Response to Reviewers

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Granular Computing: Trends, Insights, and Bibliometric Review

We thank all reviewers for their constructive comments and insightful suggestions, which have significantly improved the quality, clarity, and rigor of the manuscript. All comments were carefully addressed, and substantial revisions were made in the updated version. Below is our point-by-point response. Excerpts from the updated manuscript are shown in italics.

Reviewer 1

General Evaluation: We thank the reviewer for the comprehensive assessment and helpful comments regarding theoretical clarity, methodological explanation, and experimental justification.

1.1. "There is a big disproportion of statistical analysis of the GrC literature in comparison with the characterization of the GrC domain. The characterization of the GrC domain should be substantially extended and elaborated."

We sincerely thank the reviewer for this insightful comment. In response, we have substantially expanded the characterization of the Granular Computing (GrC) domain to better balance conceptual foundations with bibliometric analysis. The revised manuscript now includes an extended theoretical background, a structured taxonomy of core GrC models (interval, rough, fuzzy, and shadowed sets) with formal definitions and illustrative examples, and an expanded discussion of key frameworks such as justifiable granularity and three-way decision theory (Sections 2–2.2). These revisions provide clearer theoretical grounding for the statistical analyses presented later in the paper.

Interval Analysis and Granular Computing

Interval analysis and granular computing are integrated by treating intervals as fundamental information granules and using interval arithmetic with guaranteed bounds Leite et al., 2010; H. Liu et al., 2017; Skowron, 2023a. Data and model parameters are represented as intervals (or hyperboxes), enabling uncertainty propagation, error control, and multiresolution analysis Jaulin and Walter, 2001; Skowron, 2023a. Core methods include hyperbox-based classification, interval arithmetic with enclosure techniques, branch-and-bound bounded-error estimation, distributed interval structures, justifiable granularity design, evolving interval learners, and interval-valued neural models trained with evolutionary algorithms Cimino et al., 2014; Kreinovich, 2008; Kreinovich and Aló, 2002; Leite et al., 2010, 2016; Tahayori et al., 2007; D. Wang et al., 2016. This supports robust applications in classification, regression, parameter estimation, nonlinear system solving, streaming model adaptation, sensor fusion, and optimization—offering interpretability and guaranteed uncertainty bounds Jaulin and Walter, 2001; Skowron, 2023a; Tahayori et al., 2007; D. Wang et al., 2016.

Cimino, M. G., Lazzerini, B., Marcelloni, F., & Pedrycz, W. (2014). Genetic interval neural networks for granular data regression. *Information Sciences*, *257*, 313–330.

Jaulin, L., & Walter, E. (2001). Nonlinear bounded-error parameter estimation using interval computation. In *Granular computing: An emerging paradigm* (pp. 58–71). Springer.

Kreinovich, V. (2008). Interval computations as an important part of granular computing: An introduction. *Handbook of Granular Computing*, 1–31.

Kreinovich, V., & Aló, R. (2002). Interval mathematics for analysis of multi-level granularity. *Archives of Control Sciences*, *12*(4), 323–350.

Leite, D., Costa Jr, P., & Gomide, F. (2010). Granular approach for evolving system modeling. *International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, 340–349.

Leite, D., Costa Jr, P., & Gomide, F. (2016). A review on evolving interval and fuzzy granular systems. *Learning and nonlinear models*.

Liu, H., Li, J., Guo, H., & Liu, C. (2017). Interval analysis-based hyperbox granular computing classification algorithms. *Iranian Journal of Fuzzy Systems*, *14*(5), 139–156.

Skowron, A. (2023a). Informational granules in interactive granular computing. *Computer Sciences & Mathematics Forum*, *8*(1), 39.

Tahayori, H., Pedrycz, W., & Degli Antoni, G. (2007). Distributed intervals: A formal framework for information granulation. *2007 Canadian Conference on Electrical and Computer Engineering*, 1409–1412.

