COMPUTING MODEL BASED ON INTERACTIVE GRANULAR COMPUTING & ROUGH SETS: TOWARD FOUNDATIONS OF COMPLEX INTELLIGENT SYSTEMS

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To
Professors
Helena Rasiowa
and
Zdzisław Pawlak
in memoriam

AGENDA

Motivations for

- development of computing model creating the basis for the design and analysis of Complex Intelligent Systems (IS's), i.e., Intelligent Systems dealing with complex phenomena

Interactive Granular Computing (IGrC)

- Complex granules (c-granules)
  - Specification (syntax)
  - Abstract and physical semantics
    - States and dynamics
- c-granules with control (in particular societies of such c-granules)
  - Structure of c-granules: from atomic and elementary to networks; IS's – examples of c-granules with control
  - Control as a sub-granule of a given c-granule with control
  - Modeling of perception of situations (objects) in the physical world by control: generation and management (steering) of configurations of sub-granules
  - Granular computations of c-granules with control

- IGrC and Rough Sets (RS)
  - Challenge for control of a given c-granule (IS): The discovery of adaptive complex games used to generate high quality, approximate solutions to problems along granular computations that are steered by the control of c-granule.

Summary

MOTIVATIONS FOR NEW COMPUTING MODEL

NEEDS FOR RELEVANT MODEL OF COMPUTATIONS:

EXAMPLE OF CONCEPT OF MULTI-AGENT SYSTEM

An important open issue in the Agent Based Simulation field is **the lack of an univocal definition of the term "agent"** and of the paradigms and methodology used to build models; in software-engineering, a system of independent programs is considered a multi-agent system, **although there is no clarity about what the term exactly defines**. The term "agent", deriving from the Latin "agens", identifies someone (or something) who acts; the same word can also be used to define a mean through which some action is made or caused. **The term is used in many different fields and disciplines, such as economics, physics, natural sciences, sociology and many others.**

Marco Remondino: Reactive and deliberative agents applied to simulation of socio-economical and biological systems. International Journal of Simulation 6(12-13) 1473-8031 (2025)

THE RELEVANT COMPUTING MODEL: FOUNDATIONS FOR DESIGN AND ANALYSIS OF IS's

Many partial proposals in many different domains exist, e.g., complex (adaptive) systems, multi-agent systems, machine learning, robotics, cognitive science, neuroscience, computational intelligence, web intelligence, natural computing, multisensory learning, cyber-physical systems, Internet of things,…

but we need

the relevant computing model for developing foundations of IS's.

WE PROPOSE IGrC AS SUCH A MODEL

COMPLEX SYSTEMS

Complex system: the elements are difficult to separate. This difficulty arises from the interactions between elements. Without interactions, elements can be separated. But when interactions are relevant, elements co-determine their future states. Thus, the future state of an element cannot be determined in isolation, as it co-depends on the states of other elements, precisely of those interacting with it.

Gershenson, C., Heylighen, F.: How can we think the complex? In: Richardson, K. (Ed.): Managing Organizational Complexity: Philosophy, Theory and Application, pp. 47–61. Information Age Publishing (2005)

COMPLEX SYSTEMS
MODELING PROBLEM

system state ?

H ?

$$\frac{ds}{dt} = H(s, e, t)$$

only partially known at a given moment of time; should be discovered adaptively

environment state ?

F ?

$$\frac{de}{dt} = F(s, e, t)$$

K. Egan, W. Li, R. Carvalho: Automatically discovering ordinary differential equations from data with sparse regression. Communications Physics 7(20) 2024. DOI: 10.1038/s42005-023-01516-2

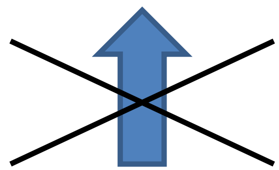
H. Sayama: Introduction to the Modeling and Analysis of Complex Systems" (2015). Milne Open Textbooks 14. https://knightscholar.geneseo.edu/oer-ost/14

COMPLEX SYSTEMS

MODELING PROBLEM

THE BEHAVIOR OF THE *WHOLE* IS NOT DEFINED BY ITS PARTS

$$\frac{ds}{dt} = H(s, e, t) \qquad \frac{de}{dt} = F(s, e, t)$$



s is not defined by $s_1, \ldots, s_k$ only
e is not defined by $e_1, \ldots, e_k$ only
H is not defined by $H_1, \ldots, H_k$ only
F is not defined by $F_1, \ldots, F_k$ only

$$\frac{ds_1}{dt} = H_1(s_1, e_1, t) \qquad \frac{de_1}{dt} = F_1(s_1, e_1, t) \quad \ldots$$

$$\frac{ds_k}{dt} = H_k(s_k, e_k, t) \qquad \frac{de_k}{dt} = F_k\,(s_k, e_k, t)$$

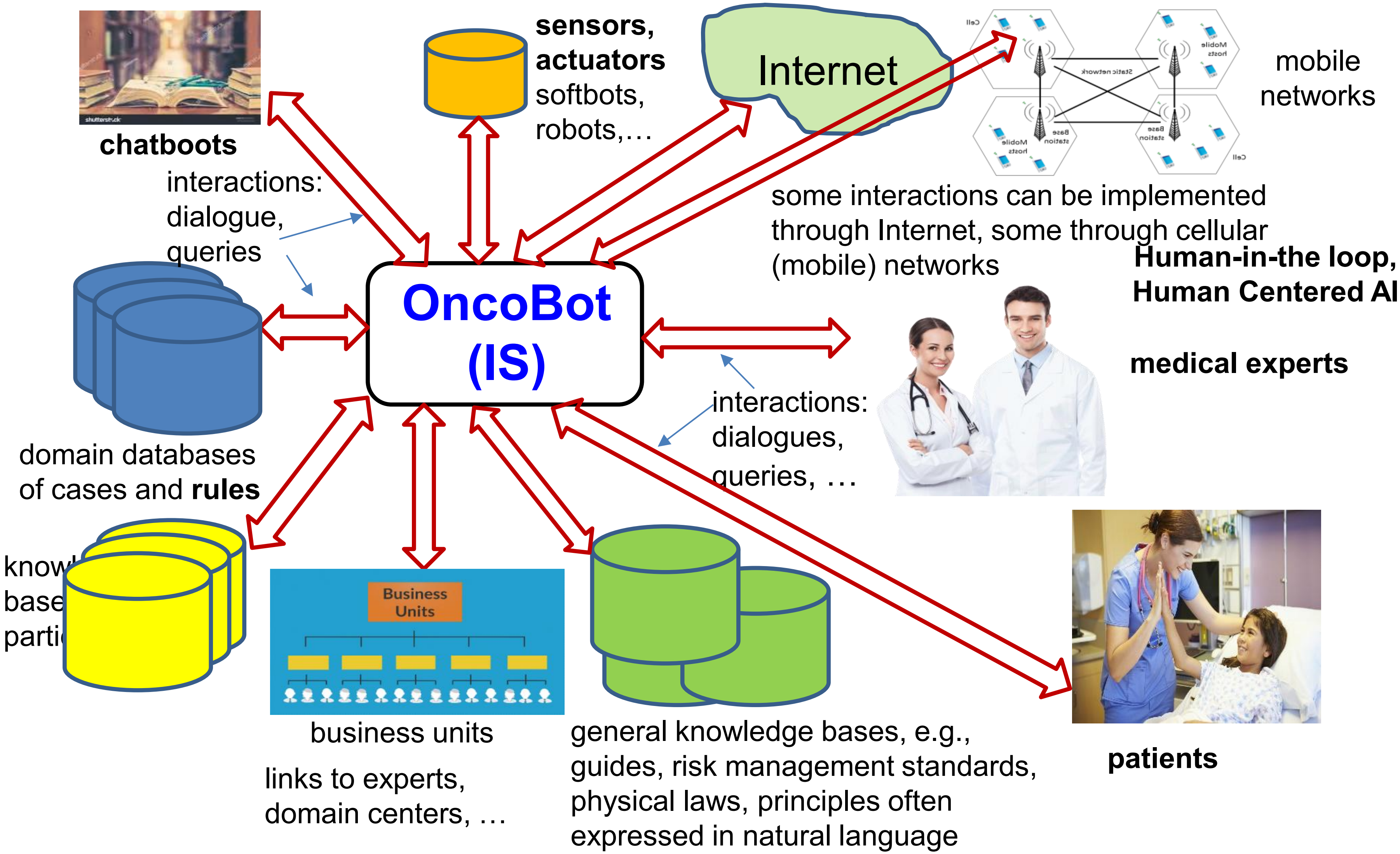
COMPLEX ADAPTIVE SYSTEMS (CAS)

[…] The niche, …, is made up of physical and virtual boundaries that determine the limits of …inter-actions.
Ecosystems, for example, have highly, diverse niches with smells and visual patterns as signals. Governments have departmental hierarchies, with memoranda as signals. Biological cells have a wealth of membranes, with proteins as signals. Markets have traders and specialists who use buy and sell orders as signals. And so it is with other complex adaptive systems. Despite a wealth of data and descriptions concerning different complex adaptive systems, we still know little about how to steer these systems.

- Exhibiting internal boundaries dividing any of such system into a diverse array of semi-autonomous subsystems called agents; agent has a *program* guiding its interactions with other agents and other parts of its environment.
- CAS are signal/boundary systems. The steering of CAS is expressed by modifying signal/boundary hierarchies.
- **Interactions are basic concepts of the approach.** Categories of interactions in signal/boundary systems: diversity, recirculation, niche, and coevolution.

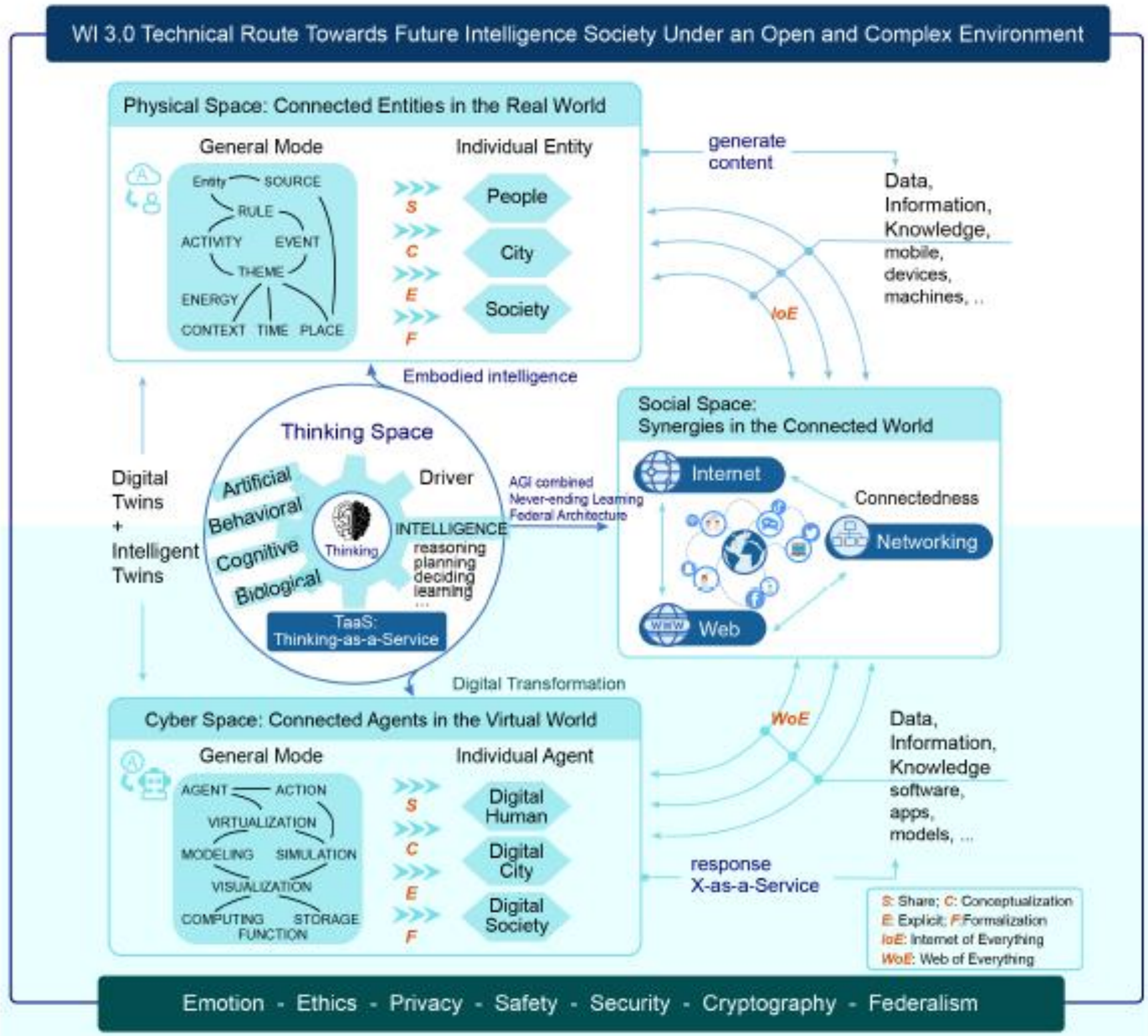
John Holland: Signals and Boundaries. Building Blocks for Complex Adaptive Systems MIT Press 2012.

COMPLEX INTELLIGENT SYSTEM (IS) i.e. INTELLIGENT SYSTEM DEALING WITH COMPLEX PHENOMENA IN THE REAL WORLD: EXEMPLARY PROJECT OncoBot

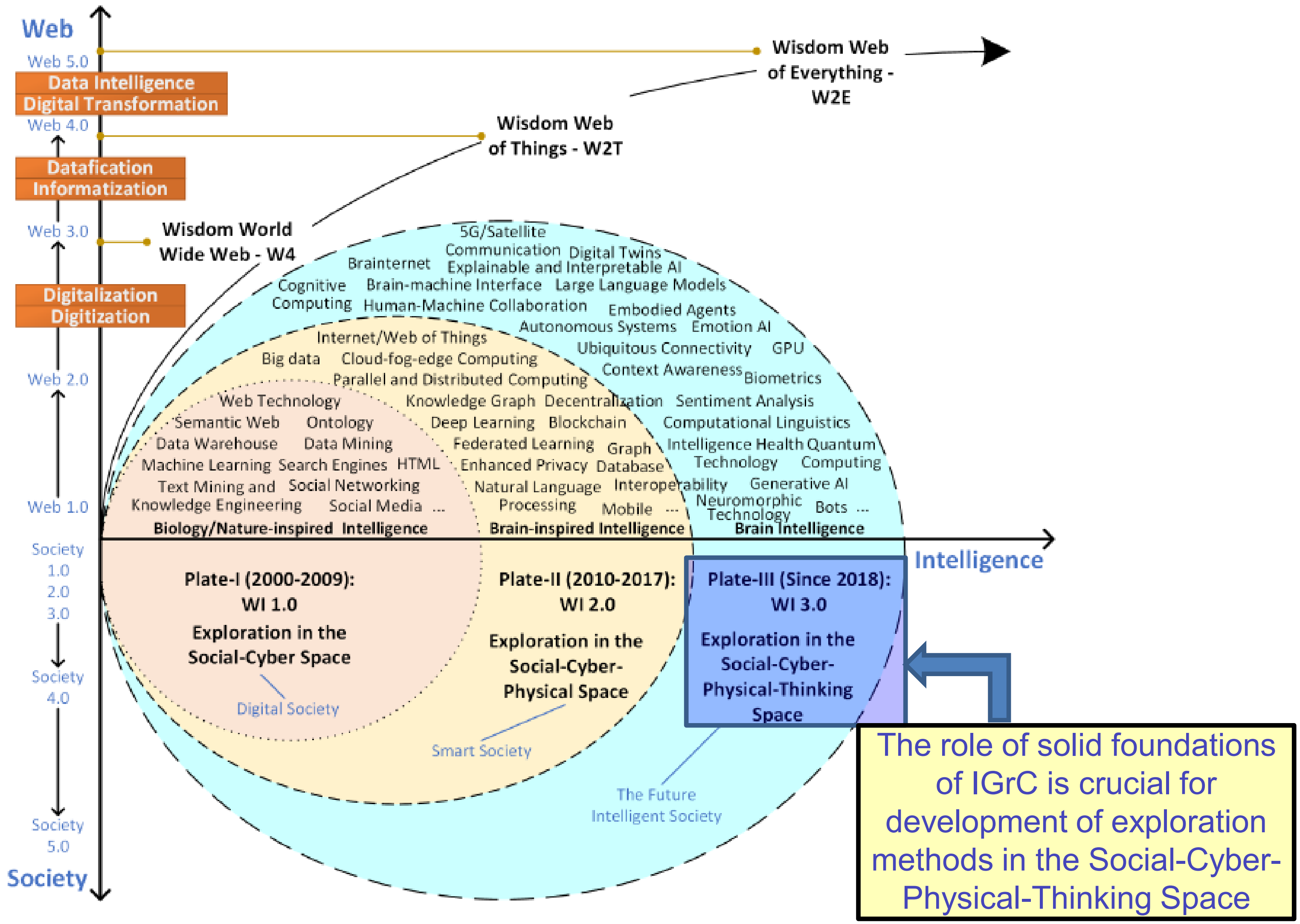


WEB INTELLIGENCE 3.0

THE SOCIAL-CYBER-PHYSICAL-THINKING SPACE: COMPLEX NETWORK OF SOCIETIES OF C-GRANULES



Kuai, H., Huang, J.X., Tao, X. *et al.:* Web Intelligence (WI) 3.0: in search of a better-connected world to create a future intelligent society. *Artif Intell Rev* **58**, 265 (2025). https://doi.org/10.1007/s10462-025-11203-z



The study of Web Intelligence (WI) 3.0 as AI in the connected world, running with the integration of web and intelligence towards the future intelligent society.

H. Kuai, Y.Y. Yao, J. Liu et al.: Web Intelligence: AI in the Connected World. The Innovation (submitted)

DEALING WITH COMPLEX PHENOMENA

Mathematics and the physical sciences made great strides for three centuries by constructing simplified models of complex phenomena, deriving, properties from the models, and verifying those properties experimentally.

This worked because the complexities ignored in the models were not the essential properties of the phenomena. **It does not work when the complexities are the essence**.

Frederick Brooks: The Mythical Man-Month: Essays on Software Engineering. Addison-Wesley, Boston, 1975. (extended Anniversary Edition in 1995).

Frederick Brooks introduced the distinction between:

Essential complexity — inherent to the problem's nature; irreducible.

Accidental complexity — due to imperfect tools; reducible.

BEYOND THE TURING TEST
&
REASONING

The Turing test, as originally conceived, focused on language and reasoning; **problems of perception and action were conspicuously absent**. The proposed tests will provide an opportunity to bring four important areas of AI research
(language, reasoning, perception, and action)
back into sync after each has regrettably diverged into a fairly independent area of research.

C. L. Ortitz Jr. Why we need a physically embodied Turing test and what it might look like. AI Magazine 37 (2016) 55–62.

PHYSICAL SEMANTICS

Constructing the **physical part of the [learning] theory** and unifying it
with the mathematical part should be considered as one of
the main goals of statistical learning theory

Vladimir Vapnik, Statistical Learning Theory, Wiley 1998,
(Epilogue: Inference from sparse data, p. 721)

EMBODIED AI

Embodied AI refers to the integration of artificial intelligence into physical systems, enabling them to interact with the physical world. These systems can include general-purpose robots, humanoid robots, autonomous vehicles (AVs), and even factories and warehouse facilities. The fusion of machine learning, sensors, and computer vision lets these systems perceive, reason, and act in real-world environments.

https://www.nvidia.com/en-us/glossary/embodied-ai/

[…] Yann LeCun, argue that LLMs alone cannot achieve AGI due to several fundamental limitations: their lack of persistent memory, reasoning and planning capabilities, and physical grounding. LeCun specifically asserts that **true intelligence requires interaction with the physical world through sensors and embodiment**. He emphasizes that while LLMs demonstrate impressive linguistic capabilities, they lack genuine understanding and even suggests that "the sensory-motor abilities of a cat surpass those of an LLM."

E. Y. Chang: Multi-LLM Agent Collaborative Intelligence: The Path to Artificial General Intelligence. Association for Computing Machinery, New York, NY 2025. doi.org/10.1145/3749421

WHAT IS A COMPUTATION ?

Two main problems of Computer Science:

What is a state?
What is a transition relation?

What's an algorithm?
Yuri Gurevich (2011)
https://www.youtube.com/watch?v=FX2J24u92GI

WHAT IS A COMPUTATION ?

It seems that we have no choice but to recognize the **dependence of our mathematical knowledge (...) on physics**, and that being so, it is time to abandon the classical view of computation as a purely logical notion independent of that of computation as a physical process

David Deutsch, Artur Ekert,and Rossella Lupacchini,
Machines, logic and quantum physics.
Bull. Symbolic Logic 6 (2000) 265–283, p. 268

ROLE OF LANGUAGE: BASIC PHILOSOPHICAL MOTIVATION

1. soul (psyche)
   → observer
2. object (referent)
   → physical object
   and/or phenomenon
3. thought (likenesses of real existing things and phenomena)
   → model (of concept)
4. symbol
   → name, word (syntax)

ARISTOTLE THETRAHEDRON & WITTGENSTEIN LANGUAGE GAMES

ARISTOTLE PYRAMID

Meaning of words is not a static label, but a dynamic process emerging from action in open world; emerging from their use within specific social activities and contexts.

Language is much more than a static relationship between the observer, the object, the imagination, and the name. Language is primarily a tool for interacting with other abstract and physical objects (people agents, units, granules) in order to achieve goals. Therefore, the meaning of a word is simply the rules of its use in a specific social context, that is, within the framework of a *language game*.

INTERACTIONS

[…] **interaction** is a critical issue in the understanding of complex systems of any sorts: as such, it has emerged in several well-established scientific areas other than computer science, like biology, physics, social and organizational sciences.

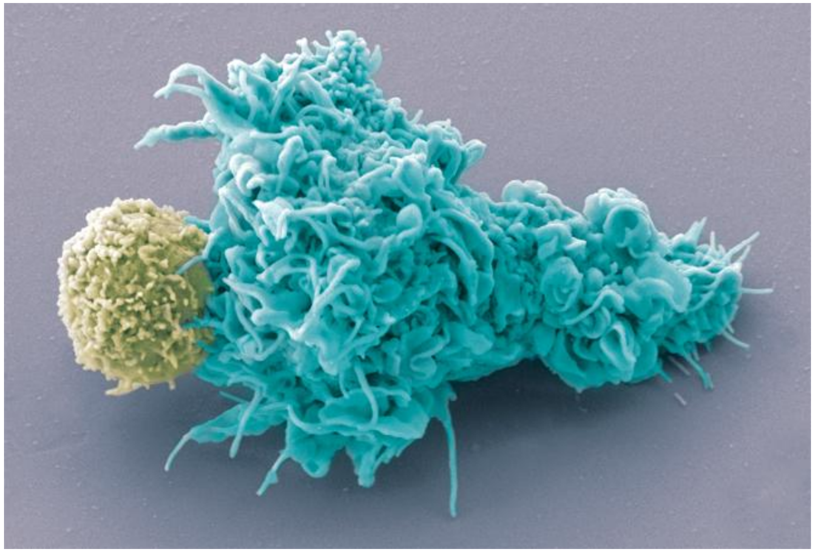
Andrea Omicini, Alessandro Ricci, and Mirko Viroli, The Multidisciplinary Patterns of Interaction from Sciences to Computer Science. In: D. Goldin, S. Smolka, P. Wagner (eds.): Interactive computation: The new paradigm, Springer 2006

INTERACTIONS & MULTISCALING

[…] One of the fascinating goals of natural computing is to understand, in terms of information processing, the functioning of a living cell. An important step in this direction is understanding of **interactions** between biochemical reactions. … the functioning of a living cell is determined by **interactions** of a huge number of biochemical reactions that take place in living cells.

Andrzej Ehrenfeucht, Grzegorz Rozenberg: Reaction Systems: A Model of Computation Inspired by Biochemistry, LNCS 6224, 1–3, 2010

B. Allen, B. C. Stacey, Y. Bar-Yam: Multiscale information theory and the marginal utility of information, Entropy 19(6): 273 (2017) doi:10.3390/e19060273.



A human dendritic cell (blue pseudo-color) in close interaction with a lymphocyte (yellow pseudo-color). This contact may lead to the creation of an immunological synapse.

The Immune Synapse by Olivier Schwartz and the Electron Microscopy Core Facility, Institut Pasteur

http://www.cell.com/Cell_Picture_Show

COMPLEX INTELLIGENT SYSTEM (IS)

CHALLENGE FOR IS CONTROL:

DEVELOPMENT OF REASONING METHODS SUPPORTING GENERATION & COORDINATION OF DIFFERENT KINDS OF INTERACTIONS WITH ABSTRACT AND PHYSICAL OBJECTS AND REASONING (EXPRESSED IN DIFFERENT LANGUAGES: FORMAL AND NATURAL) ABOUT THEIR RESULTS FOR MAKING THE RIGHT DECISIONS (i.e. FOR JUDGMENT)

Traditional statistics is strong in devising ways of describing data and inferring distributional parameters from sample.
Causal inference requires two additional ingredients:

- *a science-friendly language for articulating causal knowledge*,

and

- *a mathematical machinery for processing that knowledge, combining it with data and drawing new causal conclusions about a phenomenon*.

Judea Pearl: Causal inference in statistics: An overview. Statistics Surveys 3, 96-146 (2009)

TWO MAIN CONCERNS REGARDING COMPLEX INTELLIGENT SYSTEMS (IS)

- **COMPLEXITY:** very difficult and risky, e.g. in already complex medical environments; design and analysis of IS based on the IGrC model
- **LACK OF TRUST:** toward **Human-Centered AI, Human-in-the-Loop ML;** interactions (dialogues) with experts and chatbots based on the IGrC model

IGrC is proposed as the computing model for developing trustworthy intelligent systems that deal with complex phenomena in the real physical world.

B. Shneiderman, Human Centered AI, Oxford University Press, Oxford, UK (2022)
R. M. Monarch, Human-in-the-Loop Machine Learning. Active Learning and Annotation for Human-Centered AI, MANNING, Shelter Island, NY (2021)

THE SOLID FOUNDATIONS OF IGrC WILL HELP TO DESIGN AND ANALYSE THE BEHAVIOUR OF AI SYSTEMS ACROSS DIFFERENT DOMAINS.

PANEL: BEYOND BENCHMARKS: RETHINKING REASONING IN LANGUAGE MODELS

(NIPS 2025 https://neurips.cc/virtual/2025/loc/san-diego/128665)

Large Reasoning Models (LRMs), which generate explicit chains of thought, raise new optimism but also expose clear limits: they often underperform standard models on simple tasks, improve briefly at medium complexity, and then collapse on harder ones despite having unused compute. **Crucially, reasoning is not the same as knowledge recall, tool use, or agent-like behavior. True reasoning involves solving novel problems, decomposing them into steps, generalizing to new contexts, recombining partial results, and finally generating novel hypotheses—capabilities current systems largely lack.** Today’s evaluations, focused on final answers and contaminated benchmarks, risk giving a misleading sense of progress.

WORLD ARTIFICIAL INTELLIGENCE CONFERENCE & HIGH-LEVEL MEETING ON GLOBAL AI GOVERNANCE

(July 26-29, 2025 Shanghai)

Despite the proliferation of approximately 3,755 large language models (LLMs) globally and 1,509 in China, most are pre-trained on similar natural language corpora sourced from the Internet.

T. Huang: Innovative research paradigms, AI models, and autonomous robots at the 2025 world artificial intelligence conference. The Innovation 6(12): 101082, December 1, 2025 www.cell.com/the-innovation

FOR DEVELOPMENT OF IS INTERDISCIPLINARY COLLABORATION ACROSS DIFFERENT DOMAINS IS NEEDED

HINTS FROM COMPUTING IN NATURE

G. Rozenberg, T. Back, J.N. Kok (eds.):Handbook of Natural Computing. Springer Berlin Heidelberg, Berlin, Heidelberg (2012). doi:10.1007/978-3-540-92910-9_57

Deeper understanding of human brain and network structures can also help to unravel the mysteries of neural networks and construct more transparent yet more powerful AI models.

T. Huang, H.Xu, H. Wang et al. (2023). Artificial intelligence for medicine: Progress, challenges, and perspectives. The Innovation Medicine 1:100030. doi.org/10.59717/ j.xinnmed.2023.100030.

e.g. LMM $\rightarrow$ SMM

The potential of AI in medicine is vast, yet much remains to be explored and refined. Researchers should strive to develop more advanced AI model architectures and algorithms that can better handle the intricacies of medical data. This includes exploring novel neural network designs, optimization techniques, and learning strategies. These AI models are not merely incremental improvements but rather transformative forces that will reshape the very fabric of medical practice and research.

J., Xu, H. Xu, T. Chen et al. (2025). Artificial intelligence for medicine 2025: Navigating the endless frontier. The Innovation Medicine 3:100120. doi/10.59717/j.xinn-med.2025.100120

HINTS FROM COMPUTING IN NATURE

G. Rozenberg, T. Back, J.N. Kok (eds.):Handbook of Natural Computing. Springer Berlin, Heidelberg (2012). doi:10.1007/978-3-540-92910-9_57

Now, **AI is transitioning from a powerful analytical tool to an interactive intelligence, driving** innovation in four key areas. **Brain computer interfaces** (BCI) allow paralyzed patients to control devices with their thoughts, opening new doors for neuro rehabilitation. **Intelligent robotic systems** are revolutionizing surgery, shifting from passive assistance to autonomous decision making, enhancing precision and safety. **AI-powered reproductive technology** is optimizing IVF success rates through predictive modeling, addressing fertility challenges. **AI-driven elderly care** integrates continuous health monitoring with robotic assistance, providing personalized support for aging populations. **As AI moves from analyzing static data to real-time interaction, it's reshaping healthcare. No longer just a tool for pattern detection, AI is becoming a truly inteligent and responsive partner** — helping doctors make more informed decisions, improving patient care, and driving a future where medicine is more precise, efficient, and human-centered.

AI in Medicine: From Data-Driven Insights to Interactive Intelligence. The Innovation Medicine 3 (2025). https://www.the-innovation.org/medicine/archive

ROBUST, INTERACTIVE, AND HUMAN-ALIGNED AI SYSTEMS

[…] *the grand challenge of creating intelligent agents that can safely and seamlessly learn from and adapt to humans. To accomplish this goal, we need to continue to develop improved AI systems that can manage uncertainty over a wide range of sources, can efficiently fuse multiple forms of human feedback, and enable efficient verification and interpretability.*

[…] *Successfully and efficiently integrating human input into the study of robust AI and robotics will not only require extending existing learning techniques but will also require developing new theoretical and algorithmic techniques that will benefit from insights from fields such as human factors, causal inference, cognitive science, robust control, and formal verification.*

D. S. Brown: Toward robust, interactive, and human-aligned AI systems. AI Magazine 2025: 46:e70024. doi/org/10.1002/aaai.70024

MULTISENSORY LEARNING

[…] *my research is inspired by the way humans naturally engage with the world using all their senses. My long-term goal is to build intelligent systems that can see, hear, touch, smell, taste, and interact with their surroundings—enabling them to perceive, understand, and act within richly multisensory environments. Realizing this vision requires interdisciplinary collaboration across computer vision, robotics, natural language processing, machine learning, augmented reality, acoustic learning, and cognitive science. By integrating insights from these domains, my research aims to empower machines to emulate and enhance human multisensory capabilities, ultimately paving the way for more capable, adaptive, and intuitive artificial agents.*

R. Gao: Multisensory machine intelligence. AI Magazine 2025: 46:e70026 doi.org/10.1002/aaai.70026

TO MAKE SUCH ITERDISCIPLINARY COOPERATION FEASIBLE, IT IS NECASSARY TO GROUD IT IN THE RELEVANT COMPUTING MODEL

INTERACTIVE GRANULAR COMPUTING (IGrC) MODEL

IGrC:
A RESPONSE TO ESSENTIAL COMPLEXITY

PARADIGM SHIFT :
FROM "HOW TO DO IT? (process-focused)"
TO "WHAT MUST BE DONE? (action/purpose-focused)"

Key Idea: One cannot eliminate or constrain the very essence of complexity. One can blend into it and steer adaptively by discovering the sets of rules of interaction (complex games in IGrC).

The Role of IGrC:

- **Approximate solutions to problems constructed along granular computations**
- **Ongoing interactions** with the complex physical and abstract objects
- **Adaptive discovery** of sets of interaction rules (complex games)
- **Emergent learning** from experience
- **Evolution of models** in real time

Paradigm Shift
From purely abstract modeling →
To abstract–physical modeling grounded in real interactions

PARADIGM SHIFT :

FROM "HOW TO DO IT?" TO "WHAT MUST BE DONE?"

function to be computed is not known a priori

model of function should be learned by interacting with the environment

ECORITHMS

The algorithms I discuss in this book are special. Unlike most algorithms, they can be run in environments unknown to the designer, and they learn by interacting with the environment how to act effectively in it. After sufficient interaction they will have expertise not provided by the designer, but extracted from the environment. I call these algorithms **ecorithms**.

Valiant, L.: Probably Approximately Correct. Nature's Algorithms for Learning and Prospering in a Complex World. Basic Books, A Member of the Perseus Books Group, New York (2013)

VAGUENESS IN PHILOSOPHY

PARADIGM SHIFT : FROM "HOW TO DO IT?" TO "WHAT MUST BE DONE?"

Discussion on vague (imprecise) concepts includes the following :

1. The presence of borderline cases.
2. **Boundary regions of vague concepts are not crisp.**
3. Vague concepts are susceptible to sorites paradoxes.

Keefe, R. (2000) Theories of Vagueness. Cambridge Studies in Philosophy, Cambridge, UK)

Concepts that trigger actions and plans are often complex and vague, concerning situations in the real physical world. Therefore, models of these concepts can only temporarily be of high quality and must continuously adapt through interaction with the physical world.

GRANULES & PERCEPTION

Leslie Valiant, of Harvard University, has been named the winner of the 2010 Turing Award for his efforts to develop computational learning theory.
http://www.techeye.net/software/leslie-valiant-gets-turing-award#ixzz1HVBeZWQL

Current research of Professor Valiant
http://people.seas.harvard.edu/~valiant/researchinterests.htm

A fundamental question for artificial intelligence is to characterize the computational building blocks that are necessary for cognition.

COMPLEX GRANULES

IGrC MODEL BASED ON INTERACTION BETWEEN ABSTRACT AND PHYSICAL OBJECTS

In IGrC, we study a new type of (behavioral) rules using the idea that the development of Complex Intelligent Systems (IS) (i.e., Intelligent Systems that deal with complex phenomena), such as OnkoBot, should be based on new rules originating from interactions between abstract and physical objects rather than on simple rules that are confined to the abstract space (as in the case of cellular automata).

Stephen Wolfram: A New Kind of Science
https://www.wolframscience.com/nks/
https://www.wolframscience.com/resources/
https://publications.stephenwolfram.com/foundations-mathematics-mathematica/

[...] *nature follows definite laws, definite rules: otherwise we couldn't do science at all.*
[…] *The question is what kinds of rules one uses.*
[…] *I think what's happened is that mathematics has ended up using only rather special kinds of rules.* […] *But in fact there's a vast universe of other rules out there, which mathematics has essentially never looked at.*

Thesis: all of mathematics (and physics) can emerge from simple iterative rules.

TWO PERSPECTIVES ON THE NATURE OF COMPLEXITY
IN MATHEMATICS AND COMPUTER SCIENCE

accepting irreducible complexity
(in the sense defined by Frederick Brooks)
vs.
seeking simple sources of complexity
(Wolfram)

Challenge:

How IGrC can help to discover such simple rules ?

Stephen Wolfram: A New Kind of Science
https://www.wolframscience.com/nks/
https://www.wolframscience.com/resources/
https://publications.stephenwolfram.com/foundations-mathematics-mathematica/

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GRANULAR COMPUTING (GrC):

GRANULES CLOSED
IN THE ABSTRACT SPACE ONLY

INTERACTIVE GRANULAR COMPUTING (IGrC):

GRANULES
IN THE ABSTRACT AND PHYSICAL SPACES
FOR PRCEPTION MODELING

FROM GrC TO IGrC

GrC, with abstract information granules as basic objects, is generalized to IGrC through the introduction of complex granules (c-granules), which are the fundamental objects of IGrC. These combine abstract and physical objects, enabling the perception of their properties through the control of c-granules. The IGrC model differs from the classical Turing model. In the IGrC model, both language and reasoning issues as well as actions and perception are significant. Research on IGrC utilizes existing partial results from various fields, such as multi-agent systems, perception and action, machine learning, natural language processing, and more.

INFORMATION GRANULES OF GrC AS SPECIAL CASES OF C-GRANULES OF IGrC

The IGrC model substantially extends the GrC model. GrC granules can be considered special cases of c-granules of IGrC. Any information granule g_o from GrC can be identified as a c-granule g consisting of an i-layer that encodes g_o and two transformation specifications: *store* and *read* (labeled by the address that points to physical memory). The control of the c-granule g can use these specifications to either store g_o in the p-layer of g or read g_o from it into the i-layer of g (without losing any information).

COMPLEX GRANULES
(C-GRANULES)

GRANULAR COMPUTING (GrC)
&
INTERACTIVE GRANULAR COMPUTING (IGrC)

g =(syn(g), sem(g)) – granule

syn(g) – syntax *sem(g) – semantics*

(only in IGrC)

abstract *physical*

<u>Remarks</u>.

