alpha}_{\beta}(x)$, $NE^{\alpha}_{\beta}(x)$ and $AL^{\alpha}_{\beta}(x)$ of $x$ are defined as follows:
(1) $CO^{\alpha}_{\beta}(x) = \{y \in U | \rho_A(x, y) > \alpha\}$;
(2) $NE^{\alpha}_{\beta}(x) = \{y \in U | \beta \le \rho_A(x, y) \le \alpha\}$;
(3) $AL^{\alpha}_{\beta}(x) = \{y \in U | \rho_A(x, y) < \beta\}$,
where $\rho_A(x, y)$ is the distance function for $x, y \in U$. Then $(S, \alpha, \beta)$ is called a model of conflict analysis on $S$.

**Example 2.8.** Table 1 shows an IS for the Middle East conflict. Here $x_1, x_2, \cdots, x_6$ represent six agents and $c_1, c_2, \cdots, c_5$ denote five issues. Specifically, $x_1, x_2, x_3, x_4, x_5$ and $x_6$ denote Israel, Egypt, Palestine, Jordan, Syria and Saudi Arabia respectively. In addition, $c_1$ means Autonomous Palestinian state on the West Bank and Gaza; $c_2$ stands for Israeli military outpost along the Jordan River; $c_3$ implies Israel retains East Jerusalem; $c_4$ is Israeli military outposts on the Golan Heights; $c_5$ represents Arab countries grant citizenship to Palestinians who choose to remain within their borders. For convenience, we use $c_j(x_i)$ to denote the attitude of the agent $x_i$ about the issue $c_j$. For example, $c_1(x_2) = +1$ denotes the agent $x_2$ is positive about the issue $c_1$, and $c_2(x_2) = 0$ represents the agent $x_2$ is central about the issue $c_2$, $c_3(x_2) = -1$ implies the agent $x_2$ is negative about the issue $c_3$.

Example 2.8 is given by Pawlak [25], and the IS is three-valued. Lang et al. [13] investigated three-way group conflict analysis based on Pythagorean fuzzy information system (PFIS).

**Definition 2.9.** [34] Let $U$ be an arbitrary non-empty set, and a Pythagorean fuzzy set $P$ is a mathematical object of the form $P = \{< x, P(\mu_P(x), \nu_P(x)) > | x \in U\}$, where $\mu_P(x), \nu_P(x) : U \to [0, 1]$ such that $\mu^2_P(x) + \nu^2_P(x) \le 1$. For every $x \in U$, $\mu_P(x)$ and $\nu_P(x)$ denote the membership degree and the non-membership degree of the element $x \in U$ in $P$, respectively. For convenience, we denote the Pythagorean fuzzy number (PFN) as $\gamma = P(\mu_\gamma, \nu_\gamma)$.

**Definition 2.10.** [13] A PFIS is a quadruple $S = (U, A, V, f)$, where $U = \{x_1, x_2, \cdots, x_s\}$ is a finite set of objects, $A = \{c_1, c_2, \cdots, c_t\}$ is a finite set of attributes, $V = \bigcup\{V_{c_j} | c_j \in A\}$, where $V_{c_j}$ is the set of values of attribute $c_j$, all attribute values are PFNs. Moreover, $f$ is a function from $U \times A$ into $V$.

In Definition 2.10, if $U$ and $A$ are finite sets of agents and issues, $V_{c_j}$ is the set of values of issue $c_j$, and the meaning of $f$ is interpreted as follows.

$$f(x_i, c_j) = \begin{cases} P(\mu_{ij}, \nu_{ij}) \text{ and } \mu_{ij} < \nu_{ij}, & \text{the agent } x_i \text{ is negative about the issue } c_j; \\ P(\mu_{ij}, \nu_{ij}) \text{ and } \mu_{ij} = \nu_{ij}, & \text{the agent } x_i \text{ is central about the issue } c_j; \\ P(\mu_{ij}, \nu_{ij}) \text{ and } \mu_{ij} > \nu_{ij}, & \text{the agent } x_i \text{ is positive about the issue } c_j, \end{cases}$$

where $\mu_{ij}$ and $\nu_{ij}$ denote the positive and negative degrees of the agent $x_i$ about the issue $c_j$, then the PFIS is called a PFIS for conflict analysis.

**Table 2**
The PFIS for the Middle East conflict.

| $U$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $x_1$ | $P(1.0, 0.0)$ | $P(0.9, 0.3)$ | $P(0.8, 0.2)$ | $P(0.9, 0.1)$ | $P(0.9, 0.2)$ |
| $x_2$ | $P(0.9, 0.1)$ | $P(0.5, 0.5)$ | $P(0.1, 0.9)$ | $P(0.3, 0.8)$ | $P(0.1, 0.9)$ |
| $x_3$ | $P(0.1, 0.9)$ | $P(0.1, 0.9)$ | $P(0.2, 0.8)$ | $P(0.1, 0.9)$ | $P(0.5, 0.5)$ |
| $x_4$ | $P(0.5, 0.5)$ | $P(0.1, 0.9)$ | $P(0.3, 0.7)$ | $P(0.5, 0.5)$ | $P(0.1, 0.9)$ |
| $x_5$ | $P(0.9, 0.2)$ | $P(0.4, 0.6)$ | $P(0.1, 0.9)$ | $P(0.1, 0.9)$ | $P(0.3, 0.9)$ |
| $x_6$ | $P(0.0, 1.0)$ | $P(0.9, 0.1)$ | $P(0.2, 0.9)$ | $P(0.5, 0.5)$ | $P(0.8, 0.4)$ |

**Table 3**
Comparisons of two conflict analysis models.

| Models | IS | N-A | C-A | P-A |
|---|---|---|---|---|
| Pawlak's ([25]) | Pawlak IS | $-1$ | 0 | $+1$ |
| Lang et al.'s ([13]) | PFIS | $\mu < \nu$ | $\mu = \nu$ | $\mu > \nu$ |

**Definition 2.11.** [13] Let $S = (U, A, V, f)$ be a PFIS for conflict analysis, $\alpha$ and $\beta$ are two thresholds. Then the positive, central and negative alliances are defined as follows:
(1) $\mathrm{POA}_{(\bullet,\alpha,\beta)}(U) = \{x \in U \mid \bullet(R(x)) \geq \alpha\}$;
(2) $\mathrm{CTA}_{(\bullet,\alpha,\beta)}(U) = \{x \in U \mid \beta < \bullet(R(x)) < \alpha\}$;
(3) $\mathrm{NEA}_{(\bullet,\alpha,\beta)}(U) = \{x \in U \mid \bullet(R(x)) \leq \beta\}$,
where $\bullet$ denotes a function for PFNs, and $R(x)$ represents a PFN with weight vector. Then $(S, \alpha, \beta)$ is called a model of conflict analysis on $S$.

**Example 2.12.** Table 2 gives a PFIS for the Middle East conflict. Here $x_1, x_2, \cdots, x_6$ and $c_1, c_2, \cdots, c_5$ are explained in Example 2.8. $c_5(x_6) = P(0.8, 0.4)$ represents the agent $x_6$ is positive about the issue $c_5$, where 0.8 and 0.4 denote the positive and negative degrees of the agent $x_6$ to the issue $c_5$.

**Remark 2.13.** (1) We remark here to compare Pawlak's model and Lang et al.'s model (Table 3), where N-A, C-A and P-A denote a negative attitude, a central attitude and a positive attitude, respectively. And we will use the three symbols throughout the paper.

(2) Note that Pawlak's model only used $-1$, 0 and $+1$ to represent three different attitudes, leaving a pity that the degrees of these three attitudes were not described. Although Lang et al.'s model depicted the degrees of the attitudes to some extent, it failed to consider the agents' caution in the decision-making processes. In other words, out of prudence, the degree of an agent's attitude about an issue tends to fluctuate within a range instead of a real number, which can be well expressed by TFNs. Therefore, in section 3, we will establish a TFIS for conflict analysis to better describe the attitudes of agents about issues involved in actual conflict problems.

## 3. Triangular fuzzy information systems (TFISs)

In this section, we first propose the concept of TFISs, and then establish a TFIS for conflict analysis. Furthermore, we investigate the established TFIS to obtain more information about the corresponding conflict problem. And the study will be carried out in the following two aspects: (1) depict the negative and positive degrees of TFNs in the TFIS; (2) determine the concrete attitudes represented by the TFNs.

**Definition 3.1.** A TFIS for conflict analysis is a quadruple $S = (U, A, V, f)$, where $U = \{x_1, x_2, \cdots, x_s\}$ is a finite set of agents, $A = \{c_1, c_2, \cdots, c_t\}$ is a finite set of issues, $V = \bigcup\{V_{c_j} | c_j \in A\}$, where $V_{c_j}$ is a set of TFNs, which shows all possible attitudes on $c_j$, and $|V_{c_j}| > 1$, $f$ is a function from $U \times A$ into $V$.

In the following, we shall divide $V$ into three disjoint parts, i.e. $V_N, V_C$ and $V_P$, which denote the sets of negative, central and positive attitudes of agents to issues respectively. Moreover, $f$ is interpreted as follows.

$$f(x_i, c_j) = \begin{cases} (l,m,u) \text{ and } (l,m,u) \in V_N, & \text{the agent } x_i \text{ is negative about the issue } c_j; \\ (l,m,u) \text{ and } (l,m,u) \in V_C, & \text{the agent } x_i \text{ is central about the issue } c_j; \\ (l,m,u) \text{ and } (l,m,u) \in V_P, & \text{the agent } x_i \text{ is positive about the issue } c_j. \end{cases}$$

Specifically, we in this paper limit the TFNs to $[0, 1]$. (i.e. $l, m, u \in [0, 1]$)

In fact, for a conflict problem, $V_N, V_C$ and $V_P$ contain three different attitudes of agents about issues. Given a TFIS for conflict analysis, different people may take different approaches to defining $V_N, V_C$ and $V_P$. In other words, there may be many methods to depict the attitudes represented by TFNs. We in this paper use $\Delta S$ to measure the attitudes represented by TFNs in a TFIS (see Definition 3.4), which will be studied in detail in the second aspect of this section.