Wang, D., Pedrycz, W., & Li, Z. (2016). Design of granular interval-valued information granules with the use of the principle of justifiable granularity and their applications to system modeling of higher type. *Soft Computing*, *20*(6), 2119–2134.

Rough Sets and Granular Computing

Rough sets integrate with granular computing by treating lower and upper approximations as operations over information granules within multilevel frameworks Artiemjew, 2020; Sun et al., 2014; Zadeh, 2007. Objects are grouped into granules (e.g., equivalence classes, neighborhoods, fuzzy relations), and rough approximations are generalized for probabilistic and fuzzy uncertainty modeling Inuiguchi et al., 2003; Sun et al., 2014. Multilevel zooming and quotient-space methods enable reasoning at different granular scales, while granule-based rule induction and reduct computation produce compact, interpretable decision models Skowron and Ślęzak, 2022; Yao, 1999; Zadeh, 2007. Entropy and granulation-based measures guide uncertainty quantification and attribute selection in incomplete systems Skowron and Stepaniuk, 2008. Applications include classification, rule extraction, feature reduction, multiscale knowledge representation, and engineering tasks like sensor fusion and pattern analysis Inuiguchi et al., 2003; Lin et al., 2013; Skowron and Ślęzak, 2022; Skowron and Stepaniuk, 2008.

Artiemjew, P. (2020). About granular rough computing—overview of decision system approximation techniques and future perspectives. *Algorithms*, *13*(4), 79.

Inuiguchi, M., Hirano, S., & Tsumoto, S. (2003). *Rough set theory and granular computing* (Vol. 125). Springer.

Lin, T. Y., Yao, Y. Y., & Zadeh, L. A. (2013). *Data mining, rough sets and granular computing* (Vol. 95). Physica.

Skowron, A., & Ślęzak, D. (2022). Rough sets turn 40: From information systems to intelligent systems. *2022 17th Conference on Computer Science and Intelligence Systems (FedCSIS)*, 23–34.

Skowron, A., & Stepaniuk, J. (2008). Rough sets and granular computing: Toward rough-granular computing. *Handbook of Granular Computing, John Wiley & Sons*, 425–448.

Sun, L., Xu, J., & Xu, T. (2014). Information entropy and information granulation-based uncertainty measures in incomplete information systems. *Applied Mathematics & Information Sciences*, *8*(4), 2073.

Yao, Y. (1999). Rough sets, neighborhood systems and granular computing. *Engineering solutions for the next millennium. 1999 IEEE Canadian conference on electrical and computer engineering (Cat. No. 99TH8411)*, *3*, 1553–1558.

Zadeh, L. A. (2007). Granular computing and rough set theory. *International Conference on Rough Sets and Intelligent Systems Paradigms*, 1–4.

Fuzzy Sets and Granular Computing

Fuzzy sets serve as information granules for representing imprecise, user-centric concepts, with granular computing offering multilevel abstraction, organization, and processing to manage uncertainty and complexity Pedrycz, 2008; Zadeh, 1997a. Their integration happens via fuzzy granulation, fuzzy-rough hybrid models, granular-ball representations, and granule-based preprocessing, enabling interpretable, efficient learning Maji and Pal, 2012; Xia et al., 2022. Fuzzy-rough systems combine graded membership, boundary approximations, and entropy-based measures to boost robustness and rule extraction Bello et al., 2008; Maji and Pal, 2012, while granular-ball fuzzy models enhance scalability and noise tolerance in classification tasks like SVMs Xia et al., 2022. Applications include classification, pattern recognition, data mining, and control systems, where granule compression, multilevel reasoning, and uncertainty quantification improve robustness, interpretability, and efficiency Pedrycz and Vukovich, 1999; Stefanini et al., 2008; Xia et al., 2022.

Bello, R., Falcón, R., & Pedrycz, W. (2008). *Granular computing: At the junction of rough sets and fuzzy sets* (Vol. 224). Springer Science & Business Media.

