Existence and regularity results for an important PDE system

Piotr Biler, Piotr.Biler@math.uni.wroc.pl

Michal Kowalczyk, kowalczy@dim.uchile.cl

Piotr Rybka, rybka@mimuw.edu.pl

Abstract

We consider a three-dimensional version of the fancy system from [1]. For every smooth and divergence-free initial condition $v_0(x) := v(x,0)$ satisfying the additional assumption:

for every multi-index α and any $K > 0$ there exists a constant $C = C(\alpha, K)$ such that $|\partial^\alpha v_0(x)| \leq \frac{1}{(1+|x|)^K}$ for all $x \in \mathbb{R}^3$

we prove that there exists a smooth and globally defined solution $v(x,t)$ of the system of two equations:

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\nabla p + \nu \Delta v \qquad \text{and} \qquad \nabla \cdot v = 0.$$

In addition we show that there is a constant $E > 0$ such that $\int_{\mathbb{R}^3} |v(x,t)|^2 < E$ for all $t \geq 0$.

References

[1] Au. Thor, *The Most Important Equations of the Universe,* A Noble Publishing Co, Pleasant City, 1969.