**Table 4**
A TFIS for the Middle East conflict.

| $U$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $x_1$ | (0.11,0.22,0.33) | (0.67,0.78,0.89) | (0.42,0.50,0.72) | (0.35,0.70,0.84) | (0.31,0.36,0.95) |
| $x_2$ | (0.33,0.50,0.85) | (0.10,0.50,0.90) | (0.21,0.50,0.61) | (0.12,0.55,0.65) | (0.23,0.37,0.50) |
| $x_3$ | (0.42,0.72,0.83) | (0.15,0.21,0.57) | (0.26,0.50,0.63) | (0.13,0.51,0.62) | (0.40,0.50,0.60) |
| $x_4$ | (0.50,0.50,0.50) | (0.37,0.50,0.53) | (0.15,0.22,0.55) | (0.31,0.50,0.69) | (0.32,0.50,0.50) |
| $x_5$ | (0.50,0.56,0.61) | (0.22,0.50,0.61) | (0.16,0.23,0.63) | (0.20,0.55,0.61) | (0.31,0.50,0.62) |
| $x_6$ | (0.22,0.41,0.93) | (0.50,0.50,0.73) | (0.12,0.50,0.61) | (0.11,0.61,0.72) | (0.35,0.37,0.87) |

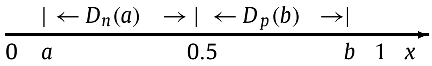


**Fig. 2.** $D_n(a)$ and $D_p(b)$.

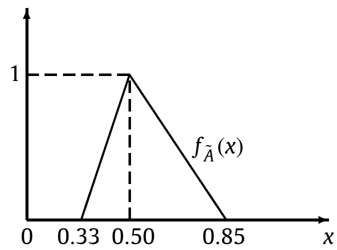


**Fig. 3.** TFN $(0.33, 0.50, 0.85)$.

We now establish a TFIS for the Middle East conflict problem. And the TFIS in this example will be discussed throughout the paper.

**Example 3.2.** Table 4 shows a TFIS for the Middle East conflict. Here agents and issues are explained as in Example 2.8. $c_j(x_i)$ implies the attitude of the agent $x_i$ to the issue $c_j$. And values in $[0, 0.5)$ and $(0.5, 1]$ indicate N-As and P-As, respectively. Besides, 0.5 represents a C-A. For example, $c_1(x_2) = (0.33, 0.50, 0.85)$, where 0.33, 0.50 and 0.85 denote a N-A, a C-A and a P-A of the agent $x_2$ about the issue $c_1$.

In Example 3.2, we make rough explanations of values in Table 4, and we still don't know the negative and positive degrees of these values, nor the concrete attitudes denoted by these TFNs. In other words, for every TFN (denoted by $(l, m, u)$) in the table, there exist two questions: one is to describe the negative and positive degrees of $l, m$ and $u$, another is to determine the concrete attitude of $(l, m, u)$. After solving the two questions, we can obtain more effective information of Table 4, which also lays a solid foundation for section 4.

Now we begin to consider the first question from the definitions of $D_n(a)$ and $D_p(b)$.

**Definition 3.3.** Let $a \in [0, 0.5)$ and $b \in (0.5, 1]$, then the *negative degree of* $a$ and the *positive degree of* $b$ can be defined as $D_n(a) = 0.5 - a$ and $D_p(b) = b - 0.5$ (Fig. 2).

By Definition 3.3, we can obtain the negative and positive degrees of the values in Table 4. For instance, we have $c_1(x_2) = (0.33, 0.50, 0.85)$, where $l = 0.33 \in [0, 0.5), u = 0.85 \in (0.5, 1]$. Thus $D_n(0.33) = 0.17$ and $D_p(0.85) = 0.35$. And the negative and positive degrees of other data in Table 4 can be calculated similarly.

For a TFN $(l, m, u)$, Definition 3.3 only presents the negative and positive degrees of $l, m$ and $u$, but does not consider the different memberships of $l, m$ and $u$. Next we reinterpret the negative and positive degrees of $l, m$ and $u$ combining membership.

Consider TFN $(0.33, 0.50, 0.85)$ in Fig. 3. $f_{\tilde{A}}(x)$ increases on $[0.33, 0.50)$, while decreases on $[0.50, 0.85]$. And the change of membership seems to reflect the change of probability. Thus, we have reason to explain the membership as probability. Note that $D_n(a)$ is decreasing on $[0, 0.5)$, while $D_p(b)$ is increasing on $(0.5, 1]$. That is, $D_n(a)$ and $D_p(b)$ increase when $a$ and $b$ stay away from 0.5. Therefore, for TFN $(0.33, 0.50, 0.85)$, 0.17 denotes possible maximum negative degree, 0.35 implies possible maximum positive degree.

For other TFNs in Table 4, we can analyze similarly. The meanings of $l$, $m$ and $u$ can be summarized in Table 5.

Although we have known the negative and positive degrees of TFNs in Table 4, it seems that we are more concerned about the concrete attitudes of these TFNs. Thus, we now consider the second question. For this purpose, we first propose the concept of relative area as follows.

**Table 5**
The meanings of $l$, $m$ and $u$ in Table 4.

| Points | $\in (0, 0.5)$ | $= 0.5$ | $\in (0.5, 1)$ |
|---|---|---|---|
| $l$ | possible maximum negative degree | C-A | possible minimum positive degree |
| $m$ | the most possible negative degree | C-A | the most possible positive degree |
| $u$ | possible minimum negative degree | C-A | possible maximum positive degree |

**Table 6**
The ten types of TFNs in Table 4.

| $U$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $x_1$ | $\boxed{(0.11,0.22,0.33)}$ | $\boxed{(0.67,0.78,0.89)}$ | (0.42,0.50,0.72) | $\boxed{(0.35,0.70,0.84)}$ | (0.31,0.36,0.95) |
| $x_2$ | $\boxed{(0.33,0.50,0.85)}$ | (0.10,0.50,0.90) | (0.21,0.50,0.61) | (0.12,0.55,0.65) | $\boxed{(0.23,0.37,0.50)}$ |
| $x_3$ | (0.42,0.72,0.83) | $\boxed{(0.15,0.21,0.57)}$ | (0.26,0.50,0.63) | (0.13,0.51,0.62) | (0.40,0.50,0.60) |
| $x_4$ | $\boxed{(0.50,0.50,0.50)}$ | (0.37,0.50,0.53) | (0.15,0.22,0.55) | (0.31,0.50,0.69) | $\boxed{(0.32,0.50,0.50)}$ |
| $x_5$ | $\boxed{(0.50,0.56,0.61)}$ | (0.22,0.50,0.61) | (0.16,0.23,0.63) | (0.20,0.55,0.61) | (0.31,0.50,0.62) |
| $x_6$ | (0.22,0.41,0.93) | $\boxed{(0.50,0.50,0.73)}$ | (0.12,0.50,0.61) | (0.11,0.61,0.72) | (0.35,0.37,0.87) |

**Definition 3.4.** Let $\tilde{A}$ be a TFN. The straight line $x = 0.5$ divides the triangle $T_{\tilde{A}}$ into two smaller regions. Let $S_L$ (resp. $S_R$) be the area of the left (resp. right) region. The *relative area*, denoted by $\Delta S$, can be defined as $\Delta S = S_R - S_L$. Then $\tilde{A}$ denotes

(1) a N-A if $\Delta S = S_R - S_L < 0$;
(2) a C-A if $\Delta S = S_R - S_L = 0$;
(3) a P-A if $\Delta S = S_R - S_L > 0$.

From Definition 3.4, we know $\Delta S$ achieves mapping TFNs into real numbers. In other words, $\Delta S$ determines the concrete attitudes of TFNs in Table 4. If $\Delta S < 0$, then the larger the $\Delta S$, the weaker the negative degree. Conversely, if $\Delta S > 0$, then the larger the $\Delta S$, the stronger the positive degree. To obtain the $\Delta S$ of the TFIS in Table 4, we first need to discuss the position relationships between the TFNs and the straight line $x = 0.5$.

**Example 3.5.** (Continued from Example 3.2) Consider the TFIS in Table 4. Based on the position relationships between the TFNs and the straight line $x = 0.5$, we find the TFNs in Table 4 contain ten types, which are boxed in Table 6. And compared with the boxed data, there are only numerical differences in the data that are not boxed.

The TFNs (i.e. $(l, m, u)$) boxed in Table 6 actually contain four situations.

(1) $l, m, u \neq 0.5$: $(0.11, 0.22, 0.33)$, $(0.67, 0.78, 0.89)$, $(0.15, 0.21, 0.57)$, $(0.35, 0.70, 0.84)$;
(2) $l = 0.5$ or $m = 0.5$ or $u = 0.5$: $(0.50, 0.56, 0.61)$, $(0.33, 0.50, 0.85)$, $(0.23, 0.37, 0.50)$;
(3) $l, m = 0.5$ or $m, u = 0.5$: $(0.50, 0.50, 0.73)$, $(0.32, 0.50, 0.50)$;
(4) $l, m, u = 0.5$: $(0.50, 0.50, 0.50)$.

If we ignore the concrete values of TFNs in the four situations above, these TFNs can be represented as follows.

(1) $l, m, u \neq 0.5$: $(l_{<0.5}, m_{<0.5}, u_{<0.5})$, $(l_{>0.5}, m_{>0.5}, u_{>0.5})$, $(l_{<0.5}, m_{<0.5}, u_{>0.5})$, $(l_{<0.5}, m_{>0.5}, u_{>0.5})$;
(2) $l = 0.5$ or $m = 0.5$ or $u = 0.5$: $(0.50, m_{>0.5}, u_{>0.5})$, $(l_{<0.5}, 0.50, u_{>0.5})$, $(l_{<0.5}, m_{<0.5}, 0.50)$;
(3) $l, m = 0.5$ or $m, u = 0.5$: $(0.50, 0.50, u_{>0.5})$, $(l_{<0.5}, 0.50, 0.50)$;
(4) $l, m, u = 0.5$: $(0.50, 0.50, 0.50)$,

where "$l_{<0.5}$" "$m_{<0.5}$" and "$u_{<0.5}$" represent $l, m, u \in [0, 0.5)$. Similarly, "$l_{>0.5}$", "$m_{>0.5}$" and "$u_{>0.5}$" denote $l, m, u \in (0.5, 1]$.

From Example 3.5, we summarize the four situations of TFNs in Table 4. Now we analyze these four situations to determine the concrete attitudes represented by them.

**Situation 1:** $(l, m, u \neq 0.5)$ TFNs $(l_{<0.5}, m_{<0.5}, u_{<0.5})$ and $(l_{>0.5}, m_{>0.5}, u_{>0.5})$ are shown as Fig. 4 and Fig. 5.

Obviously, $\Delta S((l_{<0.5}, m_{<0.5}, u_{<0.5})) = \frac{l-u}{2} \leq 0$, and when $l_{<0.5} = m_{<0.5} = u_{<0.5}$ (for example, $(0.12, 0.12, 0.12)$), $\Delta S((l_{<0.5}, m_{<0.5}, u_{<0.5})) = 0$. By Definition 3.4, $(l_{<0.5}, m_{<0.5}, u_{<0.5})$ represents a N-A or a C-A. Similarly, $\Delta S((l_{>0.5}, m_{>0.5}, u_{>0.5})) = \frac{u-l}{2} \geq 0$, and when $l_{>0.5} = m_{>0.5} = u_{>0.5}$ (for example, $(0.65, 0.65, 0.65)$), $\Delta S((l_{>0.5}, m_{>0.5}, u_{>0.5})) = 0$. From Definition 3.4, $(l_{>0.5}, m_{>0.5}, u_{>0.5})$ denotes a P-A or a C-A.