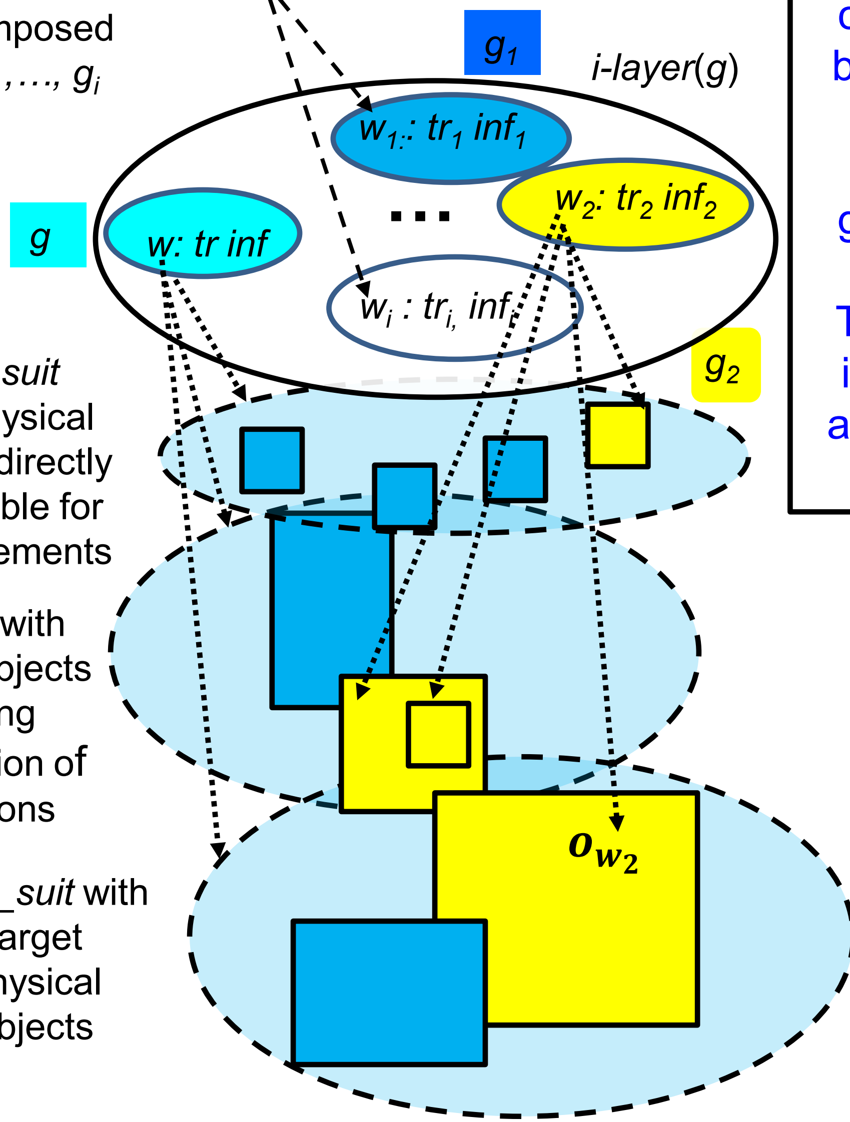
- The semantics of granules in IGrC should be based on constructive definitions of the universe of objects (i.e., granules obtained by sensory measurements or constructed prior to considered granules) and constructive methods that generalize the classical concept of an algorithm (see ecorithms by Leslie Valiant, e.g., which are based on learning). These methods aim to determine the membership of objects from the universe in considered granules.
- Physical semantics -- realized by an implementation module (IM), which is a sub-granule of the c-granule control.

C-GRANULE: INTUITION

specifications of (families of) spatio-temporal windows realized by control as physical pointers to the corresponding parts of the physical space

w – scope of granule *g* composed out of family of granules $g_1, \ldots, g_i$

$||w||$ - realization of *w* with the family of physical objects in *p-layer*(*g*)



p-layer(*g*)

soft_suit with physical objects directly accessible for measurements

link_suit with physical objects providing transmission of interactions

hard_suit with target physical objects

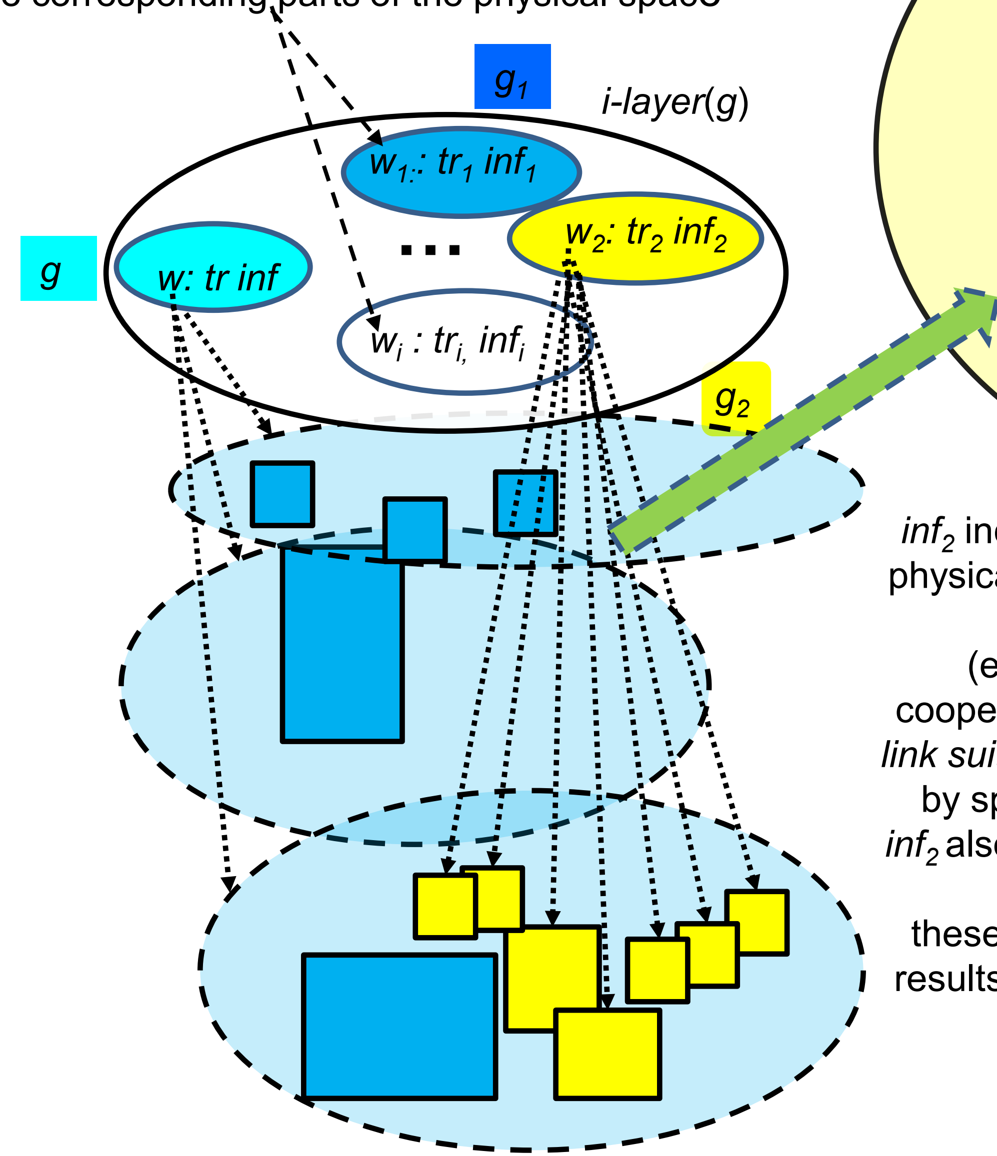
see: signals in the book by Holland

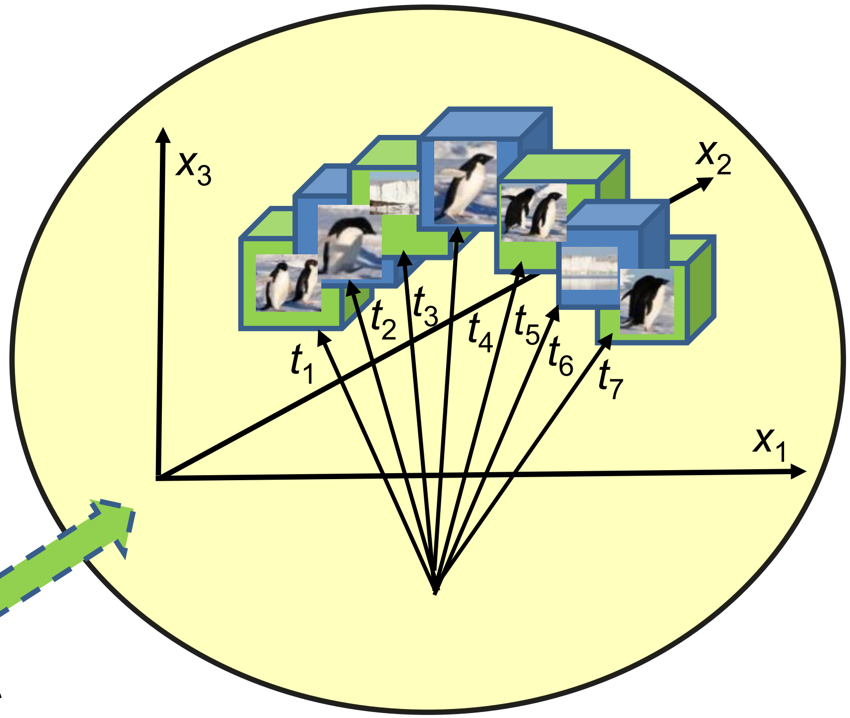
properties of physical objects from *hard_suit, link_suit* and their interactions encoded in inf_i are based on already perceived properties, granule structure, physical laws and/or knowledge bases

C-granules may have many sub-granules. State of c-granule is changing with local time of c-granule. The dynamics is steered by control of c-granule aiming by selection and realization of transformations (associations) to satisfy goals (needs, specification of problem to be solved). This is based on perceived information about physical and abstract objects as well as their interactions.

C-GRANULE: INTUITION

specifications of (families of) spatio-temporal windows realized by control as physical pointers to the corresponding parts of the physical space





inf$_2$ includes video information gathered during physical semantics realization of specification of transformation *tr*$_2$
(e.g. tracking objects what may require cooperation with cameras placed in *soft_suit* or *link suit*) from parts of the physical space pointed by spatio-temporal window specification w_2;
inf$_2$ also includes information about the expected results of *tr*$_2$ realization;
these results may be different from the actual results due to interactions with the environment

LANGUAGE OF C-GRANULES

PHILOSOPHICAL MOTIVATION FOR LANGUAGE OF C-GRANULE

1. observer
2. object
3. model
4. name

ARISTOTLE THETRAHEDRON & WITTGENSTEIN LANGUAGE GAMES

Meaning of words is not a static label, but a dynamic process emerging from action in open world; emerging from their use within specific social activities and contexts.

ARISTOTLE PYRAMID

C-GRANULE

1. observer → c-granule with control aiming to achieve goals (satisfy needs) by interaction with the abstract and physical world
2. object → physical object
3. model → abstract model represented in i-layer (e.g., classifier)
4. name → name, expression (syntax) used by control of c-granule for communication with c-granules

&

5. language games (rules of interaction and use) → stored in i-layer of c-granule specifications of associations for rules of interaction and their use realized by physical semantics in the open world

EXAMPLE OF LANGUAGE GAME: CHESS

Let’s consider a game of chess played by an intelligent system (IS) — treated as a specific c-granule — against a human. Of course, one might try to limit the operation of the IS control to an abstract space that takes into account the rules of chess and various knowledge bases. However, this proves to be insufficient. In a specific game, the IS should take into account, e.g., the opponent's class, her/his current predisposition, emotional state (e.g., whether her/his is currently fully focused on the game), which will require the IS to interact with the current opponent (being a physical object!) and perceive her/his skills and other traits caused by interactions with other objects in the real world. It is easy to see that an analogous situation can occur in applications related to medical diagnostics.

IGrC & THE BIOLOGY OF LANGUAGE:
TOWARD COMPUTERS PUTING REASONING LANGUAGE, PERCEPTION AND ACTION INTO SYNC

[…] *goal in this book is to provide a coherent view on the relation between language and the brain based on the multifold empirical data coming from different research fields*

Angela D. Friederici: Language in Our Brain: The Origins of a Uniquely Human Capacity. MIT Press 2017

[…] ***processing and perception access a common "knowledge base," core internal language, a computational system that yields an unbounded array of structured expressions.***

PROPOSAL:
IGrC as the basic tool for modeling such computational systems

[…] *Friederici's most striking conclusions concern specific regions of Broca's area (BA 44 and BA 45) and the white matter dorsal fiber tract that connects BA 44 to the posteriori temporal cortex.*

N. Chomsky: Preface. In: *Angela D. Friederici: Language in Our Brain: The Origins of a Uniquely Human Capacity. MIT Press 2017*

LANGUAGE OF I-LAYER OF C-GRANULE FOR EXPRESSING RELATIONS IN GrC and IGrC

BASIC POSTULATE

The language in the information layer of the c-granule allows for the expression of 'relations' (called in IGrC *associations*) between abstract and physical objects. These types of relations are not purely mathematical in the sense of set theory. C-granule control attempts to construct models of these associations with physical semantics using models defined in the abstract world. Interaction with the physical world supports the construction of these models and makes their adaptation possible. Thus, we have two worlds of relations: those that allow for the expression of properties of abstract objects and those that do not fit within classical set theory. The focus is on how these abstract relations can 'approximate' the latter, i.e. associations.

IS's are aiming to discover or learn such approximations of associations. This process is related to many challenges.

LANGUAGE OF I-LAYER OF C-GRANULE FOR EXPRESSING RELATIONS IN GrC and IGrC

GrC:

Formal or (fragments of natural) language for expressing relations between abstract objects only.

Remark: As long as the specific representation of the granules in a finite granular space is not important, one can base the language of granules on any enumeration of its granules. Often, specifying such granules uses only their semantic part.

IGrC:

Language allows us to use relations over abstract and physical objects due to the need to express relations between them in the perception process. Physical objects can only be partially perceived, which implies that the control of the considered c-granule may only have partial and incomplete information about the physical semantics of realized specifications in the physical world (e.g., arguments *for* or *against* their satisfiability in the currently perceived situation concerning the currently diagnosed patient in the physical world).

LANGUAGE OF C-GRANULE:
DESCRIPTION IN I-LAYER OF C-GRANULE BEHAVIOR

LANGUAGE for description of c-granule behavior, e.g.,

- behavior of the control of c-granule, including its different module
- sub-granules constructed by the c-granule representing properties of abstract and physical objects as well as relations and associations between them
- properties of perceived so far physical and abstract objects and/or interactions between them
- properties of behavioral patterns
- specifications of spatio-temporal windows
- specifications of transformations (associations)
- expected and/or real results of realization of transformations
- information about performed or planned reasoning processes and reasoning results about stored information
- aggregation operations and their results
- information about local time of c-granule; period for transformation realization; initialization (suspension, resumption) moment of transformation realization; cancelling moment of c-granules pointed by control

LANGUAGE OF C-GRANULE:
DESCRIPTION IN I-LAYER OF C-GRANULE BEHAVIOR (Cont.)

- communication protocols with, e.g., knowledge bases, experts, sensors, actuators, and other c-granules
- communication expressions used by c-granule for encoding information into the external world, e.g., messages with important information, queries, orders, calls for cooperation
- communication expressions used by c-granule in decoding information from the external world, e.g., messages with important information, queries, orders, calls for cooperation
- information about languages used by other c-granules in the environment
- information about behavior of other c-granules (teams of c-granules or societies of c-granules) from the environment, e.g., behavioral patters
- usefulness of other c-granules in achieving the goals (needs) by the considered c-granule
- trustworthiness of other granules relative to the goals (needs) of the considered c-granule
- …

LANGUAGE OF FORMULAS AND THEIR INTERPRETATION IN I-LAYER

L – language of formulas over a set of constants (representing names of objects, situations, results of measurements, …), functional and relational symbols.

<u>Interpretation *I*</u>: A finite set of formulas from *L* (intuition: true facts in the current situation).

In GrC, we are unaware of *how, when,* or *why* they are obtained, but in IGrC, we are.

EVOLVING LANGUAGE OF C-GRANULES

constants

interpretation *I*

$a(x_1) = v_1 \quad b(x_1) = v_1'$

$a(x_2) = v_2 \quad b(x_2) = v_2'$

... ...

functional symbols

information system

	a	b	
x_1	v_1	v_1'	
x_2	v_2	v_2'	
...	...	...	

variable

$a(\boldsymbol{x}) = v$

formula with free variable $\boldsymbol{x}$
satisfied on x_1 , x_2 ,…

... new expressions (formulas)

EVOLVING LANGUAGE OF C-GRANULES

information system

	a	b	…
x_1	v_1	v'_1	
x_2	v_2	v'_2	
…	…	…	…

information system

	α	…	β	…
x_1	0	…	1	
x_2	1	…	0	
…	…		…	…

…

relational system
(of a given signature τ)

$(V_a, \leq, \ldots), (V_b, \varrho, \ldots), \ldots$

language of formulas
(with signature τ)

$L_a, L_b, \ldots$

interpretation of the language in the relational systeem (of signature τ)

$I_a, I_b, \ldots$

new information granules

$(\alpha, I_a(\alpha))$, where $\alpha \in L_a,\ I_a(\alpha) \subseteq U = \{x_1, x_2, \ldots\}$

$(\beta, I_b(\beta))$, where $\beta \in L_b,\ I_b(\beta) \subseteq U = \{x_1, x_2, \ldots\}$

in information system new information granules can be defined by aggregation of attributes (formulas), e.g., using conjunction

$(\alpha \wedge \beta, I_a(\alpha) \cap I_b(\beta))$

Remark. Interpretation may be fuzzy, not necessarily crisp.

EVOLVING LANGUAGE OF C-GRANULES

elementary granules in rough sets: indiscernibility classes represented by vectors of attribute values

information system

	a	b	…
x_1	v_1	v_1'	
x_2	v_2	v_2'	
…	…	…	…

information system

	a	b	…
$[x_1]$	v_1	v_1'	
$[x_2]$	v_2	v_2'	
…	…	…	…

…

information system

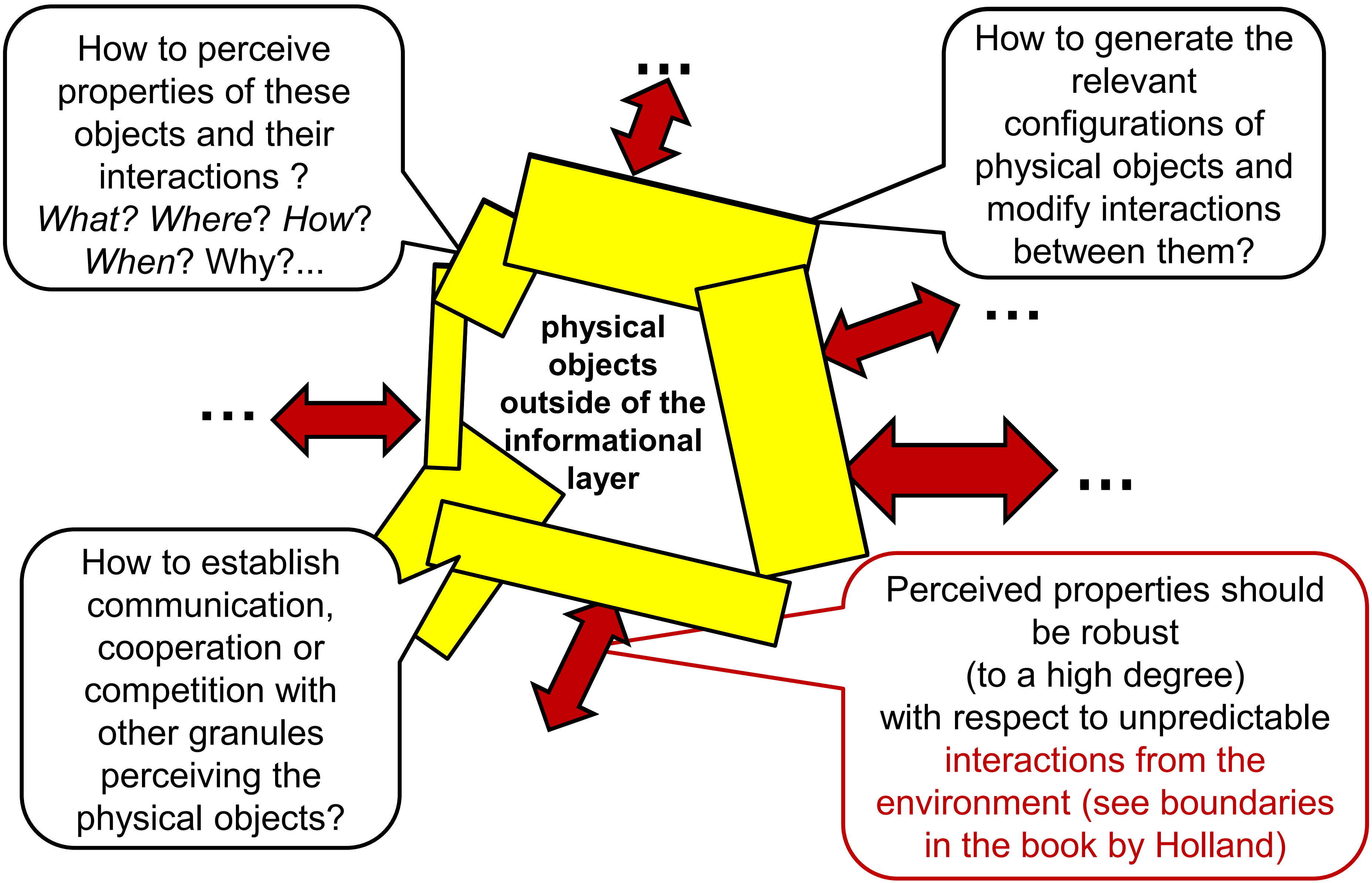
	α	…	β	…
g_1	0	…	1	
g_2	1	…	0	
…	…		…	…

…

Discovery of new languages in which relevant attributes can be defined and interpreted in relational systems over new granules is necessary

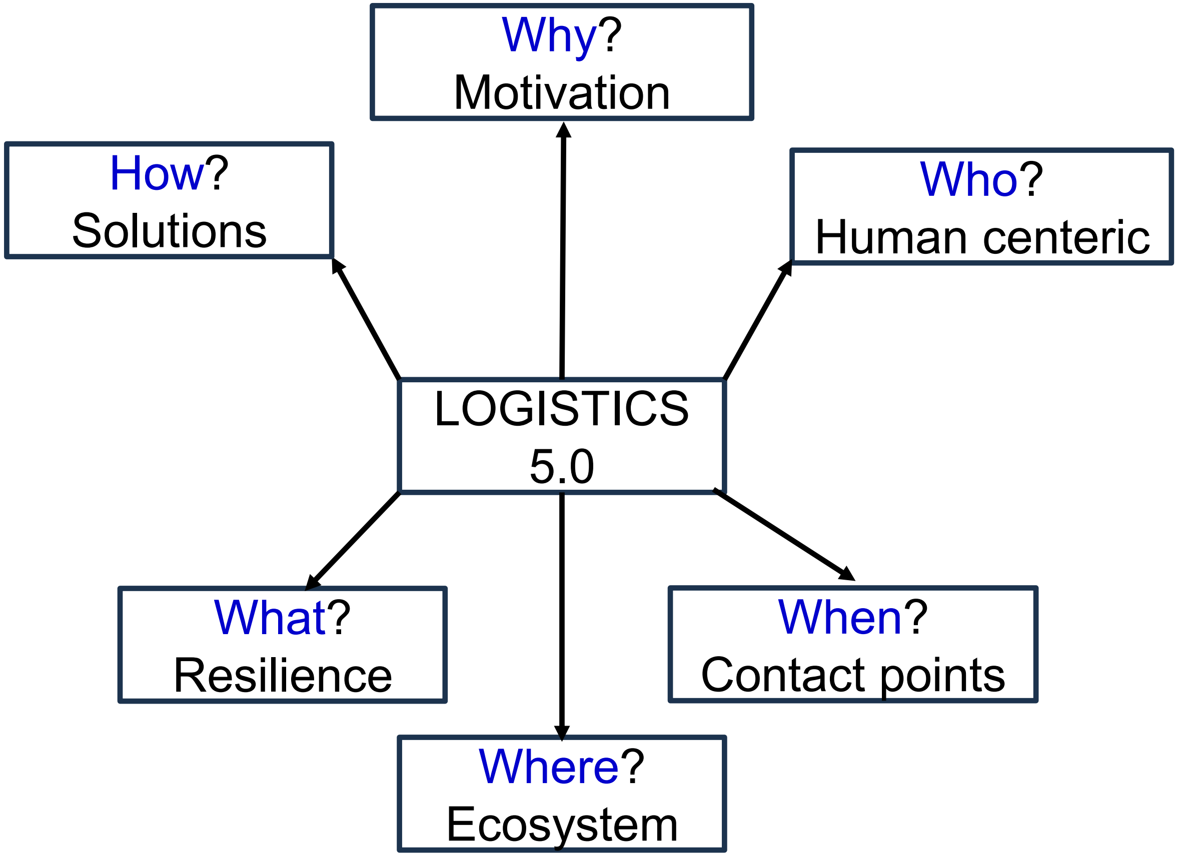
More general granules,
e.g., tolerance (similarity) classes, time windows, classifiers, granulation of already constructed granules (including structural objects defined by aggregation of already defined granules with constraints or dynamic granules)

HOW C-GRANULES ARE GENERATED, MODIFIED AND MANAGED?



ANSWERS TO QUERRIES *What? Where? How? When?...* DEPEND ON THE CONTEXT EXAMPLE: AI in LOGISTICS 5.0

All these concepts require further multi-level decompositions dependent on the context to make them understandable by AI system.
For example,
AI resilience is an AI system's ability to adapt to disruptions or shocks and recover its performance and functionality without permanent changes.
This concerns: Operations, Inventory, Warehouse, Packaging, Transportation, Staging, Return.
Next, e.g., staging is a process in which products or shipments are manually removed and placed in separate, consolidated areas for inspection.

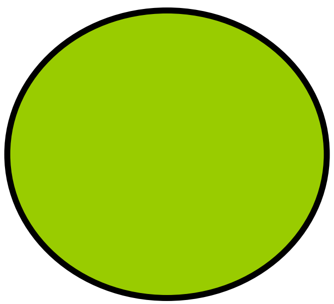


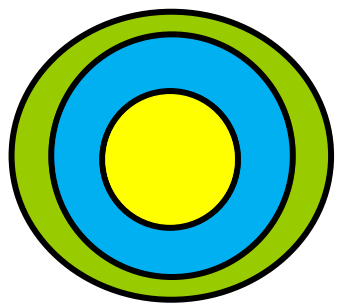
B. Nicoletti: Artificial Intelligence for Logistics 5.0. From Foundation Models to Agentic AI. Springer (2025)
E. Y. Chang: Multi-LLM Agent Collaborative Intelligence: The Path to Artificial General Intelligence. Association for Computing Machinery, New York, NY 2025. doi.org/10.1145/3749421

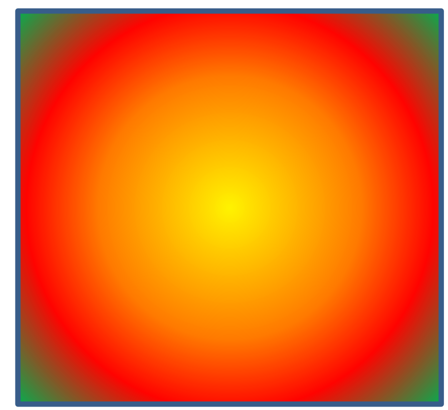
EXAMPLES OF GRANULAR ALGEBRAS

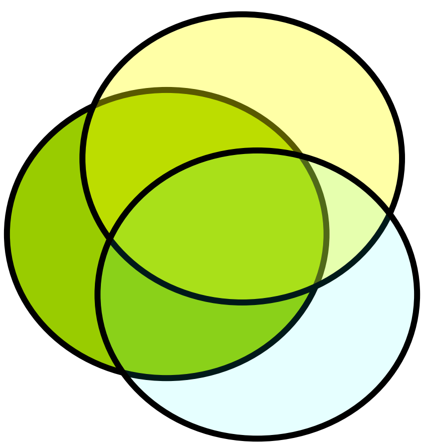
GRANULAR ALGEBRA
=
ATOMIC GRANULES
+
OPERATIONS ON GRANULES

Examples of families of elementary granules









...

g =(syn(g), sem(g)) – granule

syn(g) – syntax of g expressed in a language L

sem(g) – semantics of g: crisp (or fuzzy) set of objects (already defined granules)

ATOMIC ABSTRACT GRANULES FROM INFORMATION SYSTEMS

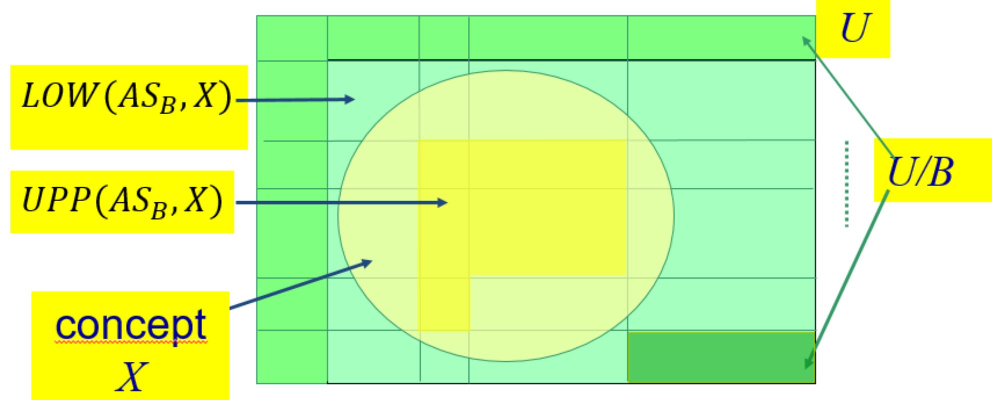
LOWER AND UPPER APROXIMATION

	a_1	a_2	…	a_m
x_1	v_1	v_2	…	v_m
	•••	•••	•••	•••

$X \subseteq U, B \subseteq A,\ AS_B = (U, IND(B))$

$LOW(AS_B, X) = \cup\{Y \in U/B : Y \subseteq X\}$

$UPP\ (AS_B, X) = \cup\{Y \in U/B : Y \cap X \neq 0\}$



BOUNDARY REGION

$Bd(AS_B, X) = UPP(AS_B, X) \backslash LOW(AS_B, X)$

- indiscernibility classes of (subsets of) attributes
- partitions defined by attributes
- partitions defined by subsets of attributes
- granules defined by calculi over the above granules
- …

computational building blocks for concept approximation in the rough set approach

INDISCERNIBILITY RELATIONS OF INFORMATION SYSTEM

Information system (data table)

$$IS = (U, A)$$

$$U=\{x_1,\ldots, x_n\},\ A=\{a_1,\ldots, a_m\},\ a_i : U \rightarrow V_{a_i}$$

$$B \subseteq A$$

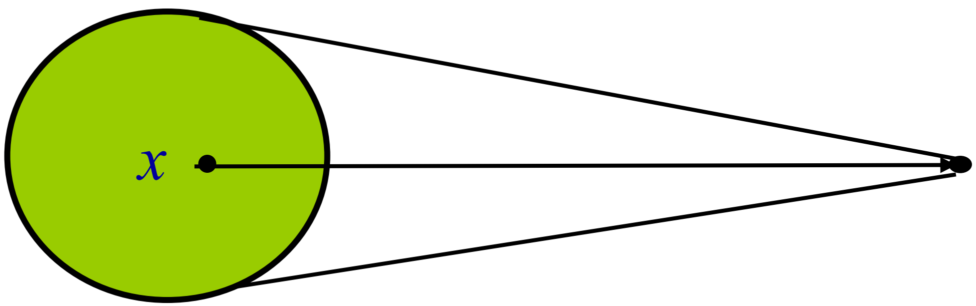
$$B - indiscernibility\ relation\ of\ IS$$

$$xIND(B)y \ \ iff\ Inf_B(x) = Inf_B(y)$$

B

	a_1	a_2	…	a_m
x_1	v_1	v_2	…	v_m
	…	…	…	…

$$N_B(x) = [x]_{IND(B)} = [x]_B = \{y \epsilon U : xIND(B)y\}$$



$u=Inf_B(x)$ signature of x

$$Inf_B(x) = Inf_B(y)$$

τ

tolerance or similarity

neighborhood of x

elementary granule

$$U/B = \{[x]_B : x \epsilon U\}$$

EXAMPLES OF DEFINABLE GRANULES IN THE PAWLAK MODEL OF ROUGH SETS

i−layer of the considered c−granule g consists
of information granules representing:
$IS = (U, A), B \subseteq A, \; AS_B = \big(U, IND(B)\big), X \subseteq U$
decision attribute: d: U → V_d; decision class: $X_v = \{x \epsilon U: d(x) = v\}$
decision system: $DS = (U, B, d)$

Examples of granules definable (generated) from elementary granules (i.e., indiscernibility classes of *IND*(*B*)) by their interaction (realized in the abstract space) with the relevant information granules represented by algorithms provided by the control of *g*. Construction of the algorithms is supported by a sub-granule of control of *g,* called reasoning module.

$B-$lower approximation of X: $LOW(AS_B, X)$
$B-$upper approximation of X: $UPP(AS_B, X)$
$B-$boundary region of X: $Bd(AS_B, X)$
B-positive region of classification $\{X_v: v \epsilon V_d\}$:
$POS(AS_B, \{X_v: v \epsilon V_d\}) = \bigcup_{v \epsilon V_d} LOW(AS_B, V_d)$

EXAMPLE OF OPTIMIZATION PROBLEM:

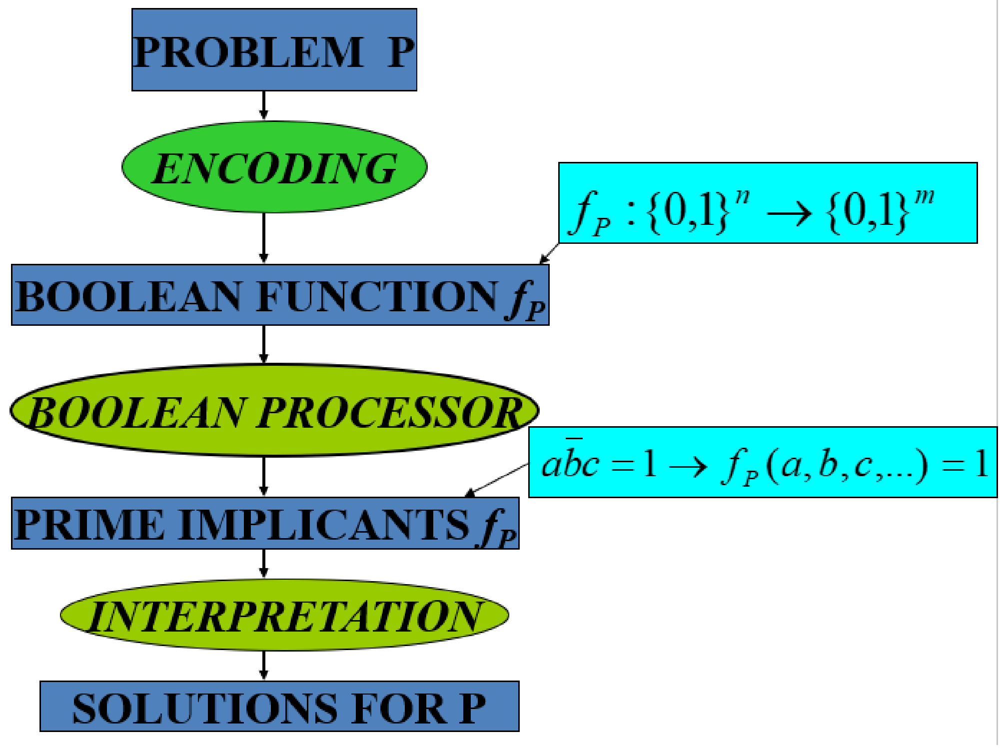
For given $IS = (U, A)$, $X \subseteq U$, and $tr \epsilon$ (0,0.5) find (if exists) a minimal $B \subseteq A$ such that $|Bd(AS_B, X)| \leq tr \; |U|$.

OPTIMIZATION PROBLEMS

- feature selection
- data reduction
- discretization
- symbolic value grouping
- decision rules
- …

BOOLEAN REASONING

George Boole (1815-1864)



Z. Pawlak, A. Skowron: Rough Sets and Boolean Reasoning. Inf. Sci. 177(1) 41-73 (2007)

ILLUSTRATIVE EXAMPLES OF GRANULAR ALGEBRAS

$IS = (U, A)$ – information system

GA_{IS} – granular algebra defined by *IS*

ATOMIC GRANULES OF GA_{IS}:

Example 1. partitions defined by $IND(a)$ for $a \in A$

Example 2. partitions defined by coarsening of partitions defined by $IND(a)$ for $a \in A$ (where a coarsening of the partition defined by $a \in A$ relative to any partition $\boldsymbol{V}$ of V_a is the partition of V_a defined by relation $x\, r_{\boldsymbol{V}}\, y$ iff $a(x), a(y) \in V$ for some $V \in \boldsymbol{V}$).

DEFINABLE GRANULES OF GA_{IS}

(GENERATED FROM ATOMIC GRANULES AND OPERATIONS ON GRANULES)

Example 1&2: partitions of U generated from atomic granules and their intersections.

Remark. In the case of discretization: $V_a \subseteq R$ (where R is the set of reals) and $\boldsymbol{V}$ is a finite (and relevant for the problem) family of partitions of R (or interval of reals) into intervals. This allows for inducing classification for unseem cases so far.

Z. Pawlak, A. Skowron: Rough Sets and Boolean Reasoning. Inf. Sci. 177(1) 41-73 (2007)

EXAMPLE OF OPTIMIZATION PROBLEM: SEARCHING FOR THE OPTIMAL PARTITION (GRANULE) FROM A GIVEN FAMILY OF PARTITIONS (GRANULES) APPROXIMATING DECISION C-GRANULE

Extended decision system relative to universe of granular calculus GA_{IS} defined as, e.g., in Example 2.

Decision system as granular algebra linking two granular algebras defined by *IS* and *d*

$$DS = (U, GA_{IS}, d),\ d: U \rightarrow V_d$$

Examples of **optimization** problems (e.g., discretization, symbolic value grouping):

Find a partition *part* from GA_{IS} providing the relevant balance. This balance should be between (i) the description length of the *part* and (ii) the quality of the *part* as an approximation of the classification realized by *d*.

EXAMPLE OF OPTIMIZATION PROBLEM: C-GRANULES AS MINIMAL DECISION RULES (Cont.)

For given

$DS = (U, GA_{IS}, d)$ – generalized decision system

where $IS=(U, A)$,

$d : U \rightarrow V_d$ – decision;

GA_{IS} algebra of granules generated from partitions of U defined for $a \in A$ by $IND(a)$ and their intersections

find

$\{[x]_B \in {}^{U}/_{IND(B)} :\ B$ is a minimal subset of A such that $d([x]_B) = d([x]_A)\ \&\ x \in U\}$

where

$[x]_B$ is the eqivalence class of $IND(B)$ defined by $x \in U$;

$d([x]_B) = \{d(y) : y \in [x]_B\}$.