Maji, P., & Pal, S. K. (2012). Rough-fuzzy hybridization and granular computing.

Pedrycz, W. (2008). Fuzzy sets as a user-centric processing framework of granular computing. *Handbook of Granular Computing*, 97–139.

Pedrycz, W., & Vukovich, G. (1999). Granular computing in the development of fuzzy controllers. *International journal of intelligent systems*, *14*(4), 419–447.

Stefanini, L., Sorini, L., Guerra, M. L., Pedrycz, W., Skowron, A., & Kreinovich, V. (2008). Fuzzy numbers and fuzzy arithmetic. *Handbook of granular computing*, *12*, 249–284.

Xia, S., Lian, X., Wang, G., Gao, X., & Shao, Y. (2022). Granular-ball fuzzy set and its implementation in svm. *arXiv preprint arXiv:2210.11675*.

Zadeh, L. A. (1997a). Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic. *Fuzzy sets and systems*, *90*(2), 111–127.

Shadowed Sets and Granular Computing

Shadowed sets are three-valued abstractions of fuzzy sets classifying elements into full membership, nonmembership, and an intermediate shadow (uncertainty) region Pedrycz, 2005a, 2009. Introduced by Pedrycz to localize uncertainty and simplify fuzzy information, they transform fuzzy membership through an optimization process yielding a concise and interpretable three-way representation. Research has established properties and developed algorithms for stable partitions Pedrycz, 2009; Zhou et al., 2019. Extensions such as entropy-based approximations and complex shadowed sets expand decision-making and clustering applications Campagner et al., 2020; H. Wang et al., 2018. Shadowed sets facilitate three-way decision models, fuzzy clustering, preprocessing, and hybrid fuzzy systems, providing efficient, uncertainty-aware granular computing applications Pedrycz, 2005b; Yang, Wang, et al., 2024.

Campagner, A., Dorigatti, V., & Ciucci, D. (2020). Entropy-based shadowed set approximation of intuitionistic fuzzy sets. *International Journal of Intelligent Systems*, *35*(12), 2117–2139.

Pedrycz, W. (2005a). Granular computing with shadowed sets. *International Workshop on Rough Sets, Fuzzy Sets, Data Mining, and Granular-Soft Computing*, 23–32.

Pedrycz, W. (2005b). Granular computing with shadowed sets. *Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing: 10th International Conference, RSFDGrC 2005, Regina, Canada, August 31-September 3, 2005, Proceedings, Part I 10*, 23–32.

Pedrycz, W. (2009). From fuzzy sets to shadowed sets: Interpretation and computing. *international journal of intelligent systems*, *24*(1), 48–61.

Wang, H., He, S., Pan, X., & Li, C. (2018). Shadowed sets-based linguistic term modeling and its application in multi-attribute decision-making. *Symmetry*, *10*(12), 688.

Yang, J., Wang, X., Wang, G., Zhang, Q., Zheng, N., & Wu, D. (2024). Fuzziness-based three-way decision with neighborhood rough sets under the framework of shadowed sets. *IEEE Transactions on Fuzzy Systems*, *32*(9), 4976–4988.

Zhou, J., Gao, C., Pedrycz, W., Lai, Z., & Yue, X. (2019). Constrained shadowed sets and fast optimization algorithm. *International Journal of Intelligent Systems*, *34*(10), 2655–2675.

1.2. "The current stage of the GrC foundations should be better discussed. In particular, the attempts to formalize the concept of granules should be discussed, rather than being restricted to examples of granules in different domains. Goals of searching for relevant granules should be discussed emphasizing problems and challenges."