TFNs $(l_{<0.5}, m_{<0.5}, u_{>0.5})$ and $(l_{<0.5}, m_{>0.5}, u_{>0.5})$ can be shown in Fig. 6 and Fig. 7.

For $(l_{<0.5}, m_{<0.5}, u_{>0.5})$, we have $S_L = \frac{u-l}{2} - \frac{(0.5-u)^2}{2(u-m)}$, $S_R = \frac{(0.5-u)^2}{2(u-m)}$. Thus

$$\Delta S = S_R - S_L = \frac{u^2 - u(2 - m - l) - lm + 0.5}{2(u - m)},$$

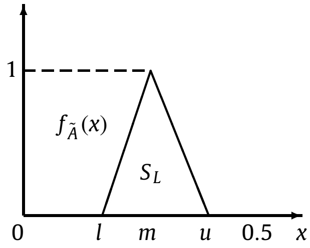


**Fig. 4.** $(l_{<0.5}, m_{<0.5}, u_{<0.5})$.

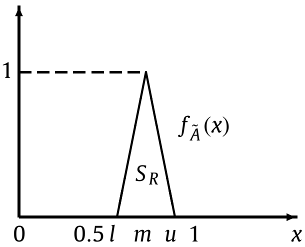


**Fig. 5.** $(l_{>0.5}, m_{>0.5}, u_{>0.5})$.

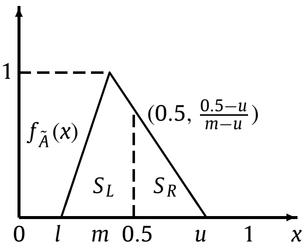


**Fig. 6.** $(l_{<0.5}, m_{<0.5}, u_{>0.5})$.

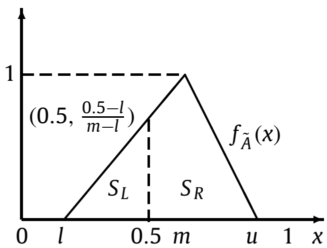


**Fig. 7.** $(l_{<0.5}, m_{>0.5}, u_{>0.5})$.

if $u^2 - u(2-m-l) - lm + 0.5 < 0$ $(=0, >0)$, then $\Delta S < 0$ $(=0, >0)$. From Definition 3.4, $(l_{<0.5}, m_{<0.5}, u_{>0.5})$ implies a N-A (a C-A, a P-A). For $(l_{<0.5}, m_{>0.5}, u_{>0.5})$, we have $S_L = \frac{(0.5-l)^2}{2(m-l)}$, $S_R = \frac{u-l}{2} - \frac{(0.5-l)^2}{2(m-l)}$. Thus

$$\Delta S = S_R - S_L = \frac{-l^2 - l(u+m-2) + um - 0.5}{2(m-l)},$$

if $-l^2 - l(u+m-2) + um - 0.5 < 0$ $(=0, >0)$, then $\Delta S < 0$ $(=0, >0)$. From Definition 3.4, $(l_{<0.5}, m_{>0.5}, u_{>0.5})$ implies a N-A (a C-A, a P-A).

**Situation 2:** $(l = 0.5$ or $m = 0.5$ or $u = 0.5)$ TFNs $(0.5, m_{>0.5}, u_{>0.5})$, $(l_{<0.5}, 0.5, u_{>0.5})$ and $(l_{<0.5}, m_{<0.5}, 0.5)$ can be shown in Fig. 8, Fig. 9 and Fig. 10.

For TFN $(0.5, m_{>0.5}, u_{>0.5})$, $S_L = 0$, $S_R = \frac{u-0.5}{2}$. That is, $\Delta S = \frac{u-0.5}{2} > 0$. By Definition 3.4, $(0.5, m_{>0.5}, u_{>0.5})$ denotes a P-A. For TFN $(l_{<0.5}, 0.5, u_{>0.5})$, $S_L = \frac{0.5-l}{2}$, $S_R = \frac{u-0.5}{2}$, $\Delta S = S_R - S_l = \frac{u+l-1}{2}$. If $u + l < 1$ $(=1, >1)$, $\Delta S < 0$ $(=0, >0)$. From Definition 3.4, $(l_{<0.5}, 0.5, u_{>0.5})$ implies a N-A (a C-A, a P-A). And for TFN $(l_{<0.5}, m_{<0.5}, 0.5)$, we have $S_L = \frac{0.5-l}{2}$, $S_R = 0$.

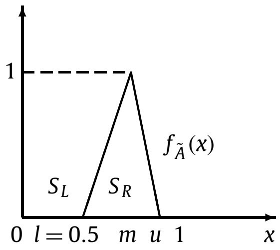


**Fig. 8.** $(0.5, m_{>0.5}, u_{>0.5})$.

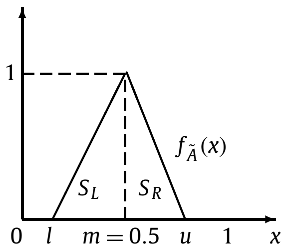


**Fig. 9.** $(l_{<0.5}, 0.5, u_{>0.5})$.

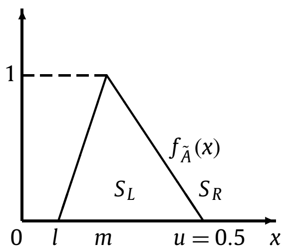


**Fig. 10.** $(l_{<0.5}, m_{<0.5}, 0.5)$.

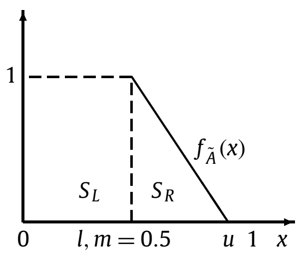


**Fig. 11.** $(0.5, 0.5, u_{>0.5})$.

That is, $\triangle S = \frac{l-0.5}{2} < 0$. Thus $(l_{<0.5}, m_{<0.5}, 0.5)$ represents a N-A.

**Situation 3:** $(l, m = 0.5$ or $m, u = 0.5)$ TFNs $(0.5, 0.5, u_{>0.5})$ and $(l_{<0.5}, 0.5, 0.5)$ can be shown in Fig. 11 and Fig. 12.

For TFN $(0.5, 0.5, u_{>0.5})$, we have $S_L = 0$, $S_R = \frac{u-0.5}{2}$. That is, $\triangle S = \frac{u-0.5}{2} > 0$. By Definition 3.4, $(0.5, 0.5, u_{>0.5})$ represents a P-A. For TFN $(l_{<0.5}, 0.5, 0.5)$, $S_L = \frac{0.5-l}{2}$, $S_R = 0$, then $\triangle S = \frac{l-0.5}{2} < 0$. Thus $(l_{<0.5}, 0.5, 0.5)$ denotes a N-A.

**Situation 4:** $(l, m, u = 0.5)$ TFN $(0.5, 0.5, 0.5)$ is in fact a real number, thus $\triangle S((0.5, 0.5,\ 0.5)) = 0$. From Definition 3.4, $(0.5, 0.5, 0.5)$ implies a C-A.

After analyzing the above four situations, we further compile the $\triangle S$ of them as shown in Table 7.

Note that N-A, C-A, and P-A in Table 7 all correspond to six types of TFNs, which is consistent with the decision-making processes in actual conflict situations to some extent. In addition, we in Definition 3.1 have limited TFNs to $[0, 1]$, thus we can obtain the range of $\triangle S$ in Table 7 as follows.

**Proposition 3.6.** *Let $\tilde{A} = (l, m, u)$ be a TFN, $\triangle S(\tilde{A})$ the relative area of $\tilde{A}$, then $\triangle S(\tilde{A}) \in [-0.29, 0.29]$. And*

(1) *If $\tilde{A} = (0, 0, 0.71)$, $\triangle S((0, 0, 0.71)) = -0.29$;*

(2) *If $\tilde{A} = (0.29, 1, 1)$, $\triangle S((0.29, 1, 1)) = 0.29$.*

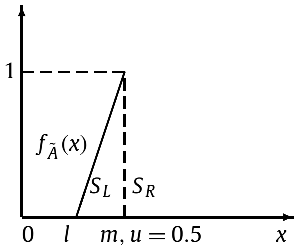


**Fig. 12.** $(l_{<0.5}, 0.5, 0.5)$.

**Table 7**
The $\Delta S$ of ten types of TFNs in Table 4.

| Types | $\Delta S$ | attitude |
|---|---|---|
| $(l_{<0.5}, m_{<0.5}, u_{<0.5})$ | $(l-u)/2 < 0\ (=0)$ | N-A (C-A) |
| $(l_{>0.5}, m_{>0.5}, u_{>0.5})$ | $(u-l)/2 > 0\ (=0)$ | P-A (C-A) |
| $(l_{<0.5}, m_{<0.5}, u_{>0.5})$ | $\frac{u^2-u(2-m-l)-lm+0.5}{2(u-m)} < 0\ (=0, >0)$ | N-A (C-A,P-A) |
| $(l_{<0.5}, m_{>0.5}, u_{>0.5})$ | $\frac{-l^2-l(u+m-2)+um-0.5}{2(m-l)} < 0\ (=0, >0)$ | N-A (C-A,P-A) |
| $(0.5, m_{>0.5}, u_{>0.5})$ | $(u-0.5)/2 > 0$ | P-A |
| $(l_{<0.5}, 0.5, u_{>0.5})$ | $(l+u-1)/2 < 0\ (=0, >0)$ | N-A (C-A,P-A) |
| $(l_{<0.5}, m_{<0.5}, 0.5)$ | $(l-0.5)/2 < 0$ | N-A |
| $(0.5, 0.5, u_{>0.5})$ | $(u-0.5)/2 > 0$ | P-A |
| $(l_{<0.5}, 0.5, 0.5)$ | $(l-0.5)/2 < 0$ | N-A |
| $(0.5, 0.5, 0.5)$ | $=0$ | C-A |

**Table 8**
The $\Delta S$ of the TFIS in Table 4.

| $\Delta S$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $x_1$ | −0.11 | 0.11 | 0.07 | 0.18 | 0.02 |
| $x_2$ | 0.09 | 0 | −0.09 | −0.07 | −0.14 |
| $x_3$ | 0.18 | −0.20 | −0.06 | −0.12 | 0 |
| $x_4$ | 0 | −0.05 | −0.19 | 0 | −0.09 |
| $x_5$ | 0.06 | −0.09 | −0.19 | −0.05 | −0.04 |
| $x_6$ | 0 | 0.12 | −0.14 | 0 | 0.01 |