Every such $[x]_B$ defines a (minimal) decision rule

$$\wedge\{a = a(x) : a \in B\} \Longrightarrow d \in \delta_B(x)$$

where

$$\delta_B(x) = \{v \in V_d : \exists y \in [x]_B : d(y) = v\} = d([x]_B).$$

EXAMPLE OF OPTIMIZATION PROBLEM:
C-GRANULES AS COMPUTATIONAL BUILDING BLOCKS FOR APPROXIMATION OF CONCEPTS IN THE FORM OF MINIMAL DECISION RULES IN DOMINANCE ROUGH SETS (Cont.)

For given

$DS = (U, GA_{IS}, d)$ – generalized decision system

where IS=(U, A), $a: U \rightarrow V_a \subseteq N$ for $a \epsilon A$ (N the set of natural numbers),

$d : U \rightarrow V_d \subseteq N$ – decision attribute,

for $a \in At = A \cup \{d\}$: $R_a = (V_a, \geq_a)$, $\geq_a$ - natural order in N,

V_a interpreted as a set of preferences of a,

$v \geq_a v'$: v preferred at least as much as v',

$\geq_a (v) = \{v' \in V_a : v' \geq_a v\}$ – neighborhood of values of a dominating v,

GA_{IS} algebra of granules generated from neighborhoods defined for $a \in A$

by $\tau_a(x) = \{y \epsilon U: a(y) \geq_a a(x)\}$ for $x \epsilon U$ and their intersections

and a distinguished decision value v_d from V_d,

find $\{[x]_B$: $B \neq \emptyset$ is a minimal subset of A such that $[x]_A \subseteq D_{v_d}$ implies $[x]_B \subseteq D_{v_d}\}$,

where $[x]_B = \bigcap_{a \in B} \tau_a(x)$ for $B \subseteq A$ and $D_{v_d} = \tau_d(z), where\ d(z) = v_d$.

Every such $[x]_B$ defines a granule as computational building blocks for the lower approximation of D_{v_d} in the form of a (minimal) decision rule

$$\bigwedge_{a \in B} a \in \geq_a (a(x)) \Longrightarrow d \in \geq_d (v_d).$$

S. Greco, B. Matarazzo, R. Słowiński: Rough sets theory for multi-criteria decision analysis. European Journal of Operational Research 129(1):1-47 (2001)

EXAMPLE OF OPTIMIZATION PROBLEM: ROUGH-FUZZY APPROXIMATION OF THE DECISION RELATIVE TO A GIVEN ALGEBRA OF GRANULES

For given

$DS = (U, GA_{IS}, d)$,

where $d: U \rightarrow [0,1]$ – fuzzy decision;

GA_{IS} algebra of granules generated from partitions of U defined for $a \in A$ by $IND(a)$ and their intersections (i.e. the family of partitions defined by $IND(B)$, where $B \subseteq A$);

and a quality threshold $\varepsilon \in (0,0.5]$

find, if possible,

the ε –approximation of d in the from of two fuzzy membership functions from U into [0,1]: $(UPP(part_B, d), LOW(part_B, d))$, i.e. a *minimal* $B \subseteq A$ with quality defined by

$$max_{x \in U} |UPP(part_B, d)(x) - LOW(part_B, d)(x)| < \varepsilon$$

$$where \;\; UPP(part_B, d)(x) = \max\{d(y) : y \in [x]_B\} = \max(d[x]_B)$$

$$LOW(part_B, d)(x) = \min\{d(y) : y \in [x]_B\} = \min(d[x]_B)$$

$part_B$ is defined by $IND(B)$

$[x]_B$ is the eqivalence class of $IND(B)$ defined by $x \in U$.

A. M. Radzikowska, E. E. Kerre: A comparative study of fuzzy rough sets. Fuzzy Sets and Systems 126 (2002) 137–155

EXAMPLE OF OPTIMIZATION PROBLEM: ROUGH-FUZZY DECISION RULES RELATIVE TO A GIVEN ALGEBRA OF GRANULES

For given

$DS = (U, GA_{IS}, d)$,

where $d : U \rightarrow [0,1]$ – fuzzy decision;

GA_{IS} algebra of granules generated from partitions of U defined for $a \in A$ by $IND(a)$ and their intersections (i.e. the family of partitions defined by $IND(B)$, where $B \subseteq A$);

and a quality threshold $\varepsilon \in (0,0.5]$

find

$\{[x]_B \in U/IND(B): B$ is a minimal such that

$|UPP(part_B, d)(x) - LOW(part_B, d)(x)| < \varepsilon\}$

where $UPP(part_B, d)(x) = \max\{d(y): y \in [x]_B\} = \max(d[x]_B)$

$LOW(part_B, d)(x) = \min\{d(y): y \in [x]_B\} = \min(d[x]_B)$

$part_B$ is defined by $IND(B)$

$[x]_B$ is the eqivalence class of $IND(B)$ defined by $x \in U$.

The value of approximated decision d for objects from $[x]_B$ is the interval

$[LOW(part_B, d)(x), UPP(part_B, d)(x)]$.

J. G. Bazan, A. Osmólski, A. Skowron, D. Ślęzak, M. S. Szczuka, J. Wróblewski: Rough Set Approach to the Survival Analysis. In: RSCTC Proceedings, LNCS 2475, pp. 522—529, Springer, 2002. doi.org/10.1007/3-540-45813-1_69

DISCOVERY OF GRANULAR CALCULUI

Unlike classical mathematical logic, where they are a priori given, granular calculi composed out of granular relational system and granular languages should be discovered by control of c-granules using data obtained through interaction with the physical and abstract worlds including human experts and chatbots.

Although the aforementioned discovery is challenging, it is important for explainability and scalability, unlike LLM, which is based on the iteration of the neural scheme.

It is worth mentioning an analogy with mathematical proofs, which are based on the discovery of new concepts and their properties, making the proofs feasible and understandable.

GRANULAR CALCULUS

Granular relational system

- granular algebra consists of
  - atomic granules generated on the basis of granules
    - from a preceding granular level relative to the considered one or
    - generated through interactions with the physical world (e.g., by performing sensory measurements or actions or dialogues with users)
  - aggregation granular operations (e.g., intersection)
- granular relations (expressing properties of granules or relations between granules)

Granular language

allows granules from the granular calculus to be described. It is used to search for granules from the granular calculus that are expected to be computational building blocks relevant to cognition [using terminology that Leslie Valiant would use], (e.g., the approximation of target concepts or classifications). For example, these can be elementary granules in rough sets, which are defined as conjunctions of atomic formula (descriptors of the form $a=v$) and are interpreted in the granular relational system.

EXAMPLE OF GRANULAR CALCULUS : DECISION SYSTEMS WITH INCLUSION MEASURES

Relational system defined by

decision system $DS = (U, GA_{IS}, d)$ *and incusion relations* $\{\nu_{tr}\}_{tr\epsilon(0.5,1]}$

where $IS = (U, A)$, $d\colon U \to V_d$ $-$ decision attribute

inclusion measure: $\nu : P(U) \times P(U) \to [0,1]$

$$\text{standard inclusion measure: } \nu(X,Y) = \begin{cases} \frac{|X \cap Y|}{|X|} & if\ X \neq \emptyset \\ 1 & if\ X = \emptyset \end{cases}$$

$$\text{inclusion to a degree at least } tr\text{: } \nu_{tr}(X,Y) = \begin{cases} 1 & if\ \nu(X,Y) \geq tr \\ 0 & otherwise \end{cases}$$

Language: granules can be defined by intersection of granules expressed in the language using conjunction.

Optimization concerns not only selection of the relevant granules (partitions) from GA_{IS}, but also the relevant inclusion measure. The optimization criterion is based on the quality of the classifier induced from attributes defined by the selected partition and the selected inclusion measure. This may be based on tuning of a proper balance between the description length and approximation quality of the given concept (or classification) based on the made selection (Minimum Description Length Principle (MDL) in Machine Learning). Optimization process is supported by reasoning module of the control of c-granule.

GRANULAR CALCULI DEFINED BY DECISION SYSTEMS EXTENDED BY VECTORS OF FAMILIES OF RELATIONS

e.g., parameterized decision systems, parameterized tolerance/similarity decision systems, parameterized quality relations etc.

APPROXIMATION SPACE OVER GRANULAR RELATIONAL SYSTEM OF GRANULAR CALCULUS

$$\left(U, \underbrace{GA_{IS}, d}_{\text{family of granules}}, \underbrace{\{\nu_{tr}\}_{tr\epsilon[0.5,1]}, \mathcal{F}_1, \ldots, \mathcal{F}_k}_{\text{vector of families of relations}}\right)$$

family of granules (partitions of U defined by the granular algebra GA_{IS} and the decision d)

vector of families of relations expressing properties of granules and relations between them used to define new granules in the language of granular calculus

<u>Language</u>: allowing to define new granules by intersection of already defined granules, and aggregation of, e.g., tolerance/ similarity relations or weighted quality measures.

Calculi of granules and granular relational systems over them in the case of tasks related to approximation of concepts or classifications are called **approximation spaces**.

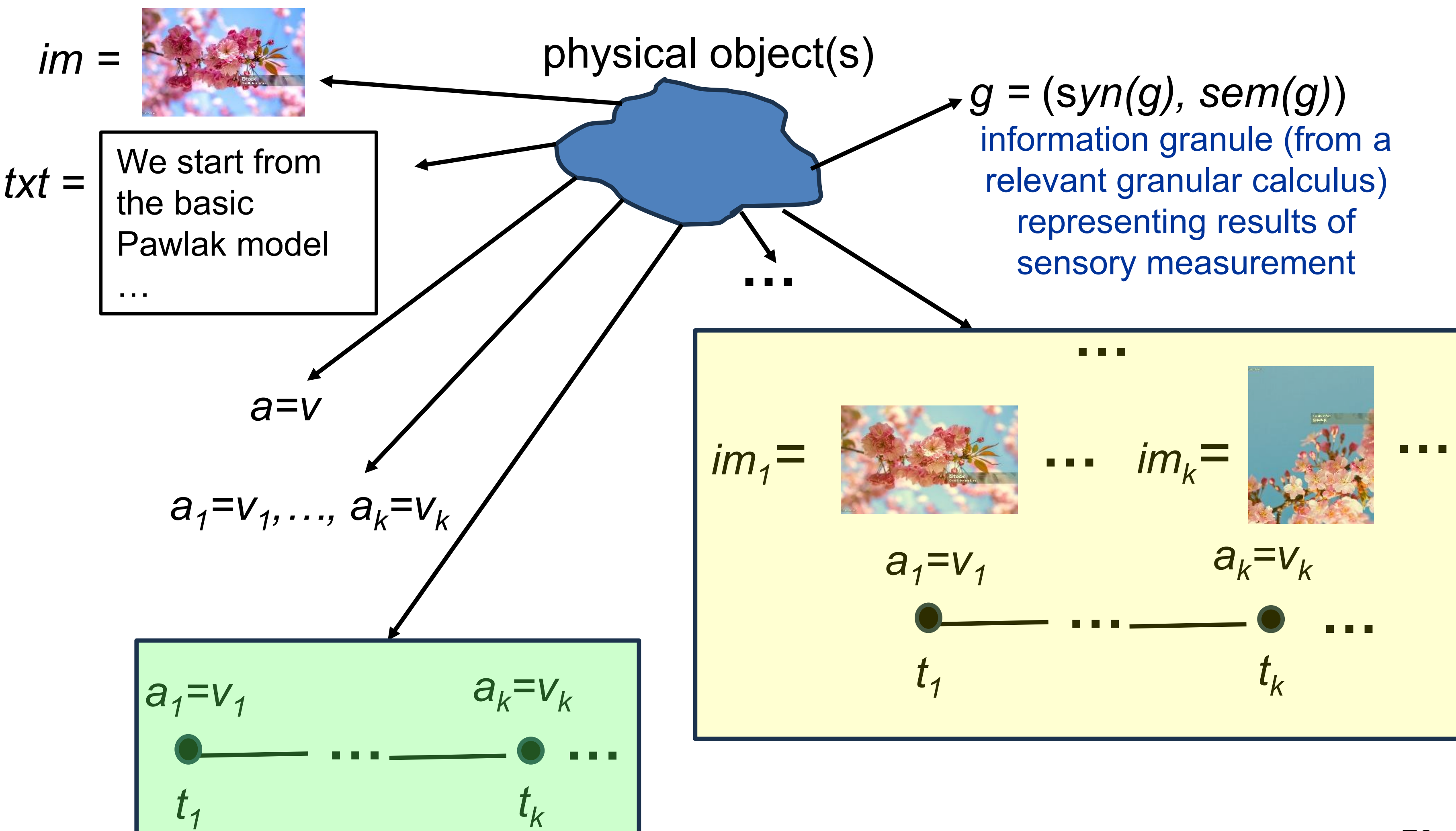
<u>OPTIMIZATION PROBLEMS</u>:

Searching for (semi-)optimal solutions of the specified problems (concerning of, e.g., approximation of concepts or classifications) relative to a given calculus of granules (with quality measures).

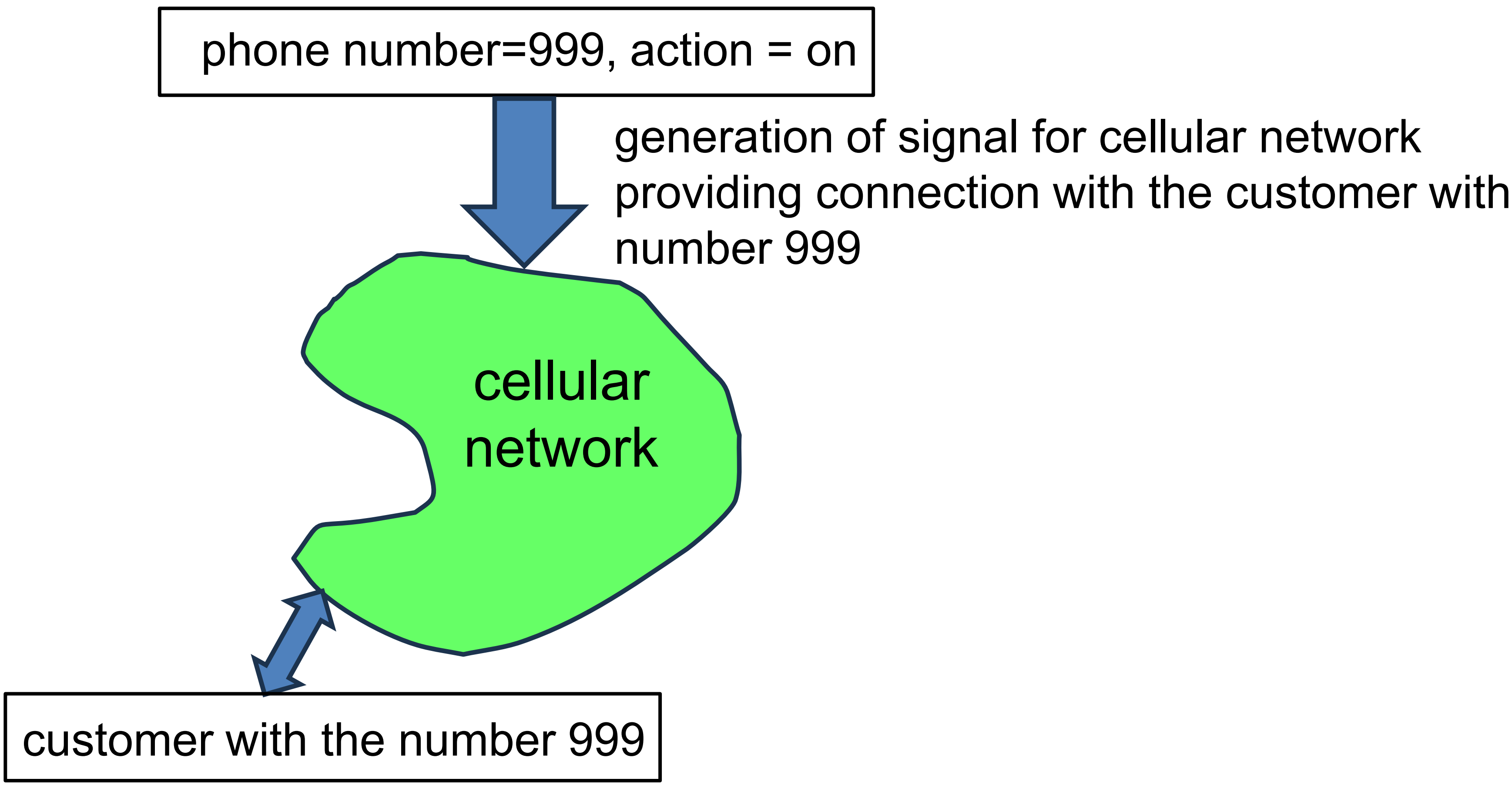
ATOMIC GRANULES LINKING ABSTRACT AND PHYSICAL OBJECTS

- encoding information from information granules into physical objects
- decoding results of sensory measurements and actions from physical objects into information granules

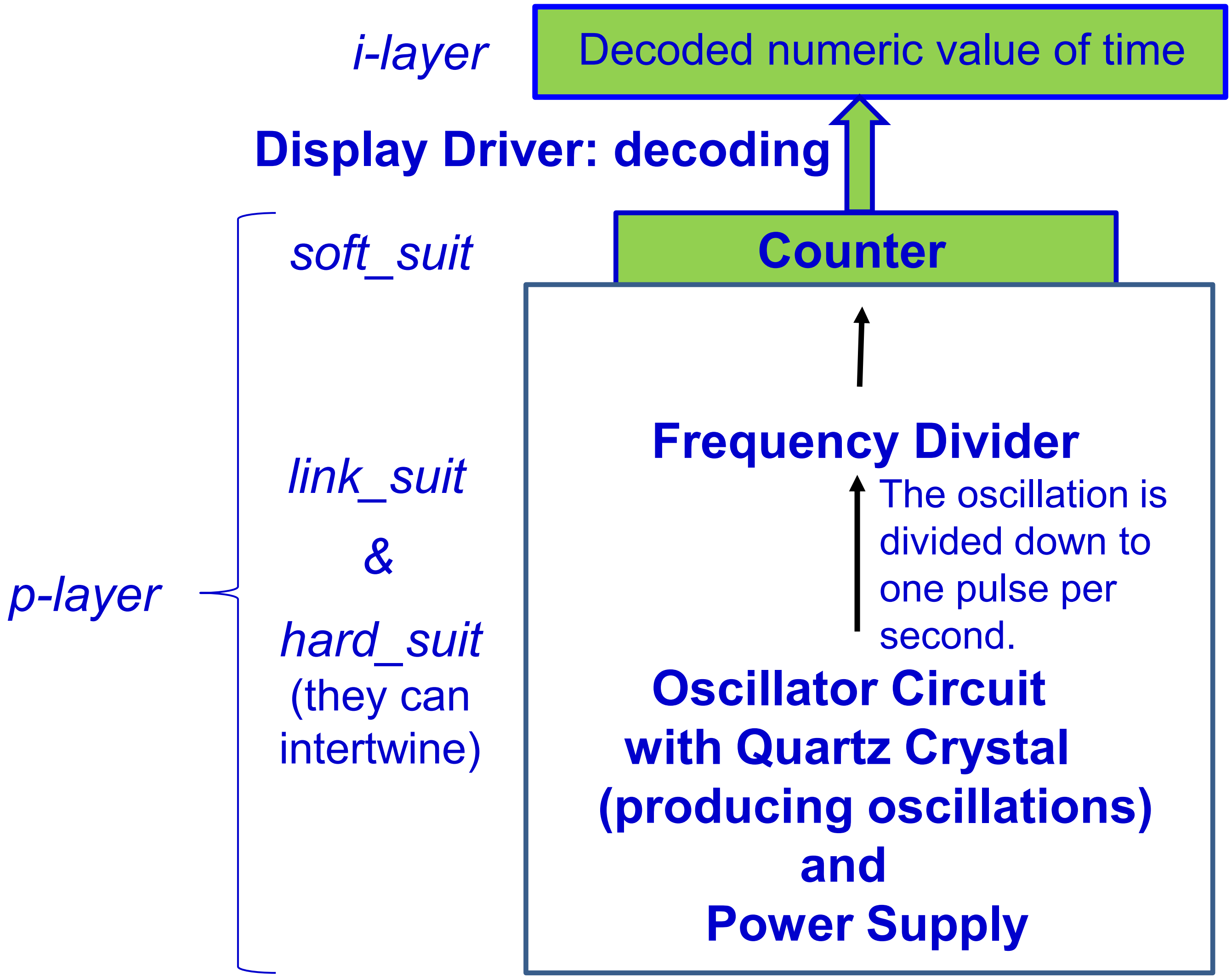
DECODING RESULTS OF SENSORY MEASUREMENTS AND ACTIONS BY C-GRANULES FROM PHYSICAL OBJECTS INTO INFORMATION LAYERS OF C-GRANULES: EXAMPLES



ENCODING INFORMATION FROM INFORMATION GRANULES INTO PHYSICAL OBJECTS: EXAMPLE



DECODING INFORMATION FROM PHYSICAL OBJECTS INTO I-LAYER: EXAMPLE: QUARTZ CLOCK



ATOMIC GRANULES DEFINED ON THE BASIS OF ATTRIBUTES AS ATOMIC BUILDING BLOCKS FOR CONCEPT FORMATION

- relational structures over V_a
- optimization problems in discovery of relevant attributes

$$a: U \rightarrow V_a$$

$R_a = (V_a, =)$
$xIND(a)y$ iff $a(x) = a(y)$
atomic granules:
$[x]_a = \{y \epsilon U: xIND(a)y\}$
indiscernibility classes
partitioning of U
$U/IND(a) = \{[x]_a: x \epsilon U\}$

$R_a = (V_a, \tau)$
$x\tau_a y$ iff $a(x)\ \tau\ a(y)$
τ – tolerance or similarity relations
atomic granules:
$[x]_\tau = \{y \epsilon U: x\tau_a y\}$
covering of U by tolerance classes
$U/\tau = \{[x]_\tau: x \epsilon U\}$
possible partition of U from τ_a, e.g.
$\{P_x^\tau: \text{x} \in U\}$ where
$P_x^\tau = \{y \in U: \forall z \in U (x\tau z \text{ iff } y\tau z)\}$

combination of these two approaches may help to construct better granules, e.g., construct granules for recognition regions with different certainty for decision

L. Ma, M. Li: Covering rough set models, fuzzy rough set models and soft rough set models induced by covering similarity. Information Sciences 689 (2025)doi.org/10.1016/j.ins.2024.121520
Z. Pawlak: Rough sets. Theoretical Aspects of Reasoning About Data. Kluwer (1991). doi.org/10.1007/978-94-011-3534-4

ATOMIC GRANULES DEFINED ON THE BASIS OF ATTRIBUTES AS ATOMIC BUILDING BLOCKS FOR CONCEPT FORMATION

optimization problems in discovery of relevant attributes

$$a: U \to V_a$$

$$d: U \to V_d$$

V_a - a finite set of values of attribute a

P - a partition of V_a

a^P constructed from a

$$a^P: U \to P$$

$$a^P(x) \ni a(x), \quad \text{for } x \epsilon U$$

a large set of partitions P

<u>optimization problem</u>: selection of P with (semi-) minimal $|P|$ satisfying a given optimization criterion based on *purity* of

$$d(P) = \{d(a^{-1}(p)): p \in P\}$$

ATOMIC GRANULES DEFINED ON THE BASIS OF ATTRIBUTES AS ATOMIC BUILDING BLOCKS FOR CONCEPT FORMATION (cont.)

optimization problems in discovery of relevant attributes

$$a: U \rightarrow V_a$$

<u>structural objects</u>

$a(x) = ag_a[a_1(\pi_1(x)), \ldots, a_k(\pi_k(x))]$

$\pi_1, \ldots, \pi_k$ - decompositions of objects into parts selected from Π_a; $\pi_i: U \rightarrow U_i$

$a_1, \ldots, a_k$ - attributes of parts of objects selected from At_a; $a_i: U_i \rightarrow V_{a_i}$

ag_a- aggregation of attribute values of parts selected from Ag_a;

$ag_a: \rightarrow V_{a_1} \times \cdots \times V_{a_k} \rightarrow V_a$

<u>optimization problem</u>: selection of (semi-) optimal π_i, a_i, ag_a from Π_a, At_a, Ag_a relative to a given optimization criterion

Discovery of Π_a, At_a *and* Ag_a necessary !

TWO PARADIGMS OF LOGIC: JUSTIFICATION & DISCOVERY

Bocheński says that

[…] *one can ask two different basic questions:*

(1) What follows given premises?

(2) From what premises can a given sentence (conclusion) be deduced?

***Aristotle** primarily considered the first question,*

justification,

[…] but *poses also the second*,

discovery, and tries to show

how the premises of a syllogism must be constructed in order to yield a given conclusion,

discovery is

[…] *not essentially a matter of formal logic.*

Bocheński, J. (1961). A history of formal logic. University of Notre Dame Press.

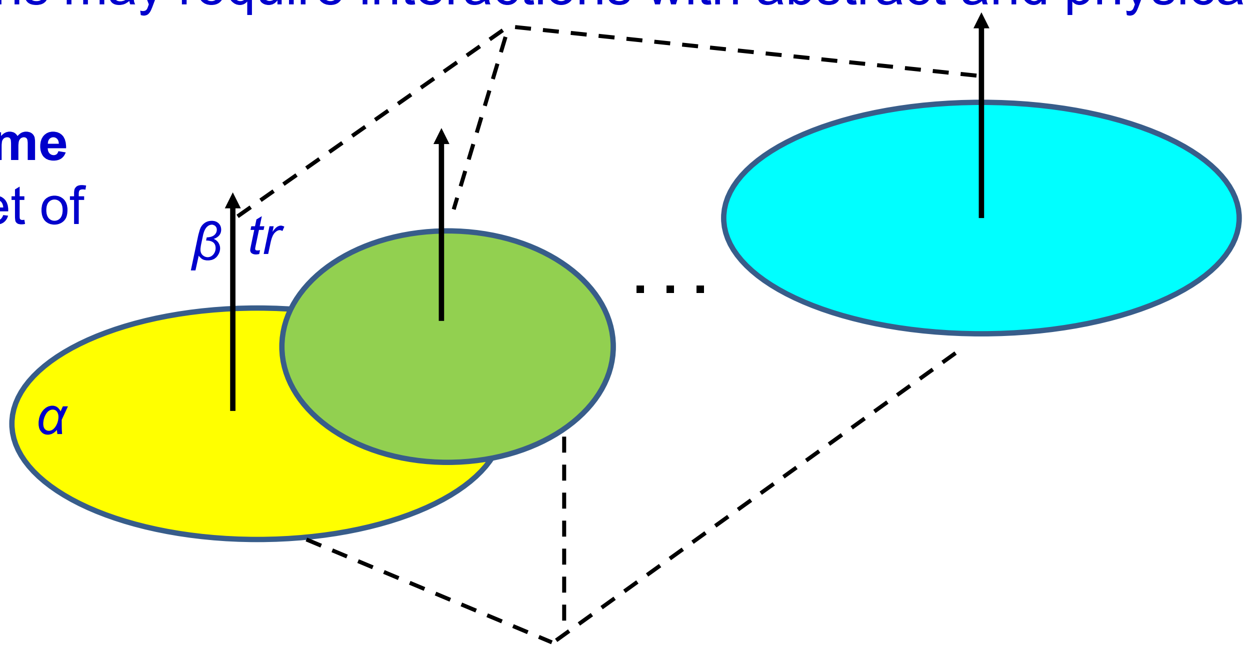
E. Ippoliti,.F. Sterpetti (eds.): The Heuristic View. Logic, Mathematics, and Science. Springer (2025)

For IS to be grounded in IGrC, generation of relevant granular computations, especially through the discovery of complex games, is essential for developing high-quality approximate solutions.

DISCOVERY AND EVOLUTION OF COMPLEX GAMES INTERACTING WITH THE ABSTRACT AND PHYSICAL WORLDS

transformations (associations) (*tr*), with specifications concerning their expected results (*β*), aiming to perform the relevant measurements/ actions/ plans toward achieving the expected target goals; the implementation of transformations may require interactions with abstract and physical objects

complex game
i.e., a finite set of rules
$r: \alpha \rightarrow_{tr} \beta$



complex vague concepts (*α*) triggering actions/plans;
their approximations by classifiers may require interactions with abstract and physical objects

DISCOVERY OF PREMISSES FOR GIVEN CONSEQUENCES OF RULES OF COMPLEX GAMES (PD)

The c-granule *g* (in particular, IS) with control (and sub-granules consisting of, in particular testing samples) should be able to solve the following PD problem:

for given quality measure γ over granular computations and rule consequence (*tr, β*), where

- γ allows to check if a granular computation of *g* has a given property (to a degree) and
- *tr* is a specification of transformation (association, action, plan, procedure) from a given set *TR* (of specifications of transformations of *g*) such that the realization of *tr* by *g* in the form of granular computation is expected to satisfy property *β* (to satisfactory degree)

discover a concept (property, classifier) α_{tr} (with acceptable support) such that

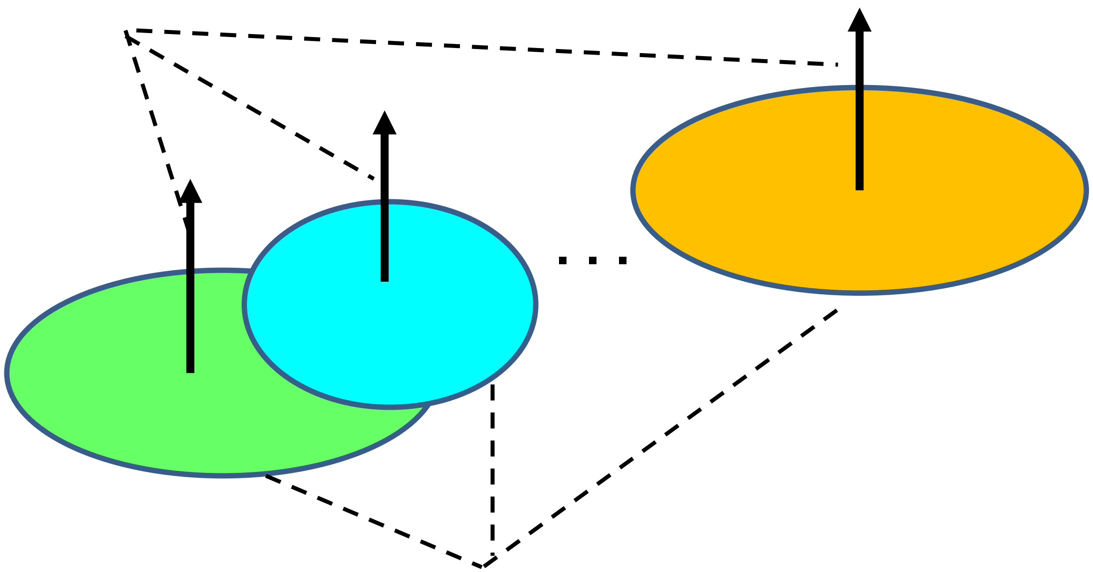
if α_{tr} is satisfied in the currently perceived situation/object by *g* to a satisfactory degree in comparison to satisfiability degrees of concepts discovered by *g* for transformations from $TR \setminus \{tr\}$

then

granular computations of *g* starting at a granular network for the currently perceived situation satisfying α_{tr} and ending at the granular network obtained after realization of *tr* are satisfying property *β* to a satisfactory degree what is checked using γ (and estimated relative to, e.g., a set of testing samples).

DISCOVERY
OF COMPLEX ADAPTIVE GAMES
INTERACTING
WITH THE ABSTRACT AND PHYSICAL WORLDS

complex games for situations with the relevant properties



complex vague concepts
triggering complex games

DISCOVERY OF ADAPTATION STRATEGIES FOR COMPLEX GAMES STEERING (CGA PROBLEM)

The c-granule *g* (in particular, IS) with control (and sub-granules consisting of, in particular a set of testing samples) should be able to solve the following CGA problem:

Given

- property *α* of the perceived situation (object) by the c-granule *g,*
- class od complex games ***CCG****,*
- quality measure γ for checking if a granular computation of *g* is of the relevant quality (to a satisfactory degree),

discover

- complex game $CG = \{\alpha_1 \rightarrow_{tr_1} \beta_1, \ldots, \alpha_k \rightarrow_{tr_k} \beta_k\} \in$ ***CCG*** and
- an adaptation strategy *S* over ***CCG****,*

  assigning to

  the current granular network *GN* (with the current complex game CG_{cr} encoded in *GN*) of the currently realized granular computation *Gcom* (starting at CG)

  the relevant game from ***CCG*** on the basis of the quality of *GCom* (estimated using γ) and properties of history of granular computations encoded in *GN*, such that

the granular computations of *g,* starting from a granular network with encoded CG, satisfying the property *α,* and steered by the strategy *S*, are of acceptable quality (estimated relative, e.g., to a set of testing samples) defined by γ*.*

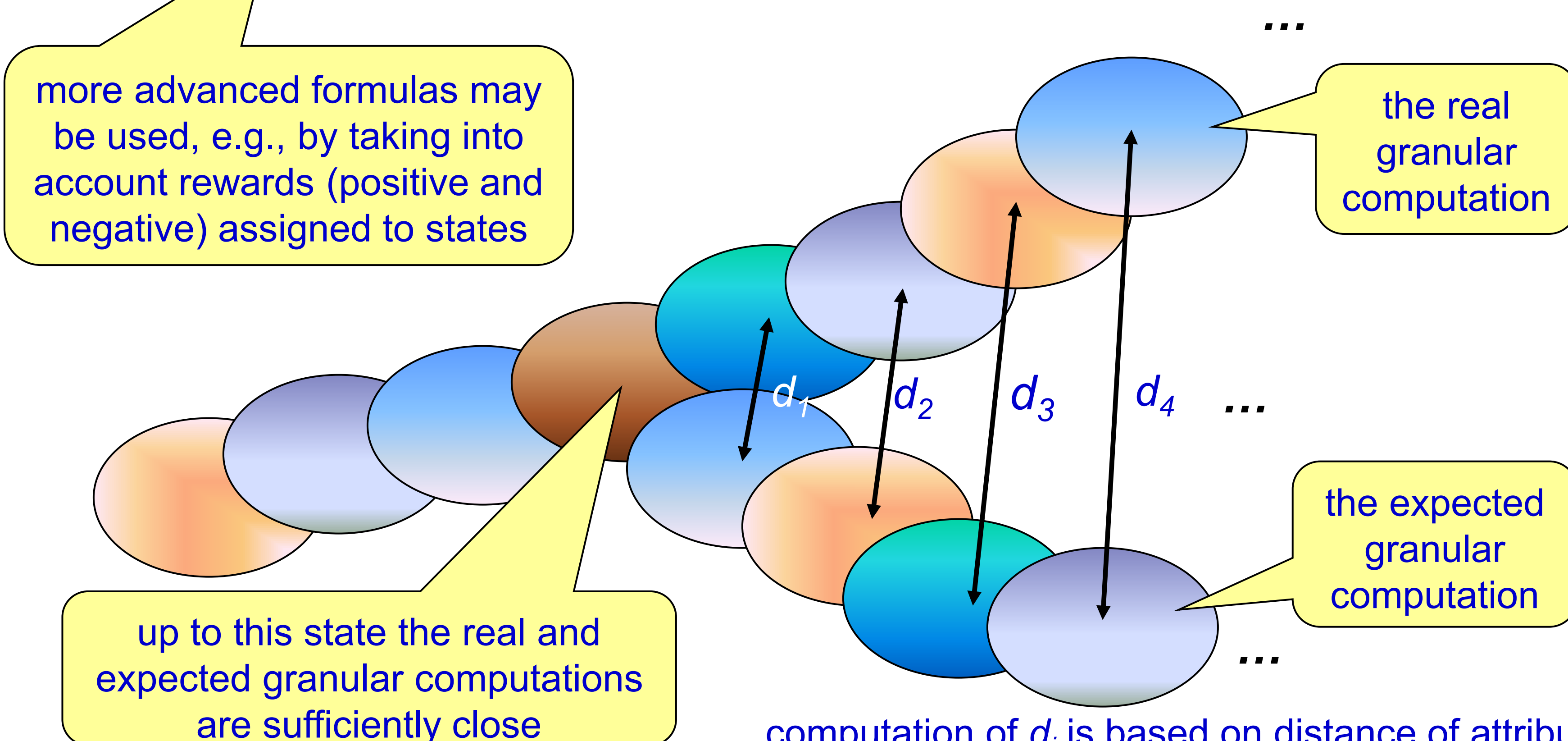
QUALITY OF GRANULAR COMPUTATION: EXAMPLE

Quality measure based on a distance between the real granular computation and the expected one derived on the basis of a given plan

Illustrative simple example:

$If\ \sum d_i > tr$ (tr – threshold)

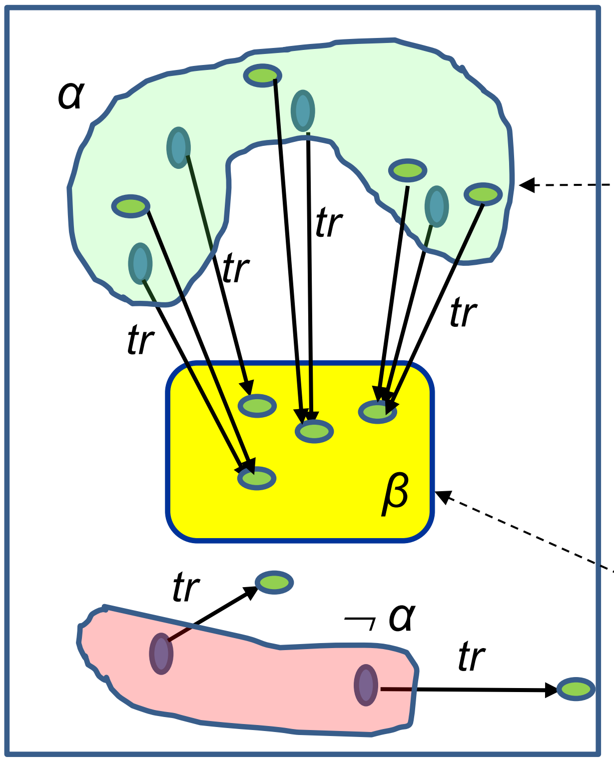
then modify the current complex game using the adaptive strategy.



computation of d_i is based on distance of attribute value vectors characterizing the states of granular computation

DISCOVERY OF PREMISES OF RULES: EXEMPLARY SCHEME

$$Given: tr, \beta.\ Discover: \alpha\ (with\ some\ \ additional\ requirements)\ such\ that\ \alpha \rightarrow_{tr} \beta$$



Objects are generated from objects with property β (defined by a classifier) using the **deviation** of the vectors of attribute values of objects satisfying β, which is properly tuned.

