

Imagine a two dimensional surface whose boundary is just a circle. It is intuitively obvious that among such surfaces the flat disc is the one with minimal area but a formal proof is not that simple. Moreover, we consider certain anisotropic spaces where the value of area depends on the slope of the surface. This means that the area of a flat disc varies as we rotate it about some axis. To make things even more interesting we place everything in an ambient space of arbitrary high dimension n and we also consider “surfaces” of arbitrary dimension k . The boundary is always fixed and has to be a $(k - 1)$ -dimensional sphere. With all these generalisations the problem to describe minimisers of area becomes really hard (which fact makes mathematicians rather happy). For $k = 2$ and $n \geq 4$ the problem was solved only in 2012. For other choices of k the problem is open.

Actually, there are even more nuances. Given a function (a norm) that computes an anisotropic length of a vector (i.e. rotations may change the length) one can construct many sensible and meaningful notions of k -dimensional area. My first goal is to describe those anisotropic notions of k -area for which the flat k -disc minimises k -area among k -surfaces having a fixed $(k - 1)$ -sphere as boundary. I shall call them *elliptic*. **Understanding the deep nature of ellipticity is the main far-reaching objective of the project** and may lead to answering other long-standing open questions some of which are outlined below.

Any function F computing some type of (anisotropic) k -area of a surface gives rise (by a differentiation process) to the notion of *mean F -curvature*. Assume a k -surface Σ lies inside a region U and touches the boundary of U at some point p . Intuitively, a notion of curvature describes how fast the surface bends so Σ should have bigger curvature than the boundary of U at p . This type of comparison is called the *maximum principle* and is known to hold for the classical notion of curvature. In the anisotropic case the maximum principle is proven in *co-dimension one* only, i.e., if $n - k = 1$. In higher co-dimension the problem is open – especially that the notion of mean F -curvature depends also on the dimension k and it is not clear how to compare curvatures of objects of different dimensions.

If the mean F -curvature of a surface Σ is globally bounded, then the surface should not bend too fast at any point. This possibly excludes long and very thin tentacles or at least should give a bound on their thickness. If this is the case, there is a radius $R > 0$ such that the intersection of Σ with any ball centred at Σ of radius $0 < r < R$ must have k -area proportional to r^k . This is true for the usual notion of k -area but is not known for other anisotropic k -areas. It is actually a very big question that holds off many other developments.

A bit more subtle open problem, is the very definition of the mean F -curvature in case F is not differentiable. This is especially important since such F are used to model, e.g., liquid crystals.

REFERENCES