Corrigendum to "Uniqueness of critical points of the anisotropic isoperimetric problem for finite perimeter sets"

Antonio De Rosa Sławomir Kolasiński Mario Santilli

February 18, 2021

Remark 5.6 in [DRKS20] should be replaced with the following.

Remark 5.6. We briefly show that whenever $A \subseteq \mathbf{R}^{n+1}$ is closed and F is a norm, r_A^F is upper semicontinuous; in particular, r_A^F is a Borel function. Assume this is not true, i.e. there exist A, F, and for each $i \in \mathscr{P}$ there is $(a_i, u_i) \in N^F(A)$ such that

$$\lim_{i\to\infty} a_i = a \in A\,, \quad \lim_{i\to\infty} u_i = u \in \mathbf{S}^n\,, \quad (a,u) \in N^F(A)\,,$$
$$\text{and} \quad r_A^F(a,u) < \lim_{i\to\infty} r_A^F(a_i,u_i)\,.$$

Let $s \in \mathbf{R}$ be such that

$$0 < r_A^F(a,u) < s \leq r_A^F(a_i,u_i) \quad \text{for } i \in \mathscr{P}\,.$$

Since $r_A^F(a,u) < s$ we can find $b \in A$ such that $F^*((a+su)-b) < s$. Let $\varepsilon \in \mathbf{R}$ be such that

$$0 < \varepsilon < s \quad \text{and} \quad 0 < F^*((a+su)-b) < s - \varepsilon\,.$$

Let $i \in \mathscr{P}$ be so big that $F^*(a_i - a) \leq 2^{-3}\varepsilon$ and $F^*(u_i - u) \leq 2^{-3}s^{-1}\varepsilon$. Then

$$F^*((a_i + su_i) - (a+su)) \leq F^*(a_i - a) + sF^*(u_i - u) \leq 2^{-2}\varepsilon\,.$$

Since $r_A^F(a_i,u_i) \geq s$ we get a contradiction

$$\begin{aligned} s = \boldsymbol{\delta}_A^F(a_i + su_i) &\leq F^*((a_i+su_i)-b) \\ &\leq F^*((a_i+su_i)-(a+su)) + F^*((a+su)-b) \leq 2^{-2}\varepsilon + s - \varepsilon < s\,. \quad \lightning \end{aligned}$$

References

[DRKS20] Antonio De Rosa, Sławomir Kolasiński, and Mario Santilli. Uniqueness of critical points of the anisotropic isoperimetric problem for finite perimeter sets. *Arch. Ration. Mech. Anal.*, 238(3):1157–1198, 2020. URL: `https://doi.org/10.1007/s00205-020-01562-y`,

Antonio De Rosa
Courant Institute of Mathematical Sciences, New York University,
251 Mercer Street, New York, NY 10012, USA
derosa@cims.nyu.edu

Sławomir Kolasiński
Instytut Matematyki, Uniwersytet Warszawski
ul. Banacha 2, 02-097 Warszawa, Poland
s.kolasinski@mimuw.edu.pl

Mario Santilli
Institut für Mathematik, Universität Augsburg,
Universitätsstr. 14, 86159, Augsburg, Germany,
mario.santilli@math.uni-augsburg.de