The aim of this tuning is to obtain objects that, after being transformed by *tr*, are satisfying β with a high chance.

These generated objects are then used as training samples to construct a classifier for the concept α, consisting of objects satisfying β after transformation by *tr*.

Objects with the property β (recognized by the relevant classifier induced from training sample).

<u>Remarks</u>.

In searching: (i) large support of rules and/or closeness to the initial conditions are preferred , (ii) domain knowledge can help.

Note that β describes the expected results of the (physical) realization of *tr*, but the actual results may differ due to interactions with the environment, including physical objects.

DISCOVERY OF PLANS: EXEMPLARY SCHEME

$$Given\colon\ \alpha, \beta.\ Discover\ rules\ r_k\,, \ldots\,, r_2, r_1\ such\ that$$
$$the\ realization\ of\ tr_k\,, \ldots, tr_2, tr_1\ leads\ from\ \alpha\ to\ \beta$$

Bottom-up
(from β toward α)
discovery
of a sequence
of rules with
constraints between
them defining a plan
realized by
$tr_k\,, \ldots\,, tr_2, tr_1$
***

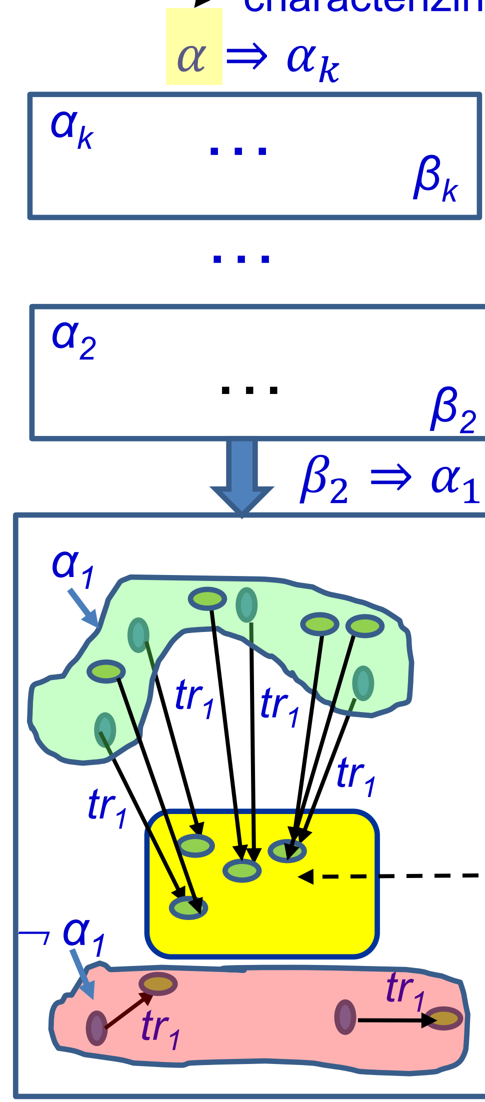
<u>Discovery challenge</u>:
Hitting α

e.g., conjunction of conditions characterizing the analyzed case(s)

$r_k\colon \alpha_k \rightarrow_{tr_k} \beta_k$

$r_2\colon \alpha_2 \rightarrow_{tr_2} \beta_2$

$r_1\colon \alpha_1 \rightarrow_{tr_1} \beta$



<u>Remarks</u>.
In searching:
(i) large support of rules is preferred,
(ii) backtracking strategies are often necessary,
(iii) domain knowledge can help
(iv) possible parallel search.

objects with property β

<u>Hint</u>: Search for ways to decrease the distance (measured between vectors of attribute values) between cases covered by the currently constructed rule premise (transformed into cases satisfying its consequence) and those covered by α.

CHAIN-OF-THOUGHT (CoT) REASONING

Adaline Labs : https://labs.adaline.ai/p/inside-reasoning-models-openai-o3

[…] Reasoning models in AI are designed to emulate human-like logical thinking through a step-by-step process rather than relying solely on pattern matching. **This approach allows the model to generate intermediate reasoning steps, resulting in more transparent and interpretable solutions.**

But there are still large gaps in understanding human logical thinking. For example, there is still much to learn about commonsense reasoning and reasoning by analogy, etc.

M. Mitchell: Abstraction and Analogy-Making in Artificial Intelligence, Annals Reports of the New York Academy of Sciences 1505(1), 79-101 (2021)

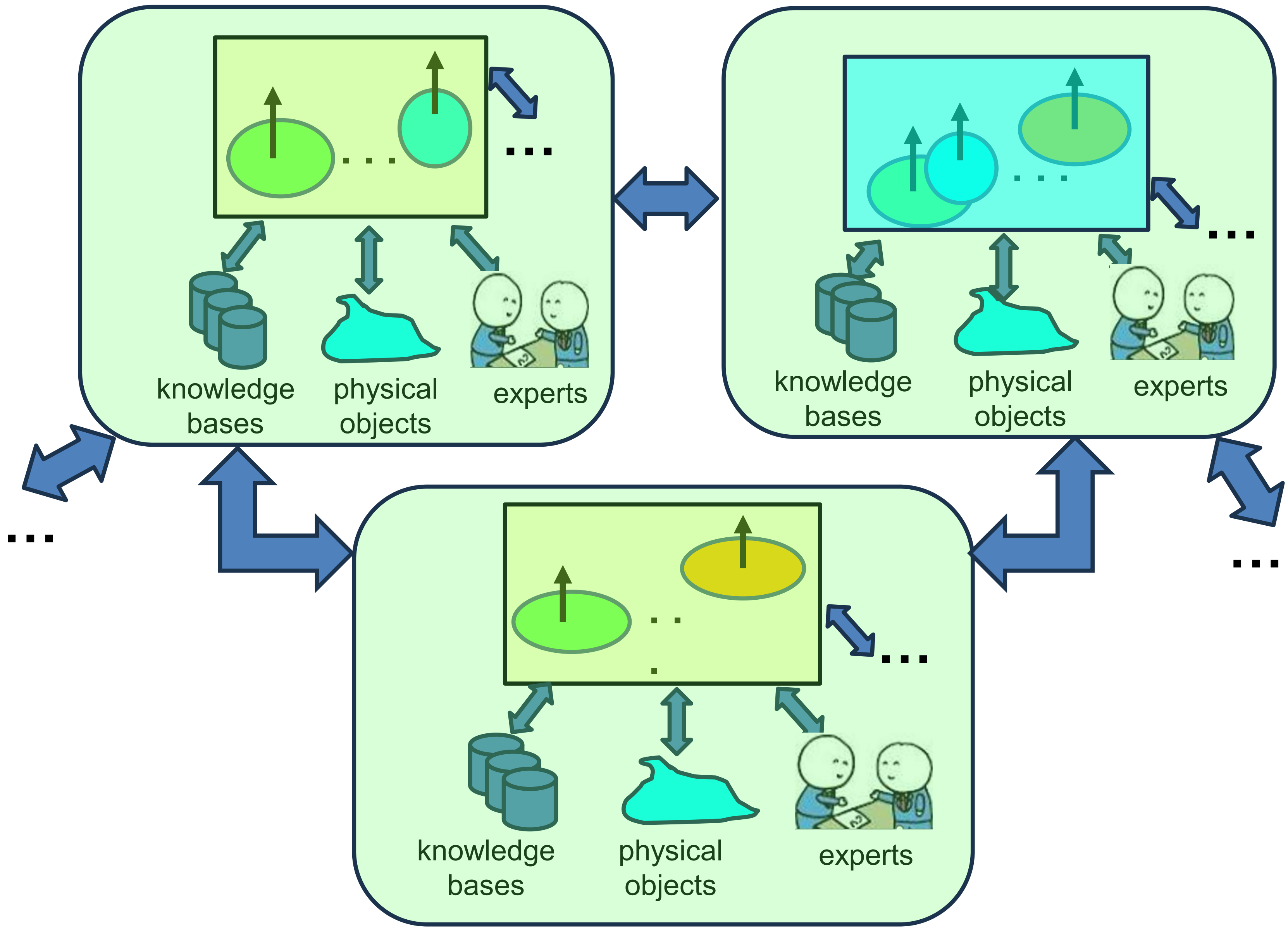
Decomposition of complex vague concepts

[…] Information granulation plays a key role in implementation of the strategy of divide-and-conquer in human problem-solving.

L. A. Zadeh: Foreword. In: S. K. Pal, L. Polkowski, A. Skowron (eds.) Rough-Neural Computing. Techniques for Computing with Words. Springer 2004.

[…] OpenAI's o3 and DeepSeek's already-released DeepSeek R1 are set to redefine AI reasoning. o3 leverages innovative test-time search to achieve high-performance reasoning, while DeepSeek R1 has captured attention for its cost-efficient design, transparent "aha moment," and ability to tackle math, coding, and logic challenges at a fraction of traditional costs.

DISCOVERING
OF FAMILIES OF INTERACTING COMPLEX GAMES



GRANULAR NETWORKS
AS OBJECTS ON WHICH GRANULAR COMPUTATIONS IN IGrC ARE REALIZED:

EXAMPLES

LINKING GRANULAR NETWORKS BY INTERFACES

PRODUCTS WITH CONSTRAINTS OF ALREADY CONSTRUCTED GRANULAR NETWORKS

GN_1 granular network

- elementary (atomic) granules
- transformations for constructing granules
- methods (algorithms) for generation new granules, modifying existing ones and reasoning about computations over them
- can have nested structure
- …

Interface

$Inter(GN_1, GN_2)$

- relations between granules from GN_1 and GN_2
- transformations of granules from GN_1 to GN_2
- rules for reasoning about properties of granules from GN_2 on the basis of properties of granules from GN_1 transformed to granules from GN_2
- methods for generation of new samples of granules in GN_1 and GN_2
- …

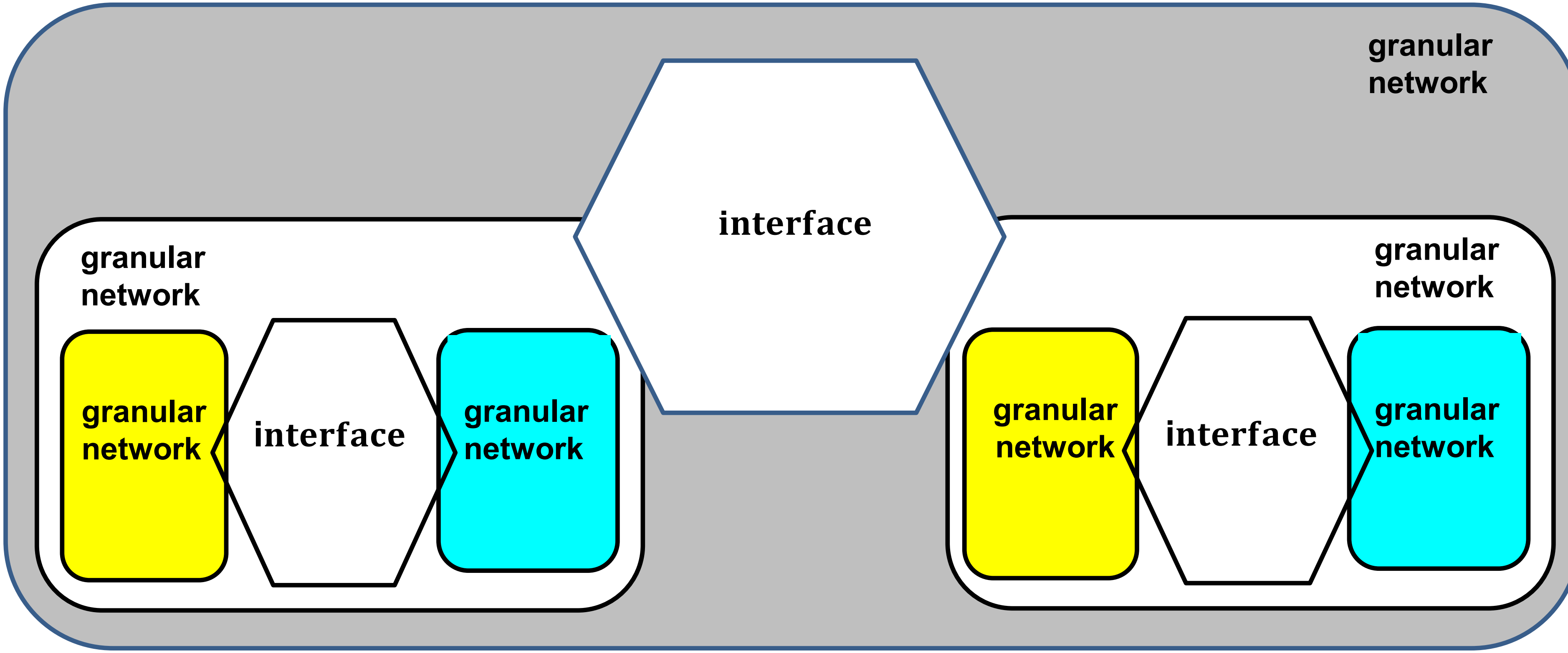
GN_2 granular network

- elementary (atomic) granules
- transformations for constructing granules
- methods (algorithms) for generation new granules, modifying existing ones and reasoning about computations over them
- can have nested structure
- …

LINKING GRANULAR NETWORKS BY INTERFACES

PRODUCTS WITH CONSTRAINTS OF ALREADY CONSTRUCTED GRANULAR NETWORKS

granular network can have nested structure:



NETWORK OF APPROXIMATION SPACES LINKED BY INTERFACES: THE PAWLAK ROUGH SET MODEL

$G_R = \{g_x : x \in U\}$

$R \subseteq U \times U$ – *equivalence relation*

$g_x = (f([x]_R), [x]_R)$, $x \in U$

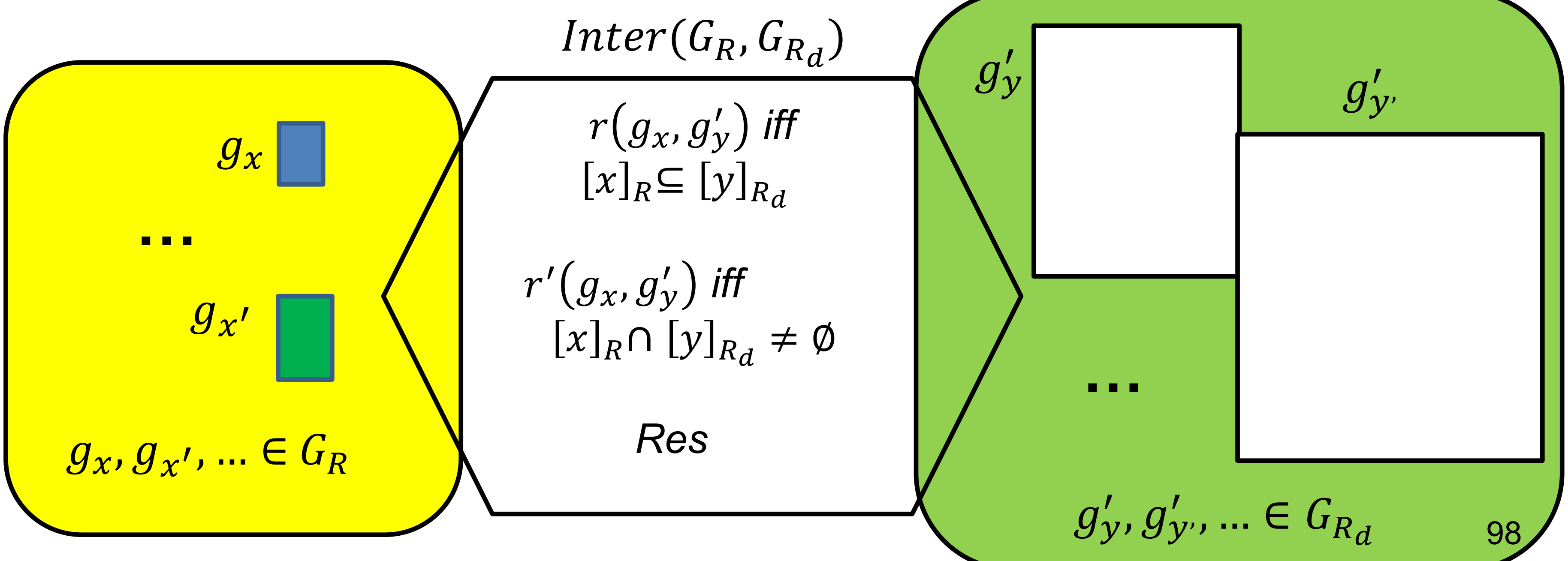
$f: U/R \rightarrow \{1, \ldots, |U/R|\}$ - *bijection*

$G_{R_d} = \{g'_y : y \in U\}$

$R_d \subseteq U \times U$ – *equivalence relation*

$g'_y = (h([y]_{R_d}), [y]_{R_d})$, $y \in U$

$h: U/R_d \rightarrow \{1, \ldots, \lceil U/R_d \rceil\}$ - *bijection*



NETWORK OF APPROXIMATION SPACES LINKED BY INTERFACES: THE PAWLAK ROUGH SET MODEL

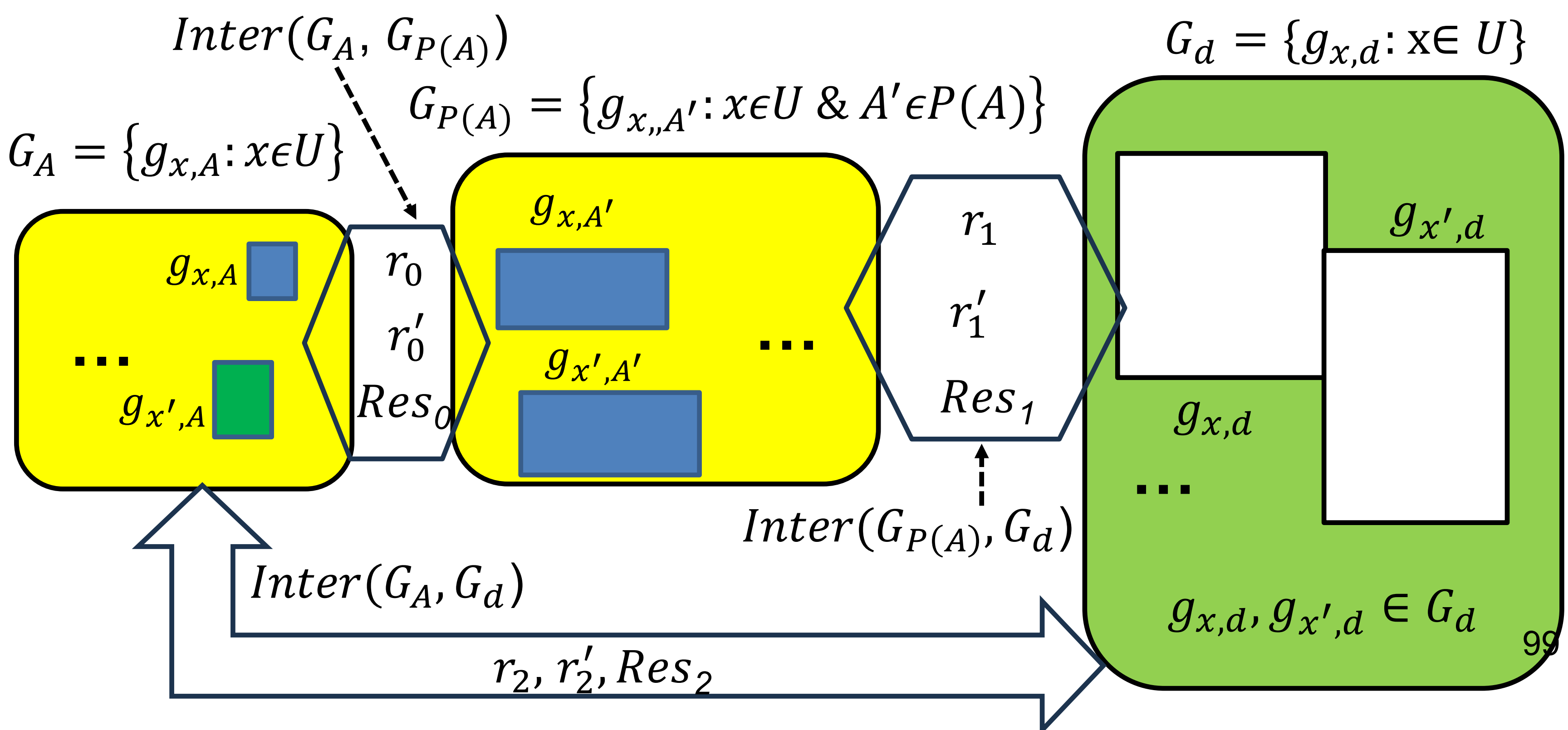
$$DS = (U, A, d)$$

$$g_{x,,A'} = (inf_{A'}(x)), [x]_{IND(A')}); \; g_{x,,d} = (d = d(x), [x]_{IND(d)})$$

$$A' \epsilon P(A) = \{A' : A' \subseteq A\}$$

$$r_i(g, g') \text{ iff } sem(g) \subseteq sem(g')$$

$$r_i'(g, g') \text{ iff } sem(g) \cap sem(g') \neq \emptyset, \; i = 0,1,2$$



NETWORK OF APPROXIMATION SPACES LINKED BY INTERFACE:THE PAWLAK ROUGH SET MODEL OPTIMIZATION OF CLASSIFICATION RELATIVE TO A FAMILY OF INDISCERNIBILITY RELATIONS

$G_I = \{g_i : i \in I\}$

$R_i \subseteq U \times U$ *– equivalence relation (or partition defined by* R_i*); U-finite set*

$g_i = (f_{R_i}, R_i)$, *i* $\in I$

f_{R_i}: *U* $\rightarrow \{1, \dots, |U/R_i|\}$ *- bijection*

$G_{R_d} = \{g'_y : y \in U\}$

$R_d \subseteq U \times U$ *– equivalence relation*

U/R_d *- classification*

g'_y=(*h*($[y]_{R_d}$), $[y]_{R_d}$), *y* $\in$*U*

h: U/R_d $\rightarrow \{1, \dots, \lceil U/R_d \rceil\}$ *- bijection*

$Inter(G_I, G_{R_d})$

$r_\gamma(G_I, G_{R_d})$ *iff*

$sup_{i \epsilon I} POS_{R_i}(R_d) \geq \gamma$

where

$$POS_{R_i}(R_d) = \frac{\left| \cup \{\underline{R_i}[y]_{R_d} : y \epsilon U\} \right|}{|U|}$$

$\underline{R_i}[y]_{R_d} =$

$= \cup \{z \epsilon\, U/R_i :\ z \subseteq [y]_{R_d}\}$

$\gamma \epsilon (0,1]$

Res

g_i

...

g_j

$g_i, g_j, \dots \in G_I$

g'_y

$g'_{y'}$

...

$g'_y, g'_{y'}, \dots \in G_{R_d}$

NETWORK OF APPROXIMATION SPACES FOR PAWLAK'S ROUGH SET MODEL: INCLUSION EXAMPLE

$$R \subseteq U \times U, R^* \subseteq U^* \times U^* - equivalence\ (indiscernibility)\ relations$$

$$U \subseteq U^*, R^* \cap (U \times U) = R$$

$$G_R = \{g_x : x \in U\}$$

$$g_x = (f([x]_R), [x]_R), x \in U$$

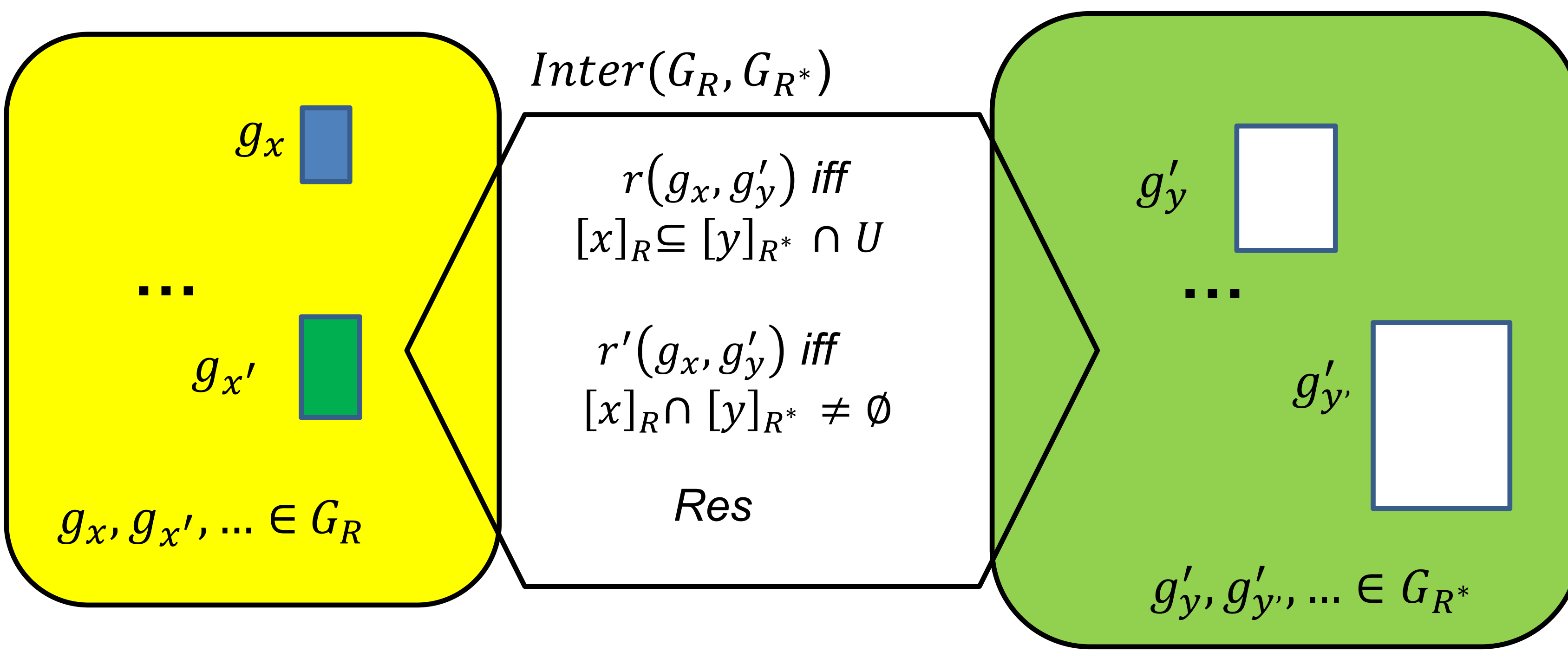
$f: U/R \to \{1, \ldots, |U/R|\}$ - *bijection*

$$G_{R^*} = \{g'_y : y \in U^*\}$$

$$g'_y = (h([y]_{R^*}), [y]_{R^*}), y \in U^*$$

$h: U^*/R^* \to \{1, \ldots, |U^*/R^*|\}$ - *bijection*

$h([y]_{R^*}) = f([y]_R)$ *for* $y \in U$



REASONING: EXAMPLE (induction, conflict resolution)

$$x \in U^* \setminus U$$
$$IND(A) \subseteq U^* \times U^*$$

$$[x]_{IND(A)} \cap U \neq \emptyset$$

$$d(x) = d(x_0)$$
for some $x_0 \in [x]_{IND(A)} \cap U$

$$[x]_{IND(A)} \cap U = \emptyset$$

--- similarity

--- reducts

--- rules

--- conflict resolution

…

NETWORK OF APPROXIMATION SPACES INDUCED FROM THE NETWORK OF APPROXIMATION SPACES FOR PAWLAK'S ROUGH SET MODEL: EXAMPLE

$R^* \subseteq U^* \times U^*, U \subseteq U^*, R = R^* \cap (U \times U), G_{R^*} = \{g^*_x : x \in U^*\}$

$g^*_x = (f^*([x]_{R^*}), [x]_{R^*})$, $x \in U^*$

$f^*: U^*/R^* \rightarrow \{1, \ldots, |U^*/R^*|\}$- *bijection,*

$f^*([x]_{R^*}) = f([x]_R)$ *for* $x \in U$

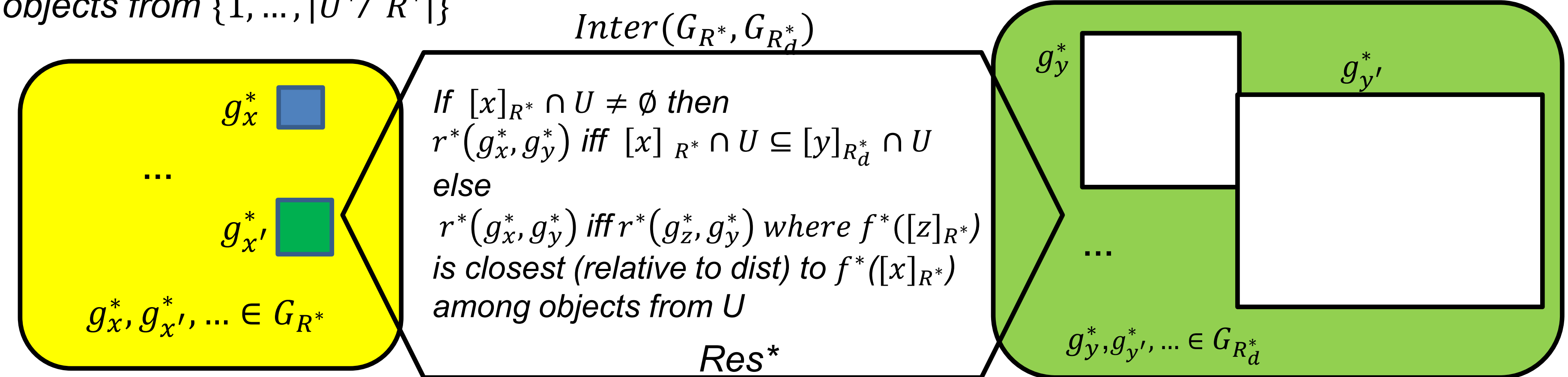
dist – distance function between objects from $\{1, \ldots, |U^*/R^*|\}$

$R^*_d \subseteq U^* \times U^*$ $G_{R^*_d} = \{g^*_y : y \in U^*\}$

$g^*_y = (h([y]_{R^*_d}), [y]_{R^*_d})$, $y \in U^*$

$h^*: U^*/R^*_d \rightarrow \{1, \ldots, [U^*/R^*_d]\}$ *– bijection,*

$h^*([x]_{R^*}) = h([x]_R)$ *for* $x \in U$



inductve extension

$R \subseteq U \times U,\ G_R = \{g_x : x \in U\}$

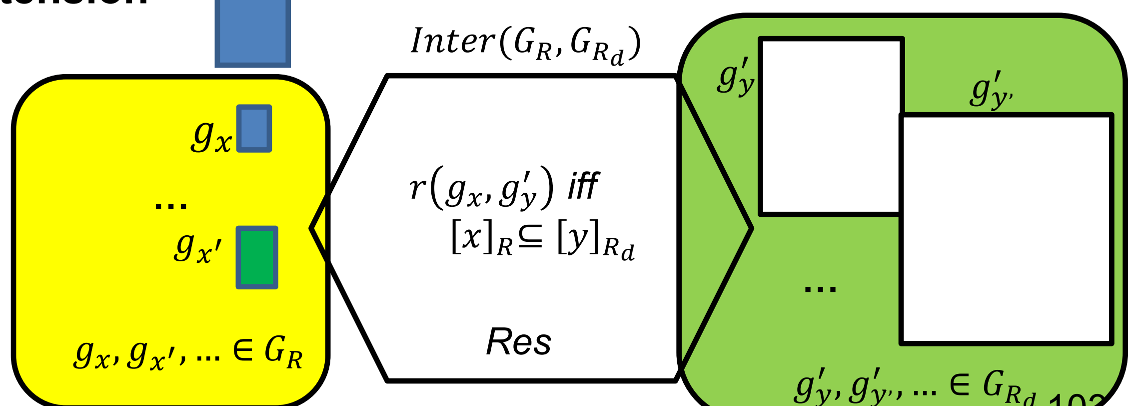
$g_x = (f([x]_R), [x]_R)$, $x \in U$

f: $U/R \rightarrow \{1, \ldots, |U/R|\}$ *- bijection*

$R_d \subseteq U \times U, G_{R_d} = \{g'_y : y \in U\}$

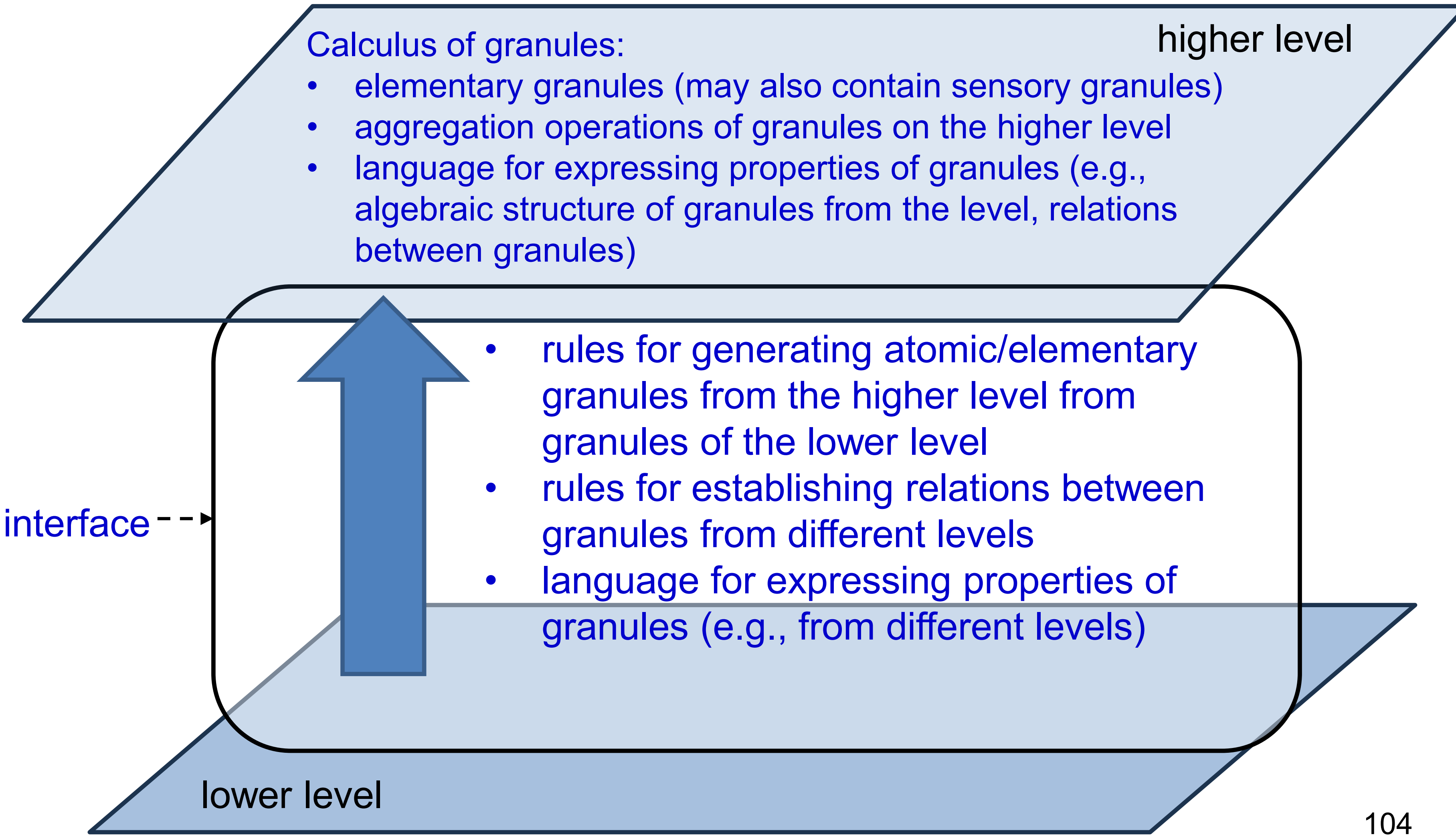
$g'_y = (h([y]_{R_d}), [y]_{R_d})$, $y \in U$

