

Existence of 1-harmonic flow

Michał Łasica

University of Warsaw and Sapienza University of Rome
lasica@mimuw.edu.pl

Abstract

We consider the functional of total variation of maps from a domain Ω in \mathbb{R}^m into a Riemannian manifold \mathcal{N} embedded in \mathbb{R}^n , defined as a suitable relaxation of the functional given by

$$\int_{\Omega} |\nabla \mathbf{u}|$$

for $\mathbf{u} \in C^1(\Omega, \mathcal{N})$.

We report on recent progress in the theory of existence of evolutions under formal L^2 -gradient flow of this functional. We note that the functional in general is not geodesically convex, hence existence is not provided by standard methods of semigroup generation. Thus, we construct the evolutions as solutions to a system of PDE, obtained as limits of regular approximations.

For Lipschitz initial data, we prove local existence of unique solutions in convex domains. We discuss conditions under which the solutions can be continued indefinitely.

In the case $m = 1$, we obtain suitably defined global solutions for any datum of bounded variation whose jumps are not too big.

This is joint work with L. Giacomelli and S. Moll.