**Proof.** (1) Except for types $(l_{<0.5}, m_{<0.5}, u_{>0.5})$ and $(l_{<0.5}, m_{>0.5}, u_{>0.5})$ in Table 7, it is easy to see that the ranges of $\Delta S$ of the other eight types are subsets of $[-0.25, 0.25]$. Now we focus on analyzing $(l_{<0.5}, m_{<0.5}, u_{>0.5})$ (i.e. $l, m \in [0, 0.5)$, $u \in (0.5, 1]$) to determine the TFN corresponding to the minimum of $\Delta S$ (denoted by $\Delta S_{min}$ for short).
1) If there are two TFNs $(0, m_{<0.5}, u_{>0.5})$ and $(l'_{<0.5}, m'_{<0.5}, u'_{>0.5})$, where $l'_{<0.5} \neq 0$, $m_{<0.5} = m'_{<0.5}$ and $u'_{<0.5} = u_{>0.5}$. From Table 7, we have $\Delta S((0, m_{<0.5}, u_{>0.5})) < \Delta S((l'_{<0.5}, m'_{<0.5}, u'_{>0.5}))$. Therefore, $\Delta S_{min}(l_{<0.5}, m_{<0.5}, u_{>0.5})$ means a TFN $(0, m_{<0.5}, u_{>0.5})$.
2) By Table 7, $\Delta S((0, m_{<0.5}, u_{>0.5})) = \frac{u^2-u(2-m)+0.5}{2(u-m)}$. We assume $f(m, u) = \frac{u^2-u(2-m)+0.5}{2(u-m)}$, then $f_m(m, u) = \frac{(2u-1)^2}{4(u-m)^2} > 0$, where $f_m(m, u)$ denotes the partial derivative of $f(m, u)$ with respect to $m$. That is, if $u$ is fixed, $\Delta S((0, m_{<0.5}, u_{>0.5}))$ becomes larger when $m$ increases. Thus $\Delta S_{min}(l_{<0.5}, m_{<0.5}, u_{>0.5})$ further implies a TFN $(0, 0, u_{>0.5})$.
3) From Table 7, $\Delta S((0, 0, u_{>0.5})) = \frac{u^2-2u+0.5}{2u}$. We assume $f(u) = \frac{u^2-2u+0.5}{2u}$, then $f'(u) = \frac{2u^2-1}{4u^2}$, where $f'(u)$ denotes the derivative of $f(u)$. Since $u \in (0.5, 1]$, then $\Delta S((0, 0, u_{>0.5}))$ decreases in $(0.5, \sqrt{1/2})$ and increases in $(\sqrt{1/2}, 1]$ (for convenience, we approximate $\sqrt{1/2}$ to 0.71). So $\Delta S_{min}(l_{<0.5}, m_{<0.5}, u_{>0.5})$ corresponds to TFN $(0, 0, 0.71)$, and $\Delta S((0, 0, 0.71)) = -0.29$.

(2) Similar to (1). □

At last of this section, we expect to determine the concrete attitudes represented by the TFNs of Table 4. For this purpose, we employ an example to calculate the $\Delta S$ of these TFNs.

**Example 3.7.** From Table 7, we can calculate $\Delta S$ of the TFIS in Table 4, see Table 8. We in fact determine the concrete attitudes of TFNs in Table 4. For instance, $\Delta S(c_1(x_1)) < 0$ implies the agent $x_1$ has a N-A about the issue $c_1$, while $\Delta S(c_2(x_1)) > 0$ denotes the agent $x_1$ shows a P-A about the issue $c_2$. Besides, $\Delta S(c_2(x_2)) = 0$ represents the agent $x_2$ has a C-A about the issue $c_2$. We also know the agent $x_1$ has the strongest and weakest positive degrees about issues $c_4$ and $c_5$. And the agent $x_2$ has the strongest and weakest negative degrees about issues $c_5$ and $c_4$. In addition, all $\Delta S$ in Table 8 are within the range given by Proposition 3.6.

**Table 9**
Comparisons of three conflict analysis models.

| Models | IS | N-A | C-A | P-A | $D_p$ | $D_n$ |
|---|---|---|---|---|---|---|
| Pawlak's ([25]) | IS | $-1$ | 0 | $+1$ | $-$ | $-$ |
| Lang et al.'s ([13]) | PFIS | $\mu < \nu$ | $\mu = \nu$ | $\mu > \nu$ | $\mu$ | $\nu$ |
| Our | TFIS | $\Delta S < 0$ | $\Delta S = 0$ | $\Delta S > 0$ | $D_p(b) = b - 0.5$ | $D_n(a) = 0.5 - a$ |

**Table 10**
The importance comparison scales between $c_j$ and $c_k$.

| Scales | the corresponding meaning |
|---|---|
| 0.10 | the issue $c_j$ is slightly more important than the issue $c_k$ |
| 0.13 | the issue $c_j$ is more important than the issue $c_k$ |
| 0.16 | the issue $c_j$ is as important as the issue $c_k$ |
| 0.19 | the issue $c_j$ is very more important than the issue $c_k$ |
| 0.22 | the issue $c_j$ is absolutely more important than the issue $c_k$ |

**Remark 3.8.** We remark here to present the comparisons of our model and the two models in subsection 2.2 (Table 9). Here N-A, C-A and P-A denote a negative attitude, a central attitude and a positive attitude, respectively. And $D_p$ and $D_n$ represent positive degree and negative degree of agents about issues. Besides, "$-$" indicates Pawlak's model doesn't involve $D_p$ and $D_n$.

After comparing, we now summarize the differences and connections of these three conflict analysis models in Table 9 as follows.
(1) The research objects of the three conflict analysis models are information systems.
(2) In terms of describing agents' attitudes, the three conflict analysis models are quite different. Pawlak used $-1$, 0 and 1 to depict agents' negative, central and positive attitudes about issues. However, this model failed to consider the degrees of agents' attitudes about issues. Lang et al. succeeded representing positive degree and negative degree of an agent's attitude about an issue by the membership degree and the non-membership degree. We not only define positive and negative degrees of an agent's attitude, but make specific explanations of positive and negative degrees in Table 5. In addition, we use $\Delta S$ to reasonably determine the concrete attitudes of agents.

In this section, we have known the positive and negative degrees of agents' attitudes about issues. But we seem to be more concerned about the total attitude of each agent to all issues. Because in this way we can further explore the relationships among agents. In the next section, the total attitude is defined based on the aforementioned $\Delta S$.

## 4. Three-way conflict analysis on TFISs

This section shall present three alliances according to the total attitudes of agents to issues. In section 4.1, we consider the fuzzy weights of issues to lay the groundwork for three-way conflict analysis. In section 4.2, we introduce the model of three-way conflict analysis on TFISs and provide an approach to thresholds based on the theory of triangular fuzzy decision-theoretic rough sets.

### *4.1. Fuzzy weights of issues*

Due to the fact that the importance of an issue is different for agents, they may attach different fuzzy weights to the same issue. In this subsection, we study fuzzy weights of issues from the perspective of agents. First, we construct the triangular fuzzy symmetric judgment matrices (TFSJMs). Second, we compute the fuzzy weights of issues based on the aforementioned TFSJMs.

To construct the TFSJMs, we first propose the concept of 0.10-0.22 scales.

**Definition 4.1.** Let $S = (U, A, V, f)$ be a TFIS for conflict analysis, and $c_j, c_k \in A$ two issues $(j, k = 1, 2, \cdots, t)$. Then *the importance comparison scales between $c_j$ and $c_k$* can be defined as shown in Table 10.

If there are $n$ issues, we have $n(n-1)$ pairs consisting of different agents. By symmetry, we only have to compare $\frac{n(n-1)}{2}$ times. According to 0.10-0.22 scales, we propose the concept of TFSJMs as follows.

**Table 11**
The comparisons of TFSJMs and TFCJMs.

| Matrices | comparison times | fuzzy weight |
|---|---|---|
| TFSJMs | $\frac{n(n-1)}{2}$ | $\tilde{\omega}_{ij}^{Sr}=\tilde{\omega}_{ij}^{Sc}$ |
| TFCJMs | $n(n-1)$ | $\tilde{\omega}_{ij}^{Cr}\neq\tilde{\omega}_{ij}^{Cc}$ |

**Definition 4.2.** A TFSJM, denoted by $P_i^s=[(l_{jk}^i,m_{jk}^i,u_{jk}^i)]_{t\times t}$, can be defined as

$$P_i^s=\begin{pmatrix} (l_{11}^i,m_{11}^i,u_{11}^i) & (l_{12}^i,m_{12}^i,u_{12}^i) & \cdots & (l_{1t}^i,m_{1t}^i,u_{1t}^i) \\ (l_{21}^i,m_{21}^i,u_{21}^i) & (l_{22}^i,m_{22}^i,u_{22}^i) & \cdots & (l_{2t}^i,m_{2t}^i,u_{2t}^i) \\ \vdots & \vdots & & \vdots \\ (l_{t1}^i,m_{t1}^i,u_{t1}^i) & (l_{t2}^i,m_{t2}^i,u_{t2}^i) & \cdots & (l_{tt}^i,m_{tt}^i,u_{tt}^i) \end{pmatrix}$$

where $0\le l_{jk}^i,m_{jk}^i,u_{jk}^i\le 0.22$, $l_{jk}^i=l_{kj}^i$, $m_{jk}^i=m_{kj}^i$, $u_{jk}^i=u_{kj}^i$, and $l_{jj}^i=m_{jj}^i=u_{jj}^i=0.16$. $(i=1,2,\cdots,s;\ j,k=1,2,\cdots,t.)$

**Remark 4.3.** (1) $(l_{jk}^i,m_{jk}^i,u_{jk}^i)$, given by the agent $x_i$, represents the importance degree of the issue $c_j$ compared with the issue $c_k$, where $m_{jk}^i$ implies the most possible importance degree, $l_{jk}^i$ and $u_{jk}^i$ denote the possible minimum and maximum importance degrees, respectively. Obviously, the order of the TFSJM is the number of the issues.

(2) In general, we only observe the diverse importance of issues, and tend to ignore uneven situations among agents. The TFSJMs, given by each agent, are exactly considering various situations in agents.