We Thank the reviewer on their insight. Speaking about the challenges of using granular computing on a broader scope is an important addition, and it is an oversight on our side not to mention it. A new section 2.3 was added to briefly outline the major issues and challenges that could be encountered in this field. We are aware that there are further and deeper discussions to be had, but for the sake of brevity, we contained this discussion in a part of the significant topics section on page 8. **New additions:**

As noted earlier, the use of granular computing has evolved through multiple stages of development and application-specific definitions. However, these definitions consistently hold that granules are clusters of information that possess properties such as cardinality, specificity, and coverage Yao, 2008. Definitions also maintain that granules provide approximations of incomplete, vague, imprecise, or uncertain data, thereby deepening comprehension and facilitating better pattern recognition Kang and Miao, 2016, which is the ultimate goal of granulation and granular computing.

Achieving these benefits is not without challenges. Chief among them is the need for compatibility among data, methods, and applications to achieve the desired results Ross, 2025. Furthermore, granulation can produce overlapping and uncertain class boundaries, particularly in iterative situations Muda, 2022. Another major issue is the lack of standardization in the field of GrC, resulting in systems that lack interoperability and regulation. Achieving the benefits does not come without challenges. Chief among them is the compatibility between data, method, and application, necessary to achieve the desired results Ross, 2025. Furthermore, granulation can face and sometimes produce overlap and uncertainty of classes, particularly in iterative situations Muda, 2022. Another major issue faced is the lack of standardization in the field of GrC, resulting in systems that lack interoperability and regulation D. Liu et al., 2023.

Kang, X., & Miao, D. (2016). A study on information granularity in formal concept analysis based on concept-bases. *Knowledge-Based Systems*, *105*, 147–159.

Liu, D., Shangguan, X., Wei, K., Wu, C., Zhao, X., Sun, Q., Zhang, Y., & Bai, R. (2023). Research on the standardization strategy of granular computing. *International Journal of Cognitive Computing in Engineering*, *4*, 340–348.

Muda, M. Z. (2022). *Interpretability studies in granular computing* [Doctoral dissertation, University of Sheffield].

Ross, O. M. (2025). Foundations of quantum granular computing with effect-based granules, algebraic properties and reference architectures. *arXiv preprint arXiv:2511.22679*.

Yao, Y. (2008). Granular computing: Past, present and future. *2008 IEEE International Conference on Granular Computing*, 80–85.

1.3. "The emerging domain of Interactive Granular Computing should be discussed."

We thank you for the observation. As our intention was to highlight significant recurring topics appearing in our thematic analysis, as well as recurring keywords. The use of interactive granules is an interesting topic that will add to our paper. We have added a new topic to our discussion of significant topics section on page 8 as follows. **New additions:**

Interactive GrC expands on traditional granular computing by including interactions between imperfect, granular information (information granules) and physical entities (considered over time or space), called "hunks" Skowron and Jankowski, 2016a. It models complex intelligent systems through interactive processes using structures known as complex granules (c-granules). These c-granules comprise three components: the soft-suit (information granules like descriptors or fuzzy sets), the link-suit (relations connecting information to physical entities), and the hard-suit (physical hunks). Computations occur within agent societies in interactive intelligent systems (IIS), approximating vague concepts via rough-set-inspired methods amid uncertainty, vagueness, and noise. IGrC enables modeling complex adaptive systems, process mining, risk management, and IoT optimization by handling human-like granularity in problem-solving. It combines rough sets, fuzzy sets, and multi-agent systems to facilitate perceptual approximations during real-time interactions. In data science, it links sensor data, implementational granules, and information-processing layers within a perception-grounded computing paradigm Skowron, 2023b; Skowron and Wasilewski, 2011. Future directions include formalizing dynamic attributes and evolving approximations, integrating rough-set foundations with interaction mechanisms, and developing control and prediction strategies Jankowski, 2017; Skowron and Jankowski, 2016b. Efforts also include implementing distributed c-granule networks and deploying IGrC decision-support tools in real-world environments Dutta, 2019; Skowron, 2023b. IGrC supports modeling complex adaptive systems, process mining, risk management, and IoT optimization by handling human-like granularity in problem-solving. It integrates rough sets, fuzzy sets, and multi-agent systems for perceptual approximations in real-time interactions .

