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Nonlinear and Nonlocal Degenerate Diffusions on Bounded Domains

Abstract. We investigate quantitative properties of nonnegative solutions $u(t, x) \geq 0$ to the nonlinear fractional diffusion equation, $\partial_t u + LF(u) = 0$ posed in a bounded domain, $x \in \Omega \subset R^N$, with appropriate homogeneous Dirichlet boundary conditions. As L we can use a quite general class of linear operators that includes the three most common versions of the fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, in a bounded domain with zero Dirichlet boundary conditions; many other examples are included. The nonlinearity F is assumed to be increasing and is allowed to be degenerate, the prototype being $F(u) = |u|^{m-1}u$, with $m > 1$.

We will shortly present some recent results about existence, uniqueness and a priori estimates for a quite large class of very weak solutions, that we call weak dual solutions.

We will devote special attention to the regularity theory: decay and positivity, boundary behavior, Harnack inequalities, interior and boundary regularity, and asymptotic behavior. All this is done in a quantitative way, based on sharp a priori estimates. Although our focus is on the fractional models, our techniques cover also the local case s = 1 and provide new results even in this setting.

A surprising instance of this problem is the possible presence of nonmatching powers for the boundary behavior: for instance, when $L = (-\Delta)^s$ is a spectral power of the Dirichlet Laplacian inside a smooth domain, we can prove that, whenever $2s \geq 1 - 1/m$, solutions behave as $dist^{1/m}$ near the boundary; on the other hand, when $2s < 1 - 1/m$, different solutions may exhibit different boundary behaviors even for large times. This unexpected phenomenon is a completely new feature of the nonlocal nonlinear structure of this model, and it is not present in the semilinear elliptic case, for which we will shortly present the most recent results. The above results are contained on a series of recent papers in collaboration with A. Figalli, Y. Sire, X. Ros-Oton and J. L.Vazquez.