(3) We construct the TFSJMs according to 0.10-0.22 scales. 0.10 and 0.22 are just the minimum and maximum values of the scales, and they don't contain any particular meaning. Other scales of course can be used if they fit our model well. Different scales only correspond to different fuzzy ranges. For example, the fuzzy range of 0.1-0.9 scales [33] is 0.8. Evidently, the 0.1-0.9 scales provide a larger fuzzy range than our scales. However, if we use the 0.1-0.9 scales to construct TFSJMs, the total attitudes (see Definition 4.8) will be beyond the range of TFNs we specified in Definition 3.1 (i.e. we limit TFNs to $[0,1]$). Because this result is easy to obtain, we omit the verification process.

Based on TFSJMs, we further propose the concept of fuzzy weight as follows.

**Definition 4.4.** Let $P_i=[(l_{jk}^i,m_{jk}^i,u_{jk}^i)]_{t\times t}$ be the TFSJMs, then *fuzzy weight*, denoted by $\tilde{\omega}_{ij}\approx(l_{ij},m_{ij},u_{ij})$, can be defined as follows.

$$\tilde{\omega}_{ij}=\Big(\sum_{k=1}^{t}l_{jk}^i,\sum_{k=1}^{t}m_{jk}^i,\sum_{k=1}^{t}u_{jk}^i\Big)\otimes\Big(\sum_{j=1}^{t}\sum_{k=1}^{t}l_{jk}^i,\sum_{j=1}^{t}\sum_{k=1}^{t}m_{jk}^i,\sum_{j=1}^{t}\sum_{k=1}^{t}u_{jk}^i\Big)^{-1}$$

$$\approx\left(\frac{\sum_{k=1}^{t}l_{jk}^i}{\sum_{j=1}^{t}\sum_{k=1}^{t}u_{jk}^i},\frac{\sum_{k=1}^{t}m_{jk}^i}{\sum_{j=1}^{t}\sum_{k=1}^{t}m_{jk}^i},\frac{\sum_{k=1}^{t}u_{jk}^i}{\sum_{j=1}^{t}\sum_{k=1}^{t}l_{jk}^i}\right)\triangleq(l_{ij},m_{ij},u_{ij}),$$

where $\tilde{\omega}_{ij}$ represents the fuzzy weight of the issue $c_j$ given by the agent $x_i$, and $0\le l_{ij},m_{ij},u_{ij}\le 1$. $(i=1,2,\cdots,s;\ j,k=1,2,\cdots,t.)$

**Remark 4.5.** (1) $m_{ij}$ implies the most possible fuzzy weight, $l_{ij}$ and $u_{ij}$ denote the possible minimum and maximum fuzzy weights, respectively.

(2) When applying 0.10-0.22 scales, the triangular fuzzy complementary judgment matrix (TFCJM) [42] can be expressed as $P_i^c=[(l_{jk}^i,m_{jk}^i,u_{jk}^i)]_{t\times t}$, where $0\le l_{jk}^i,m_{jk}^i,u_{jk}^i\le 0.22$, $l_{jk}^i+u_{kj}^i=0.32$, $m_{jk}^i+m_{kj}^i=0.32$, $u_{jk}^i+l_{kj}^i=0.32$, and $l_{jj}^i=m_{jj}^i=u_{jj}^i=0.16$. $(i=1,2,\cdots,s;\ j,k=1,2,\cdots,t.)$ Now we compare TFSJMs with TFCJMs in Table 11. Here $n$ is the number of issues in a conflict problem. $\tilde{\omega}_{ij}^{Sr}$ and $\tilde{\omega}_{ij}^{Sc}$ represent the fuzzy weights by summing up the rows and columns of TFSJMs. Similarly, $\tilde{\omega}_{ij}^{Cr}$ and $\tilde{\omega}_{ij}^{Cc}$ denote the fuzzy weights by summing up the rows and columns of TFCJMs.

From Table 11, we know TFSJM means fewer comparisons times than TFCJM. In addition, by symmetry, we have $\tilde{\omega}_{ij}^{Sr}=\tilde{\omega}_{ij}^{Sc}$, which is consistent with actual situations. In other words, whether you compare the issue $c_i$ with the issue $c_j$ or the issue $c_j$ with the issue $c_i$, the fuzzy weight should be the same. However, TFCJM doesn't possess this advantage. Therefore, TFSJM is a better method than TFCJM to obtain fuzzy weight in a way.

With the concept of fuzzy weight, we now introduce the concept of fuzzy weight matrix.

**Definition 4.6.** A *fuzzy weight matrix*, denoted by $W = [\tilde{\omega}_{ij}]_{s\times t}$, is defined as

$$W = \begin{pmatrix} \tilde{\omega}_{11} & \tilde{\omega}_{12} & \cdots & \tilde{\omega}_{1t} \\ \tilde{\omega}_{21} & \tilde{\omega}_{22} & \cdots & \tilde{\omega}_{2t} \\ \vdots & \vdots & & \vdots \\ \tilde{\omega}_{s1} & \tilde{\omega}_{s2} & \cdots & \tilde{\omega}_{st} \end{pmatrix}$$

where $\tilde{\omega}_{ij} \approx (l_{ij}, m_{ij}, u_{ij})$ represents the fuzzy weight of the issue $c_j$ given by the agent $x_i$, and $0 \le l_{ij}, m_{ij}, u_{ij} \le 1$, $\sum_{j=1}^{t} m_{ij} = 1$ $(i = 1, 2, \ldots, s;\ j = 1, 2, \ldots, t)$.

At last of this subsection, we employ an example to illustrate how to obtain a fuzzy weight matrix.

**Example 4.7.** Based on the TFIS in Table 4, we now construct its fuzzy weight matrix. By definition, we need to consider the six TFSJMs respectively.
(1) The TFSJM of the agent $x_1$ is

$$P_1^s = \begin{pmatrix} (0.16,0.16,0.16) & (0.11,0.12,0.13) & (0.14,0.15,0.16) & (0.10,0.14,0.15) & (0.11,0.13,0.15) \\ (0.11,0.12,0.13) & (0.16,0.16,0.16) & (0.12,0.13,0.14) & (0.15,0.16,0.17) & (0.16,0.18,0.19) \\ (0.14,0.15,0.16) & (0.12,0.13,0.14) & (0.16,0.16,0.16) & (0.14,0.15,0.17) & (0.11,0.13,0.16) \\ (0.10,0.14,0.15) & (0.15,0.16,0.17) & (0.14,0.15,0.17) & (0.16,0.16,0.16) & (0.12,0.13,0.15) \\ (0.11,0.13,0.15) & (0.16,0.18,0.19) & (0.11,0.13,0.16) & (0.12,0.13,0.15) & (0.16,0.16,0.16) \end{pmatrix}$$

From Definition 4.4, we can calculate the fuzzy weights of five issues given by the agent $x_1$. That is, $\tilde{\omega}_{1j} \approx (l_{1j}, m_{1j}, u_{1j})$. And

$$\sum_{j=1}^{5}\sum_{k=1}^{5} l_{jk}^1 = 3.32,\ l_{11} = \frac{\sum_{k=1}^{5} l_{1k}^1}{\sum_{j=1}^{5}\sum_{k=1}^{5} u_{jk}^1} \approx 0.16,$$
$$\sum_{j=1}^{5}\sum_{k=1}^{5} m_{jk}^1 = 3.64,\ m_{11} = \frac{\sum_{k=1}^{5} m_{1k}^1}{\sum_{j=1}^{5}\sum_{k=1}^{5} m_{jk}^1} \approx 0.19,$$
$$\sum_{j=1}^{5}\sum_{k=1}^{5} u_{jk}^1 = 3.94,\ u_{11} = \frac{\sum_{k=1}^{5} u_{1k}^1}{\sum_{j=1}^{5}\sum_{k=1}^{5} l_{jk}^1} \approx 0.23.$$

Thus $\tilde{\omega}_{11} \approx (l_{11}, m_{11}, u_{11}) \approx (0.16, 0.19, 0.23)$. And $\tilde{\omega}_{1j} \approx (l_{1j}, m_{1j}, u_{1j})$ can be calculated similarly by the formula $(j = 2, \cdots, 5)$

$$\tilde{\omega}_{1j} \approx \Big(\frac{\sum_{k=1}^{5} l_{jk}^1}{\sum_{j=1}^{5}\sum_{k=1}^{5} u_{jk}^1}, \frac{\sum_{k=1}^{5} m_{jk}^1}{\sum_{j=1}^{5}\sum_{k=1}^{5} m_{jk}^1}, \frac{\sum_{k=1}^{5} u_{jk}^1}{\sum_{j=1}^{5}\sum_{k=1}^{5} l_{jk}^1}\Big).$$

So we have $\tilde{\omega}_{12} \approx (0.18, 0.21, 0.24)$, $\tilde{\omega}_{13} \approx (0.17, 0.20, 0.24)$, $\tilde{\omega}_{14} \approx (0.17, 0.20, 0.24)$, $\tilde{\omega}_{15} \approx (0.17, 0.20, 0.24)$.
(2) The TFSJM of the agent $x_2$ is

$$P_2^s = \begin{pmatrix} (0.16,0.16,0.16) & (0.12,0.14,0.15) & (0.11,0.13,0.14) & (0.12,0.13,0.16) & (0.10,0.11,0.12) \\ (0.12,0.14,0.15) & (0.16,0.16,0.16) & (0.13,0.14,0.16) & (0.12,0.13,0.17) & (0.14,0.15,0.17) \\ (0.11,0.13,0.14) & (0.13,0.14,0.16) & (0.16,0.16,0.16) & (0.13,0.14,0.18) & (0.12,0.14,0.15) \\ (0.12,0.13,0.16) & (0.12,0.13,0.17) & (0.13,0.14,0.18) & (0.16,0.16,0.16) & (0.11,0.13,0.15) \\ (0.10,0.11,0.12) & (0.14,0.15,0.17) & (0.12,0.14,0.15) & (0.11,0.13,0.15) & (0.16,0.16,0.16) \end{pmatrix}$$

Similar to (1), we have $\tilde{\omega}_{21} \approx (0.16, 0.19, 0.23)$ $\tilde{\omega}_{22} \approx (0.17, 0.21, 0.25)$, $\tilde{\omega}_{23} \approx (0.17, 0.20, 0.25)$, $\tilde{\omega}_{24} \approx (0.16, 0.20, 0.26)$, $\tilde{\omega}_{25} \approx (0.16, 0.20, 0.23)$.
(3) The TFSJM of the agent $x_3$ is

$$P_3^s = \begin{pmatrix} (0.16,0.16,0.16) & (0.11,0.15,0.16) & (0.12,0.13,0.15) & (0.14,0.15,0.16) & (0.11,0.12,0.13) \\ (0.11,0.15,0.16) & (0.16,0.16,0.16) & (0.12,0.14,0.15) & (0.11,0.13,0.14) & (0.15,0.16,0.17) \\ (0.12,0.13,0.15) & (0.12,0.14,0.15) & (0.16,0.16,0.16) & (0.10,0.11,0.12) & (0.10,0.13,0.14) \\ (0.14,0.15,0.16) & (0.11,0.13,0.14) & (0.10,0.11,0.12) & (0.16,0.16,0.16) & (0.11,0.14,0.15) \\ (0.11,0.12,0.13) & (0.15,0.16,0.17) & (0.10,0.13,0.14) & (0.11,0.14,0.15) & (0.16,0.16,0.16) \end{pmatrix}$$