h: $U/R_d \rightarrow \{1, \ldots, [U/R_d]\}$ *- bijection*

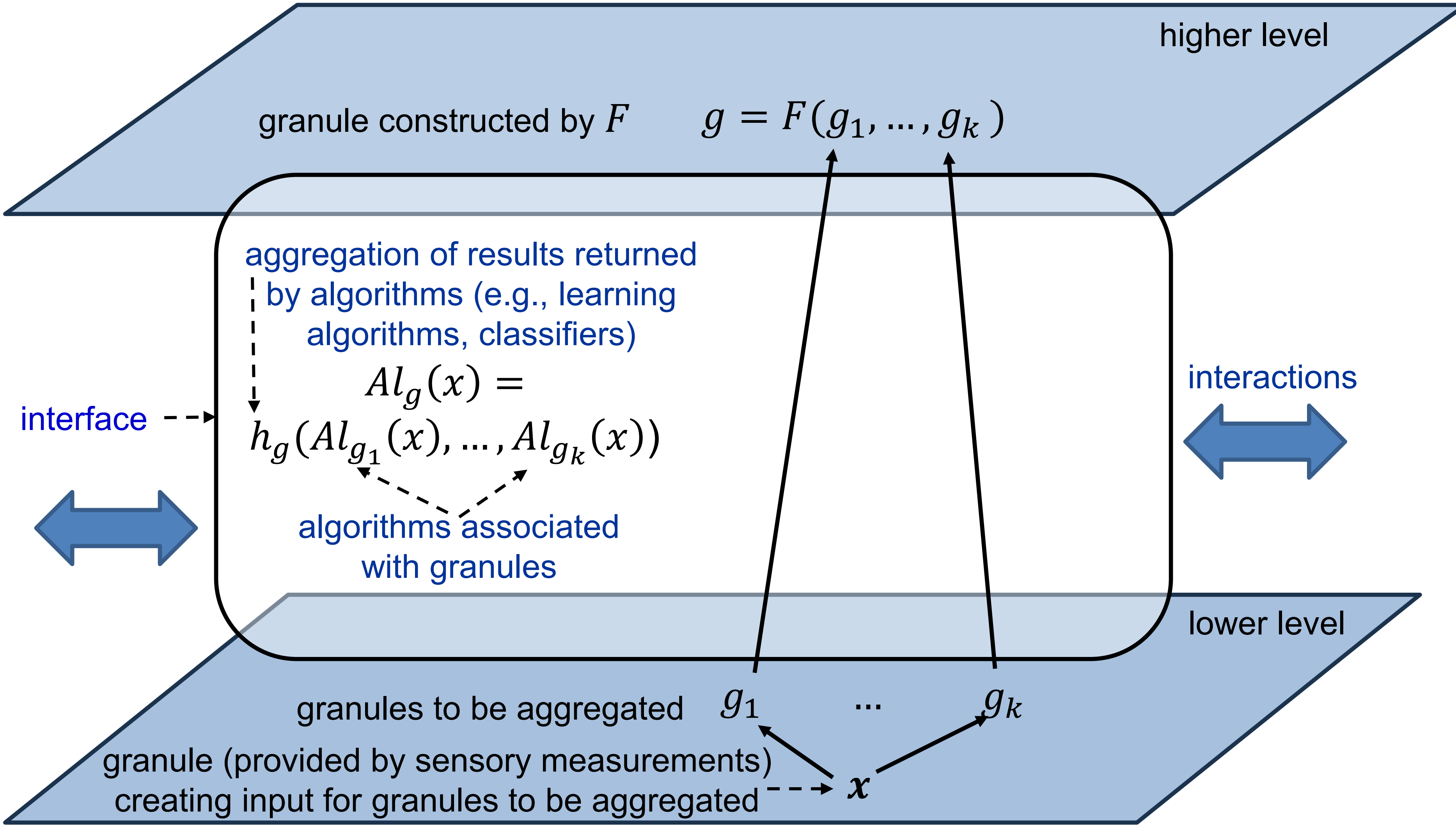


SIMPLIFIED NETWORK OF APPROXIMATION SPACES FOR PAWLAK'S ROUGH SET MODEL

INTERFACES BETWEEN GRANULAR NETWORKS

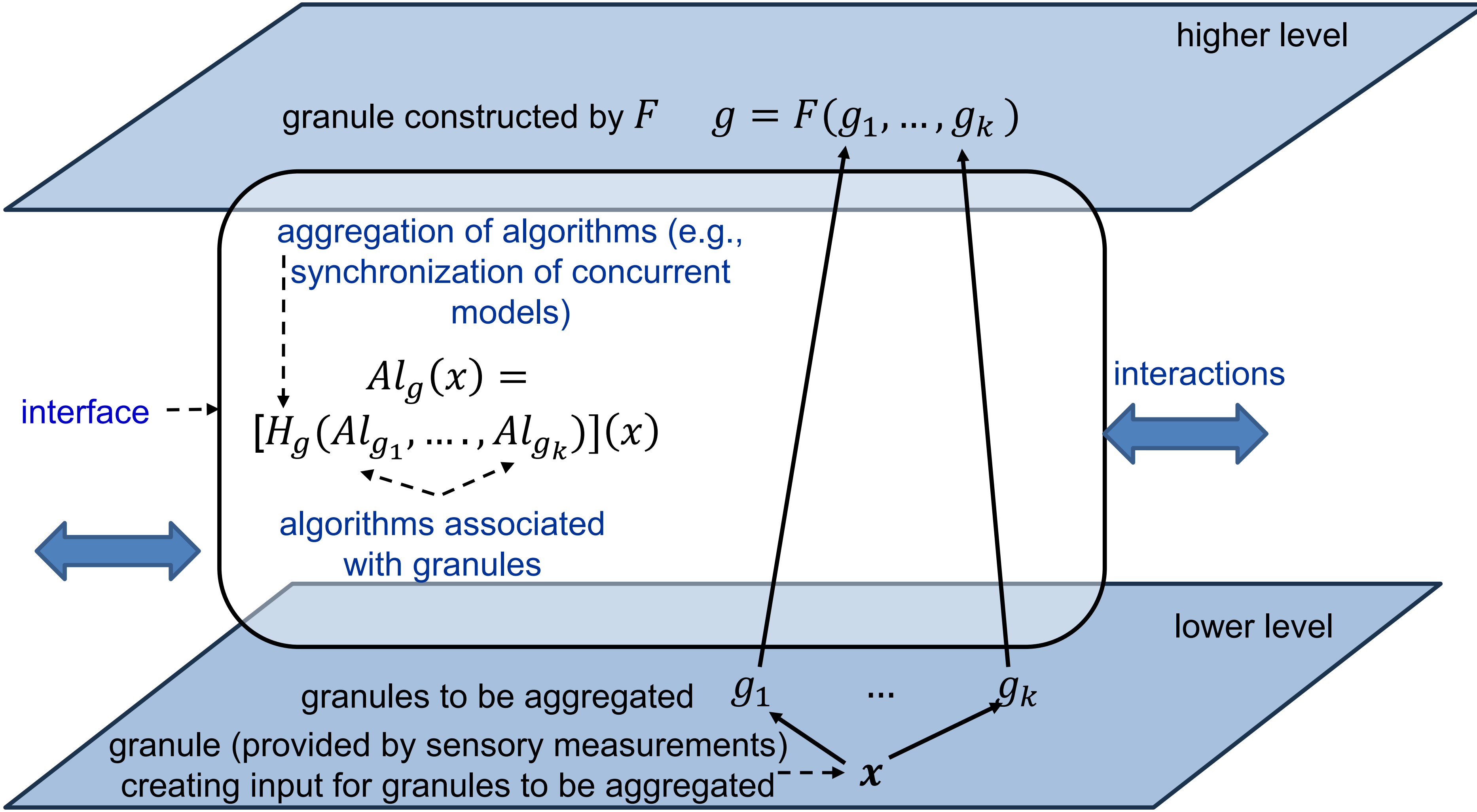


INTERFACES BETWEEN GRANULAR NETWORKS: ALGORITHMIC SEMANTICS OF GRANULES



<u>Remarks</u>. This formula is important for hierarchical reasoning about the perceived situation. Note that computations of algorithms (that can be unconventional) may be disturbed by interactions with the environment – the *real* returned results may be different from the *expected,* given by formulas.

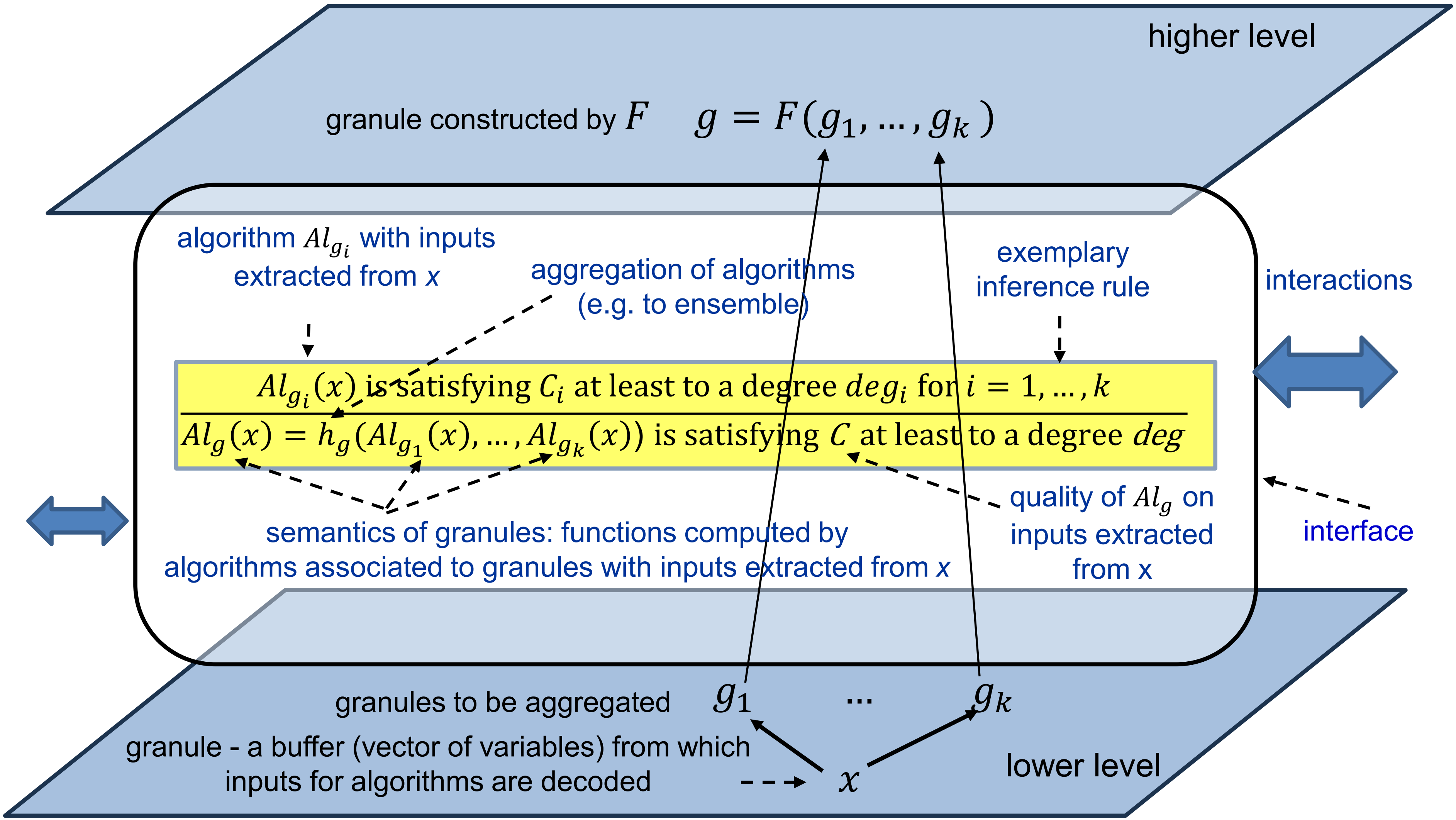
INTERFACES BETWEEN GRANULAR NETWORKS:
ALGORITHMIC SEMANTICS OF GRANULES



Remarks. This formula is important for hierarchical reasoning about the perceived situation. In some tasks H_g should be discovered from data (e.g., process mining). Note that computations of algorithms (that can be unconventional) may be disturbed by interactions with the environment – the *real* returned results may be different from the *expected,* given by formulas.

INTERFACES BETWEEN GRANULAR NETWORKS

INFERENCE RULES BASED ON ALGORITHMIC SEMANTICS OF GRANULES



L. Rokach: Ensemble Learning: Pattern Classification Using Ensemble. World Scientific (2nd Edition) 2019

INTERFACES BETWEEN GRANULAR NETWORKS: INTERACTIONS WITH THE ENVIRONMENT

Interactions may interfere with the input, construction, and aggregation of algorithms, as well as their computation.
The computations performed by the algorithms may also change interactions from the environment.

Instead $[H_g(Al_{g_1}, \ldots., Al_{g_k})](x)$ we have

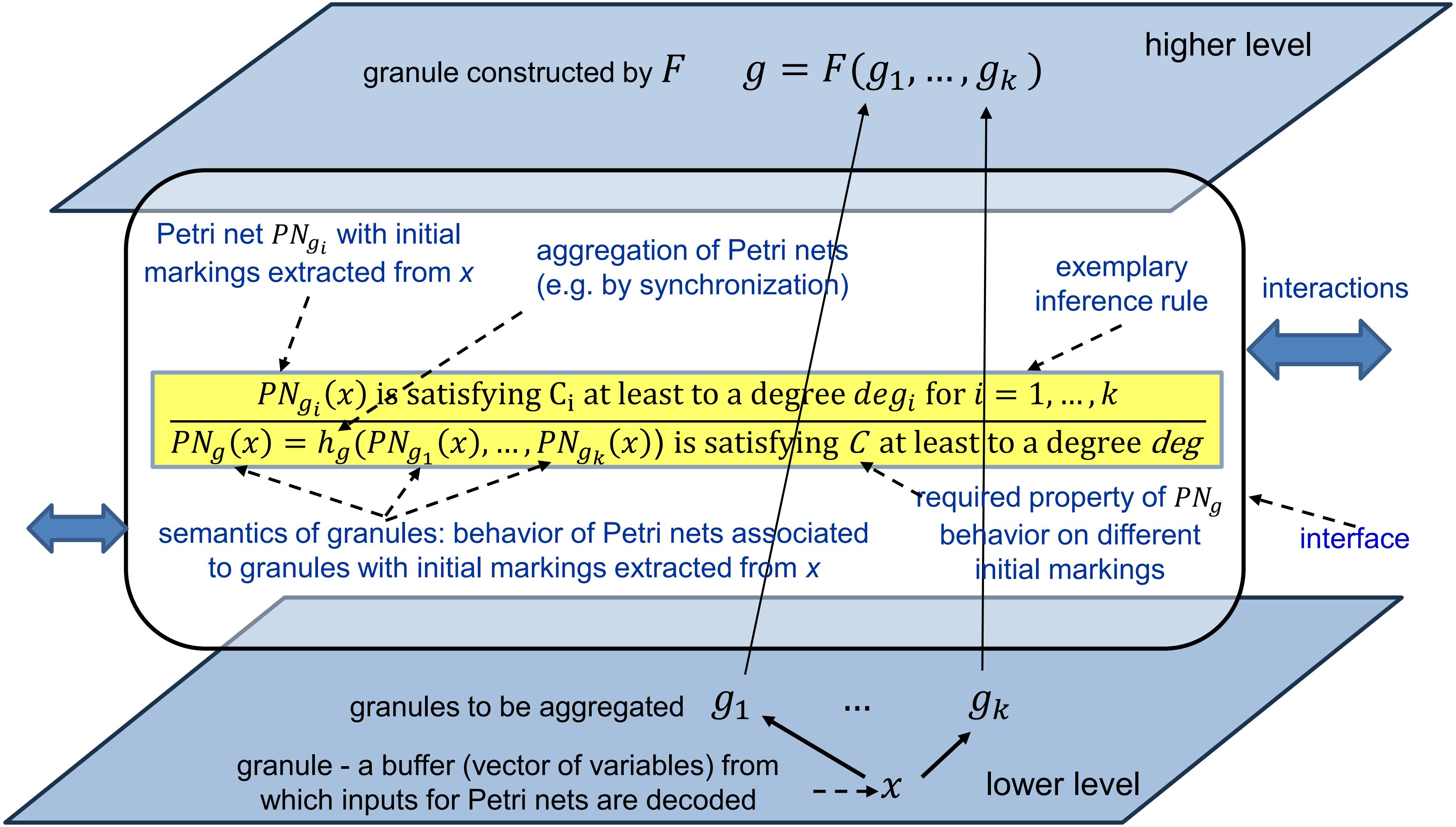
$$[\bigotimes_j (H_g, e_j)(\underbrace{\bigotimes_1 (Al_{g_1}, e_1)}, \ldots., \bigotimes_k (Al_{g_k}, e_k))](\bigotimes_i (x, e_i))$$

interaction of algorithm Al_{g_1} with the environment state *e* disturbing Al_{g_1};
uncertainty in definitions of interaction operation $\bigotimes_1$ and vector of values e_1 extracted from the global vector of values characterizing the environment – note that some values in *e* may be missing and/or *e* may not properly characterize the environment;
models for interaction operations and *e* may require adaptation due to unpredictable changes in the environment over time

Conclusion. The behavior of a granule *g* constructed from $g_1, \ldots, g_k$ cannot be defined based solely on the behavior of $g_1, \ldots, g_k$, especially when dealing with complex phenomena. In this case, taking into account the interactions with the environment is unavoidable.

INTERFACES BETWEEN GRANULAR NETWORKS

INFERENCE RULES BASED ON CONCURRENT SEMANTICS OF GRANULES



L. Reinkemeyer (ed.): Process Mining in Action: Principles, Use Cases and Outlook. Springer 2020
A.Skowron, Z. Suraj (1995). Discovery of concurrent data models from experimental data tables: A rough set approach, Proceedings of the First International Conference on Knowledge Discovery and Data Mining, Montreal, August, 1995, AAAI Press, Menlo Park CA 1995, 288-293

INTERFACES BETWEEN GRANULAR NETWORKS

INFERENCE RULES BASED ON CONCURRENT SEMANTICS INDUCED FROM DATA OF GRANULES

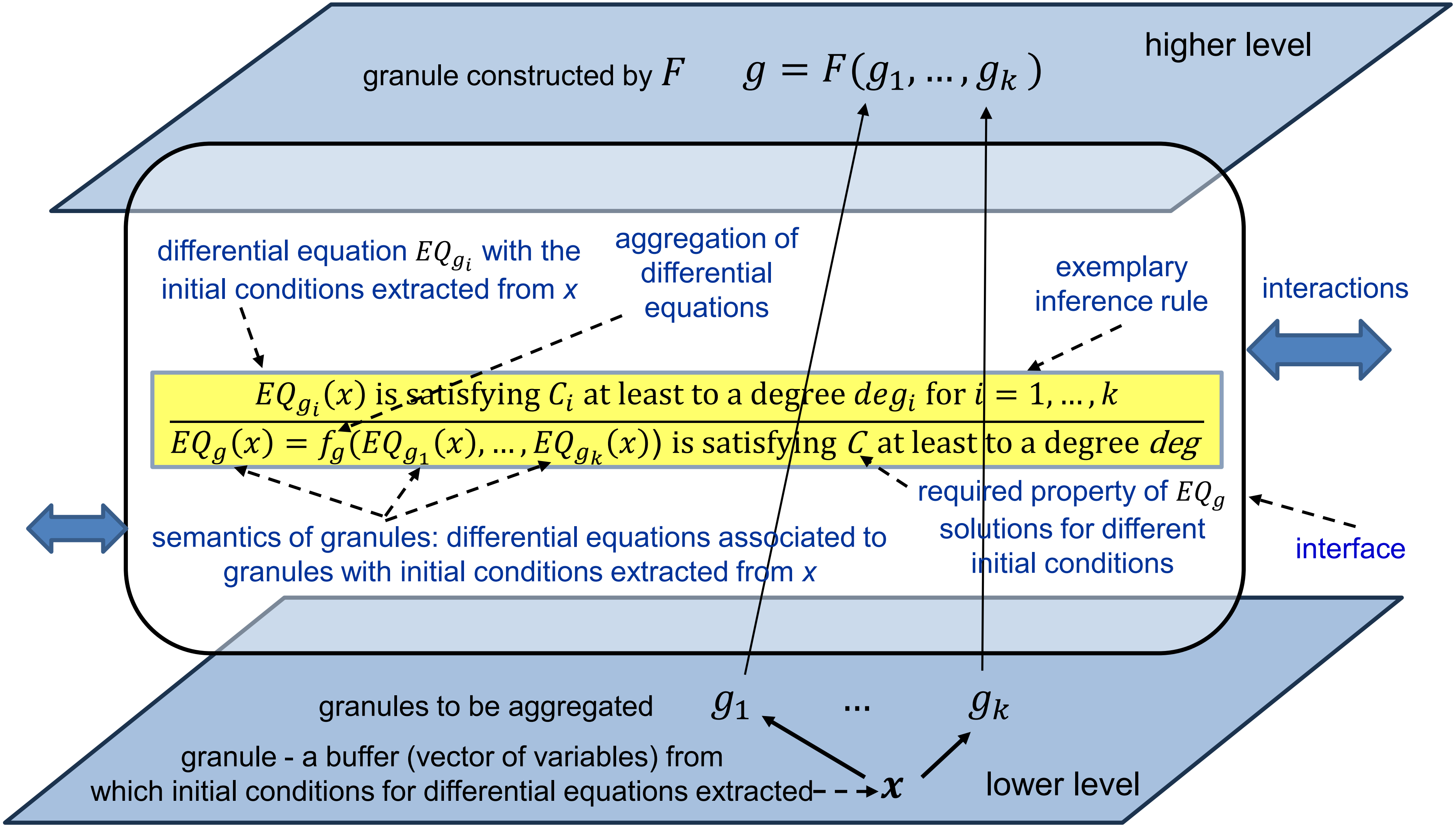
$$\frac{PN_{g_i}(x) \text{ is satisfying } \mathrm{C_i} \text{ at least to a degree } deg_i \text{ for } i = 1, \ldots, k}{PN_g(x) = h_g(PN_{g_1}(x), \ldots, PN_{g_k}(x)) \text{ is satisfying } C \text{ at least to a degree } deg}$$

PN_{g_i} induced from data may change according to changes in perceived data.

One may consider to characterize external interactions by additional to *x* vector *e* of values of some attributes. However, in *e* some values may be missing and *e* may not completely characterize the impact of the environment on the construction of concurrent models and/or x. Hence, adaptation of PN_{g_i} may be needed.

INTERFACES BETWEEN GRANULAR NETWORKS

INFERENCE RULES BASED ON SEMANTICS OF GRANULES DEFINED BY DIFFERENTIAL EQUATIONS INDUCED FROM DATA

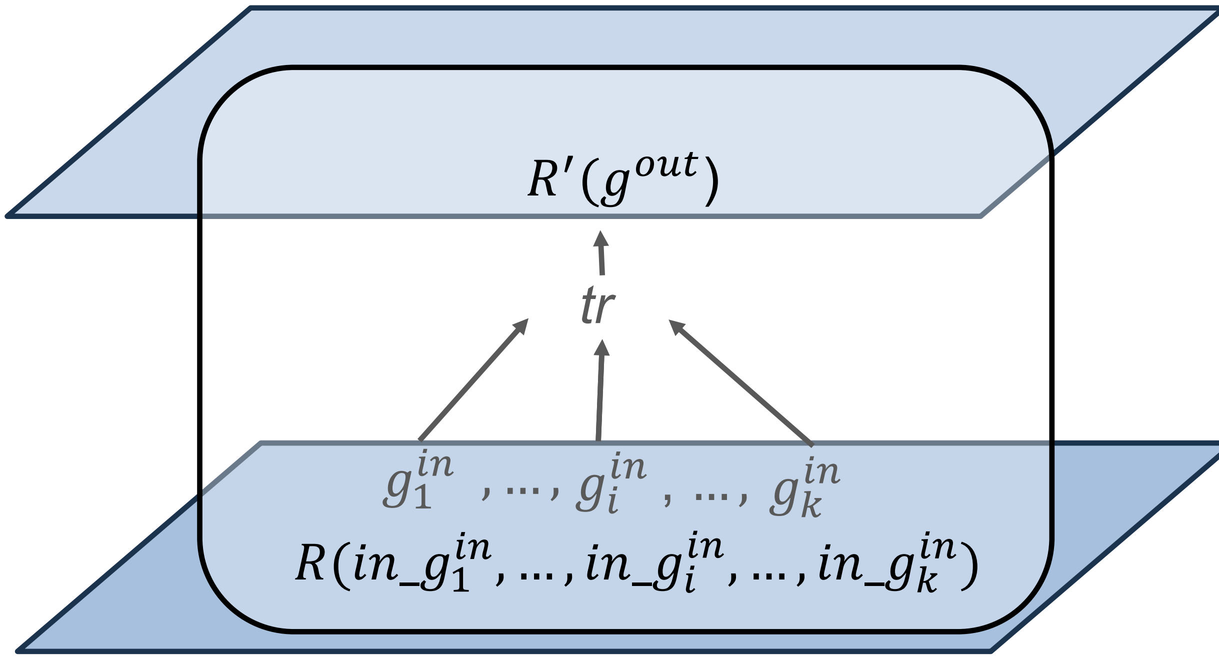


Xu, H., Chen, Y., Cao, R. et al. Generative discovery of partial differential equations by learning from math handbooks. Nature Communications (16) 10255 (2025). doi.org/10.1038/s41467-025-65114-2

INTERFACES BETWEEN GRANULAR NETWORKS OVER DIFFERENT UNIVERSES

generation of new types of granules (e.g., atomic) from given ones

g^{out} g_i^{in} *tr*
-- often represented by relevant algorithms

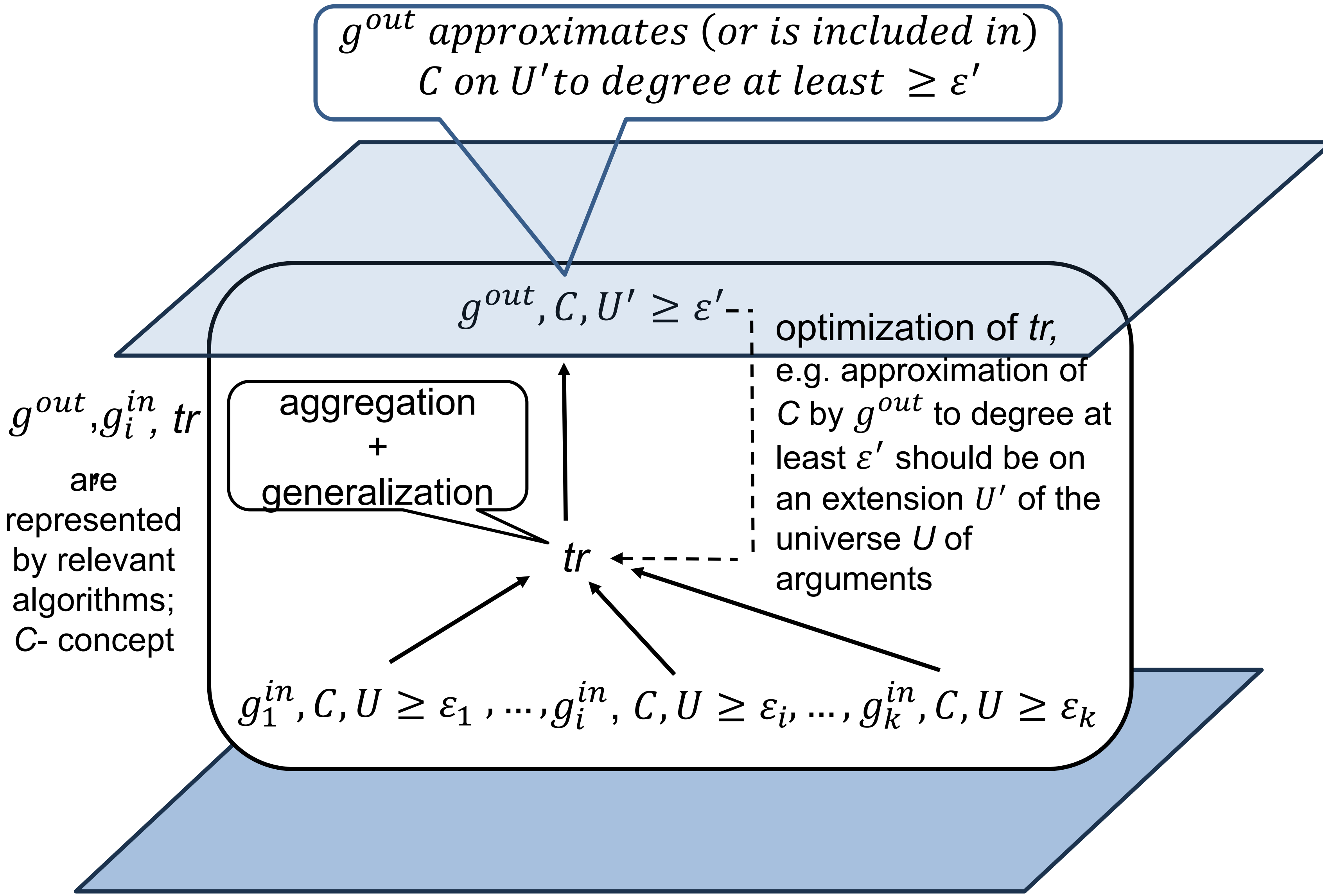


<u>Example</u>: *tr* - learning algorithm constructing an ensemble of classifiers from given classifiers $g_1^{in}, \ldots, g_i^{in}, \ldots, g_k^{in}$; *R*' - the quality measure of constructed classifier. <u>Problem</u>: Discover a constraint *R* (satisfied on sufficiently large data set) on inputs

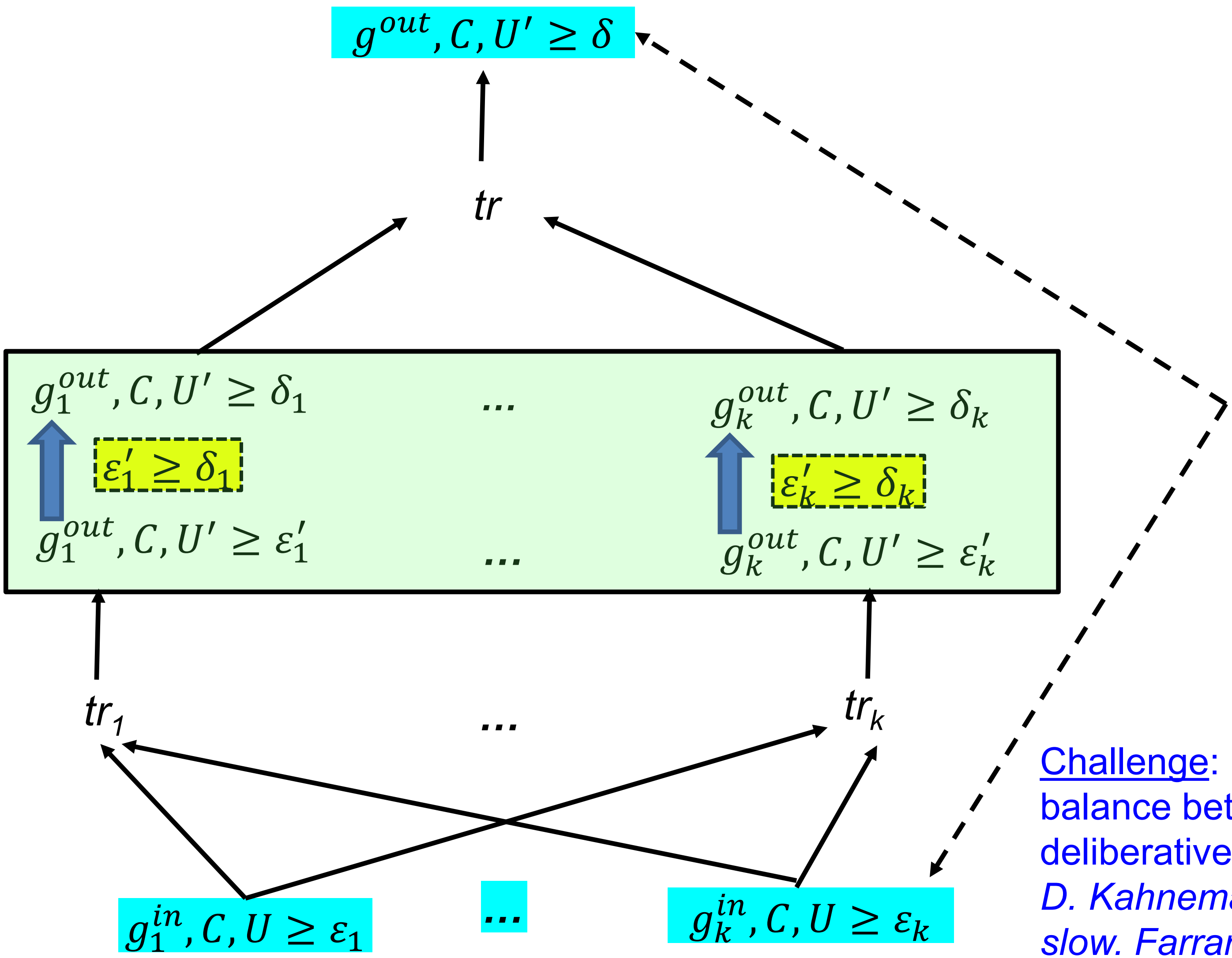
$$in\_g_1^{in}, \ldots, in\_g_i^{in}, \ldots, in\_g_k^{in} \text{ of } g_1^{in}, \ldots, \ g_i^{in}, \ldots, g_k^{in}$$

such that aggregation g^{out} by *tr* of $g_1^{in}, \ldots, g_i^{in}, \ldots, g_k^{in}$ satisfying *R* satisfies *R'* on testing data.

INTERFACES BETWEEN GRANULAR NETWORKS: RULES CONCERNING ROBUSTNESS



INTERFACES BETWEEN GRANULAR NETWORKS: REASONING THROUGH LEVELS

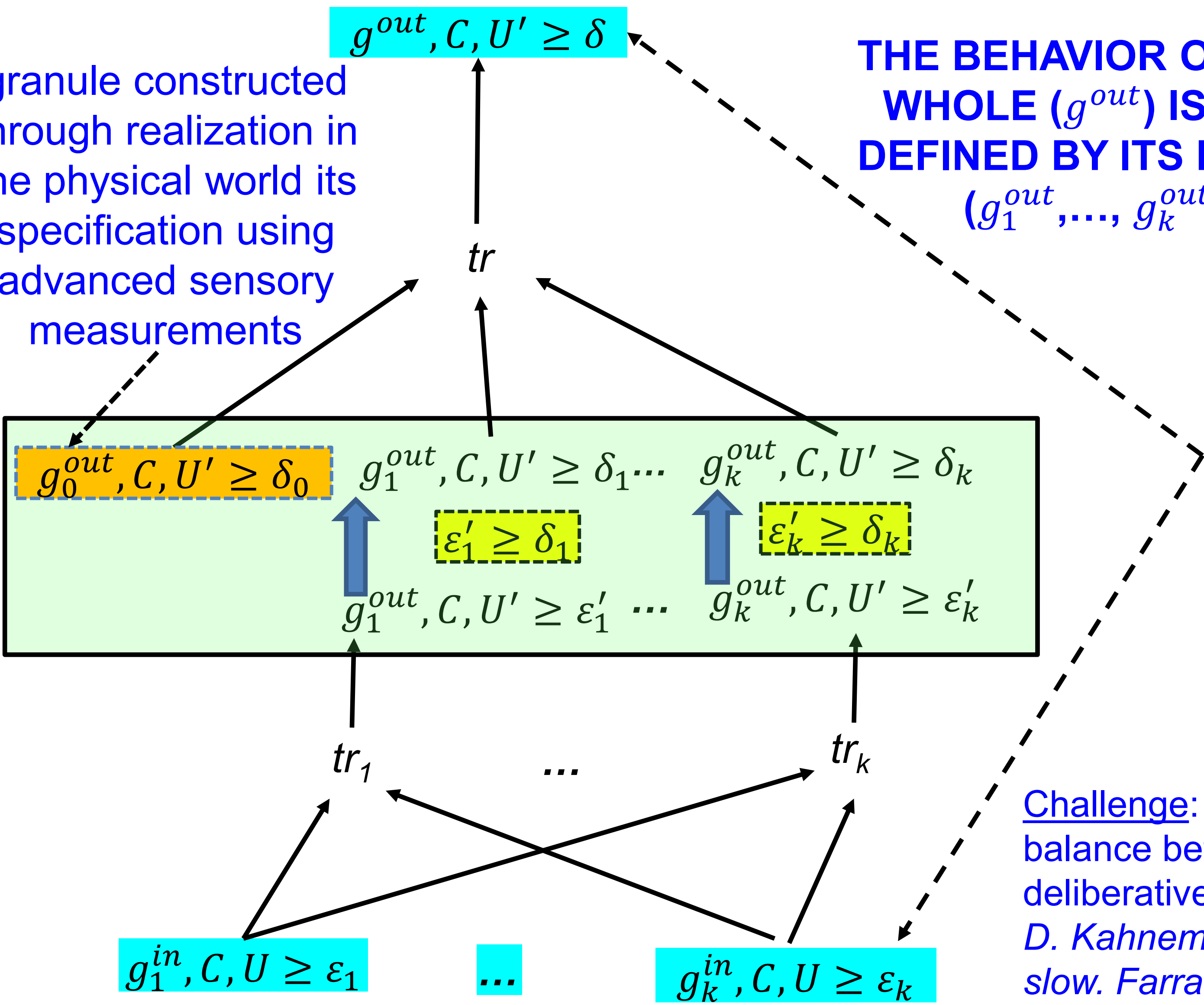


rule in the interface between the bottom and top layers constructed by composition of rules

<u>Challenge</u>: Keeping a proper balance between reactive and deliberative behavior.

D. Kahneman, Thinking, fast and slow. Farrar, Straus and Giroux (2011).

INTERFACES BETWEEN GRANULAR NETWORKS: REASONING THROUGH LEVELS (cont.)



rule in the interface between the bottom and top layers constructed by composition of rules

Challenge: Keeping a proper balance between reactive and deliberative behavior.
D. Kahneman, Thinking, fast and slow. Farrar, Straus and Giroux (2011).

