Partial differential equations I, 2011/2012

Homework 2

Deadline: May 9, 2012

1. **[2p.]** Let $c \in \mathbb{R}$ be some constant. Find a formula for the solution to the following problem

$$\begin{aligned} u_t - a^2 u_{xx} &= cu && \text{for } (x,t) \in \mathbb{R}_+ \times \mathbb{R}_+ \,, \\ u(0,t) &= 0 && \text{for } t \in \mathbb{R}_+ \,, \\ u(x,0) &= \varphi(x) && \text{for } x \in \mathbb{R}_+ \,. \end{aligned}$$

2. **[3p.]** Solve the following problem

$$\begin{aligned} u_t - u_{xx} &= e^{-4t} \sin(3\pi x) \quad \text{for } (x,t) \in (0,1) \times \mathbb{R}_+ \,, \\ u(0,t) &= u(1,t) = 0 \,, \\ u(x,0) &= 7\sin(\pi x) + 3\sin(5\pi x) \,. \end{aligned}$$

3. **[5p.]** Let $u \in C^2(\mathbb{R}^n, \mathbb{R}_+)$ be such that $\Delta u = 0$ in $\mathbb{R}^n$. Show that u has to be constant.

   Hint: §2.2.3 c. from the Evans' book might be useful.

4. **[3p.]** Let $u : \mathbb{B}(0,1) \to \mathbb{R}$ satisfy

$$\begin{aligned} -\Delta u &= f \quad \text{in } \mathbb{B}(0,1) \,, \\ u &= g \quad \text{on } \partial\mathbb{B}(0,1) \,. \end{aligned}$$

   Show that there exists a constant C depending only on n such that

$$\sup_{\mathbb{B}(0,1)} |u| \leqslant C\Big(\sup_{\partial\mathbb{B}(0,1)} |g| + \sup_{\mathbb{B}(0,1)} |f| \Big) \,.$$