Similarly, we have $\tilde{\omega}_{31} \approx (0.17, 0.20, 0.24)$, $\tilde{\omega}_{32} \approx (0.17, 0.21, 0.25)$, $\tilde{\omega}_{33} \approx (0.16, 0.19, 0.23)$, $\tilde{\omega}_{34} \approx (0.17, 0.20, 0.23)$, $\tilde{\omega}_{35} \approx (0.17, 0.20, 0.24)$.
(4) The TFSJM of the agent $x_4$ is

$$P_4^s = \begin{pmatrix} (0.16, 0.16, 0.16) & (0.12, 0.13, 0.14) & (0.10, 0.11, 0.12) & (0.13, 0.15, 0.16) & (0.12, 0.13, 0.18) \\ (0.12, 0.13, 0.14) & (0.16, 0.16, 0.16) & (0.13, 0.15, 0.16) & (0.12, 0.14, 0.15) & (0.10, 0.13, 0.14) \\ (0.10, 0.11, 0.12) & (0.13, 0.15, 0.16) & (0.16, 0.16, 0.16) & (0.11, 0.13, 0.15) & (0.12, 0.14, 0.16) \\ (0.13, 0.15, 0.16) & (0.12, 0.14, 0.15) & (0.11, 0.13, 0.15) & (0.16, 0.16, 0.16) & (0.13, 0.14, 0.17) \\ (0.12, 0.13, 0.18) & (0.10, 0.13, 0.14) & (0.12, 0.14, 0.16) & (0.13, 0.14, 0.17) & (0.16, 0.16, 0.16) \end{pmatrix}$$

Similarly, we have $\tilde{\omega}_{41} \approx (0.16, 0.19, 0.24)$, $\tilde{\omega}_{42} \approx (0.16, 0.20, 0.24)$, $\tilde{\omega}_{43} \approx (0.16, 0.20, 0.24)$, $\tilde{\omega}_{44} \approx (0.17, 0.21, 0.25)$, $\tilde{\omega}_{45} \approx (0.16, 0.20, 0.26)$.
(5) The TFSJM of the agent $x_5$ is

$$P_5^s = \begin{pmatrix} (0.16, 0.16, 0.16) & (0.13, 0.15, 0.17) & (0.14, 0.15, 0.18) & (0.10, 0.12, 0.14) & (0.11, 0.13, 0.14) \\ (0.13, 0.15, 0.17) & (0.16, 0.16, 0.16) & (0.12, 0.13, 0.15) & (0.13, 0.15, 0.16) & (0.12, 0.15, 0.17) \\ (0.14, 0.15, 0.18) & (0.12, 0.13, 0.15) & (0.16, 0.16, 0.16) & (0.13, 0.14, 0.15) & (0.13, 0.14, 0.18) \\ (0.10, 0.12, 0.14) & (0.13, 0.15, 0.16) & (0.13, 0.14, 0.15) & (0.16, 0.16, 0.16) & (0.14, 0.15, 0.17) \\ (0.11, 0.13, 0.14) & (0.12, 0.15, 0.17) & (0.13, 0.14, 0.18) & (0.14, 0.15, 0.17) & (0.16, 0.16, 0.16) \end{pmatrix}$$

Similarly, we have $\tilde{\omega}_{51} \approx (0.16, 0.20, 0.24)$, $\tilde{\omega}_{52} \approx (0.16, 0.20, 0.25)$, $\tilde{\omega}_{53} \approx (0.17, 0.20, 0.25)$, $\tilde{\omega}_{54} \approx (0.16, 0.20, 0.24)$, $\tilde{\omega}_{55} \approx (0.16, 0.20, 0.25)$.
(6) The TFSJM of the agent $x_6$ is

$$P_6^s = \begin{pmatrix} (0.16, 0.16, 0.16) & (0.11, 0.13, 0.18) & (0.13, 0.16, 0.17) & (0.12, 0.13, 0.16) & (0.13, 0.16, 0.18) \\ (0.11, 0.13, 0.18) & (0.16, 0.16, 0.16) & (0.11, 0.12, 0.14) & (0.12, 0.13, 0.14) & (0.13, 0.15, 0.16) \\ (0.13, 0.16, 0.17) & (0.11, 0.12, 0.14) & (0.16, 0.16, 0.16) & (0.11, 0.13, 0.14) & (0.13, 0.15, 0.16) \\ (0.12, 0.13, 0.16) & (0.12, 0.13, 0.14) & (0.11, 0.13, 0.14) & (0.16, 0.16, 0.16) & (0.12, 0.14, 0.15) \\ (0.13, 0.16, 0.18) & (0.13, 0.15, 0.16) & (0.13, 0.15, 0.16) & (0.12, 0.14, 0.15) & (0.16, 0.16, 0.16) \end{pmatrix}$$

Similarly, we have $\tilde{\omega}_{61} \approx (0.16, 0.21, 0.26)$, $\tilde{\omega}_{62} \approx (0.16, 0.19, 0.24)$, $\tilde{\omega}_{63} \approx (0.16, 0.20, 0.24)$, $\tilde{\omega}_{64} \approx (0.16, 0.19, 0.23)$, $\tilde{\omega}_{65} \approx (0.17, 0.21, 0.25)$.

It follows from Definition 4.6 that the fuzzy weight matrix of the TFIS in Table 4 is as follows.

$$W = \begin{pmatrix} (0.16, 0.19, 0.23) & (0.18, 0.21, 0.24) & (0.17, 0.20, 0.24) & (0.17, 0.20, 0.24) & (0.17, 0.20, 0.24) \\ (0.16, 0.19, 0.23) & (0.17, 0.21, 0.25) & (0.17, 0.20, 0.25) & (0.16, 0.20, 0.26) & (0.16, 0.20, 0.23) \\ (0.17, 0.20, 0.24) & (0.17, 0.21, 0.25) & (0.16, 0.19, 0.23) & (0.17, 0.20, 0.23) & (0.17, 0.20, 0.24) \\ (0.16, 0.19, 0.24) & (0.16, 0.20, 0.24) & (0.16, 0.20, 0.24) & (0.17, 0.21, 0.25) & (0.16, 0.20, 0.26) \\ (0.16, 0.20, 0.24) & (0.16, 0.20, 0.25) & (0.17, 0.20, 0.25) & (0.16, 0.20, 0.24) & (0.16, 0.20, 0.25) \\ (0.16, 0.21, 0.26) & (0.16, 0.19, 0.24) & (0.16, 0.20, 0.24) & (0.16, 0.19, 0.23) & (0.17, 0.21, 0.25) \end{pmatrix}$$

### *4.2. Three alliances based on TFISs*

For a TFIS, we have obtained the corresponding fuzzy weight matrix. Further, we shall study the relationships among agents based on fuzzy weights of issues and total attitudes of agents. For this purpose, we proceed the study from the following two points: (1) the total attitudes of agents to all issues are introduced; (2) all agents are divided into positive, central and negative alliances according to the idea of three-way decision theory.

**Definition 4.8.** The *total attitude* of the agent $x_i$ to all issues, denoted by $\tilde{T}(x_i)$, can be defined as

$$\tilde{T}(x_i) = c_1(x_i) \otimes \tilde{\omega}_{i1} \oplus c_2(x_i) \otimes \tilde{\omega}_{i2} \oplus \cdots \oplus c_j(x_i) \otimes \tilde{\omega}_{ij},$$

where $c_j(x_i)$ implies the attitude of the agent $x_i$ about the issue $c_j$, and $\tilde{\omega}_{ij}$ represents the fuzzy weight of the issue $c_j$ given by the agent $x_i$. $(i = 1, 2, \cdots, s;\ j = 1, 2, \cdots, t.)$

**Remark 4.9.** (1) Obviously, $\tilde{T}(x_i)$ are TFNs. That is, we actually use TFNs to denote the total attitude in Definition 4.8. We in Definition 3.4 utilize the relative area $\Delta S$ to depict a N-A, a C-A or a P-A. In the following, we consider the relative area of $\tilde{T}(x_i)$ as the specific attitude of the agent $x_i$ to all issues. Explanations of the relative area can be found in section 3.

(2) The *value* of total attitude of the agent $x_i$ to all issues is defined as $\Delta S(\tilde{T}(x_i))$ (denoted by $T(x_i)$ for short), where $x_i \in U, i = 1, 2, \cdots, s$. The detailed information of $\Delta S$ is shown in Table 7.

(3) By Proposition 3.6, we have $\Delta S(\tilde{T}(x_i)) = T(x_i) \in [-0.29, 0.29]$. For convenience, we further convert $T(x_i)$ to $\bar{T}(x_i)$ by the following mapping

**Table 12**
$\tilde{T}(x_i)$, $T(x_i)$ and $\bar{T}(x_i)$ of the TFIS in Table 4.

| $U$ | $\tilde{T}(x_i)$ | $T(x_i)$ | $\bar{T}(x_i)$ |
|---|---|---|---|
| $x_1$ | $(0.32, 0.52, 0.89)$ | 0.12 | 0.71 |
| $x_2$ | $(0.16, 0.49, 0.86)$ | 0 | 0.50 |
| $x_3$ | $(0.23, 0.48, 0.77)$ | −0.02 | 0.47 |
| $x_4$ | $(0.27, 0.44, 0.68)$ | −0.07 | 0.38 |
| $x_5$ | $(0.22, 0.47, 0.76)$ | −0.04 | 0.43 |
| $x_6$ | $(0.21, 0.47, 0.95)$ | 0.05 | 0.59 |

$$f(T(x_i)) = \frac{1}{0.58}(T(x_i) + 0.29) = \bar{T}(x_i).$$

Evidently, $\bar{T}(x_i) \in [0, 1]$, which is consistent with Table 4. (In Table 4, we use TFNs between 0 and 1 to represent the agents' attitudes to issues.)