DISCOVERY OF RELEVANT TRANSFORMATIONS *tr:* EXAMPLES

Discovery of *tr* for construction
new granules in the form of

clusters encompassing discovery of

- attributes
- distance functions on attribute values
- aggregations of distance functions defined on attribute attributes
- representations of clusters, e.g., in the form of vectors of mean values of attributes in clusters and their radii
- quality measures of constructed clusters relative to given classification
- part of the universe on which the quality of the constructed clusters is low (e.g., the object/situation under classification is not in the scope of existing clusters included in the decision classes) and requires new discoveries
- …

rule-based classifiers encompassing discovery of

- attributes
- decision rules
- quality measures of rules
- methods for estimation of matching degrees of vectors of attribute values by rules
- reasoning methods for resolving conflicts between decision rules matching objects
- quality measures of constructed classifiers relative to given classification
- part of the universe on which the quality of the constructed classifier is low (e.g. for objects/situations under classification the difference of arguments *for* and *against* the decision is *small*) and requires new discoveries
- …

LIFELONG LEARNING (LL) IN DISCOVERY OF *tr*

clusters encompassing discovery of

- …
- part of the universe on which the quality of the constructed clusters is low (e.g., the object/situation under classification is not in the scope of existing clusters included in the decision classes) and requires new discoveries

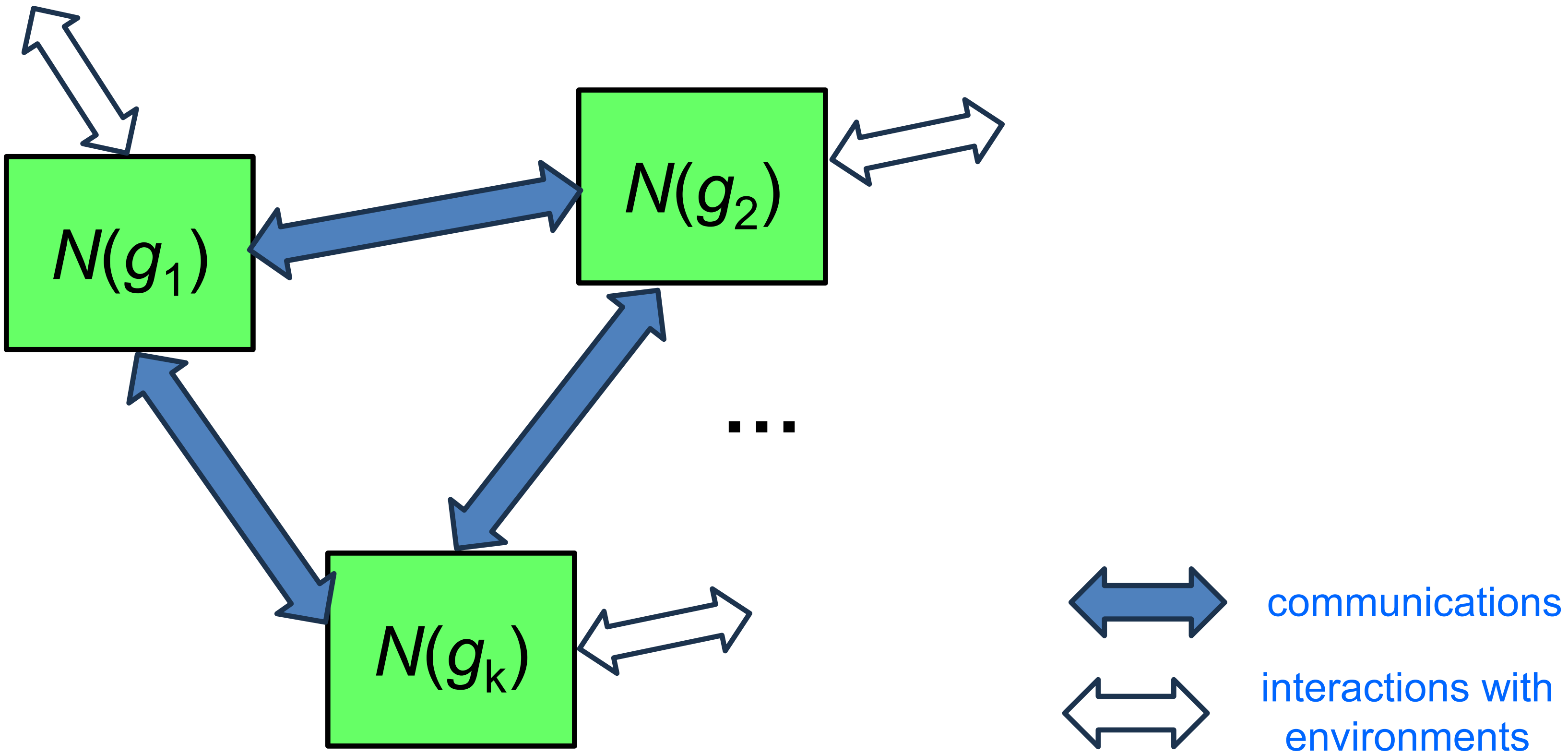
rule-based classifiers encompassing discovery of

- …
- part of the universe on which the quality of the constructed classifier is low (e.g. for objects/situations under classification the difference of arguments *for* and *against* the decision is *small*) and requires new discoveries

LL is an advanced machine learning paradigm that learns continuously, accumulates the knowledge learned in the past, and uses/adapts it to help future learning and problem solving. In the process, the learner becomes more and more knowledgeable and better and better at learning. This continuous learning ability is one of the hallmarks of human intelligence. […] humans learn effectively with a few examples and in the dynamic and open world or environment in a self-supervised manner because our learning is also very much knowledge-driven: the knowledge learned in the past helps us learn new things with little data or effort and adapt to new/unseen situations. This self-suprevised (or self-aware) learning also enables us to learn on the job in the interaction with others and with the real-world environment with no external supervision.

Z. Chen, B. Liu: Synthesis Lectures on Artificial Intelligence and Machine Learning, Morgan & Claypool Publishers (2018)

GRANULAR SOCIETIES

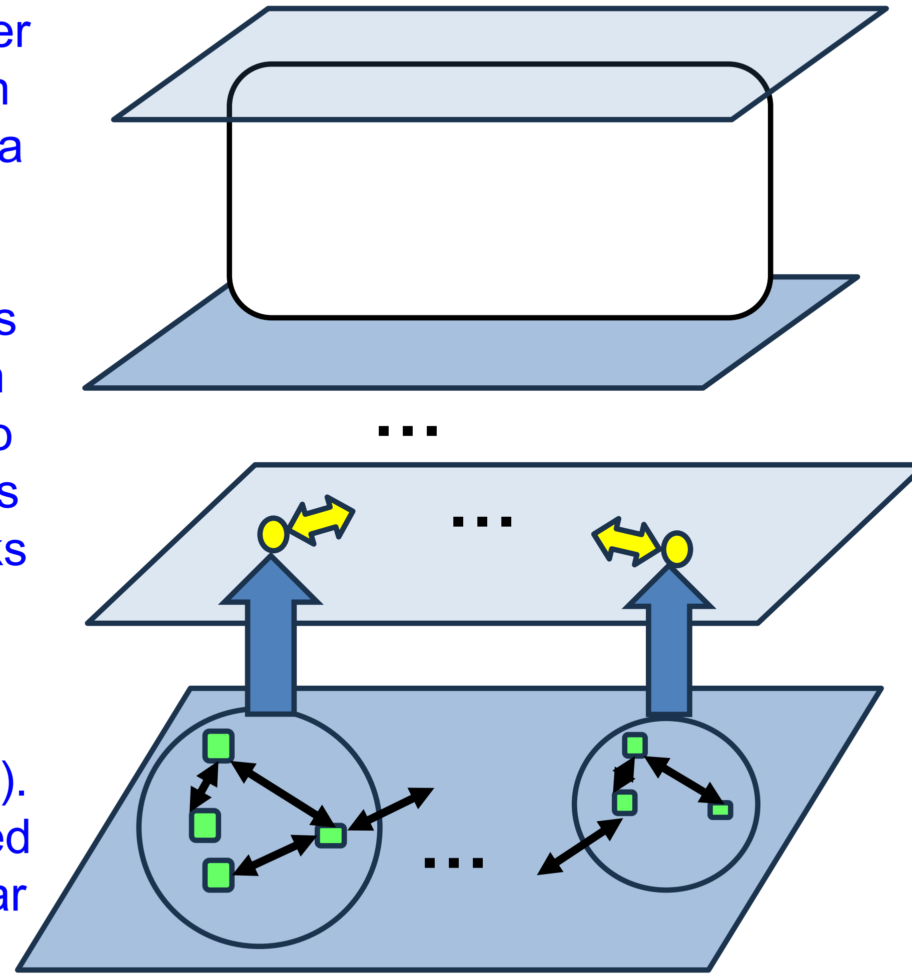


The $N(g_1)$, $N(g_2)$, …, $N(g_k)$ are granular networks represented by the granules g_1, g_2, …, g_k, and the dark arrows represent the interactions between them that realize communication in the form of cooperation or competition.

HIERARCHIES OF GRANULAR NETWORKS OF GRANULAR SOCIETIES

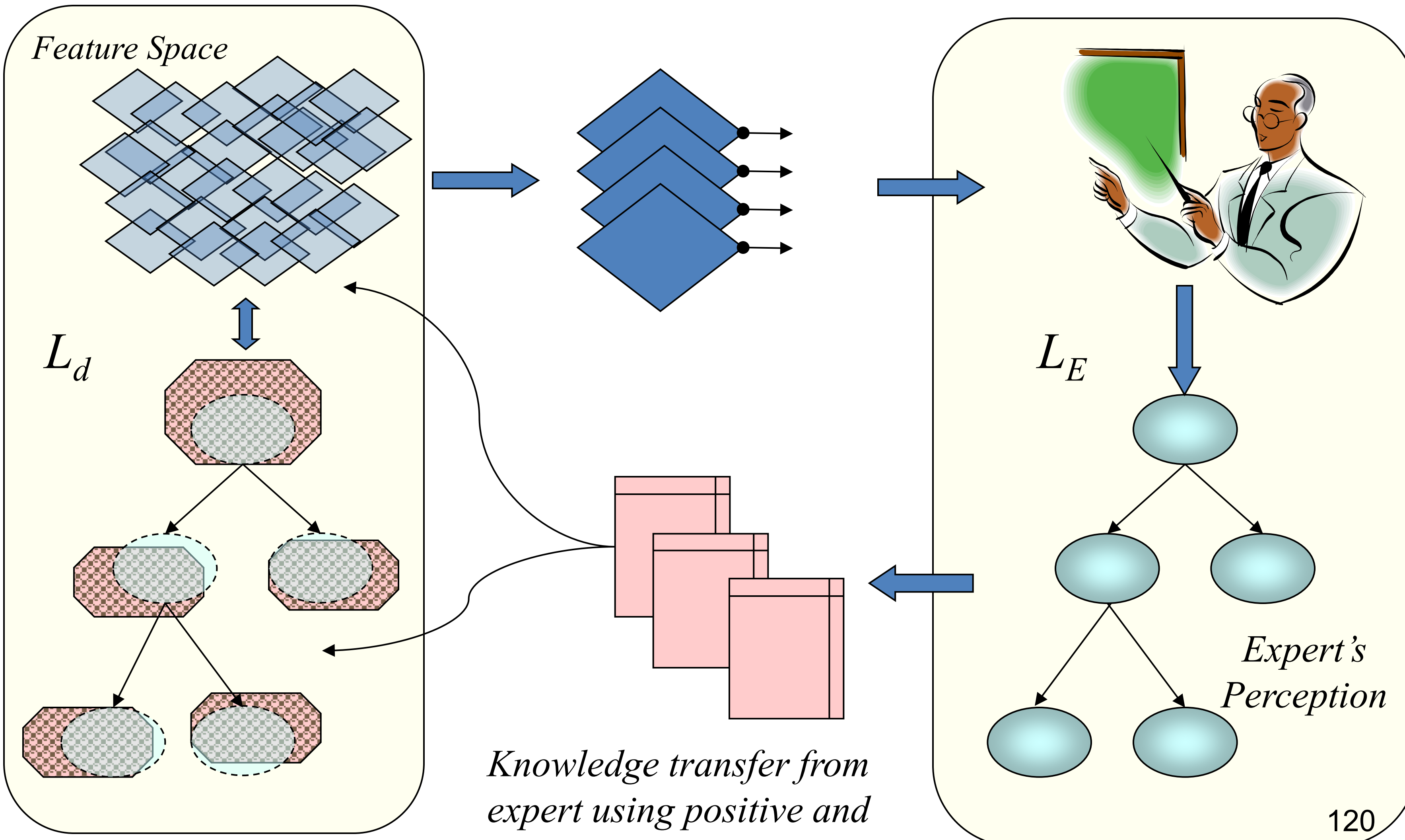
complex dynamic granules

The aim of granulation of granular (sub)societies into hierarchies is to discover computational building blocks for cognition of situations related to complex phenomena in physical spaces. This process involves identifying behavioral models of granular societies within more complex societies, as well as the relationships between them. In particular, it reveals rules that allow one to infer the properties of higher-level networks based on collections of lower-level networks that satisfy certain constraints. These constraints may be determined, e.g., by membranes or by interactions between collections of granules (see Holland's book). The hierarchical approach may be also used for reasoning about the behavior of granular society on the lowest level of hierarchy.



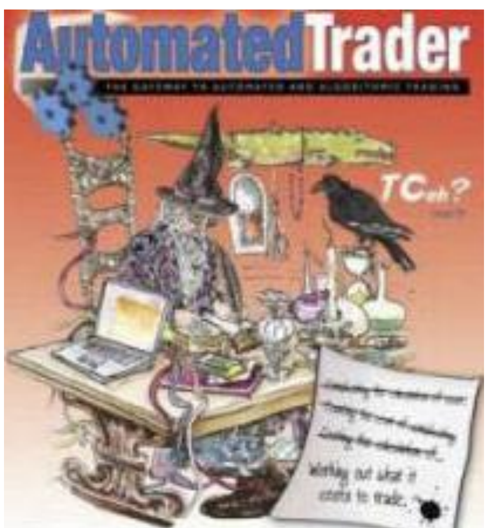
On each level, sensory measurements and actions pointing to the physical regions by the spatio-temporal windows are possible, with the expected results being granules of the relevant types for that level.

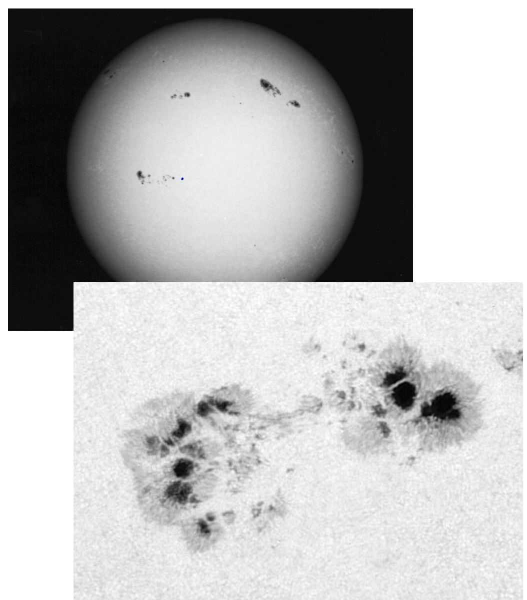
ROUGH SET BASED ONTOLOGY APPROXIMATION



Knowledge transfer from expert using positive and negative examples

APPLICATIONS :
APROXIMATION OF COMPLEX VAGUE CONCEPTS





CONTROL OF C-GRANULE *g*

(a sub-granule of *g*)

IS AIMING TO STEER GENERATED GRANULAR COMPUTATIONS IN ORDER TO ENSURE THE ACHIEVEMENT OF ITS GOALS (NEEDS) TO A SATISFACTORY DEGREE

IMPORTANCE OF PERCEPTION IN THE STATES OF REAL WORLD MODELING BY C-GRANULES

The main idea of this book is that perceiving is a way of acting. It is something we do. Think of a blind person tap-tapping his or her way around a cluttered space, perceiving that space by touch, not all at once, but through time, by skillful probing and movement. This is or ought to be, our paradigm of what perceiving is.

Alva Noë: Action in Perception, MIT Press 2004

Living and perceiving mean to participate – through embodied action (it should be purposeful and intentional).

See also:
Stephen Everso: Aristotle on Perception. Oxford University Press (1997)
Anna Marmodoro: Aristotle on perceiving objects. Oxford University Press (2014)

IMPORTANCE OF PERCEPTION IN THE STATES OF REAL WORLD MODELING BY C-GRANULES

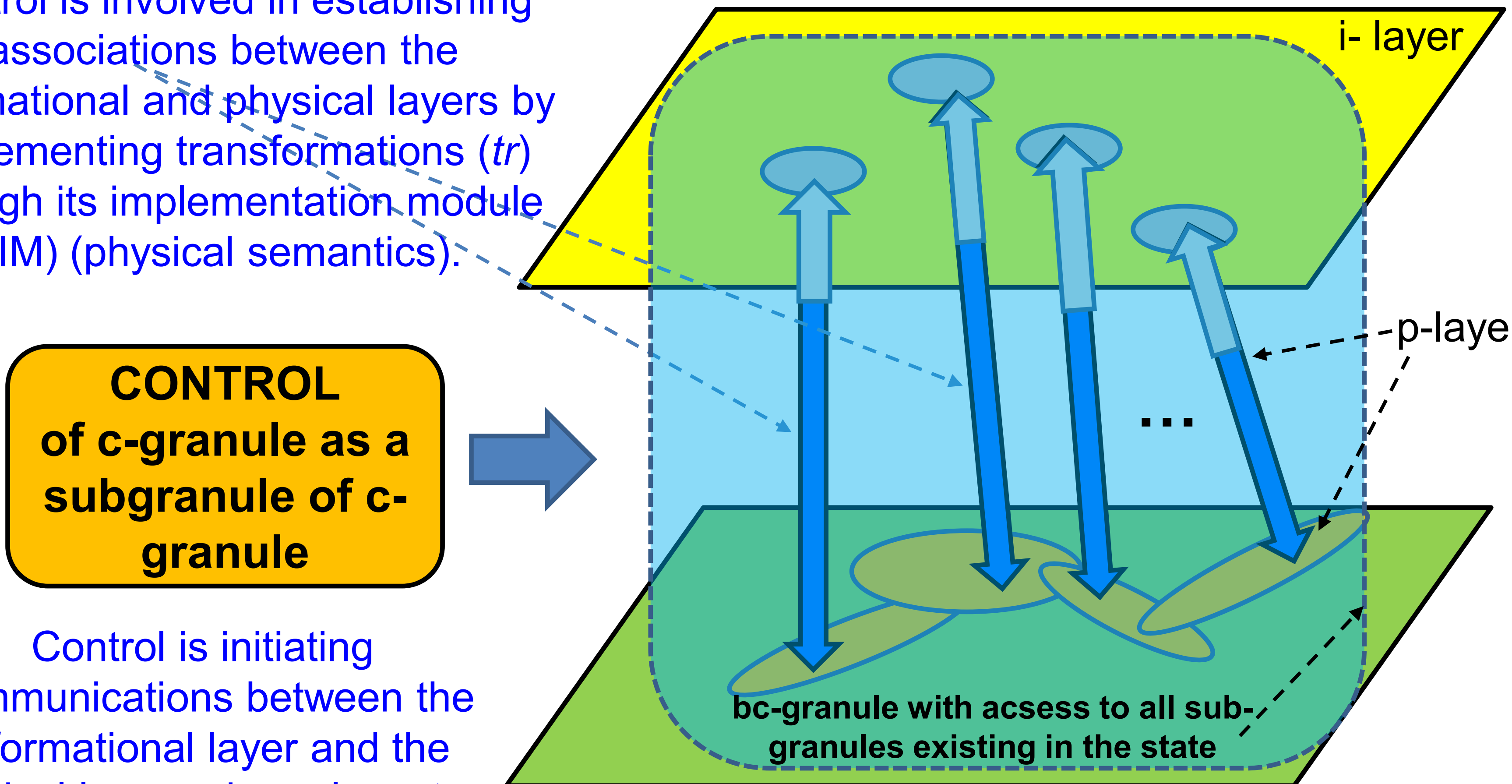
Classically, control is identified by the appropriate function on control states. How are control states determined in the case of c-granules with control? They are not given a priori as in classical control problems. To construct models of states, the c-granule control organizes a series of experiments in the real world. At any moment, the control performs some experiments on various fragments of reality, selected by that control. This is accomplished through appropriate sub-granules generated by control as kinds of pointers to the physical world. In illustration of the control idea, these sub-granules are distinguished by thick arrows. Various physical objects are involved in these experiments. In the example with the blind person, objects such as a human, a cane, and an object being tapped by the cane are highlighted. Here, the exact description of objects using possible properties of them and their parts is not relevant. The aim is to perceive such properties of these objects and their interactions (e.g., the vibrations received by human from the cane or signals from the tapping) relevant for assessing whether there is an obstacle in the blind person's path or not. In this way, in every experiment, the perception of the physical objects involved, distinguished in the real world, is essential, particularly regarding which properties the objects are perceived through and how their aggregation match the specified properties in the i-layer of the sub-granule. These experiments, conducted by the Implementation Module (IM) of control, define what is called physical semantics. Thus, in each active sub-granule of control at a given moment, a certain experiment on real objects is conducted, providing knowledge about the currently perceived state (situation). This state (situation) is perceived through the glasses of those sub-granules. As a result, these experiments form the basis of how the control perceives the current state (situation) in the real world, which serves as the foundation for making right (wise) decisions (actions, plans). In this sense, control in each of its state, also encompasses the physical objects involved in the experiments in pursuit of constructing models of states of reality that ensure the execution of appropriate actions or plans (realizations of transformations or associations in c-granule language)—thus defining input for the control function—aimed at meeting the needs or goals of the c-granule. Hence, it is also distinguished a special sub-granule of control that in each state has the scope covering objects involved in the currently runed experiments.

C-GRANULE WITH CONTROL: INTUITION

Control of c-granules implements processes aimed at understanding perceived situations in order to construct approximate solutions to problems *along* the generated granular computations. This is achieved by discovering complex games. Each complex game consists of a set of rules. The predecessor in each such rule is a classifier for often complex, vague concepts that activate the rule. If the rule is selected by the control for implementation, a realization based on the transformation specification on the right side of the rule is triggered in the physical world starting realization on the current granular network. The implementation module (IM) of c-granule control is responsible for the physical realization of the transformation specification (i.e., for the physical semantics). It may be necessary to conduct a multi-level decomposition of the transformation specification intended for implementation before it can be directly realized in the physical world.

C-GRANULE WITH CONTROL: INTUITION

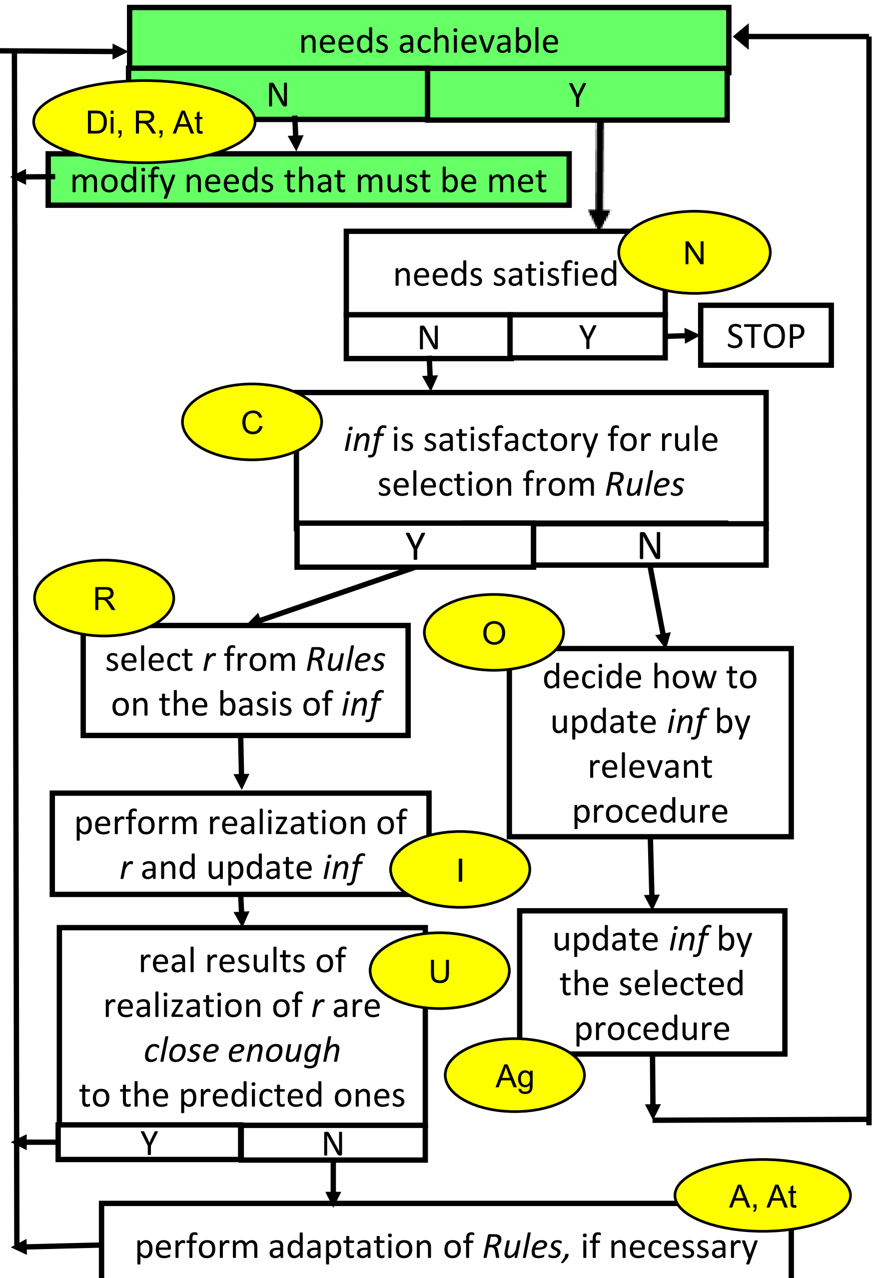
Control is involved in establishing associations between the informational and physical layers by implementing transformations (*tr*) through its implementation module (IM) (physical semantics).



Control is initiating communications between the informational layer and the physical layer using relevant c-granules (generated by its IM), allowing the collection of properties of perceived physical objects and their interactions within the informational layer.

SIMPLIFIED VIEW OF NETWORK OF SUB-GRANULES INTERACTING WITH ABSTRACT AND PHYSICAL OBJECTS GENERATED AND STEERED UNDER SUPERVISION OF A GLOBAL SUBGRANULE OF THE CONTROL RESPONSIBLE FOR BASIC CYCLE (bc-granule)

REASONING SUPPORTING THE BASIC CONTROL CYCLE OF BC-GRANULE



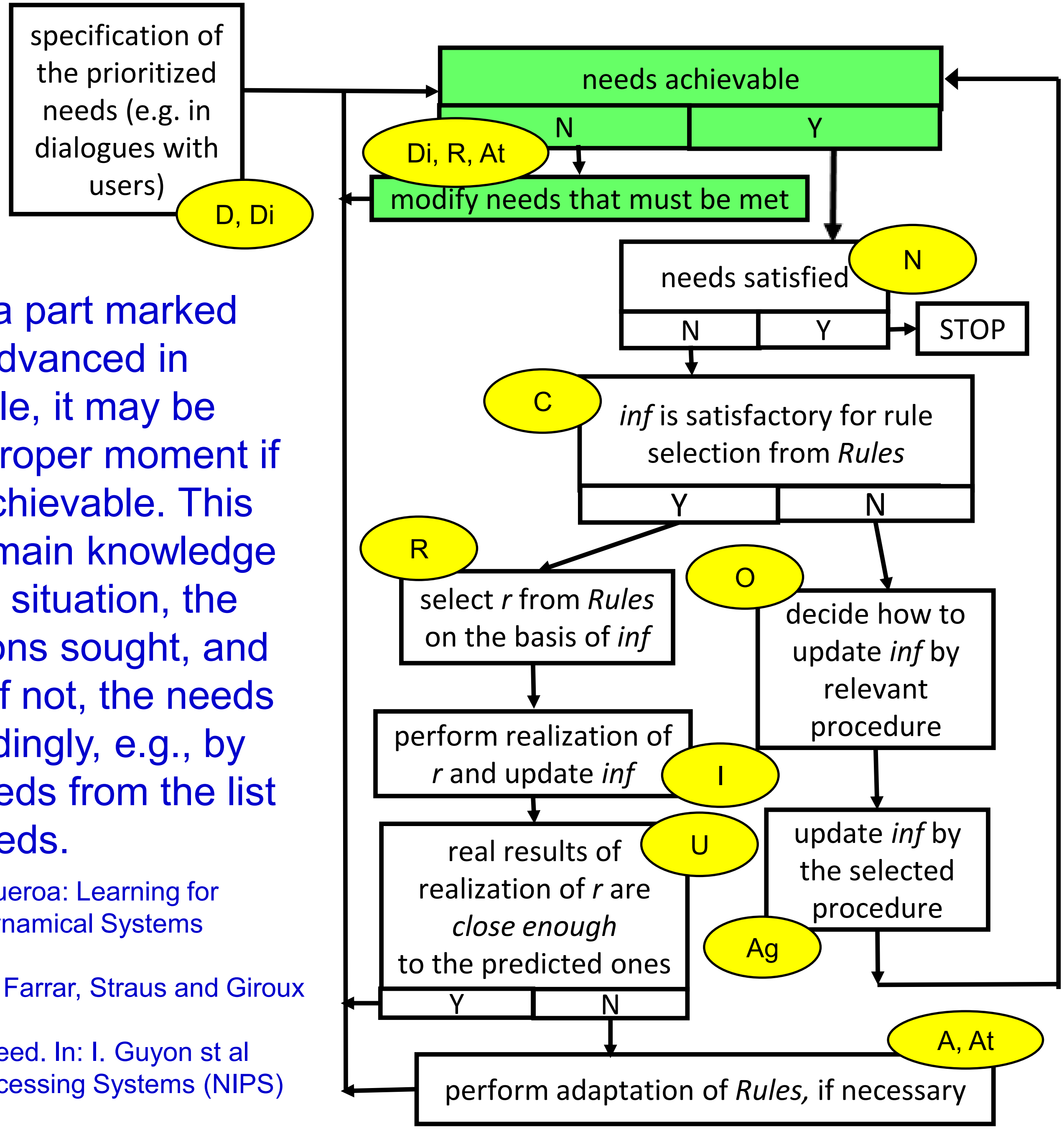
The exemplary reasoning modules (sub-granules) of the bc-granule supporting the development of procedures named in boxes:

- dialogues Di
- decomposition D
- adaptation A
- conflicting information C
- optimization O
- uncertainty U
- risk R
- aggregation Ag
- interactions I
- needs N
- attention At
- …

Many reasoning modules (sub-granules) are supporting realization of the basic cycle by bc-granule. Advanced reasoning may support two different forms of control behavior in the realization of the basic cycle (bc-granule): *reactive* and *deliberative.* For example, the control of bc-granule may recognize that there are some arguments supporting the satisfiability of a concept that signals a high risk of reaching a very dangerous state if an immediate reaction is not taken. In other situations, there is more room for deliberation, which aims to better understand the current situation by taking additional measurements, performing actions, extracting information from knowledge bases or performing adaptation (e.g., attention). One may recognize an analogy here to Kahneman's *fast* and *slow* reasoning.

inf – information in i-layer about the currently perceived situation
Rules – set of rules for transformation of sub-granules

REASONING SUPPORTING THE BASIC CONTROL CYCLE OF BC-GRANULE



The basic cycle (without a part marked *green*) may be more advanced in applications. For example, it may be necessary to check at the proper moment if the assigned needs are achievable. This would take into account domain knowledge concerning the perceived situation, the requirements for the solutions sought, and the resources used so far. If not, the needs should be modified accordingly, e.g., by selection less ambitious needs from the list of prioritized needs.

Aude Billard, Sina Mirrazavi and Nadia Figueroa: Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press (2022)
Daniel Kahneman: Thinking, fast and slow. Farrar, Straus and Giroux (1013)
Ashish Vaswani et al: Attention is All you Need. In: I. Guyon st al (eds.): Advances in Neural Information Processing Systems (NIPS) vol.30. Curran Associates, Inc. (2017) https://arxiv.org/abs/1706.03762

C-GRANULE WITH CONTROL: COMMENTS

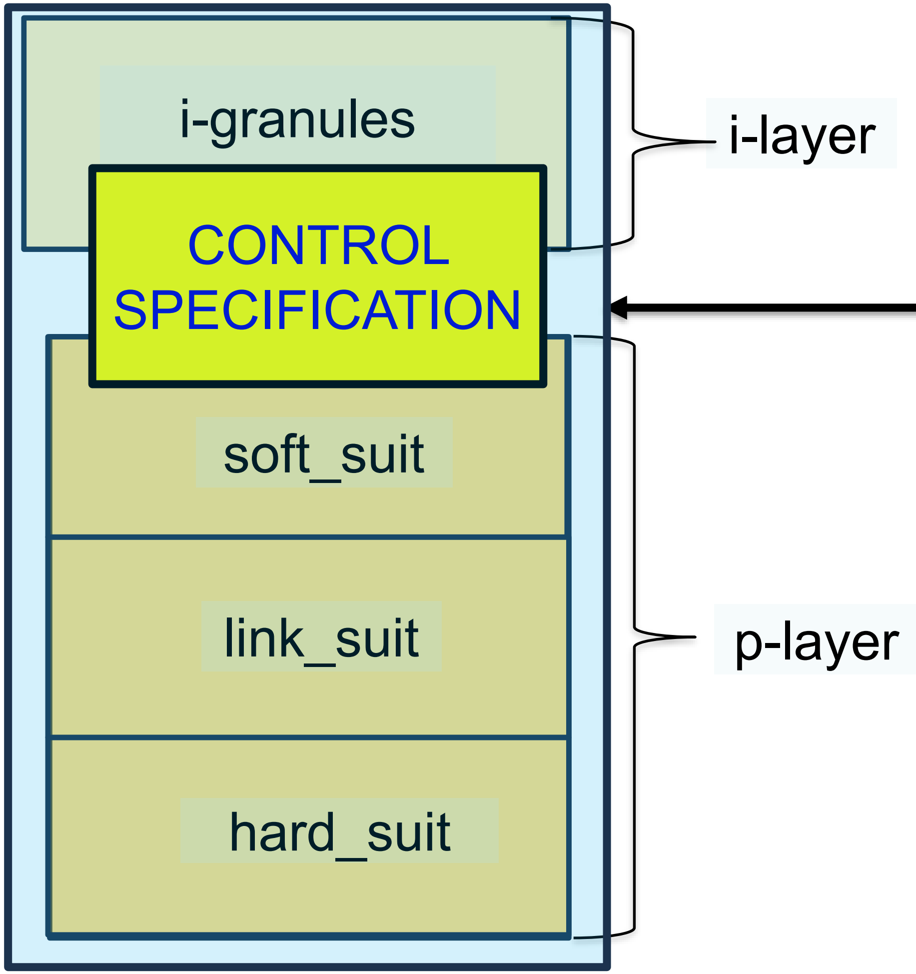
- A c-granules is a dynamic object that changes over time. Its dynamics is steered by the control of c-granule (special sub-granule of given c-granules), which aims to select and realize transformations (associations) of abstract and physical objects to satisfy goals (needs or specifications of the problem to be solved). This is based on perceived information about these objects as well as their interactions.

- Control of c-granule is using the bc-granule to supervise the behavior of the control to select and realize transformations (associations). The realization concerns generation of the network of relevant sub-granules from the currently perceived one.

- In the network of sub-granules, supervised by the control of c-granule are also sub-granules realizing of the basic cycle of control.

- cb-granule of control uses the context in which physical objects appear in networks of sub-granules, qualifying them for different regions: *soft_suit, link_suit, or hard_suit*. For example, in one sub-granule, the considered objects may be in the *link_suit* category, while in another, they may be in the *hard_suit* category.

C-GRANULE WITH CONTROL: STATE



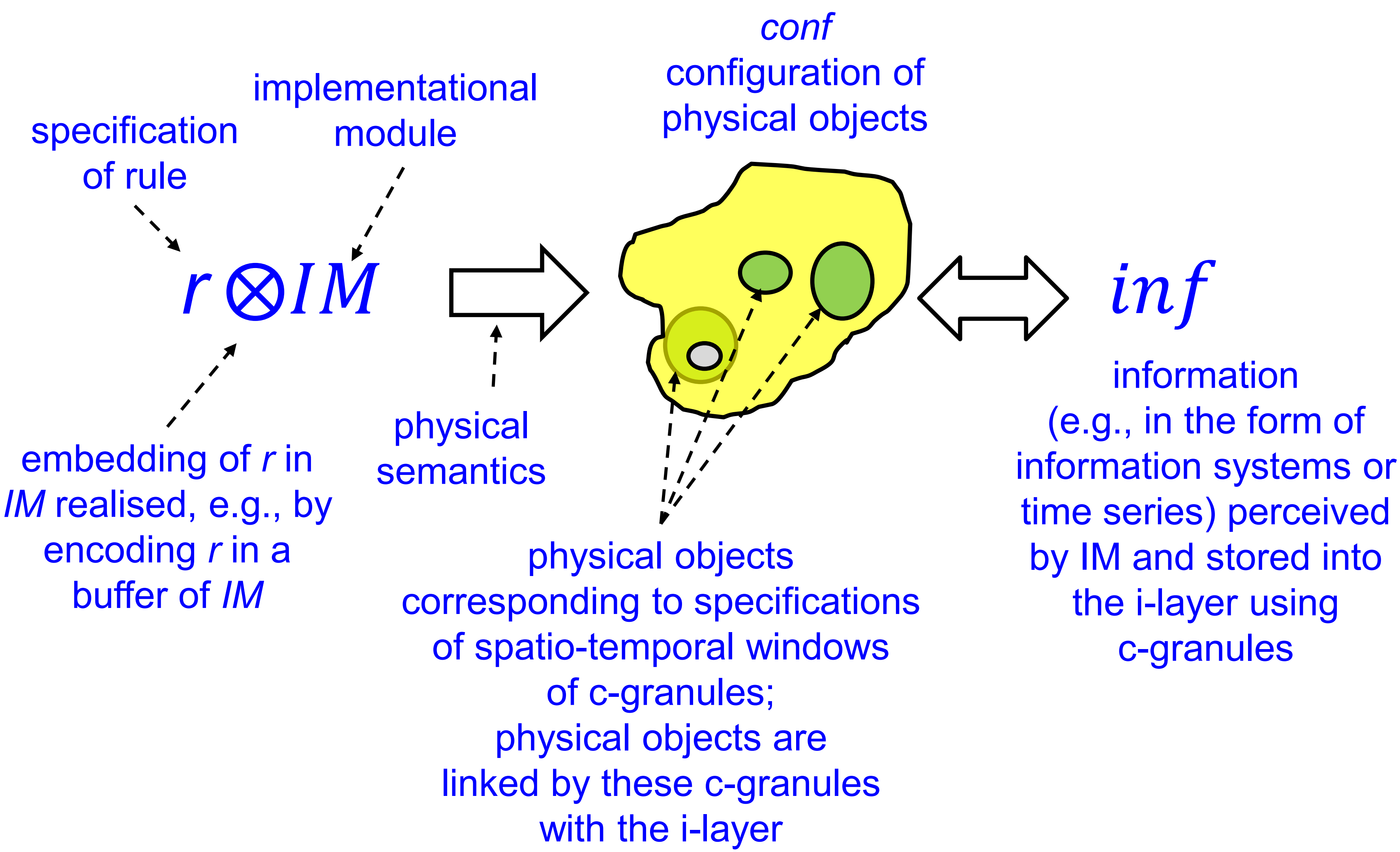
STATE (AT A GIVEN MOMENT)
OF THE CONTROL
OF C-GRANULE
contains of all sub-granules
(with abstract and physical objects)
existing at this moment
as the result of realization of the
control specification.

State should be distinguished from
the specification of control behavior
by an i-granule being a part of the
state.