In data science, it links sensor data, implementational granules, and information-processing layers as a perception-grounded computing paradigm Skowron, 2023b; Skowron and Wasilewski, 2011. Future directions include formalizing dynamic attributes and evolving approximations, integrating rough-set foundations with interaction mechanisms, and developing control and prediction strategies Jankowski, 2017; Skowron and Jankowski, 2016b, alongside efforts to implement distributed c-granule networks and deploy IGrC decision-support tools in real environments Dutta, 2019; Skowron, 2023b.

Dutta, S. (2019). Toward data science computing model: Interactive granular computing (igrc) (short paper). *Proceedings of the 28th International Workshop on Concurrency, Specification and Programming (CS&P 2019)*, *2571*, 101–105. https://ceur-ws.org/Vol-2571/CSP2019_paper_3.pdf

Jankowski, A. (2017). *Interactive granular computations in networks and systems engineering: A practical perspective* (Vol. 13). Springer International Publishing. `https://doi.org/10.1007/978-3-319-57627-5`

Skowron, A. (2023b). Informational granules in interactive granular computing. *Computer Sciences & Mathematics Forum*, *8*(1), 39. `https://doi.org/10.3390/cmsf2023008039`

Skowron, A., & Jankowski, A. (2016a). Interactive computations: Toward risk management in interactive intelligent systems. *Natural Computing*, *15*(3), 465–476.

Skowron, A., & Jankowski, A. (2016b). Rough sets and interactive granular computing. *Fundamenta Informaticae*, *147*(2-3), 371–385. `https://doi.org/10.3233/FI-2016-1413`

Skowron, A., & Wasilewski, P. (2011). Toward interactive rough-granular computing. *Control and Cybernetics*, *40*(2), 213–235.

1.4. "What are the main application domains where GrC helped to make substantial progress? How and why GrC helps solve problems in these domains? This concerns, in particular, the three-way approach."

It It was briefly discussed in sections 2.1 and 2.2 the possible additions the use of granular computing can achieve. A few additions we made in the significant topics section on page 7. **New additions:**
By taking this 3-way approach, decision-making applications gain a reduction in ambiguity, particularly that of classification Yang, Liu, et al., 2024, where this approach can manifest in the form of rejecting, accepting, or a boundary case of classification.

Yang, J., Liu, Z., Xia, S., Wang, G., Zhang, Q., Li, S., & Xu, T. (2024). 3wc-gbnrs++: A novel three-way classifier with granular-ball neighborhood rough sets based on uncertainty. *IEEE Transactions on Fuzzy Systems*, *32*(8), 4376–4387. `https://doi.org/10.1109/TFUZZ.2024.3397697`

1.5. "A discussion about challenges for which GrC can offer proper methodology or tools. For example, how GrC help to discover the relevant granules?"

We have added a new paragraph in the revised manuscript discussing the challenges for which Granular Computing (GrC) offers appropriate methodologies, particularly focusing on how GrC supports the discovery of relevant information granules. The added paragraph highlights the granulation design problem and outlines key approaches such as justifiable granularity, data-driven granulation, and multi-granulation frameworks. This addition has been incorporated in 2.3 of the revised paper.
New additions:
Granular Computing (GrC) provides a computational framework for managing complexity, uncertainty, and interpretability through the use of information granules, defined as groups of entities formed based on similarity or functional relationships Zadeh, 1997b. This paradigm is particularly suitable for high-dimensional, noisy, and multi-scale data environments. A central challenge in GrC is the discovery of relevant granules, often referred to as the granulation design problem. The goal is to construct granules that balance abstraction with representational precision. Overly coarse granules may conceal important patterns, whereas excessively fine granules may reduce generalization capability. One widely adopted approach is the principle of justifiable granularity Pedrycz, 2013, where granules are formed by optimizing the trade-off between coverage and specificity. Granule discovery may also be achieved through data-driven methods such as fuzzy clustering and rough set approximations Bargiela and Pedrycz, 2003; Pawlak, 1991, as well as optimization-based techniques that adjust granule parameters to improve interpretability and predictive performance. Additionally, multi-granulation frameworks enable knowledge representation at multiple abstraction levels, supporting hierarchical reasoning and

cross-scale analysis Pedrycz, 2013. Knowledge-guided granulation further incorporates expert insight to enhance semantic interpretability.