**Example 4.10.** (Continued from Example 4.7) Consider the TFIS in Table 4 and fuzzy weight matrix in Example 4.7. By Definition 4.8, we can compute $\tilde{T}(x_i)$. For instance,

$$\begin{aligned}\tilde{T}(x_1) =&(0.11, 0.22, 0.33) \otimes (0.16, 0.19, 0.23) \oplus (0.67, 0.78, 0.89) \otimes (0.18, 0.21, 0.24)\oplus\\ &(0.42, 0.50, 0.72) \otimes (0.17, 0.20, 0.24) \oplus (0.35, 0.70, 0.84) \otimes (0.17, 0.20, 0.24)\oplus\\ &(0.31, 0.36, 0.95) \otimes (0.17, 0.20, 0.24) = (0.32, 0.52, 0.89).\end{aligned}$$

Further, for TFN $(0.32, 0.52, 0.89)$, by Table 7 and Remark 4.9, we have

$$\begin{aligned}\bar{T}(x_1) &= \frac{1}{0.58}(T(x_1) + 0.29)\\ &= \frac{1}{0.58}(\Delta S((0.32, 0.52, 0.89)) + 0.29)\\ &= \frac{1}{0.58}(\frac{-(0.32)^2 - 0.32 \times (0.89 + 0.52 - 2) + 0.89 \times 0.52 - 0.5}{2(0.52 - 0.32)} + 0.29)\\ &\approx 0.71.\end{aligned}$$

Similarly, other cases can be computed, see Table 12. From the table, we can see $T(x_1) = 0.12 > 0$, which implies the agent $x_1$ has a P-A on all issues, while $T(x_3) = -0.02 < 0$ denotes the agent $x_3$ shows a N-A about all issues. Besides, $T(x_2) = 0$ represents the agent $x_2$ has a C-A towards all issues. We also know that in agents $x_3$, $x_4$ and $x_5$, the agents $x_4$ and $x_3$ have the strongest and weakest negative degrees about all issues, respectively. And the agent $x_1$ is more positive than the agent $x_6$ about all issues.

After the aforementioned preliminaries, we now define the model of three-way conflict analysis on TFISs, which divides the set of agents into the positive, central and negative alliances according to the total attitudes of agents to all issues.

**Definition 4.11.** Let $S = (U, A, V, f)$ be a TFIS for conflict analysis, $\alpha$ and $\beta$ two thresholds satisfying $0 \le \beta < \alpha \le 1$. The *positive, central and negative alliances of* $U$ are defined as
(1) $\mathrm{POA}_{(\alpha,\beta)}(U) = \{x_i \in U | \bar{T}(x_i) \ge \alpha\}$;
(2) $\mathrm{CTA}_{(\alpha,\beta)}(U) = \{x_i \in U | \beta < \bar{T}(x_i) < \alpha\}$;
(3) $\mathrm{NEA}_{(\alpha,\beta)}(U) = \{x_i \in U | \bar{T}(x_i) \le \beta\}$.
Then $(S, \alpha, \beta)$ is called a model of three-way conflict analysis on $S$.

In the model $(S, \alpha, \beta)$, the operator $\bar{T}$ has been explained in Remark 4.9. Thus the three alliances are determined by a pair of thresholds $\alpha$ and $\beta$. In the following, we present an approach to thresholds based on the theory of triangular fuzzy decision-theoretic rough sets [19]. First, we propose the concept of triangular fuzzy loss function which fits our model. Second, we use the area of expected loss to measure the concrete expected losses. Third, we employ a theorem to illustrate how to form decision rules and obtain the thresholds $\alpha$ and $\beta$.

**Definition 4.12.** Let $(U, A, V, f)$ be a TFIS for conflict analysis, then the *triangular fuzzy loss function* is defined as shown in Table 13.

**Remark 4.13.** (1) $X$ and $\neg X$ represent two states of the agent $x_i$, $a_P$, $a_B$ and $a_N$ denote three actions of agent $x_i$, respectively. $\tilde{\lambda}_{PP}$, $\tilde{\lambda}_{BP}$ and $\tilde{\lambda}_{NP}$ imply the losses of taking actions $a_P$, $a_B$ and $a_N$ when the agent belongs to $X$; similarly, $\tilde{\lambda}_{PN}$, $\tilde{\lambda}_{BN}$ and

**Table 13**
Triangular fuzzy loss function.

| Action | $X$ | $\neg X$ |
|---|---|---|
| $a_P$ | $\tilde{\lambda}_{PP} = (l_{PP}, m_{PP}, u_{PP})$ | $\tilde{\lambda}_{PN} = (l_{PN}, m_{PN}, u_{PN})$ |
| $a_B$ | $\tilde{\lambda}_{BP} = (l_{BP}, m_{BP}, u_{BP})$ | $\tilde{\lambda}_{BN} = (l_{BN}, m_{BN}, u_{BN})$ |
| $a_N$ | $\tilde{\lambda}_{NP} = (l_{NP}, m_{NP}, u_{NP})$ | $\tilde{\lambda}_{NN} = (l_{NN}, m_{NN}, u_{NN})$ |

$\tilde{\lambda}_{NN}$ stand for the losses of taking actions $a_P$, $a_B$ and $a_N$ when the agent belongs to $\neg X$, where $\tilde{\lambda}_{PP}$, $\tilde{\lambda}_{BP}$, $\tilde{\lambda}_{NP}$, $\tilde{\lambda}_{PN}$, $\tilde{\lambda}_{BN}$ and $\tilde{\lambda}_{NN}$ are TFNs between 0 and 1.

(2) $\tilde{\lambda}_{PP}$, $\tilde{\lambda}_{BP}$, $\tilde{\lambda}_{NP}$, $\tilde{\lambda}_{PN}$, $\tilde{\lambda}_{BN}$ and $\tilde{\lambda}_{NN}$ satisfy the following constraints

$$\begin{aligned} &l_{PP} \leq l_{BP} < l_{NP}, m_{PP} \leq m_{BP} < m_{NP}, u_{PP} \leq u_{BP} < u_{NP}; \\ &l_{NN} \leq l_{BN} < l_{PN}, m_{NN} \leq m_{BN} < m_{PN}, u_{NN} \leq u_{BN} < u_{PN}. \end{aligned} \tag{4.1}$$

$$\begin{aligned} &(u_{PN} - l_{PN}) + (l_{BN} - u_{BN}) \geq 0, (u_{BP} - l_{BP}) + (l_{PP} - u_{PP}) > 0; \\ &(u_{BN} - l_{BN}) + (l_{NN} - u_{NN}) \geq 0, (u_{NP} - l_{NP}) + (l_{BP} - u_{BP}) > 0; \\ &(u_{PN} - l_{PN}) + (l_{NN} - u_{NN}) \geq 0, (u_{NP} - l_{NP}) + (l_{PP} - u_{PP}) > 0. \end{aligned} \tag{4.2}$$

(3) Taking $\tilde{\lambda}_{PP} = (l_{PP}, m_{PP}, u_{PP})$ as an example, $m_{PP}$ represents the most possible loss degree. And $l_{PP}$ and $u_{PP}$ denote the possible minimum and maximum loss degrees, respectively. We can explain other elements of Table 13 in a similar way.

We now propose the concept of expected loss to facilitate the decision procedure.

**Definition 4.14.** *Expected loss*, denoted by $\tilde{E}(a_j|x_i)$, can be defined as

$$\tilde{E}(a_j|x_i) = \bar{T}(x_i)\tilde{\lambda}_{jP} \oplus (1 - \bar{T}(x_i))\tilde{\lambda}_{jN},$$

where $\tilde{\lambda}_{jP} = (l_{jP}, m_{jP}, u_{jP})$, $\tilde{\lambda}_{jN} = (l_{jN}, m_{jN}, u_{jN})$. $(i = 1, 2, \cdots, s;\ j = P, B, N.)$

**Remark 4.15.** (1) By Remarks 4.9 and 4.13, we have $\bar{T}(x_i) \in [0, 1]$, $\tilde{\lambda}_{jP}$ and $\tilde{\lambda}_{jN}$ are TFNs between 0 and 1. Therefore, $\tilde{E}(a_j|x_i)$ are TFNs between 0 and 1. In the following, we consider the area of $\tilde{E}(a_j|x_i)$ as the concrete expected loss.

(2) The *value* of expected loss, denoted by $E(a_j|x_i)$, is defined as

$$E(a_j|x_i) = S(\tilde{E}(a_j|x_i)) = \tfrac{1}{2}[(u_{jP} + l_{jN} - u_{jN} - l_{jP})\bar{T}(x_i) + u_{jN} - l_{jN}]. \tag{4.3}$$

In fact, the idea of $E(a_j|x_i)$ comes from $\tilde{E}(a_j|x_i)$. And on the contrary, $E(a_j|x_i)$ can effectively measure $\tilde{E}(a_j|x_i)$. Next we present how to obtain (4.3). Using the addition and scalar multiplication operations of TFNs (see Definition 2.5), we can further calculate $\tilde{E}(a_j|x_i)$ as follows.

$$\begin{aligned} \tilde{E}(a_j|x_i) =& \bar{T}(x_i)(l_{jP}, m_{jP}, u_{jP}) \oplus (1 - \bar{T}(x_i))(l_{jN}, m_{jN}, u_{jN}) \\ =& (l_{jP}\bar{T}(x_i) + l_{jN}(1 - \bar{T}(x_i)), m_{jP}\bar{T}(x_i) + m_{jN}(1 - \bar{T}(x_i)), u_{jP}\bar{T}(x_i) + u_{jN}(1 - \bar{T}(x_i))). \end{aligned}$$

We assume $l_E = l_{jP}\bar{T}(x_i) + l_{jN}(1 - \bar{T}(x_i))$, $m_E = m_{jP}\bar{T}(x_i) + m_{jN}(1 - \bar{T}(x_i))$ and $u_E = u_{jP}\bar{T}(x_i) + u_{jN}(1 - \bar{T}(x_i))$. Then $\tilde{E}(a_j|x_i) = (l_E, m_E, u_E)$ are TFNs between 0 and 1, where $m_E$ denotes the most possible expected loss, $l_E$ and $u_E$ imply the possible minimum and maximum expected losses. And the area of $\tilde{E}(a_j|x_i)$ is $S(\tilde{E}(a_j|x_i)) = E(a_j|x_i) = \frac{1}{2}[(u_{jP} + l_{jN} - u_{jN} - l_{jP})\bar{T}(x_i) + u_{jN} - l_{jN}]$, where $S(A)$ denotes the area of $A$.

Based on the aforementioned triangular fuzzy loss function and expected loss, we now present the decision rules by Theorem 4.16.