PHYSICAL SEMANTICS IS REALIZED BY IMPLEMENTATIONAL MODULE (IM)
A SUB-GRANULE
OF
C-GRANULE CONTROL

IM IS ABLE TO GENERATE OF NEW C-GRANULES WITH LINKS (POINTERS) BETWEEN ABSTRACT AND PHYSICAL OBJECTS.
BY USING THESE C-GRANULES THE CONTROL PERCEIVES PROPERTIES OF PHYSCAL OBJECTS AND THEIR INTERACTIONS (BELONGING TO THE SCOPE OF THESE C-GRANULES) IN THE PHYSICAL WORLD

PHYSICAL SEMANTICS: INTUITION



INTUITION OF RULES IN RULE MODULE (RulM), A SUBGRANULE OF CONTROL OF C-GRANULE

$$r : \alpha \Rightarrow_{tr} \beta$$

α, β may have different components related, in particular to properties of stored in i-layer information or local time of c-granule

intuition of meaning

If

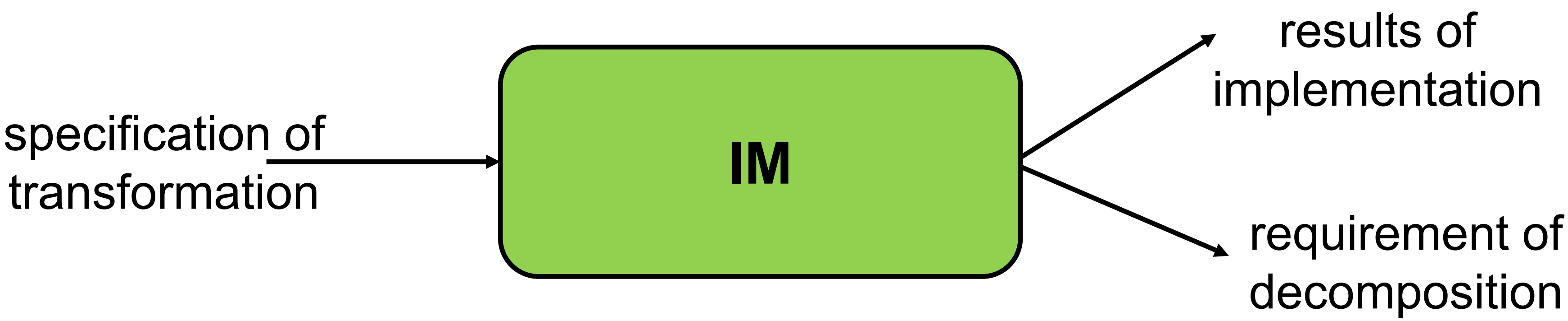
at the actual moment of local time of the c-granule,
the currently realized specification of associations by the physical semantics leads to the storage of information satisfying the property encoded in α in the i-layer

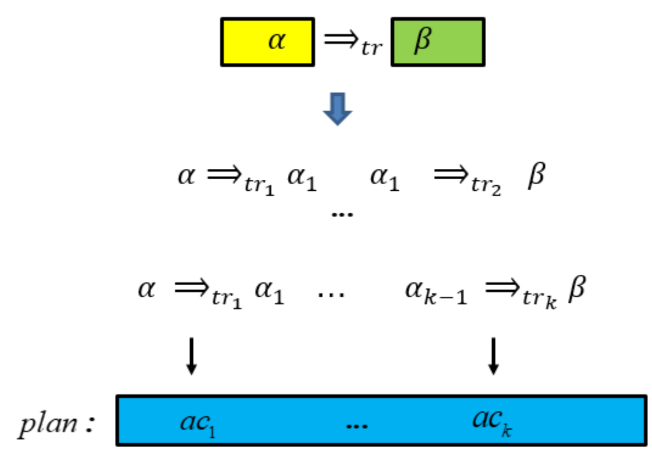
then

starting at a specified moment after that actual moment, the realization of specification of the transformation *tr* (treated as an association) by the physical semantics is expected to lead to the storage of information in the i-layer that satisfies the property encoded in β after a given period of time.

IMPLEMENTATIONAL MODULE (IM)

IM plays a crucial role in interaction of abstract and physical objects; IM realizes physical semantics

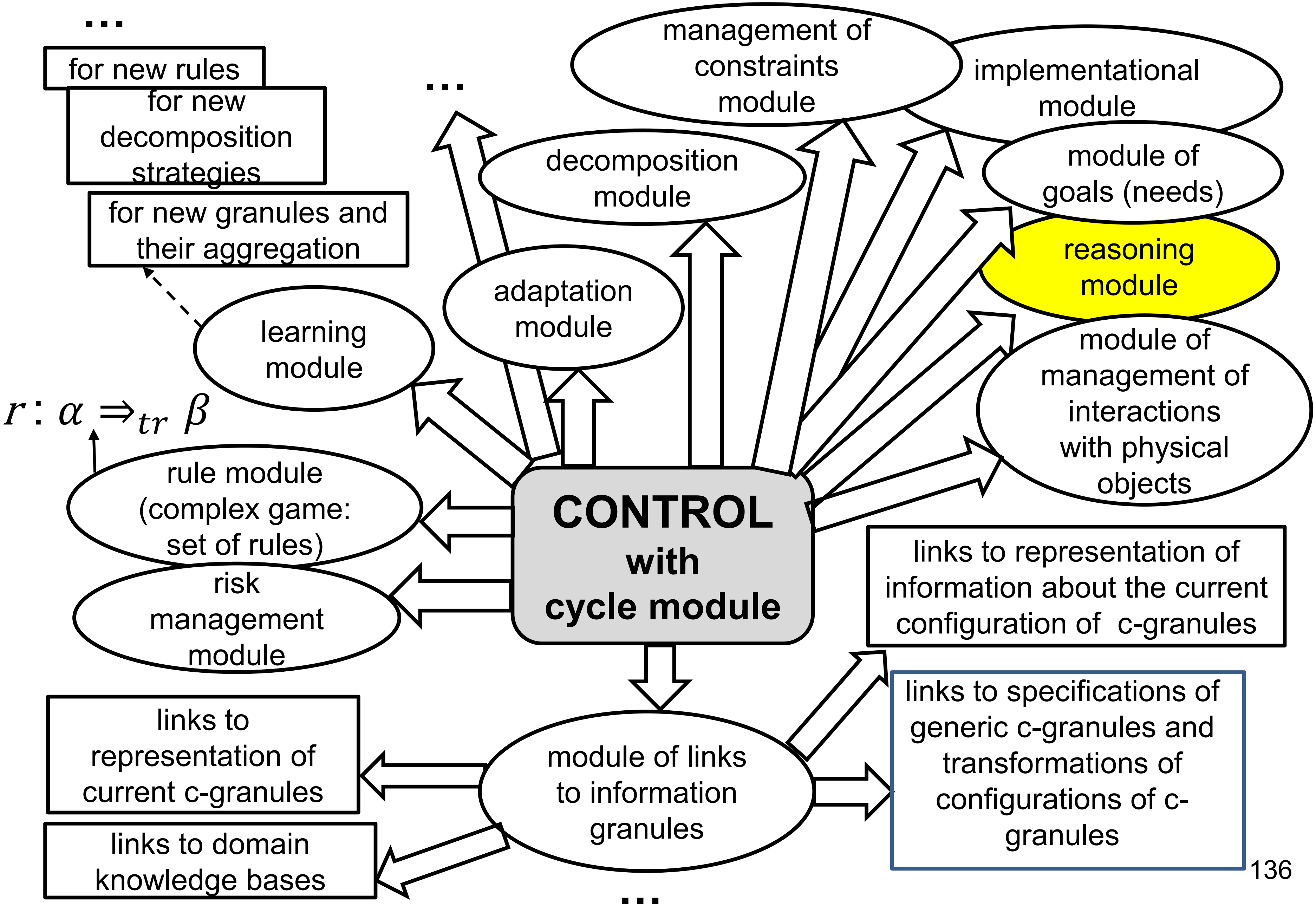




C-GRANULE CONTROL

The structure of the control sub-granules, referred to as the control module (RM) of the c-granules, may be extremely complex. The RM has many submodules (sub-granules) responsible for supporting various aspects of inference through control, interconnected by different dependencies.

MODULES OF C-GRANULE CONTROL



C-GRANULE CONTROL

For instance, the structure of the control sub-granule known as the inference module (RM) is very intricate.

JUDGMENT IN IS’s BASED ON IGrC

CHALLENGE FOR CONTROL OF IS’s:

DEVELOPMENT OF REASONING METHODS SUPPORTING GENERATION & COORDINATION OF DIFFERENT KINDS OF INTERACTIONS AND REASONING ABOUT THEIR RESULTS FOR MAKING THE RIGHT DECISIONS (I.E. FOR JUDGMENT)

[…] starting with Aristotle there was in fact a long tradition of trying to use logic as a framework for **drawing conclusions about nature.**
[…] And indeed by this point logic was viewed mostly as a possible representation of human **thought—and not as a formal system relevant to nature.**

Stephen Wolfram:
https://publications.stephenwolfram.com/foundations-mathematics-mathematica/

FROM LMM TO REASONING MODELS

[…] **Standard AI models might be good at describing the scene (listing objects), but reasoning models act like that detective. They delve deeper, performing logical deductions, solving complex problems, and planning multi-step actions.**

R. Hightower: Chapter 9: Leveraging Advanced Reasoning Models. In: Programming the OpenAI APIs with Python: A Comprehensive Guide to Building AI Applications. https://rick-hightower.notion.site/Chapter-9-Leveraging-Advanced-Reasoning-Models-1e0d6bbdbbea808190d0e3fa0f26192c?pvs=21

[…] LLMs have transformed how we process and generate text, but their success has been largely driven by statistical pattern recognition. However, new advances in reasoning methodologies now enable LLMs to tackle more complex tasks, such as solving logical puzzles and advanced math problems involving multi-step arithmetic. Moreover, **REASONING IS AN ESSENTIAL TECHNIQUE FOR MAKING "AGENTIC" AI PRACTICAL**.

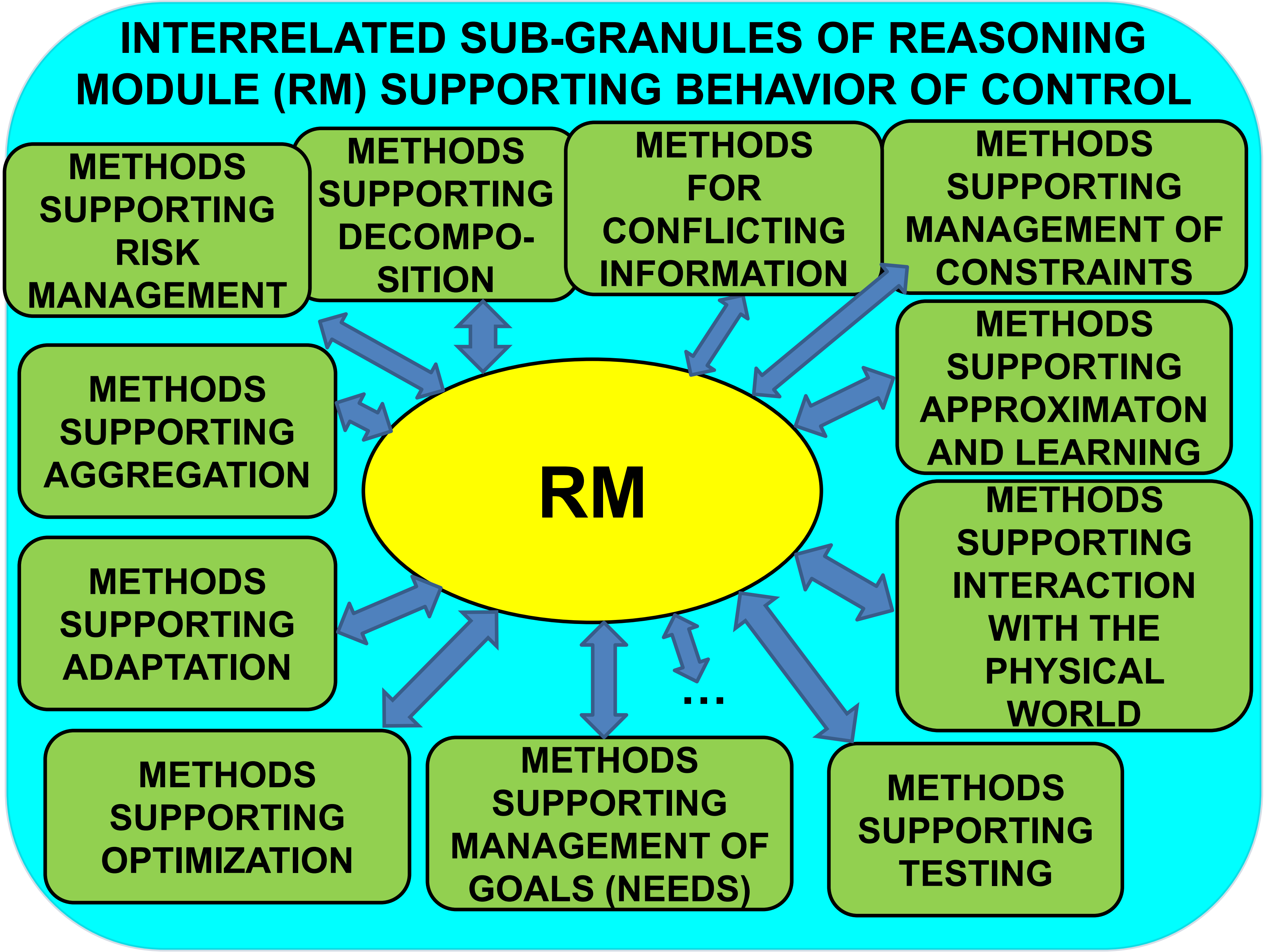
S. Raschka: Build a Reasoning Model (From Scratch). Manning 2025.

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
GPT-4o-0513	9.3	13.4	74.6	49.9	32.9	759
Claude-3.5-Sonnet-1022	16.0	26.7	78.3	65.0	38.9	717
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	**1820**
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9	1316
DeepSeek-R1-Distill-Qwen-1.5B	28.9	52.7	83.9	33.8	16.9	954
DeepSeek-R1-Distill-Qwen-7B	55.5	83.3	92.8	49.1	37.6	1189
DeepSeek-R1-Distill-Qwen-14B	69.7	80.0	93.9	59.1	53.1	1481
DeepSeek-R1-Distill-Qwen-32B	**72.6**	83.3	94.3	62.1	57.2	1691
DeepSeek-R1-Distill-Llama-8B	50.4	80.0	89.1	49.0	39.6	1205
DeepSeek-R1-Distill-Llama-70B	70.0	**86.7**	**94.5**	**65.2**	**57.5**	1633
DeepSeek-R1-Zero	**71.0**		**95.9**	**73.3**	**50.0**	**1444**
DeepSeek-R1	**79.8**		**97.3**	**71.5**	**65.9**	**2029**

Benchmark comparison of distilled versus non-distilled models. Annotated figure from the DeepSeek-R1 technical report (https://arxiv.org/abs/2501.12948).

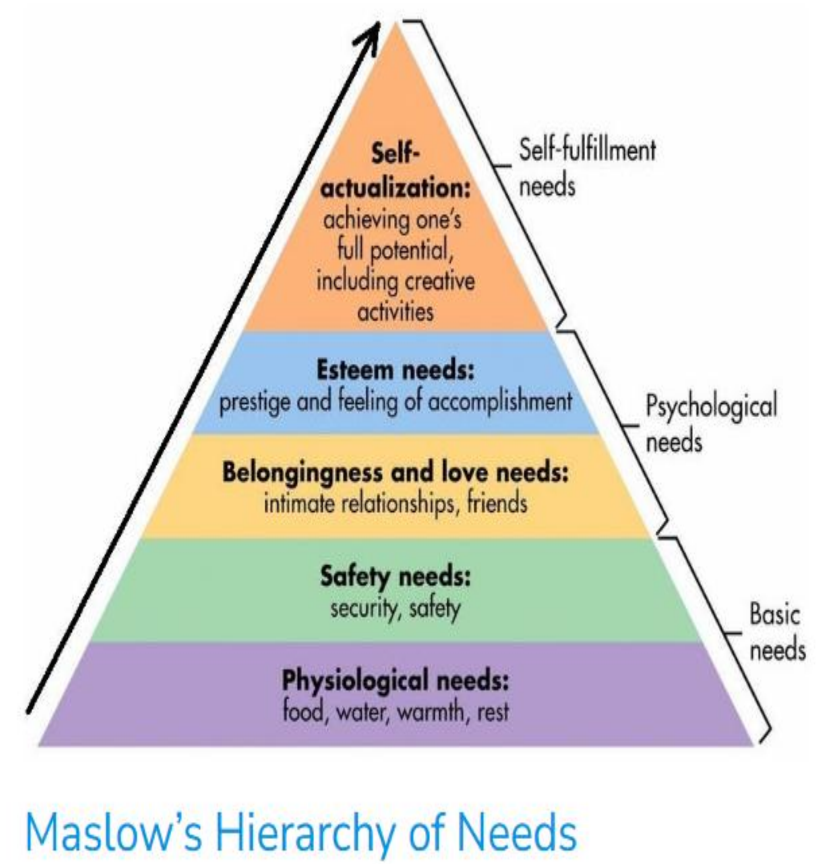
[…] [Inside Reasoning Models OpenAI o3 And DeepSeek R1](https://labs.adaline.ai/p/inside-reasoning-models-openai-o3): OpenAI’s o3 and DeepSeek’s already-released DeepSeek R1 are set to redefine AI reasoning. o3 leverages innovative test-time search to achieve high-performance reasoning, while DeepSeek R1 has captured attention for its cost-efficient design, transparent “aha moment,” and ability to tackle math, coding, and logic challenges at a fraction of traditional costs.

Adaline Labs: https://labs.adaline.ai/p/inside-reasoning-models-openai-o3


INTERRELATED SUB-GRANULES OF REASONING MODULE (RM) SUPPORTING BEHAVIOR OF CONTROL
METHODS SUPPORTING RISK MANAGEMENT
METHODS SUPPORTING DECOMPO-SITION
METHODS FOR CONFLICTING INFORMATION
METHODS SUPPORTING MANAGEMENT OF CONSTRAINTS
METHODS SUPPORTING APPROXIMATON AND LEARNING
METHODS SUPPORTING AGGREGATION
RM
METHODS SUPPORTING INTERACTION WITH THE PHYSICAL WORLD
METHODS SUPPORTING ADAPTATION
...
METHODS SUPPORTING OPTIMIZATION
METHODS SUPPORTING MANAGEMENT OF GOALS (NEEDS)
METHODS SUPPORTING TESTING

C-GRANULE CONTROL

In particular, one of its sub-granules (module of control) is responsible for supporting the management of goals (needs) (including their adaptation according to the properties of perceived objects/situations), taking into account the idea of Maslow's hier



https://www.projectmaslow.org/maslows-hierarchy-of-needs/

REASONING
(JUDGMENT SUPPORTING CONTROL OF C-GRANULES IN MAKING THE RIGHT DECISIONS)
REALIZED OVER INTERACTIVE COMPUTATIONS COMPOSED OF NETWORKS OF C-GRANULES

IN PARTICULAR, JUDGMENT SUPPORTS REALIZATION OF PERCEPTION

i.e. understanding the perceived situation to satisfactory degree for making the right decisions

SOME CHALLENGES CONCERNING REASONING

- Associative memory
- Practical judgment
- Analogy based reasoning
- Experience based reasoning
- Perception based reasoning
- Common sense reasoning

REASONING MODULE OF C-GRANULE CONTROL: EXAMPLE

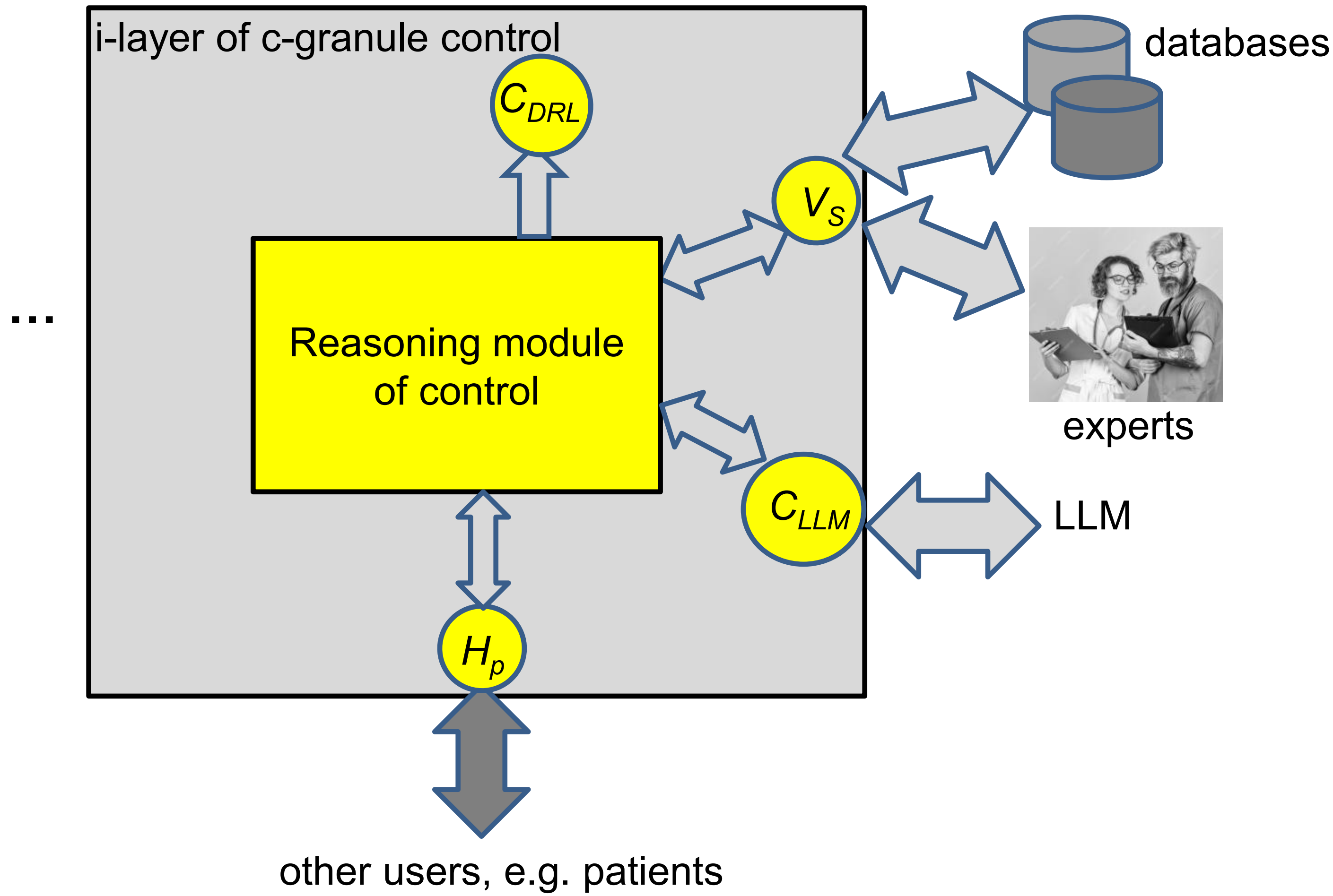
Upon receiving a task (question) from the user, the control of the c-granule initiates reasoning in the information layer of its sub-granule called the reasoning module. This reasoning is realized by granular computations involving interactions (communications) with databases, experts, or other users, and aims to provide an answer to the task.

C_{LLM} - information granule representing parameters of LLM

V_S – information granule consisting of the induced set of rules

C_{CDR} - information granule consisting of the optimized set of rules

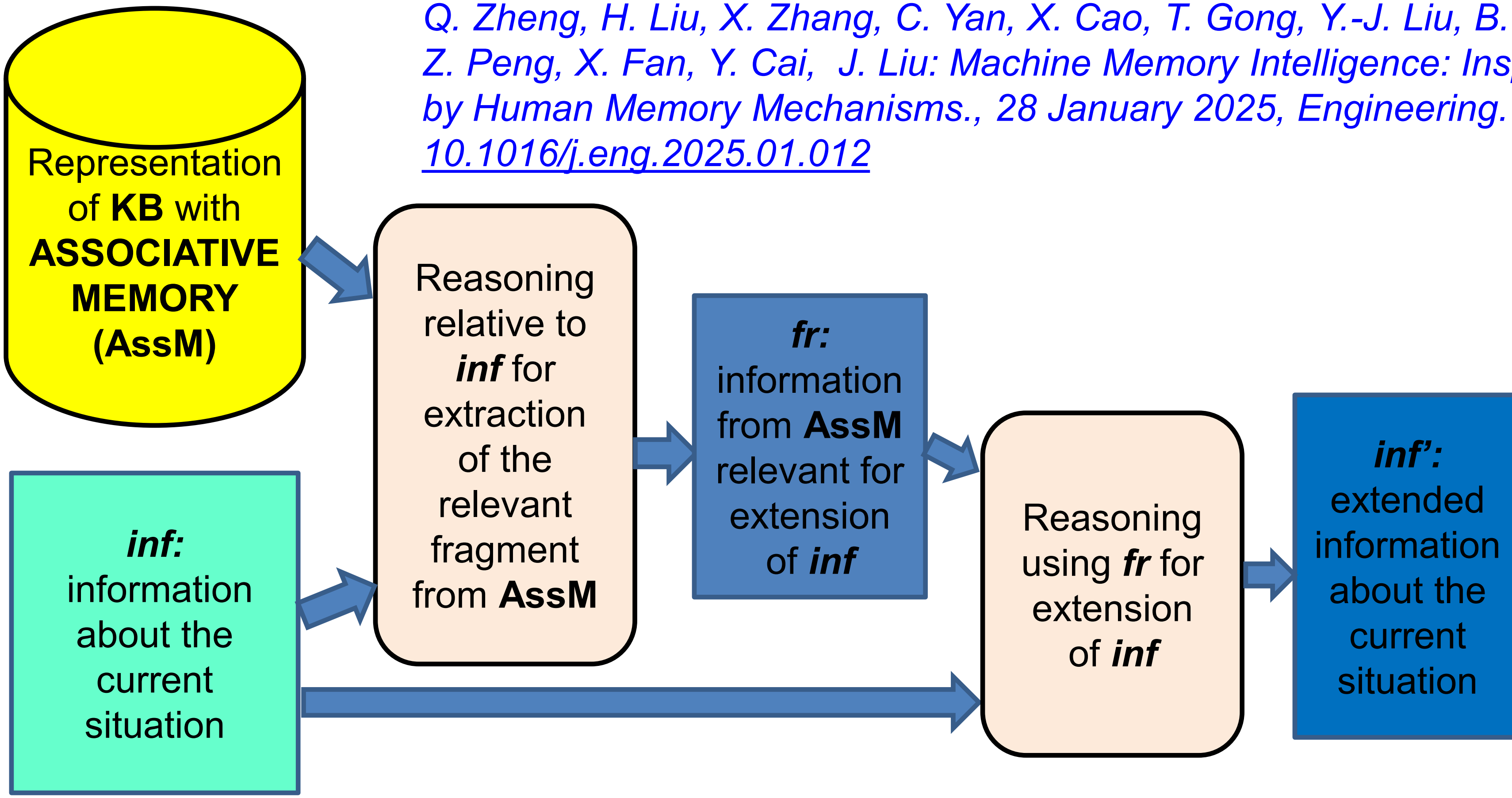
H_p - information granule representing the patient's history and the reasoning module's response regarding support for their therapy



INTERACTIONS WITH KB: ASSOCIATIVE MEMORY

Machine Memory Intelligence (M^{2}I)
framework encompassing
representation, learning, and reasoning modules and loops

Q. Zheng, H. Liu, X. Zhang, C. Yan, X. Cao, T. Gong, Y.-J. Liu, B. Shi, Z. Peng, X. Fan, Y. Cai, J. Liu: Machine Memory Intelligence: Inspired by Human Memory Mechanisms., 28 January 2025, Engineering. DOI: 10.1016/j.eng.2025.01.012



Q. Zheng, H. Liu, X. Zhang, C. Yan, X. Cao, T. Gong, Y.-J. Liu, B. Shi, Z. Peng, X. Fan, Y. Cai, J. Liu: Machine Memory Intelligence: Inspired by Human Memory Mechanisms., 28 January 2025, Engineering. DOI: 10.1016/j.eng.2025.01.012

ASSOCIATIVE MEMORY

HOPFIELD NETWORKS

T. Kohonen: Self-organization and associative memory. Springer (1989)

T.F. Burns, T. Fukai, Ch.J. Earls: Associative memory inspires improvements for in-context learning using a novel attention residual stream architecture. Transactions on Machine Learning Research (07/2025)

H. Ramsauer, B. Schäfl, J. Lehner, P. Seidl, M. Widrich, L.Gruber, M. Holzleitner, T. Adler, D. Kreil, M. K. Kopp, G. Klambauer, J. Brandstetter, S, Hochreiter: Hopfield networks is all you need. In International Conference on Learning Representations, 2021. URL https://openreview.net/forum?id=tL89RnzIiCd.

ASSOCIATIVE MEMORY

REASONING BASED ON RULES DERIVED FROM INFORMATION SYSTEMS AND INCOMPLETE VECTORS OF ATTRIBUTE VALUES

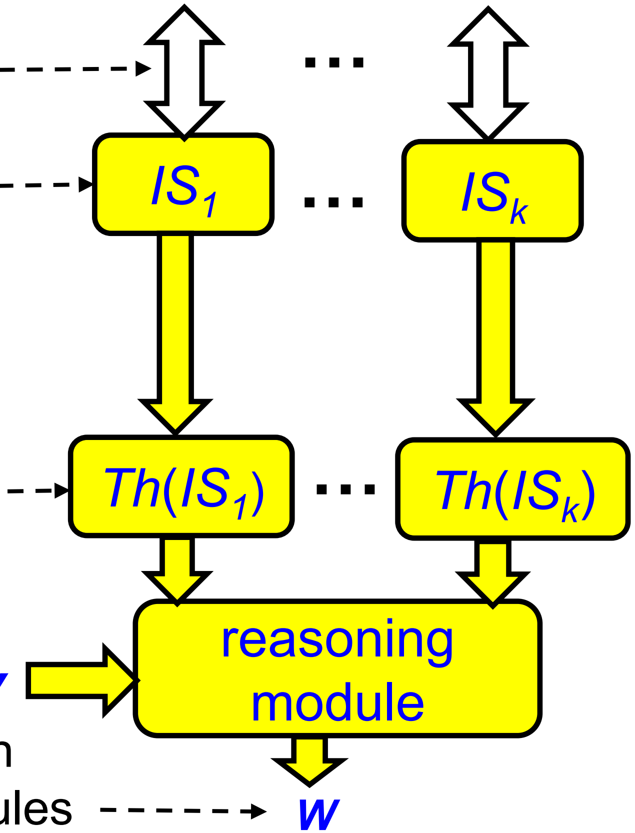
interaction with the environment
(physical world, data or knowledge bases, experts, …)

(dynamic) information systems
(obtained: (i) by decomposition of large datasets for paralel processing or relative to functionality,
(ii) from distributed data sources,
(iii) by interaction with the physical world …

(adaptive) theories of information systems:
sets of (probabilistic) rules derived from information systems

incomplete vector of attribute values

extension of ***v*** consistent (to the required degree) with (some/all) sets of rules derived from ***v*** and the sets of rules



Sets of rules derived from information systems (for discovery of concurrent models from data):

M. Moshkov, A. Skowron, Z. Suraj: Maximal consistent extensions of information systems relative to their theories. Information Sciences 178(12) (2008) 2600-2620. doi.org/10.1016/j.ins.2008.01.018

A. Skowron, Z. Suraj (1995). Discovery of concurrent data models from experimental data tables: A rough set approach, Proceedings of the First International Conference on Knowledge Discovery and Data Mining, Montreal, August, 1995, AAAI Press, Menlo Park CA 1995, 288-293

J. Barwise, J. Seligman, Information Flow: The Logic of Distributed 2339 Systems, Cambridge University Press, Cambridge, 1997. doi:10.2340 1017/CBO9780511895968.

PRACTICAL JUDGMENT

Practical judgment is not algebraic calculation. Prior to any deductive or inductive reckoning, the judge is involved in selecting objects and relationships for attention and assessing their interactions. Identifying things of importance from a potentially endless pool of candidates, assessing their relative significance, and evaluating their relationships is well beyond the jurisdiction of reason

Leslie Paul Thiele: The Heart of Judgment Practical Wisdom, Neuroscience, and Narrative. Cambridge University Press 2006

MELANIE MITCHELL

Santa Fe Institute

The quest for machines that can make abstractions and analogies is as old as the AI field itself, but the problem remains almost completely open.

Melanie Mitchell: Abstraction and Analogy-Making in Artificial Intelligence, Annals Reports of the New York Academy of Sciences 1505(1) 79-101 (2021)

We do not have yet formal reasoning for experience based reasoning working in IS's

However,

IS's on the basis of data analysis can help domain expert in this kind of reasoning.

Human experts and/or chatbots can help IS's to improve reasoning, e.g., in inducing classifiers.

Human-Centered AI, Human-in-the-Loop ML

SOME CHALLENGES CONCERNING REASONING (cont.)

Reasoning supporting

- optimization
- decomposition
- Adaptation (see co-evolution in the book by Holland)
- negotiation and conflict resolving
- searching for new relevant data and knowledge (*where*?, *what*?, *when*?, *how*?)
- discovery of granular computations according to given specifications (drug discovery, automatic design of robots, discovery of strategies on financial markets etc.)
- construction of quality measures over granular computations
- risk management in generation of approximate solutions of high quality
- …

EVOLUTION OF CONCEPT FORMATION AND LANGUAGE USED BY SOCIETY OF C-GRANULES

As granular computation progresses, the concepts and language of a society of c-granules evolve. This evolution is supported by the reasoning modules of c-granule control, which are involved in the steering of c-granules (and through this the entire society) solving tasks such as:

- adaptation of existing concepts (e.g., through relevant generalizations and/or the use of new actuators or sensors for defining concepts)
- discovery of new concepts
- discovery of backtracking strategies in searching for new concepts
- forgetting concepts not useful for the granular society
- discovery of new contexts (e.g., in the form of relational constraints or texts from language representing situations) in which the discovered concepts can be applied
- adaptation of the discovered contexts
- generalization of global contexts of the granular society.

There are many challenges on the way to developing these reasoning methods (see, e.g., https://medium.com/the-art-of-bagging/grasping-the-concept-book-v1-introduction-f8daae90af22

Y. Engeström: Concept formation in the wild. Cambridge University 2024:

> *Little is known about how new concepts are collectively created and used in workplaces, communities, and social movements. It is time to start filling this lacuna with generative ideas and solid findings.*

N. W. Morton, A. R. Preston: Concept formation as a computational cognitive process. Current Opinion in Behavioral Sciences 38 (2021) 83–89).

INFORMATION GRANULATION & COMPUTING WITH WORDS – LOTFI A. ZADEH

[…] **Information granulation plays a key role in implementation of the strategy of divide-and-conquer in human problem-solving.**

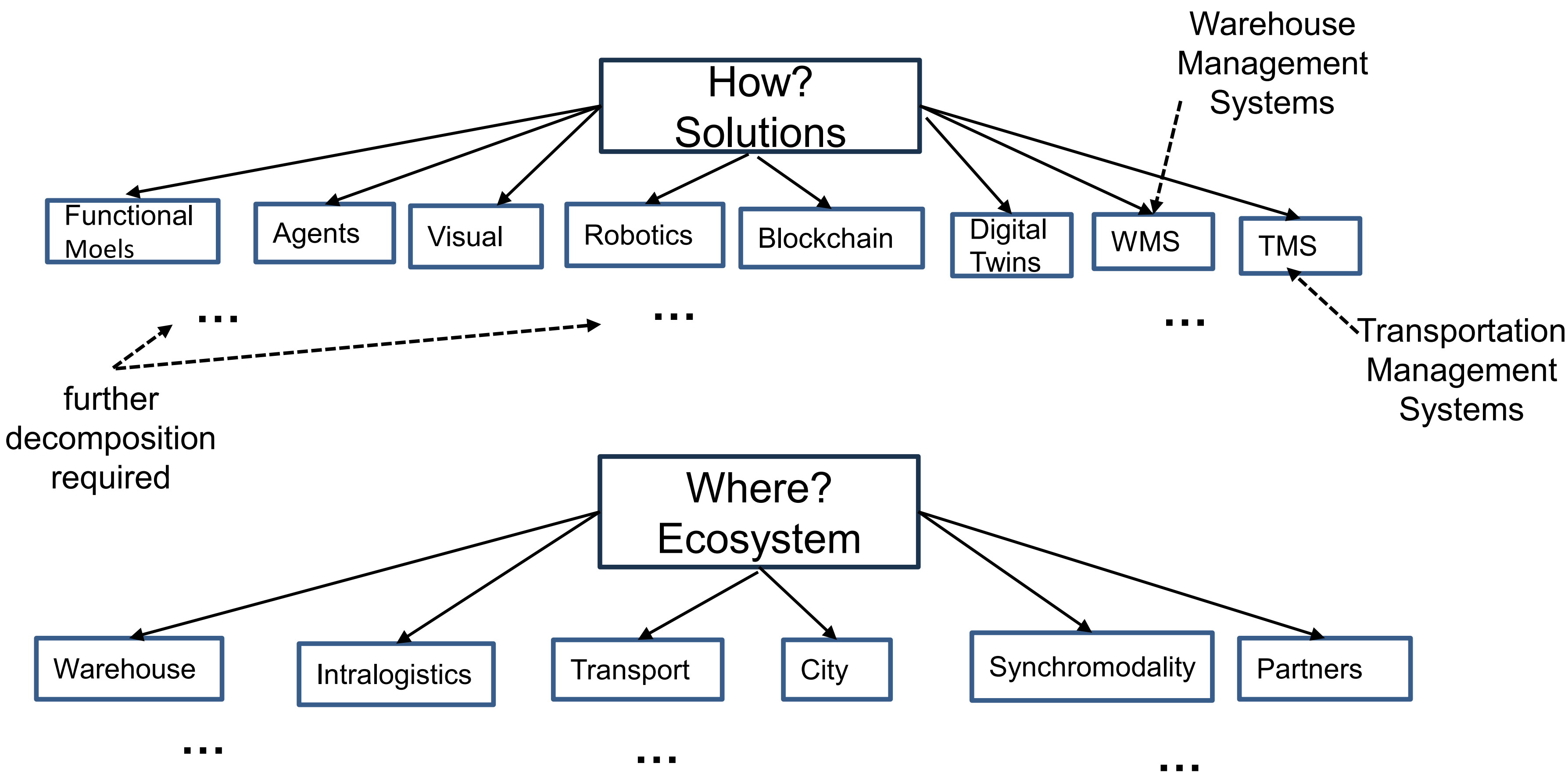
Lotfi A. Zadeh: Foreword. In: S.K. Pal, L.Polkowski, A. Skowron (eds.) Rough-Neural Computing. Techniques for Computing with Words. Springer 2004.