Bargiela, A., & Pedrycz, W. (2003). *Granular computing: An introduction.* Kluwer Academic Publishers.
Pawlak, Z. (1991). *Rough sets: Theoretical aspects of reasoning about data.* Kluwer Academic Publishers.
Pedrycz, W. (2013). *Granular computing: Analysis and design of intelligent systems.* CRC Press.
Zadeh, L. A. (1997b). Towards a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic. *Fuzzy Sets and Systems*, *90*(2), 111–127.

1.6. "References for the main domains of GrC should be completed, e.g. J. Hobbs: Granularity. Proceedings of IJCAI-85, 432-435."

We thank our reviewer for the insight. With the additions and adjustment done through the course of this editing and review process, our knowledge base was expanded greatly and now hopefully reflects the desired level of depth. The informative reference supplied was used in page 2 as so:

... years, months, days, or seconds? Each level of detail (i.e., granularity) can serve its purpose and contribute to solving the problem Hobbs, 1990. To further....

Hobbs, J. R. (1990). Granularity. In *Readings in qualitative reasoning about physical systems* (pp. 542–545). Elsevier.

Reviewer 2

General Evaluation: We thank the reviewer for identifying structural and logical flow issues. Major structural revisions have been applied.

2.1. "However, the manuscript contains critical terminology errors, methodological gaps regarding data cleaning, and significant visual/formatting issues that must be addressed before submission to ensure professional standards. ...'Some wording problems:"

We thank our reviewers for their vigilance and for taking note of our mishaps. With regards to the issues mentioned, they were adjusted accordingly. The issue with the figures axis was addressed as needed. We appreciate the reviewers' valuable feedback. Their suggestion, all the points in this portion of the review, have been fully taken into account and addressed accordingly.

2.2. Contents problems:

We thank you for your review and your requests for clarifications. In reply, here are our clarifications:

- Please clarify formulas (18) and (19)

  We thank the reviewer for the request. Upon examination, we can confirm that Equations (18) and (19) are amply defined and explained in the manuscript, including all variables. They are explained in section 3 on page 9.

  When referring to Centrality, in terms of thematic analysis, is a metric that indicates the influence of a theme within certain thematic groups Giannakos et al., 2020. Specifically, the centrality examined here measures the influence of a theme on a scale of 0-1. It is calculated using equation 1, where e refers to the number of occurrences of k, a keyword belonging to the theme, and h belonging to other themes.

  $$c = 10 \times \sum e_{kh} \tag{1}$$

Density: A theme is conceived through a clustering of keywords. The density of a theme refers to the number of ties between keywords relative to the total number of keywords. Ties here refer to instances of co-occurrences of the keywords throughout the literature. This is shown through equation 2, where i and j are keywords from the theme and T the number of keywords in the theme.

$$d = 100 \sum \frac{e_{ij}}{T}, \tag{2}$$

- The description of QIV is not clear.
  Small adjustment added to section 3 on page 9.

  The high centrality characterizing this quadrant provides a high level of inter-theme relation, but a low density describe low development of each theme internally. Considering the time period being studied, this quadrant can highlight older themes that have a strong influence on the field but lack dedicated interest, categorized as a "bandwagon" (i.e., tailing other themes) in the field.

- Please revise the English according to a native speaker.
  We appreciate the reviewers' valuable feedback. Their suggestion has been fully taken into account and addressed accordingly