**Theorem 4.16.** *Let $S = (U, A, V, f)$ be a TFIS for conflict analysis, $\alpha$ and $\beta$ two thresholds satisfying $0 \leq \beta < \alpha \leq 1$. The losses $\tilde{\lambda}_{PP}$, $\tilde{\lambda}_{PN}$, $\tilde{\lambda}_{BP}$, $\tilde{\lambda}_{BN}$, $\tilde{\lambda}_{NP}$ and $\tilde{\lambda}_{NN}$ satisfy (4.1) and (4.2). The following three assertions hold.*

*(1) If $\bar{T}(x_i) \geq \alpha$, then $x_i \in POA_{(\alpha,\beta)}(U)$;*

*(2) If $\beta < \bar{T}(x_i) < \alpha$, then $x_i \in CTA_{(\alpha,\beta)}(U)$;*

*(3) If $\bar{T}(x_i) \leq \beta$, then $x_i \in NEA_{(\alpha,\beta)}(U)$,*

*where*

$$\alpha = \frac{(u_{PN} - l_{PN}) + (l_{BN} - u_{BN})}{(u_{PN} - l_{PN}) + (l_{BN} - u_{BN}) + (u_{BP} - l_{BP}) + (l_{PP} - u_{PP})};$$
$$\beta = \frac{(u_{BN} - l_{BN}) + (l_{NN} - u_{NN})}{(u_{BN} - l_{BN}) + (l_{NN} - u_{NN}) + (u_{NP} - l_{NP}) + (l_{BP} - u_{BP})}; \tag{4.4}$$
$$\gamma = \frac{(u_{PN} - l_{PN}) + (l_{NN} - u_{NN})}{(u_{PN} - l_{PN}) + (l_{NN} - u_{NN}) + (u_{NP} - l_{NP}) + (l_{PP} - u_{PP})}.$$

**Proof.** From Definition 4.14, we have expected losses $\tilde{E}(a_P|x_i)$, $\tilde{E}(a_B|x_i)$ and $\tilde{E}(a_N|x_i)$ associated with taking the individual actions for the agent $x_i$ as follows.

$$\tilde{E}(a_P|x_i) = \bar{T}(x_i)(l_{PP}, m_{PP}, u_{PP}) \oplus (1 - \bar{T}(x_i))(l_{PN}, m_{PN}, u_{PN});$$
$$\tilde{E}(a_B|x_i) = \bar{T}(x_i)(l_{BP}, m_{BP}, u_{BP}) \oplus (1 - \bar{T}(x_i))(l_{BN}, m_{BN}, u_{BN});$$
$$\tilde{E}(a_N|x_i) = \bar{T}(x_i)(l_{NP}, m_{NP}, u_{NP}) \oplus (1 - \bar{T}(x_i))(l_{NN}, m_{NN}, u_{NN}).$$

By (4.3), the value of $\tilde{E}(a_P|x_i)$, $\tilde{E}(a_B|x_i)$ and $\tilde{E}(a_N|x_i)$ are as follows.

$$E(a_P|x_i) = \frac{1}{2}[(u_{PP} + l_{PN} - u_{PN} - l_{PP})\bar{T}(x_i) + u_{PN} - l_{PN}];$$
$$E(a_B|x_i) = \frac{1}{2}[(u_{BP} + l_{BN} - u_{BN} - l_{BP})\bar{T}(x_i) + u_{BN} - l_{BN}];$$
$$E(a_N|x_i) = \frac{1}{2}[(u_{NP} + l_{NN} - u_{NN} - l_{NP})\bar{T}(x_i) + u_{NN} - l_{NN}].$$

The Bayesian decision procedure suggests the following minimum-cost decision rules.
$(P)$ If $E(a_P|x_i) \le E(a_B|x_i)$ and $E(a_P|x_i) \le E(a_N|x_i)$, then $x_i \in \mathrm{POA}_{(\alpha,\beta)}(U)$;
$(B)$ If $E(a_B|x_i) \le E(a_P|x_i)$ and $E(a_B|x_i) \le E(a_N|x_i)$, then $x_i \in \mathrm{CTA}_{(\alpha,\beta)}(U)$;
$(N)$ If $E(a_N|x_i) \le E(a_P|x_i)$ and $E(a_N|x_i) \le E(a_B|x_i)$, then $x_i \in \mathrm{NEA}_{(\alpha,\beta)}(U)$.

Since the triangular fuzzy loss function satisfies (4.1) and (4.2), we can simplify the rules $(P)$, $(B)$, $(N)$ as follows.
$(P)$ If $\bar{T}(x_i) \ge \alpha$ and $\bar{T}(x_i) \ge \gamma$, then $x_i \in \mathrm{POA}_{(\alpha,\beta)}(U)$;
$(B)$ If $\beta < \bar{T}(x_i) < \alpha$, then $x_i \in \mathrm{CTA}_{(\alpha,\beta)}(U)$;
$(N)$ If $\bar{T}(x_i) \le \beta$ and $T(x_i) \le \gamma$, then $x_i \in \mathrm{NEA}_{(\alpha,\beta)}(U)$, where

$$\alpha = \frac{(u_{PN} - l_{PN}) + (l_{BN} - u_{BN})}{(u_{PN} - l_{PN}) + (l_{BN} - u_{BN}) + (u_{BP} - l_{BP}) + (l_{PP} - u_{PP})};$$
$$\beta = \frac{(u_{BN} - l_{BN}) + (l_{NN} - u_{NN})}{(u_{BN} - l_{BN}) + (l_{NN} - u_{NN}) + (u_{NP} - l_{NP}) + (l_{BP} - u_{BP})};$$
$$\gamma = \frac{(u_{PN} - l_{PN}) + (l_{NN} - u_{NN})}{(u_{PN} - l_{PN}) + (l_{NN} - u_{NN}) + (u_{NP} - l_{NP}) + (l_{PP} - u_{PP})}.$$

If $0 \le \beta < \gamma < \alpha \le 1$, we can further simplify the rules $(P)$, $(B)$, $(N)$ as follows.
$(P)$ If $\bar{T}(x_i) \ge \alpha$, then $x_i \in \mathrm{POA}_{(\alpha,\beta)}(U)$;
$(B)$ If $\beta < \bar{T}(x_i) < \alpha$, then $x_i \in \mathrm{CTA}_{(\alpha,\beta)}(U)$;
$(N)$ If $\bar{T}(x_i) \le \beta$, then $x_i \in \mathrm{NEA}_{(\alpha,\beta)}(U)$,
which completes the proof. □

We now continue to discuss the previous example.

**Example 4.17.** (Continued from Example 4.10) From Example 4.10, we have

$$\bar{T}(x_1) = 0.71, \bar{T}(x_2) = 0.50, \bar{T}(x_3) = 0.47, \bar{T}(x_4) = 0.38, \bar{T}(x_5) = 0.43, \bar{T}(x_6) = 0.59.$$

And the triangular fuzzy loss function for $U$ is depicted as shown in Table 14.

After verification, the triangular fuzzy loss function in Table 12 satisfies (4.1) and (4.2). And by (4.4), we can calculate thresholds $\alpha, \beta, \gamma$ as follows.

$$\alpha = \frac{(u_{PN} - l_{PN}) + (l_{BN} - u_{BN})}{(u_{PN} - l_{PN}) + (l_{BN} - u_{BN}) + (u_{BP} - l_{BP}) + (l_{PP} - u_{PP})} = 0.50;$$
$$\beta = \frac{(u_{BN} - l_{BN}) + (l_{NN} - u_{NN})}{(u_{BN} - l_{BN}) + (l_{NN} - u_{NN}) + (u_{NP} - l_{NP}) + (l_{BP} - u_{BP})} = 0.40;$$
$$\gamma = \frac{(u_{PN} - l_{PN}) + (l_{NN} - u_{NN})}{(u_{PN} - l_{PN}) + (l_{NN} - u_{NN}) + (u_{NP} - l_{NP}) + (l_{PP} - u_{PP})} = 0.43.$$

**Table 14**
Triangular fuzzy loss function for $U$.

| Action | $X$ | $\neg X$ |
|---|---|---|
| $a_P$ | $\tilde{\lambda}_{PP} = (0.22, 0.31, 0.46)$ | $\tilde{\lambda}_{PN} = (0.41, 0.45, 0.50)$ |
| $a_B$ | $\tilde{\lambda}_{BP} = (0.39, 0.49, 0.64)$ | $\tilde{\lambda}_{BN} = (0.40, 0.43, 0.48)$ |
| $a_N$ | $\tilde{\lambda}_{NP} = (0.40, 0.53, 0.68)$ | $\tilde{\lambda}_{NN} = (0.39, 0.42, 0.45)$ |

From Theorem 4.16, the positive, central, negative alliances of $U$ are as follows.

$$\mathrm{POA}_{(0.50,0.40)}(U) = \{x_i | \bar{T}(x_i) \geq 0.50\} = \{x_1, x_2, x_6\};$$

$$\mathrm{CTA}_{(0.50,0.40)}(U) = \{x_i | 0.40 < \bar{T}(x_i) < 0.50\} = \{x_3, x_5\};$$

$$\mathrm{NEA}_{(0.50,0.40)}(U) = \{x_i | \bar{T}(x_i) \leq 0.40\} = \{x_4\}.$$

According to the total attitudes of agents to all issues, three alliances are constructed. $x_1, x_2$ and $x_6$ form an alliance whose members show positive attitudes to all issues. $x_3$ and $x_5$ have similar central attitudes, so they also constitute an alliance. $x_4$ alone has a negative attitude to all issues.

## 5. Conclusions

In this paper, we establish the model of three-way conflict analysis on TFISs and further use $\Delta S$ to depict concrete attitudes of agents about issues. Second, in view of different importance of issues, we attach fuzzy weights to corresponding issues according to TFSJMs. Third, we determine total attitudes of agents to all issues, which is helpful for obtaining the tri-partition of agents. Finally, we demonstrate how to calculate the thresholds $\alpha$ and $\beta$ using the theory of triangular fuzzy decision-theoretic rough sets.

There are three points worth considering in the future: One is the measure of determining the attitudes of TFNs in the TFIS. In this paper, we consider the relative area $\Delta S$ as the measure. The main reason is that $\Delta S$ is not only intuitive, but easy to understand. Seeking other measures to depict the attitudes of TFNs is an interesting topic. Another point is related to thresholds. We in this paper compute the thresholds in light of triangular fuzzy decision-theoretic rough sets. In [18], Li et al. proposed an algorithm for computing thresholds, which provided a new idea for obtaining thresholds. Based on this, designing an algorithm to calculate the thresholds in our model is also a significant research problem. The last point is that we only study three-way conflict analysis model on complete TFISs, leaving the research on incomplete TFISs in the future.

## Compliance with ethical standards

This article does not contain any studies with human participants or animals performed by any of the authors.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgements

This work was supported by the National Natural Science Foundation of China (Nos. 61772019, 61976244, 61906154) and the Shaanxi Province Natural Science Foundation Research Project (No. 2018JQ6099).

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