[…] Manipulation of perceptions plays a key role in human recognition, decision and execution processes. As a methodology, computing with words provides a foundation for a computational theory of perceptions - a theory which may have an important bearing on how humans make - and machines might make – perception - based rational decisions in an environment of imprecision, uncertainty and partial truth.

[…] computing with words, or CW for short, is a methodology in which the objects of computation are words and propositions drawn from a natural language.

Lotfi A. Zadeh: From computing with numbers to computing with words – From manipulation of measurements to manipulation of perceptions. IEEE Transactions on Circuits and Systems 45(1), 105–119 (1999)

DECOMPOSITION OF COMPLEX VAGUE CONCEPTS IN DIALOGUE WITH HUMANS & CHATBOTS: EXAMPLES FROM LOGISTICS 5.0



Bernardo Nicoletti: Artificial Intelligence for Logistics 5.0. From Foundation Models to Agentic AI. Springer (2025)

DECOMPOSITION OF COMPLEX VAGUE CONCEPTS IN DIALOGUE WITH HUMANS & CHATBOTS: TRUSTWORTHINESS

Trustworthiness dimensions:

- human agency and oversight, fairness and non-discrimination, transparency and explainability, robustness and accuracy, privacy and security, and accountability

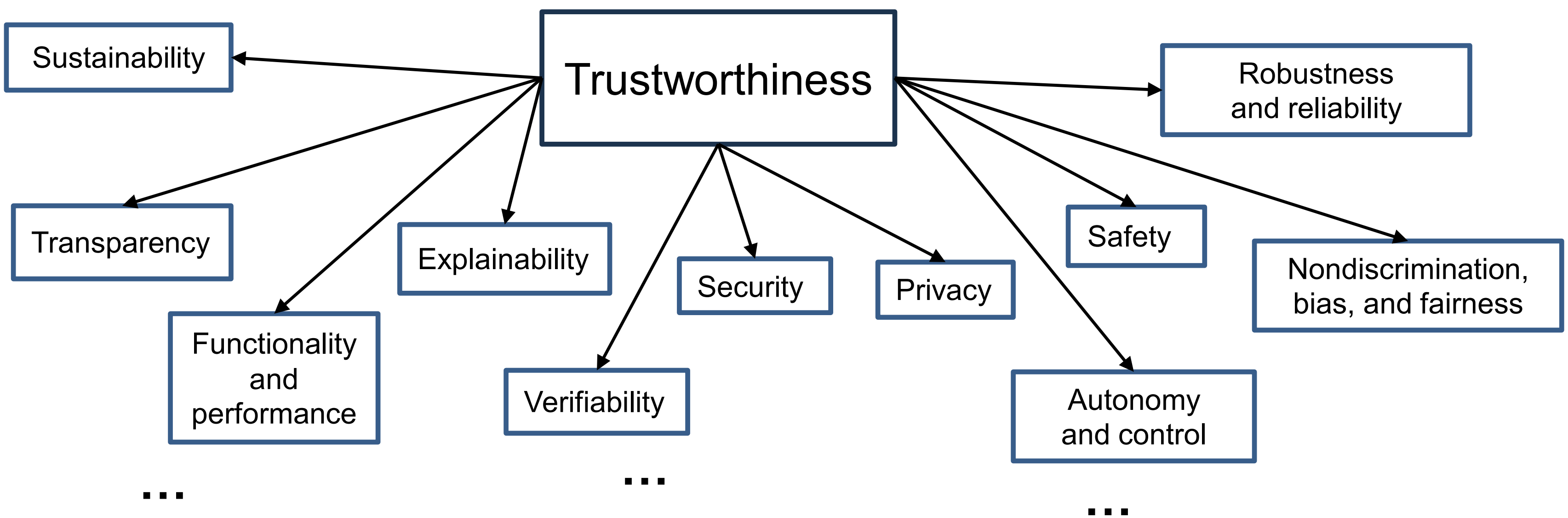
  D. Kowald et al.: Establishing and evaluating trustworthy AI: overview and research challenges. Frontiers of Big Data 7: 1467222, (2024)

- reliability, security, privacy, accountability, transparency, ethical standards and societal values, adaptability

  chatbot poe

- authenticity, fulfillment, value, reliability, safety, recurse

  https://www.linkedin.com/pulse/six-dimensions-trustworthiness-4grader-fokuf



R. Mariani et al.: Trustworthy AI. Part I. Computer 56(2) 14-18 (February 2022)

JUDEA PEARL- TURING AWARD 2011

for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning

Traditional statistics is strong in devising ways of describing data and inferring distributional parameters from sample. Causal inference requires two additional ingredients:

- *a science-friendly language for articulating causal knowledge*,

and

- *a mathematical machinery for processing that knowledge, combining it with data and drawing new causal conclusions about a phenomenon*.

Judea Pearl: Causal inference in statistics: An overview. Statistics Surveys 3, 96-146 (2009)

PHENOMENOLOGY
originated by Edmund Husserl
as a method for exploring the nature of human experience and perception

Husserl was frustrated by the idea that science and mathematics were increasingly conducted on an abstract plane [treating nature itself as a mathematical manifold] that was disconnected from human experience and human understanding, independently of questions of truth and applicability. He felt that the sciences increasingly dealt with idealized entities and internal abstractions a world apart from the concrete phenomena of daily life.

Dourish, P.: Where the Action Is. The Foundations of Embodied Interaction. The MIT Press (2004)

GRANLAR COMPUTATIONS

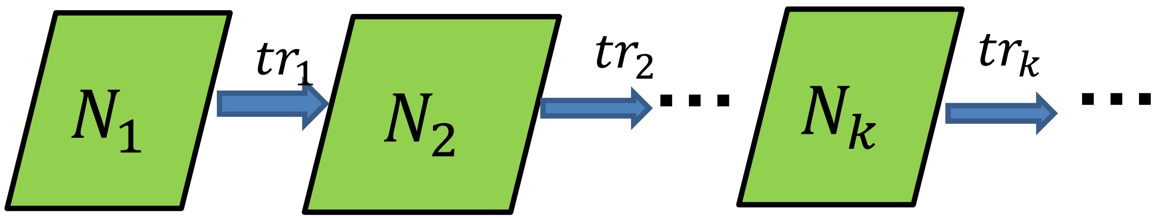
GRANULAR COMPUTATIONS IN IGrC and IGrC

GrC:

$N_1, N_2 \ldots, N_k$ -- granular networks in the abstract space

$tr_1, tr_2, \ldots, tr_k$ -- transformations realized in the abstract space

The control of c-granules, whether cooperating or competing with other c-granules, aiming to generate a granular computation along which is constructed an approximate solution of the problem to be solved to the required quality.



IGrC:

$N_1, N_2 \ldots, N_k$ -- granular networks in the abstract and physical spaces

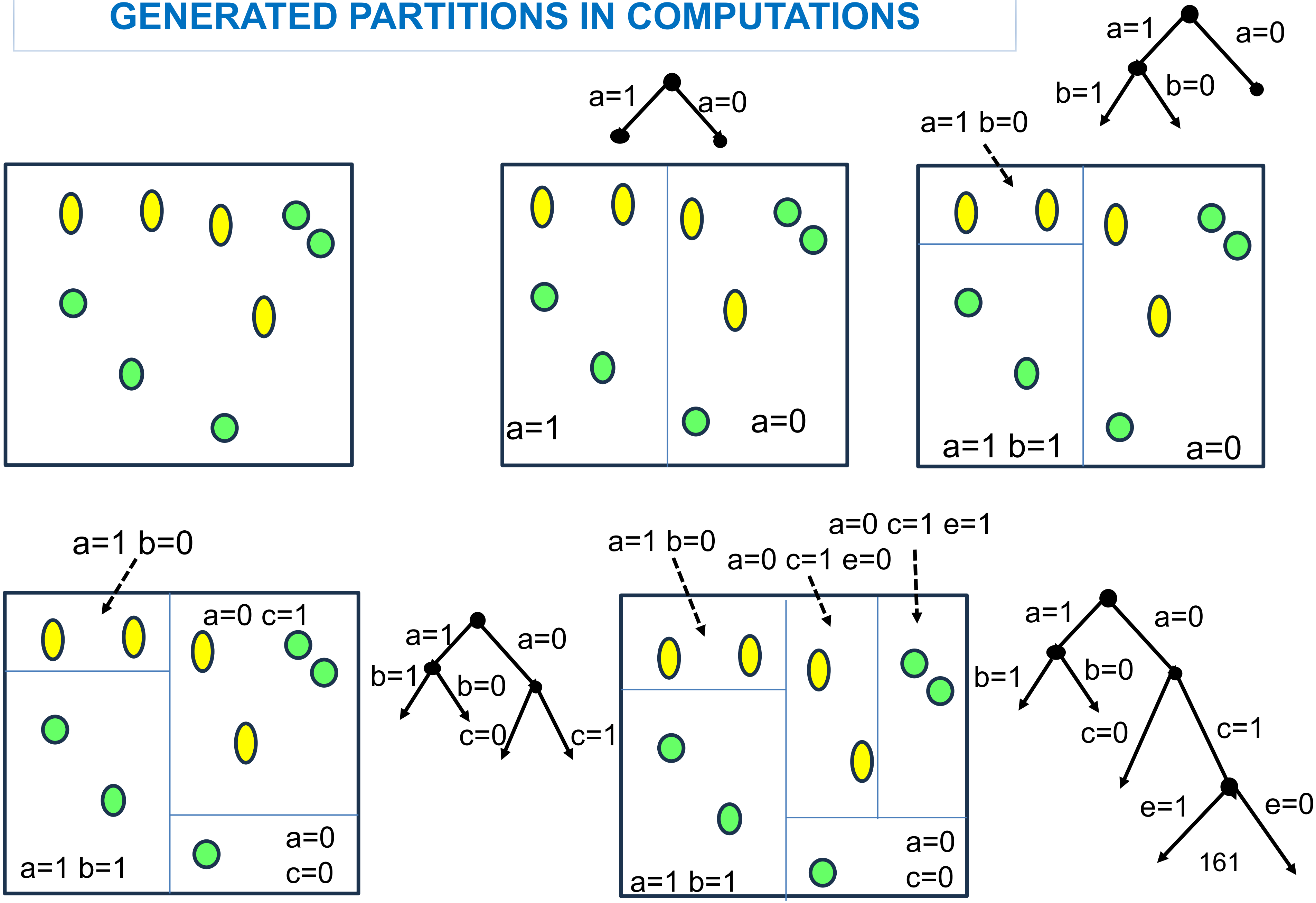
$tr_1, tr_2, \ldots, tr_k$ -- transformations realized in the abstract and physical spaces

The control of c-granules, whether cooperating or competing with other c-granules, engages in interaction with the physical space, aiming to generate a granular computation along which is constructed an approximate solution of the problem to be solved to the required quality.

ILLUSTRATIVE EXAMPLE

SEMI-OPTIMAL DECISION TREE CONSTRUCTED ALONG PARTITIONS GENERATED IN COMPUTATIONS REALIZED BY CONTROL OF C-GRANULE

EXAMPLE: DECISION TREE CONSTRUCTION ALONG GENERATED PARTITIONS IN COMPUTATIONS



DISCOVERY AS A PARADIGM OF LOGIC

When generating a decision tree along granular computation (with granules being partitions), it is important to understand the reasoning behind **discovering** a heuristic for transforming the current partition.

For example, the entropy (information) gain criterion suggests splitting an impure class in the current partition using the best attribute that has not yet been used to generate this class. The selection of the attribute is based on the difference between the impurity of the class being split and its subpartition created by that attribute.

Bocheński says that […] *one can ask two different basic questions:*

(1) What follows given premises?

(2) From what premises can a given sentence (conclusion) be deduced?

Aristotle *primarily considered the first question,* **justification**, […] but *poses also the second*, **discovery,** and tries to show ***how the premises of a syllogism must be constructed in order to yield a given conclusion***

Bocheński, J. (1961). A history of formal logic. University of Notre Dame Press.

The logic of discovery is still in its infancy. Consequently, we do not yet have satisfactory automatic discovery methods supported by such logic. Consequently, AI systems cooperate with humans (see Human-Centered AI and Human-in-the-Loop Machine Learning) and chatbots in discovery tasks.

EXAMPLES OF DOMAINS WHERE DISCOVERING HIGH-QUALITY APPROXIMATE SOLUTIONS IS IMPORTANT

- Discovery of learning algorithms and construction of classifiers
- Automatic design of robots
- Drug discovery
- Algorithmic trading
- Evaluation of information provided by LMM (hallucinations)
- Generative AI
- Modeling cognitive computers
- …

CHALLENGE:
DEVELOPING THE FOUNDATIONS BASED ON IGrC AND RS FOR THE DESIGN AND ANALYSIS OF DISCOVERY AI SYSTEMS FOR DIFFERENT DOMAINS;
E.G., COMMON CORE OF SUCH SYSTEMS BASED ON IGrC

MAIN TASKS IN IGrC

- Discovery of complex game teams over granular networks making it possible to generate granular computations providing approximate solutions of problems with high quality:
  - Discovery of spaces of granular networks
  - Discovery of components of complex games responsible for dynamics of granular networks:
    - approximations of concepts
    - actions labelling concepts
    - game structures
      - adaptation strategies of complex games
      - complex game teams
- …

ADAPTATION & LIFELONG LEARNING IN IGrC

Lifelong Machine Learning or **Lifelong Learning** (LL) is an advanced machine learning (ML) paradigm that **learns continuously**, **accumulates the knowledge** learned in the past, and **uses/adapts** it to help future learning and problem solving. In the process, the learner becomes **more and more knowledgeable and better and better at learning**. This continuous learning ability is one of the hallmarks of human intelligence.

However, the current dominant ML paradigm learns in isolation: given a training dataset, it runs a ML algorithm only on the dataset to produce a model. It makes no attempt to retain the learned knowledge and use it in subsequent learning. Although this isolated ML paradigm, primarily based on data-driven optimization, has been very successful, it requires a large number of training examples, and is only suitable for well-defined and narrow tasks in **closed environments**. In contrast, we humans learn effectively with a few examples and in the dynamic and **open world or environment** in a **self-supervised** manner because our learning is also very much knowledge-driven: the knowledge learned in the past helps us learn new things with little data or effort and adapt to new/unseen situations. This self-suprevised (or self-aware) learning also enables us to **learn on the job** in the **interaction** with others and with the real-world environment with no external supervision. LL aims to achieve all these capabilities. Applications such as chatbots, s elf-driving cars, or any AI systems that interact with humans/physical environments are calling for these capabilities because they need to cope with their dynamic and open environments which leave them with no choice but to continuously learn new things in order to function well. Without the LL ability, an AI system cannot be considered truly intelligent, i.e., **LL is necessary for intelligence** or **AGI** (artificial general intelligence).

Zhiyuan Chen and Bing Liu: Synthesis Lectures on *Artificial Intelligence and Machine Learning, Morgan & Claypool Publishers*, August, 2018

power of judging rightly and following the soundest course of action, based on knowledge, experience, understanding, …
Webster's New World College Dictionary

Aristotle's man of practical **wisdom**, the phronimos, does not ignore rules and models, or dispense
justice without criteria. He is observant of principles and, at the same time, open to their modification. He begins with nomoi – established law – and employs practical wisdom to determine how it should be applied in particular situations and when departures are warranted. Rules provide the guideposts for inquiry and critical reflection.

Leslie Paul Thiele: The Heart of Judgment Practical Wisdom, Neuroscience, and Narrative. Cambridge University Press 2006

ROUGH SETS (RS) & IGrC

- EVOLUTION OF APPROXIMATION SPACES
- GRANULES GENERATED IN INTERACTION WITH THE ABSTRACT AND PHYSICAL WORLDS
- CHALLENGE FOR FOUNDATIONS OF AI SYSTEMS: DISCOVERY OF APPROXIMATE SOLUTIONS OF PROBLEMS

EXAMPLES IN EVOLUTION OF APPROXIMATION SPACES IN THE ROUGH SET APPROACH

APPROXIMATION SPACE IN THE PAWLAK ROUGH SET MODEL: $AS = (U, r)$

MULTIGRANULAR APPROXIMATION SPACE: $AS = (U, R)$

APPROXIMATION SPACE OVER GRANULAR ALGEBRA: $DS = (U, GA_{IS}, d)$, $d: U \rightarrow V_d$

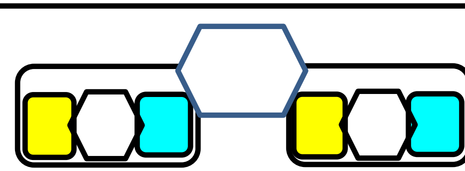
APPROXIMATION SPACE OVER GRANULAR RELATIONAL SYSTEM OF GRANULAR CALCULUS

$$(U, \underbrace{GA_{IS}, d}, \underbrace{\{\nu_{tr}\}_{tr \in [0.5,1]}, \mathcal{F}_1, \ldots, \mathcal{F}_k})$$

family of granules (partitions of *U* defined by the granular algebra GA_{IS} and the decision *d*)

vector of families of relations expressing properties of granules and relations between them used to define new granules (e.g., atomic for the next layer or quality measures of granules) in the language of granular calculus

APPROXIMATION SPACE IN IGrC:
DYNAMIC GRANULAR NETWORKS



EVOLUTION OF APPROXIMATION SPACES IN THE ROUGH SET APPROACH

APPROXIMATION SPACE IN THE PAWLAK ROUGH SET MODEL

$$AS = (U, r)$$

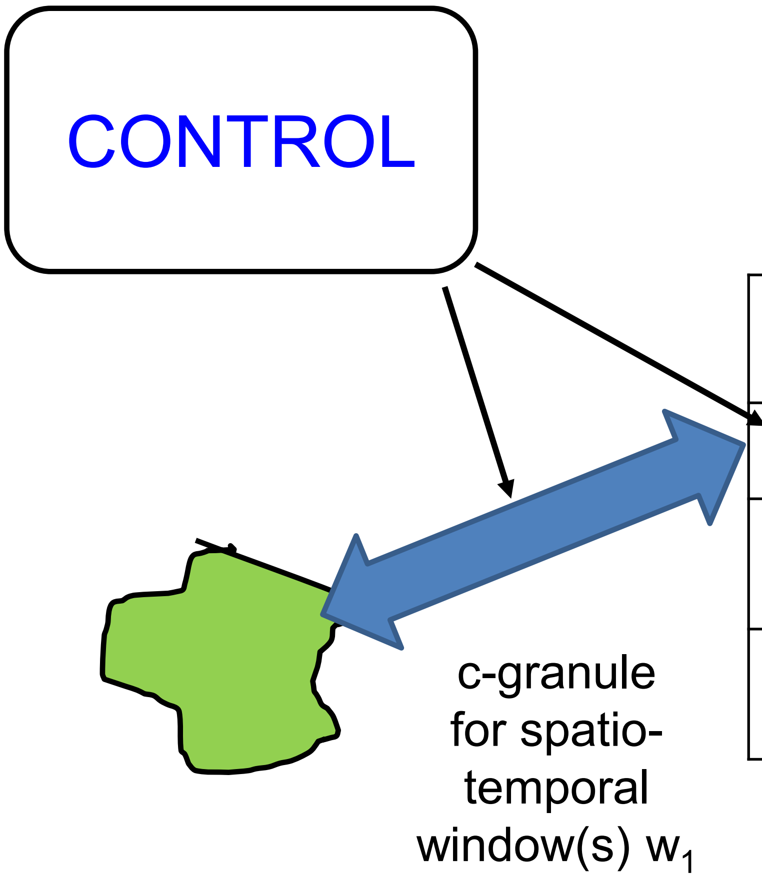
evolution

APPROXIMATION SPACE IN IGrC:
DYNAMIC GRANULAR NETWORKS

The discovery of dynamic granular networks through the control of c-granules (interacting with the environment) is the foundation for finding relevant computational building blocks [using terminology that Leslie Valiant would use] for adaptive approximations of concepts or classifications.

GRANULES DISCOVERED THROUGH INTERACTION WITH THE PHYSICAL AND ABSTRACT WORLDS

C-GRANULES WITH CONTROL AND INFORMATION SYSTEMS UNDER CONTROL OF C-GRANULES



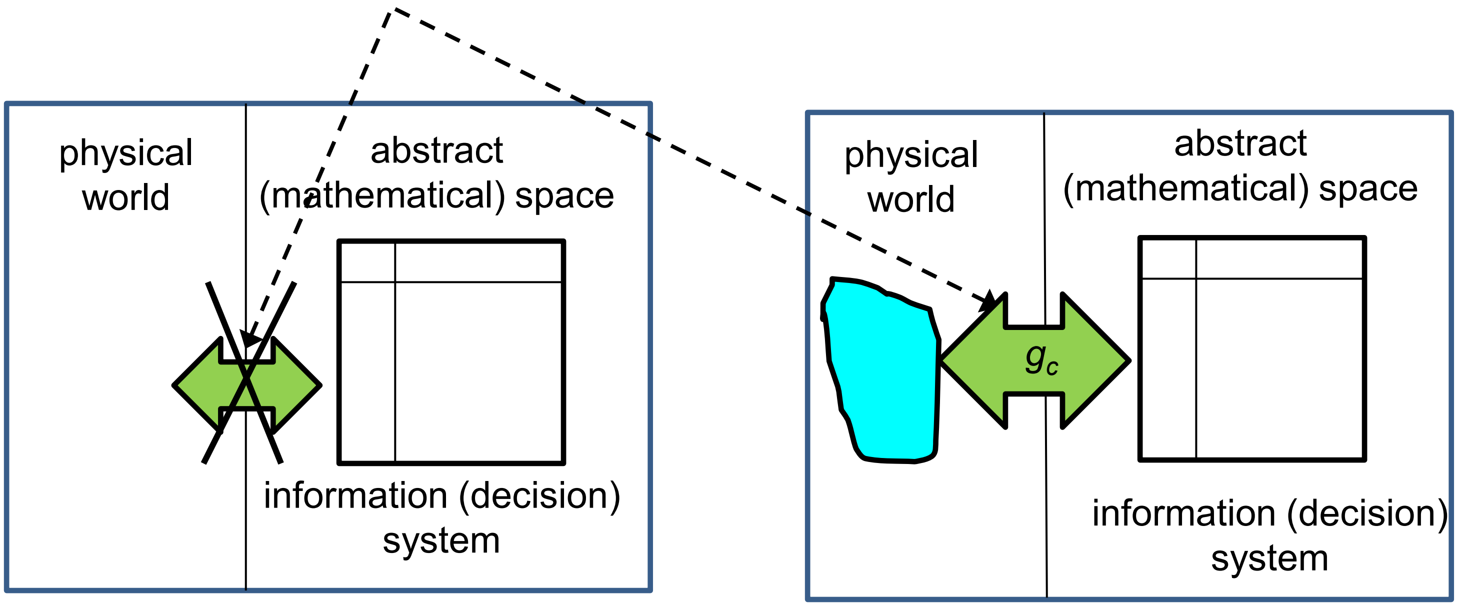
	a_1	a_2	…	a_m
w_1	v_1	v_2	…	v_m
	•••	•••	•••	•••

dynamics of information systems determined
by control and its interaction with the environment

RS: PERCEPTUAL APPROACH

Information(decision) systems are parts of dynamic objects: c-granules

Continuous interactions with the physical world during perceiving of the current situation aiming to understand this situation to a degree satisfactory for making the rights decisions



In the existing approaches to rough sets interactions with the physical world are omitted.
Information systems are **GIVEN** as pure mathematical objects.

Rough sets in IGrC (perceptual approach) based on **physical semantics**:: information (decision) systems are obtained as the result of granulation of information perceived by c-granule g_c in the physical world.

MULTISCALING FOUNDATIONS (MSF) MULTISCALE INFORMATION (DECISION) SYSTEMS (MSIS)

Recently, some progress has been made in developing the foundations of MSIS. However, for developing the MSF foundations is necessary to generalize the existing approaches to MSIS. This will require to take into account that, e.g.:

- MSIS for MSF should not be given a priori, but rather, they should be discovered through data analysis and/or dialogue with experts and/or chatbots.
- MSIS for MSF should be dynamic, not static, objects steered by control of granules interacting with the environment.
- The MSIS for MSF are complex granules that cannot be reduced to simple aggregations of their parts.
- The aggregation of granules in MSIS for MSF should be extended to vectors, which depend on the relevant context (which should be discovered).
- Advanced sensors and actions should be discovered on higher levels of modeling MSIS for MSF, together with reasoning methods that support their applications over time and space.

We propose using IGrC as the basis for modeling MSIS with networks of information systems embedded within granular networks (mappings between attribute value sets of MSIS are realized by transformations from interfaces of granular networks).

X. Ma, Y. Xiao and J. Zhan: Advancing Multiscale Information Systems: A Synthesis of Theoretical Insights, Practical Applications, and Emerging Challenges," in IEEE Transactions on Fuzzy Systems, vol. 33, no. 11, pp. 3871-3892, doi: 10.1109/TFUZZ.2025.3614637

B. Allen, B. C. Stacey, Y. Bar-Yam: Multiscale information theory and the marginal utility of information, Entropy 19(6): 273 (2017) doi:10.3390/e19060273.

J. Walpole, J. A. Papin, S. M. Peirce: Multiscale Computational Models of Complex Biological Systems. Annual Review of Biomedical Engineering 15, (2013) 137–154. doi:10.1146/annurev-bioeng-071811-150104.

R. Midya, A.S. Pawar, D.P. Pattnaik et al. Artificial transneurons emulate neuronal activity in different areas of brain cortex. Nat Commun 16, 7289 (2025). https://doi.org/10.1038/s41467-025-62151-9

Computer CL1: https://corticallabs.com/cl1.html

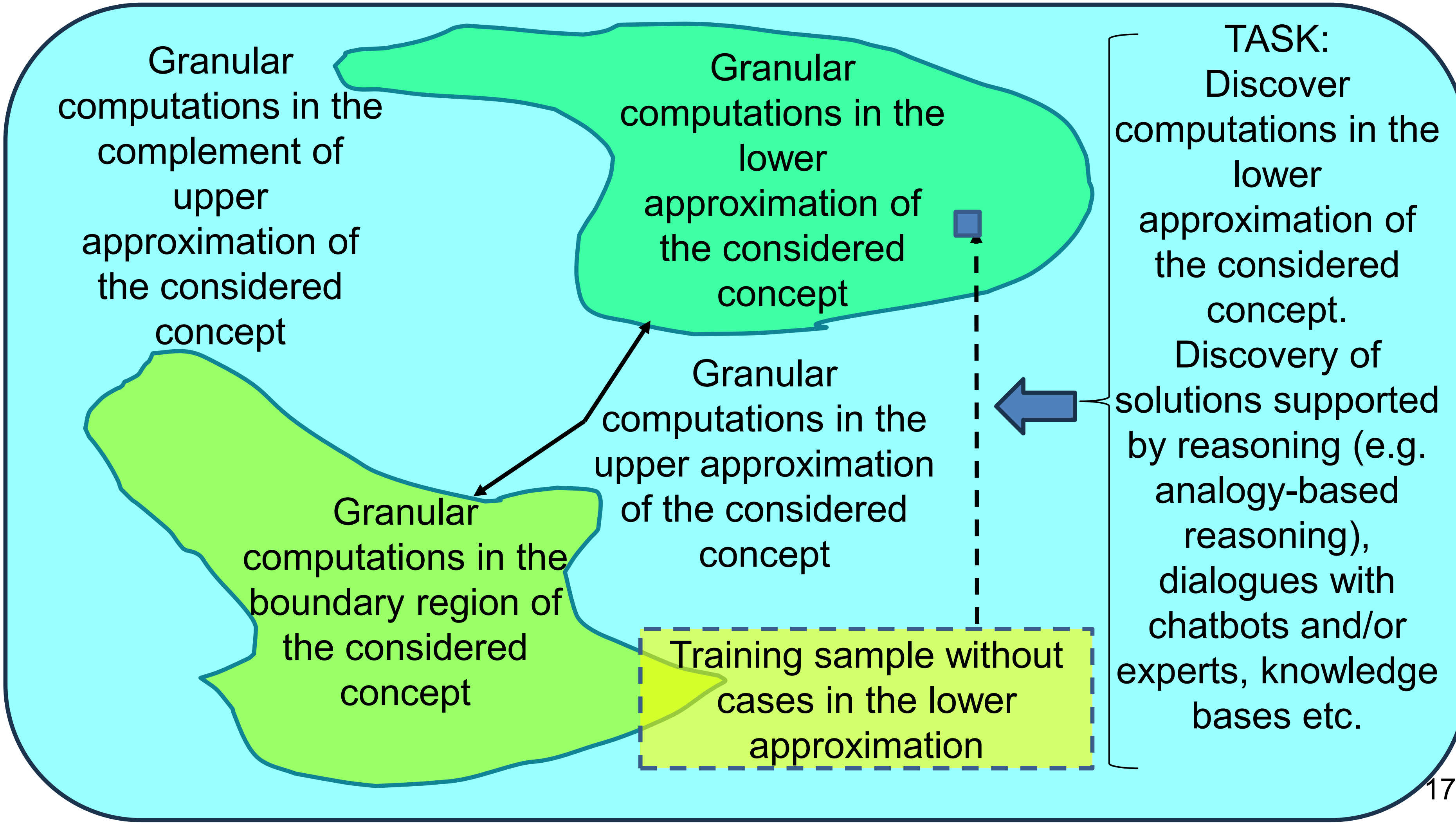
RS & IGrC IN FOUNDATIONS OF APPROXIMATE PROBLEM SOLVING IN AI SYSTEMS

The RS approach is generalized through the introduction of approximation spaces over dynamic networks of granular spaces. During the computations generated over granular networks by c-granules, new entities are discovered in interaction with the physical world, which serve as the so-called computational building blocks essential for properly understanding perceived situations in the physical world. Along with the generated computation, the control of c-granules constructs approximate solutions (of satisfactory quality) to the problems presented to the c-granules for resolution.

APPROXIMATION AREAS OF THE CONCEPT:

GRANULAR COMPUTATIONS WITH HIGH-QUALITY SOLUTIONS OF THE SPECIFIED PROBLEM

IN THE DOMAIN CONSISTING OF COMPUTATIONS OF C-GRANULE CONTROL



RS – CURRENT APPROACH

Basic concept:
Approximation space

→

Basic task:
Approximation of concepts

↓

CHALLENGE FOR RS IN IGrC

Basic concept:
Approximate reasoning processes supporting

(i) generation of rough set based granular computations in interaction with the abstract and physical worlds over dynamically discovered rough set based granular networks and

(ii) for a given problem specification to c-granule(s), construction of "abstract and/or physical" approximate solutions of appropriate quality along these granular computations in the form of granules.

→

Basic task:
Discovery of approximate solutions to problems of appropriate quality based on discovery of rough set based complex game teams over rough set based granular networks

CHALLENGES:
FOR DIFFERENT CLASSES OF PROBLEMS FOR WHICH THE HIGH QUALITY APPROXIMATE SOLUTIONS SHOULD BE DISCOVERED BY PROPERLY STEERED GRANULAR COMPUTATIONS USING ADAPTIVE COMPLEX GAMES

W. Zaremba, OpenAI (5 steps of development), https://www.youtube.com/watch?v=pX0BZwFEPiE

EXAMPLES OF CHALLENGES

related to the design methods yielding risk ratings based on a thorough examination of an AI system and its behavior

These risk degrees should clarify whether a given AI system is trustworthy and, if not, why. They should also indicate whether an LMM module of the system generates hallucinations and, if so, characterize the situations in which this occurs. Finally, they should explain whether the behavior of a given AI system is explainable and, if not, why.

S. Gerrish, How Smart Machines Think. Cambridge, MA: MIT Press, 2018. [Online]. doi.org/10.7551/mitpress/11440.001.0001

E. Y. Chang: Multi-LLM Agent Collaborative Intelligence: The Path to Artificial General Intelligence. Association for Computing Machinery, New York, NY 2025. doi.org/10.1145/3749421

R. Mariani, F. Rossi et al: Trustworthy AI—Part I. Computer 14-18, February 2023

R. Mariani, F. Rossi et al: Trustworthy AI—Part II. Computer 13-16, May 2023

Lotfi A. Zadeh: Foreword. In: S.K. Pal, L.Polkowski, A. Skowron (eds.) Rough-Neural Computing. Techniques for Computing with Words. Springer 2004

EXAMPLES OF CHALLENGES:

INTELLIGENT & COLLABORATIVE DIALOGUE OF SOCIETIES OF C-GRANULES

[…] the key to achieving AGI, characterized by versatility, adaptability, reasoning, critical thinking, planning, and ethical alignment, lies not in creating more powerful individual models, but in enabling large language models (LLMs) to engage in intelligent and collaborative dialogue. This concept, termed **Multi-LLM Agent Collaborative Intelligence** (MACI) forms the foundation of our exploration.

E. Y. Chang: Multi-LLM Agent Collaborative Intelligence: The Path to Artificial General Intelligence. Association for Computing Machinery, New York, NY 2025. doi.org/10.1145/3749421

SUMMARY
&
FUTURE RESEARCH

COMPARISON OF GrC & IGrC

	GrC	IGrC
GENERAL FEATURES OF COMPUTING MODEL		
based on the abstract Turing computation model	YES	NO
computations are pure mathematical objects	YES	NO
issues of language, reasoning, perception and action are brought into sync	NO	YES
modeling of perception of situations (objects and their interactions) in the physical world is provided	NO	YES
advanced reasoning tools based on the computing model that support the control of computations involving both abstract and physical objects are being developed and utilized in the computing model	NO	YES
MAIN FEATURES OF GRANULES		
abstract semantics of (information) granules is provided	YES	YES
physical semantics of (complex) granules is provided	NO	YES
features (attributes) of granules defined using the abstract space only	YES	NO
features (attributes) of granules dependent on interaction with physical objects	NO	YES
granules are equipped with control	NO	YES
the dynamics of granules are defined a priori in the abstract space only	YES	NO
the dynamics of granules depends on physical laws, their (internal and external) control and interactions with the environment	NO	YES
associations between abstract and physical objects related to granules are being constructed and used in the computing model	NO	YES
skills for encoding information into physical objects provided in the computing model	NO	YES
skills for decoding information from physical objects provided in the computing model	NO	YES
changes of granules are being made based on the abstract space only (they are are restricted to their abstract parts only)	YES	NO
changes of the information represented in the i-layers of granules are being made based on the abstract space only	YES	NO
changes of the information represented in granules are also being made based on interaction with physical objects	NO	YES
STATES, TRANSITION, COMPUTATIONS IN COMPUTING MODEL		
state of c-granule: c-granule (with its abstract and physical objects) at a given moment of (local) time	NO	YES
transition relation (association) defined based on information in the abstract space only	YES	NO
transition relation (association) dependent on interactions with physical objects	NO	YES
computations consist of abstract states only	YES	NO
computations depend on physical laws	NO	YES
adaptation of steering of granular computations provided by control of granules dependent on interaction with physical objects	NO	YES

COMPARISON OF GrC & IGrC

SUMMARY

	GrC	IGrC
a response to essential complexity	NO	YES
abstract–physical modeling grounded in real interactions of physical and abstract objects	NO	YES
emergent learning from experience	NO	YES
approximate solutions to problems constructed along granular computations interacting with the abstract and physical world	NO	YES
adaptive discovery of sets of interaction rules (complex games) realized in the physical and abstract worlds	NO	YES
purely abstract modeling only	YES	NO

FOUNDATIONS BASED ON IGrC & RS FOR IS's DEALING WITH COMPLEX PHENOMENA

Tomorrow, I believe, we will use
[IS's]
to support our decisions
in defining our research strategy and specific aims,
in managing our experiments,
in collecting our results, interpreting our data,
in incorporating the findings of others,
in disseminating our observations,
in extending (generalizing) our experimental observations
- through exploratory discovery and modeling -
in directions completely unanticipated

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IGrC: SUMMARY

The IGrC model was created as the basis for the design and analysis of c-granules, in particular IS's. The proposed IGrC model differs from the classical Turing model by synchronizing four components: language, reasoning, perception, and action. In the IGrC model, granular computations form the basis for reasoning that supports problem solving by c-granules.

Problem solving (or decision support) using c-granules (IS's) requires a proper understanding of real-world situations consisting of configurations of interacting objects. Therefore, the control of c-granules must include skills for perceiving situations in the physical world enabling the formation of associations between physical and abstract objects. These skills are supported by reasoning over granular computations performed by the control of c-granules. Consequently, these computations cannot be confined to the abstract space alone. Moreover, they depend on physical laws.

IGrC: SUMMARY

The generalization of GrC to IGrC was proposed to support the design of IS's that deal with complex phenomena, which can be treated in IGrC as examples of complex granules (c-granules) with control. To make such systems successful, it is necessary to enable their continuous interaction with the physical world. The control of c-granules can properly implement the physical semantics of specified transformations of c-granules in the physical world. This implementation is based on the discovery of relevant configurations of physical objects, which provides the basis for perceiving relevant data about these objects and their interactions through the control of c-granules. Furthermore, to ensure the success of the designed IS's, these configurations must adaptively change to enable the perception of relevant data that will make it possible to construct high-quality models on which the behavior of the IS's is based. Unlike information granules from GrC, the correct implementation of c-granule transformations cannot be restricted to the abstract space. An important property of the IS's discussed here is that they cannot be separated from interactions with the physical world. They cannot be confined to an abstract space.

TOWARD
BRINGING INTO SYNC
FOUR IMPORTANT AREAS OF RESEARCH ON RS:
LANGUAGE, REASONING, PERCEPTION, AND ACTION

IGrC & ESSENTIAL COMPLEXITY

Essential complexity remains an unresolved challenge.

The future of mathematics and AI must:

- learn to accept the irreducible difficulty of problems,
- develop tools that reduce accidental risks,
- support humans in managing complexity rather than eliminating it.

COMPUTATIONAL MODELING IN NEUROSCIENCE BASED ON IGRC

FURTHER RESEARCH

Computational modeling of learning in brain based on generalization of neural nets based on IGrC:

- neurons $\rightarrow$ c-granules with control
- neural nets $\rightarrow$ granular networks
- local memory of neuron $\rightarrow$ information layer of c-granule

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>
> […] ***The post-transformer era: a story of memory and reasoning***

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THANK YOU!